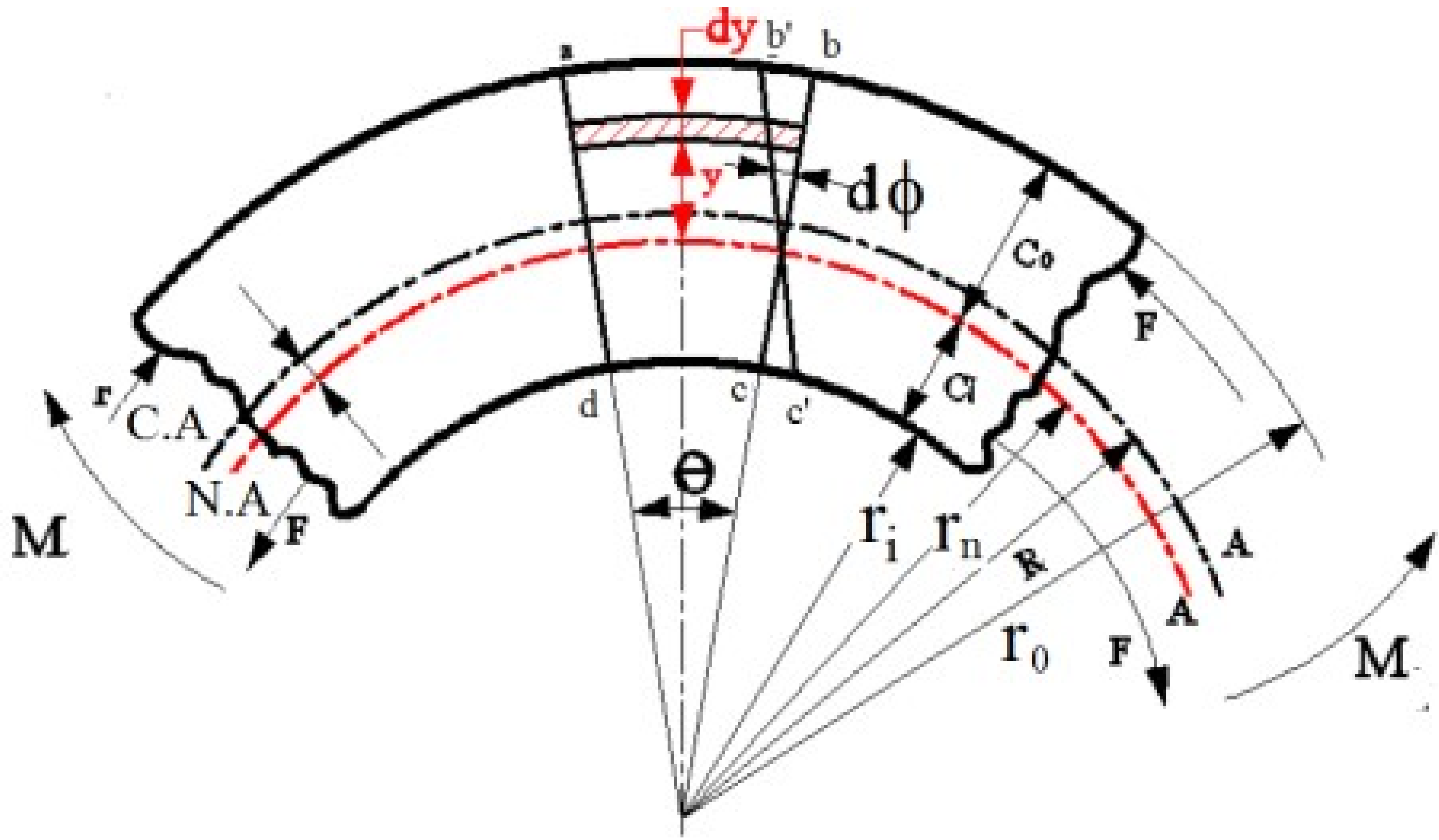
3-18 Curved Beams in Bending



Note that the NA is **always** located between the center of curvature and the centroid. The sign of moment doesn't matter.

e = distance from the centroidal axis to the neutral axis, measured towards centre of curvature, mm

C_i = distance from neutral axis to inner fiber (radius) mm

C_o = distance from neutral axis to outer fiber (radius) mm.

r_i = inner radius of curvature, mm

r_o = outer radius of curvature, mm

r_n = Radius of neutral axis, mm

R = Radius of centroidal axis, mm

A = area of section, mm^2

σ_i = Stress in the inner fiber, N/mm^2

σ_o = Stress in the outer fiber, N/mm^2

The neutral axis is the axis through a beam where the stress is zero, that is there is neither compression nor tension

The stress distribution can be found by the following relation

$$\sigma = \frac{My}{Ae(r_n - y)}$$

The critical stress occurs at the inner and outer surfaces

$$\sigma_i = \frac{Mc_i}{Aer_i}$$ inner fibre stress $y = c_i$

$$\sigma_o = -\frac{Mc_o}{Aer_o}$$ outer fibre stress $y = -c_o$

Curved Beams

- The integral may be evaluated for various cross-sectional geometries.
- The *curved-beam formula* can be used to determine the normal-stress distribution in a curved member.

$$r_n = \frac{\sum A}{\sum \int_A dA/r}$$

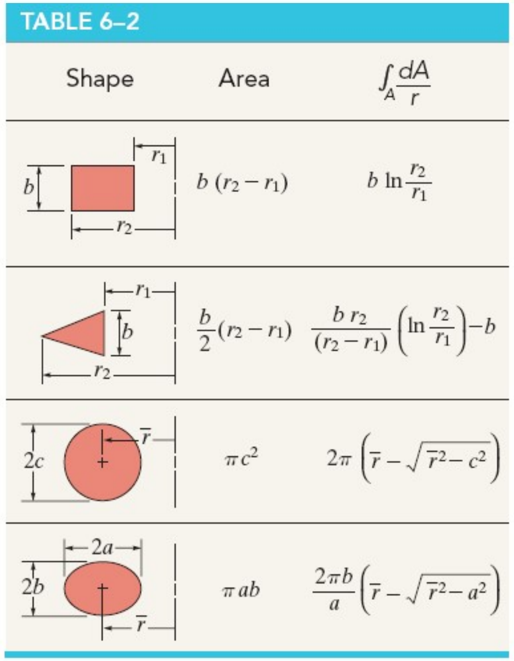
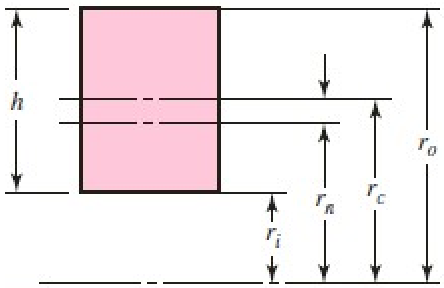


TABLE 6–2

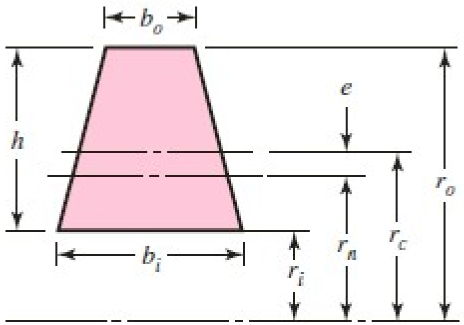
Shape	Area	$\int_A \frac{dA}{r}$
	$b\,(r_2 - r_1)$	$b \ln \frac{r_2}{r_1}$
	$\frac{b}{2}(r_2 - r_1)$	$\frac{b\,r_2}{(r_2 - r_1)}\left(\ln \frac{r_2}{r_1}\right) - b$
	πc^2	$2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$
	πab	$\frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right)$

Table 3–4



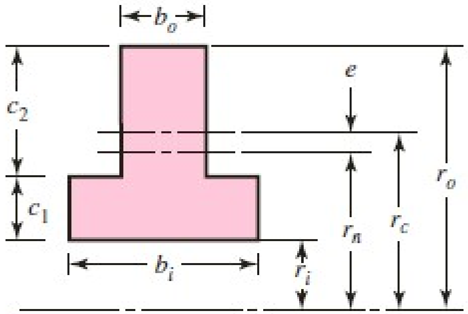
$$r_c = r_i + \frac{h}{2}$$

$$r_n = \frac{h}{\ln(r_o/r_i)}$$



$$r_c = r_i + \frac{h}{3}\frac{b_i + 2b_o}{b_i + b_o}$$

$$r_n = \frac{A}{b_o - b_i + [(b_i r_o - b_o r_i)/h]\ln(r_o/r_i)}$$



$$r_c = r_i + \frac{b_i c_1^2 + 2b_o c_1 c_2 + b_o c_2^2}{2(b_o c_2 + b_i c_1)}$$

$$r_n = \frac{b_i c_1 + b_o c_2}{b_i \ln[(r_i + c_1)/r_i] + b_o \ln[r_o/(r_i + c_1)]}$$

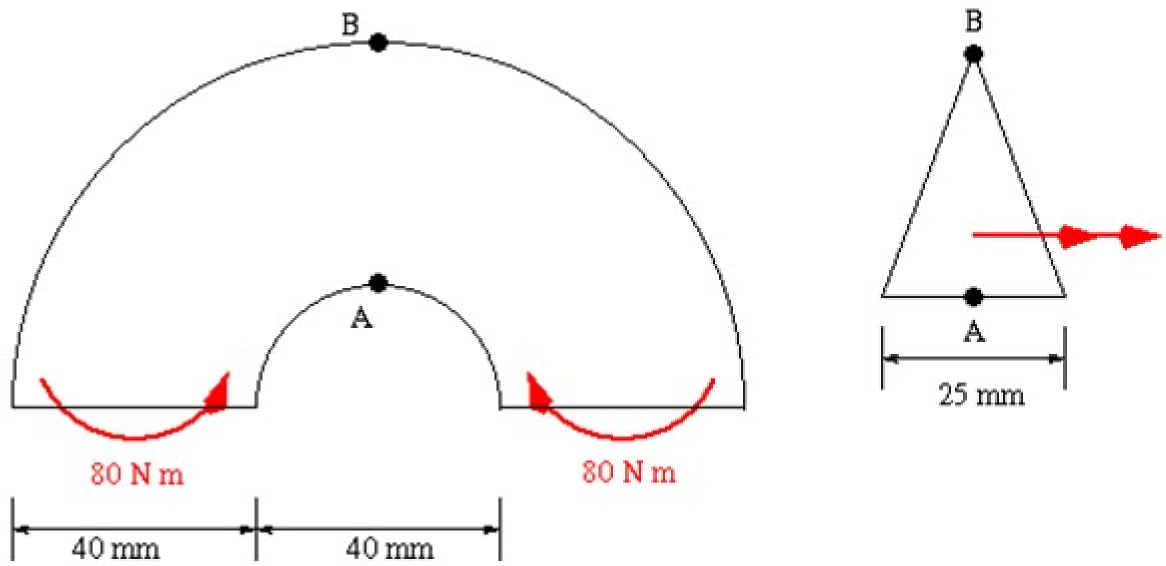
Ex:For the curved beam with triangular cross section shown, determine:

(a) the centroidal radius of curvature

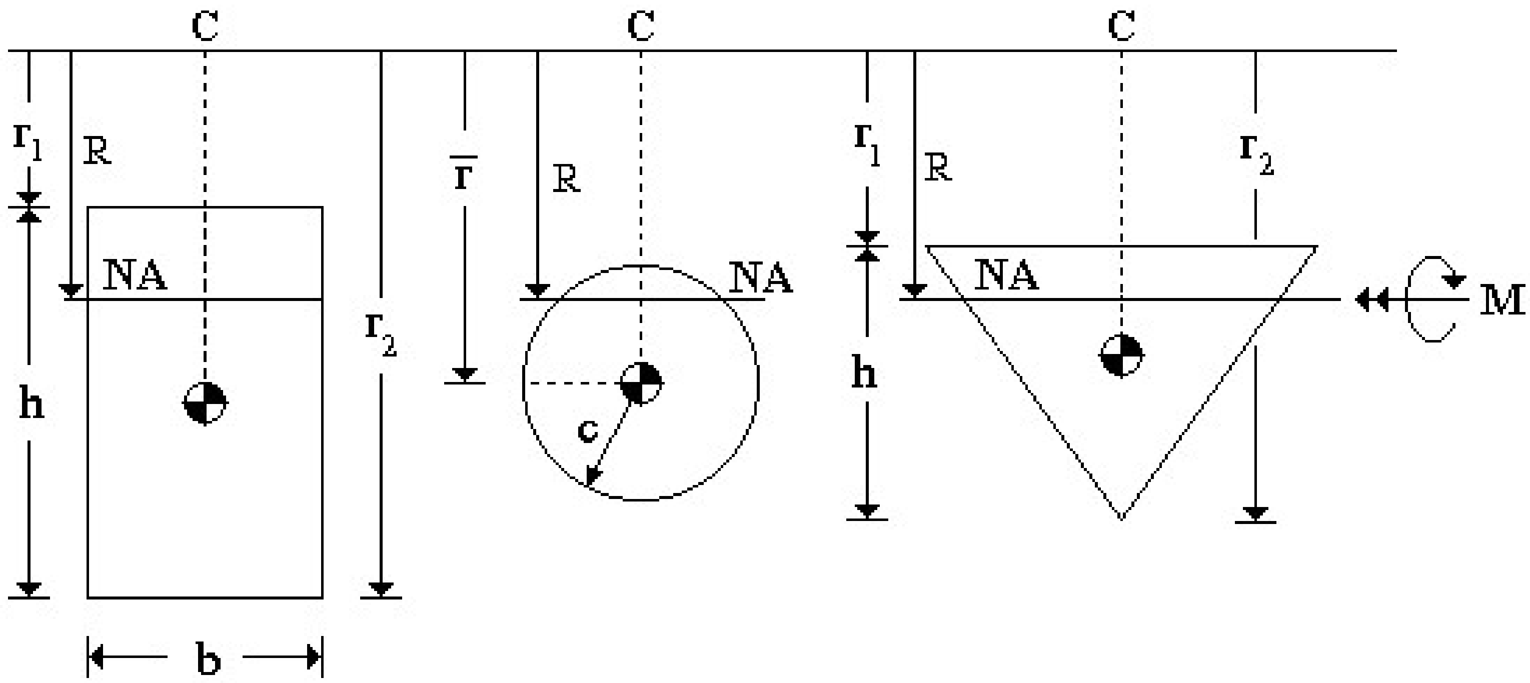
(b) the neutral axis location

(c) the maximun stress corresponding to the moment of 80 N-m

(d) the stress variation



Center of Curvature (C)



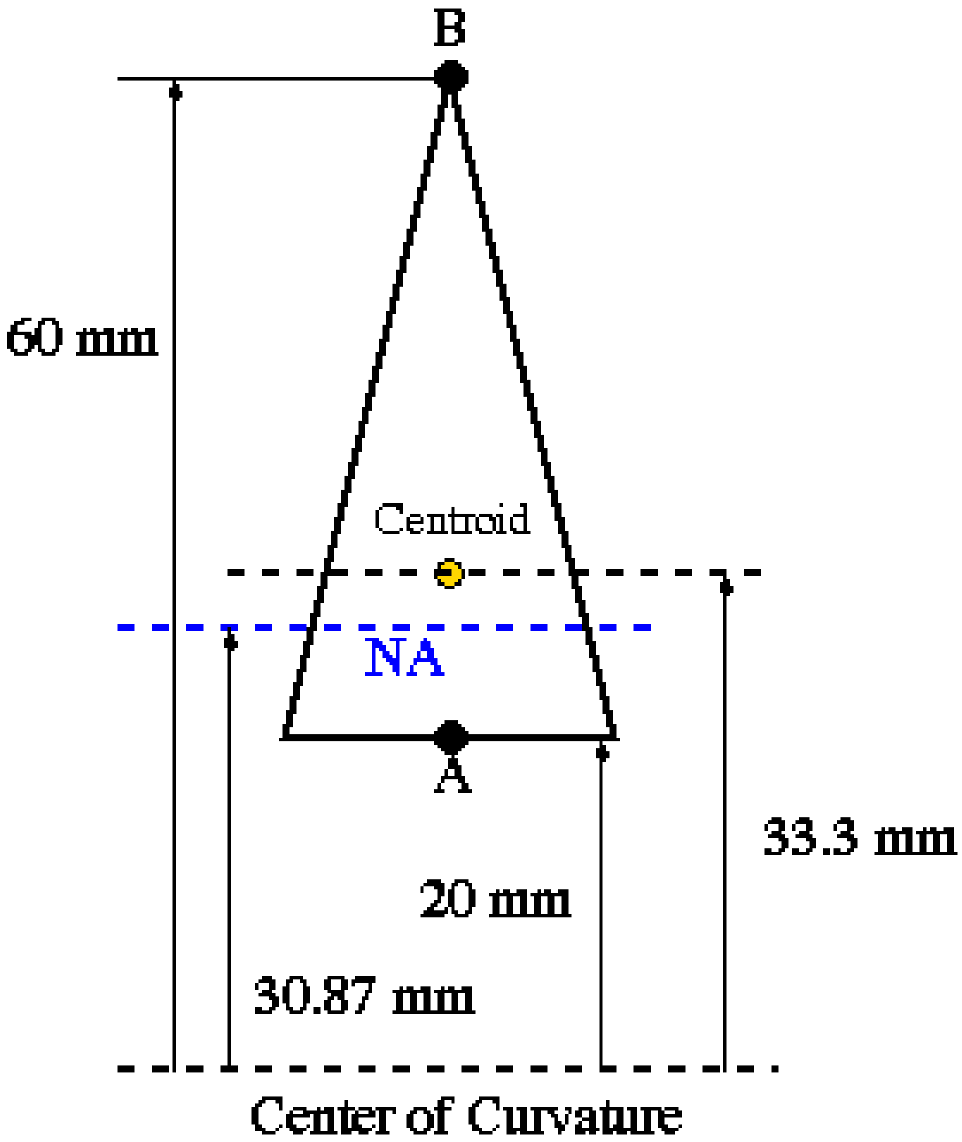
$$R = \frac{h}{\ln \frac{r_2}{r_1}} \qquad R = \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right) \qquad R = \frac{h/2}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

The radius of curvature is found as

$$\bar{r} = \frac{40}{3} + 20$$
$$= 33.3\ mm$$

(b) The neutral axis position relative to the center of curvature is found as

$$R = \frac{\frac{h}{2}}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1}$$
$$= \frac{\frac{40}{2}}{\frac{60}{h}\ln\frac{60}{20} - 1}$$
$$= 30.868\ mm$$



B
60 mm
Centroid
NA
A
33.3 mm
20 mm
30.87 mm
Center of Curvature

(c) The maximum stress will be located at either point A or point B.

$$\sigma = \frac{M(r-R)}{Aer}$$

A

$$\sigma_A = \frac{80(20x10^{-3} - 30.868x10^{-3})}{\frac{1}{2}(25x10^{-3})(40x10^{-3})(2.465x10^{-3})(20x10^{-3})}$$

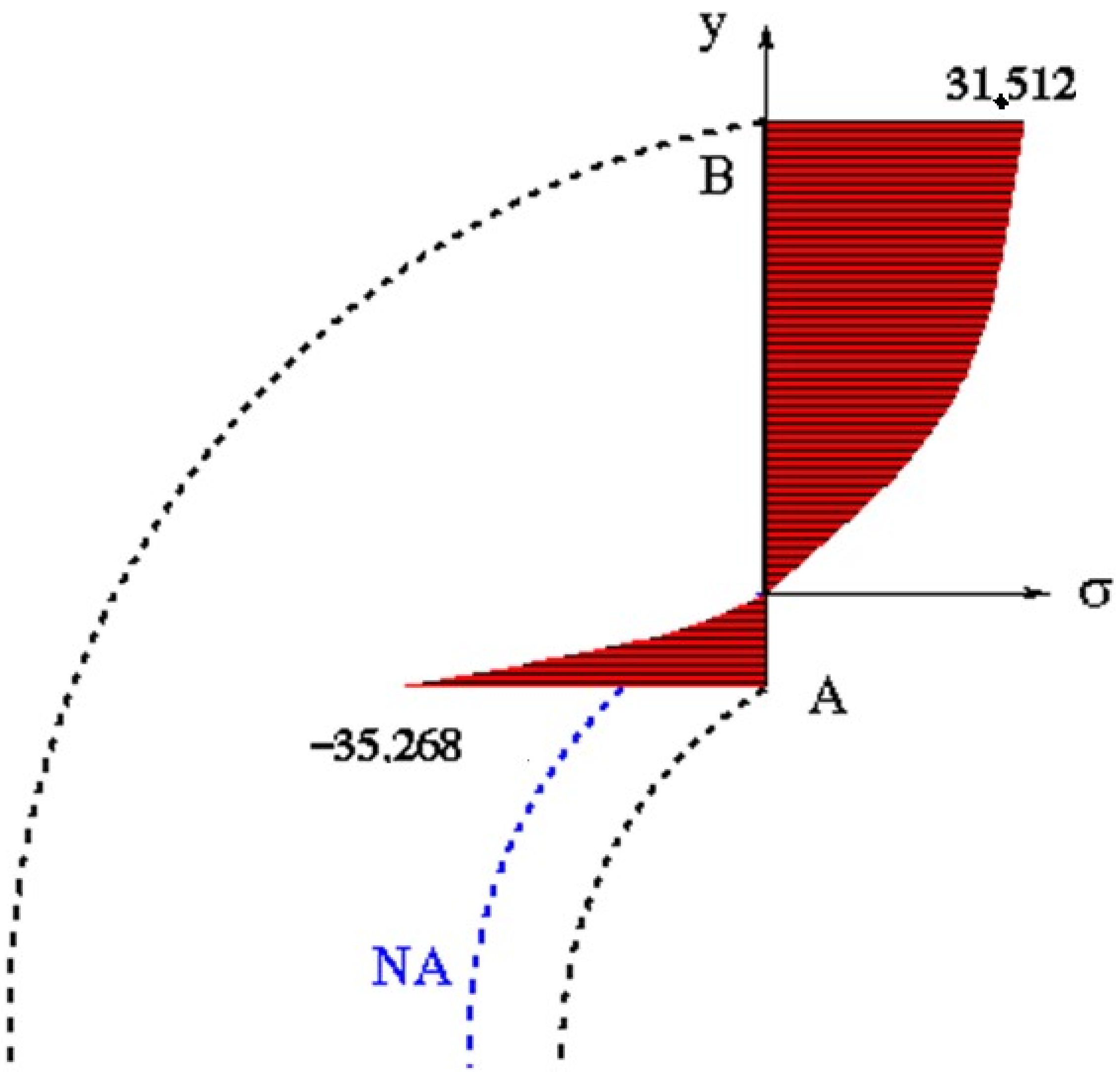
$$= -35.268 \ MPa$$

B

$$\sigma_B = \frac{80(60x10^{-3} - 30.868x10^{-3})}{\frac{1}{2}(25x10^{-3})(40x10^{-3})(2.465x10^{-3})(60x10^{-3})}$$

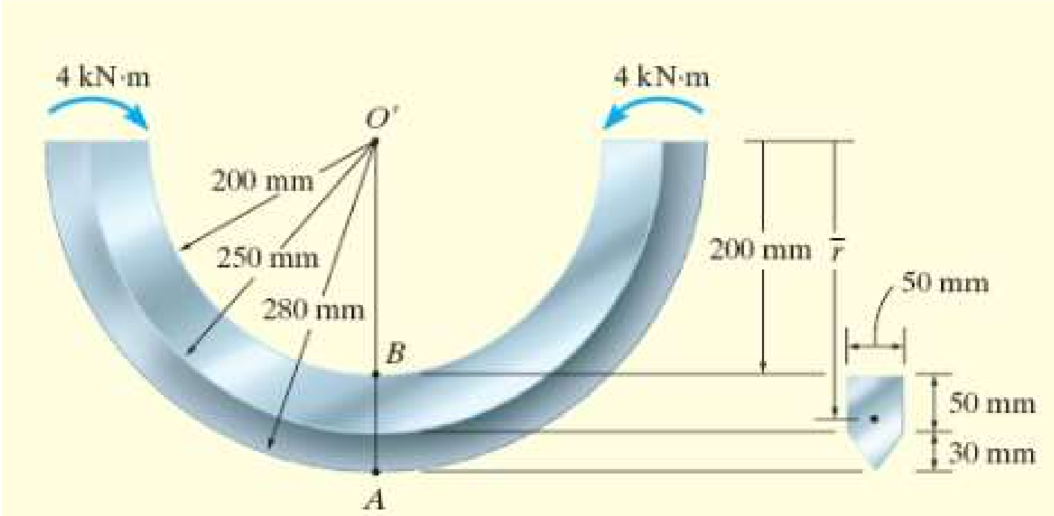
$$= 31.512 \ MPa$$

point A has the maximum stress



y
31,512
B
σ
A
-35,268
NA

EX:The curved bar has a cross-sectional area as shown. If it is subjected to bending moments of 4 kNm, determine the maximum normal stress developed in the bar.

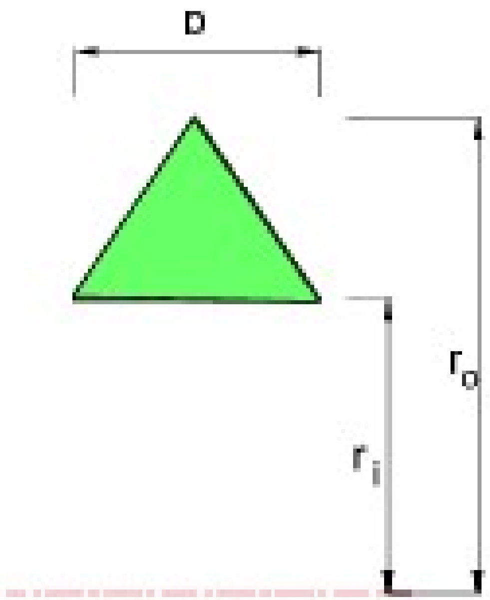


Since this moment tends to decrease the bar's radius of curvature, it is negative. Thus the total cross-sectional area is

$$\sum A = (0.05)^2 + \frac{1}{2}(0.05)(0.03) = 3.25\left(10^{-3}\right)\text{m}^2$$

The location of the centroid is determined with reference to the center of curvature, point 0',

$$r_c = \frac{\sum \tilde{r} A}{\sum A} = 0.23308\,\text{m}$$



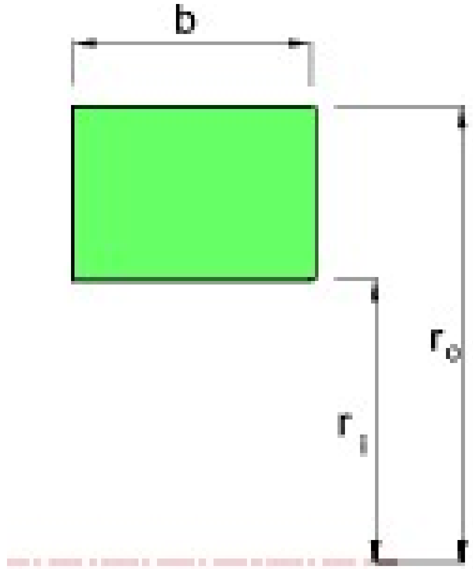
$$\int_A \frac{dA}{r} = \frac{(0.05)(0.28)}{(0.28-0.25)}\left(\ln\frac{0.28}{0.25}\right) - 0.05 = 0.0028867\ \text{m}$$

Area = $b(r_o - r_i)/2$

$$\int_A \left(\frac{dA}{r}\right) = \frac{b\, r_o}{(r_o - r_i)} \ln\left(\frac{r_o}{r_i}\right) - b$$

Radius of centroid

$r_C = r_i + (r_i - r_o) / 3$



$$\int_A \frac{dA}{r} = 0.05 \ln\frac{0.25}{0.2} = 0.011\ \text{m}$$

Area = $b(r_o - r_i)$

$$\int_A \left(\frac{dA}{r}\right) = b \ln(r_o / r_i)$$

Radius of centroid

$r_C = r_i + (r_o - r_i) / 2$

Thus the location of the neutral axis is determined from

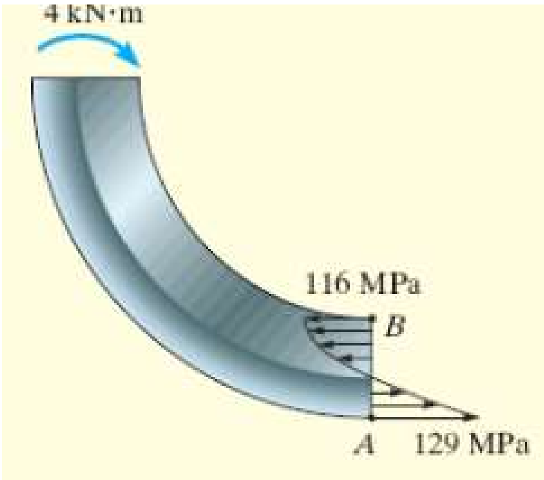
$$r_n = \frac{\sum A}{\sum \int_A dA/r} = \frac{3.25\left(10^{-3}\right)}{0.0111 + 0.0028867} = 0.23142\ \text{m}$$

e= 0.23142-0.233080=0.00166 m

Applying the curved-beam formula to calculate the normal stress at *B*,

$$\sigma_B = \frac{M(y)}{Ae(r_n - y)} = \frac{(-4)(0.23142 - 0.2)}{3.25\left(10^{-3}\right)(0.00166)(0.2)} = -116\ \text{MPa}$$

$$\sigma_A = \frac{M(y)}{Ae(r_n - y)} = -\frac{(-4)(0.23142 - 0.280)}{3.25\left(10^{-3}\right)(0.00166)(0.280)} = 129\ \text{MPa}$$



The hook is lifting a load of 25000N.

$r_c = 100mm \qquad A = 2000mm^2$

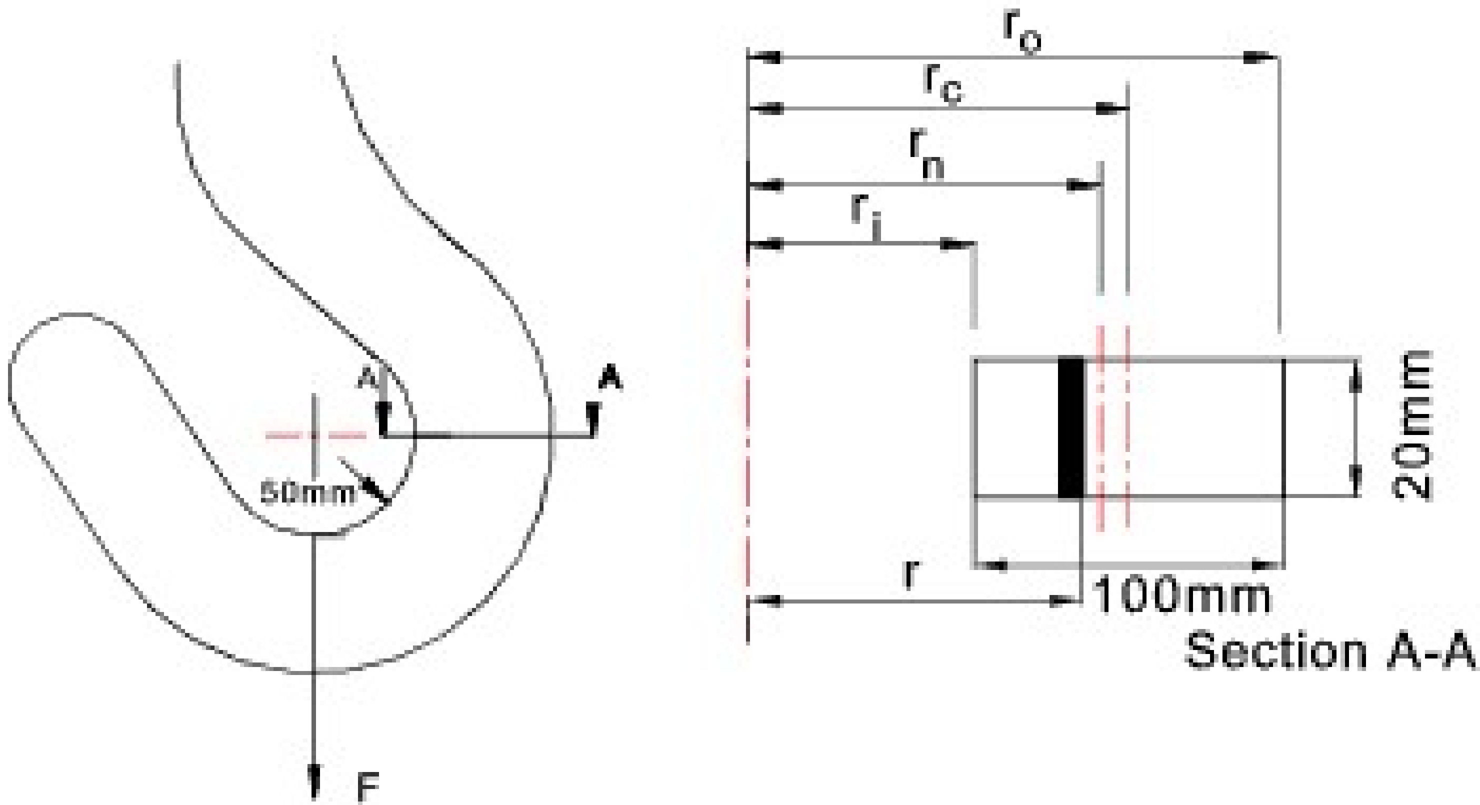
$$\int_A \left(\frac{dA}{r}\right) = b \ln\left(\frac{r_o}{r_i}\right) = 20 \ln.\left(\frac{150}{50}\right) = 21,97$$

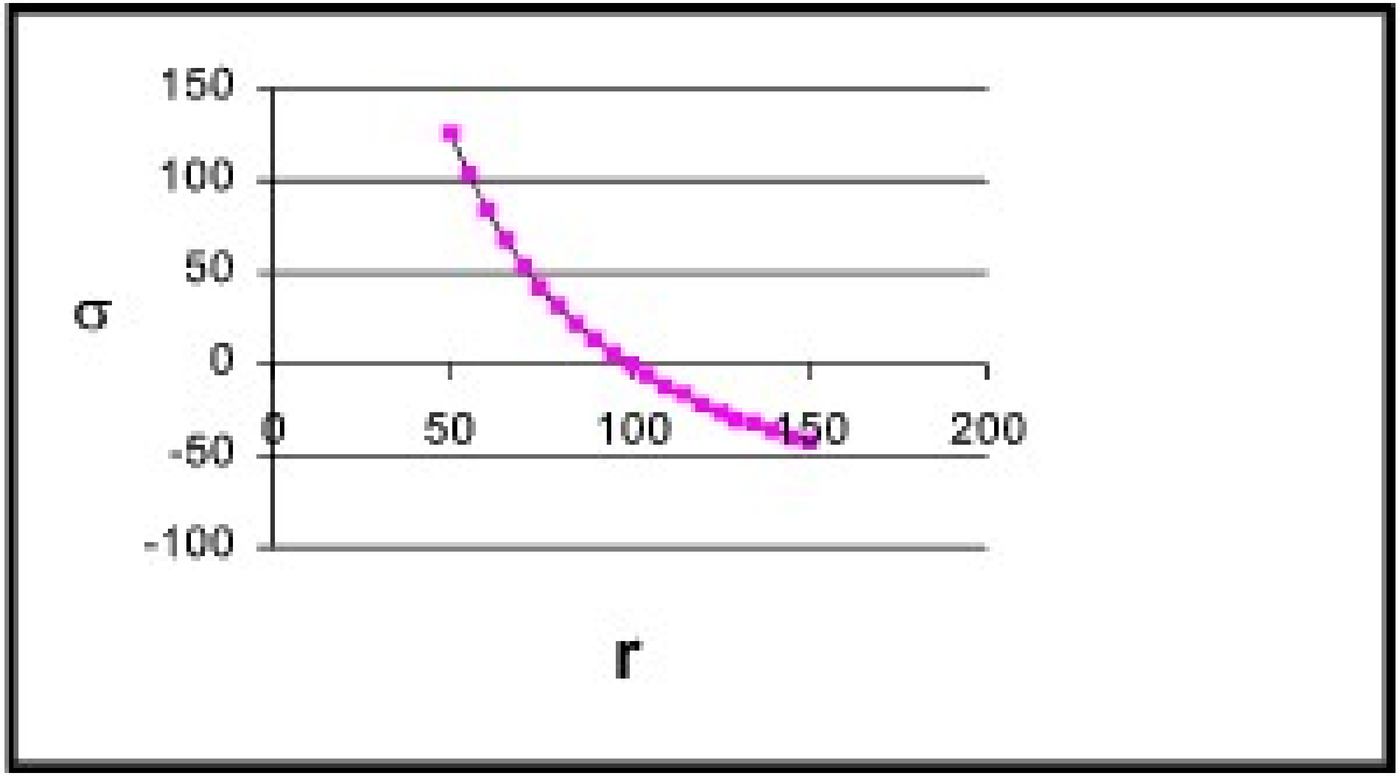
$$r_n = \frac{A}{\int_A \left(\frac{dA}{r}\right)} = \frac{2000}{21,97} = 91,024$$

Eccentricity = $e = r_c - r_n = 100 - 91,024 = 8,976mm$

Moment = 25kN.100mm = 2500kN.mm and Axial load is 25kN.

$$\sigma = \frac{F}{A} + \frac{My}{A\, e(r_n - y)} = \frac{25kN}{2000mm^2} + \frac{2500kN.mm\ (91,024 - r)mm}{2000\ .\ 8,976.\ r\ (mm)^4}$$





150
100
50
0
-50
-100
σ
0
50
100
150
200
r

Alternative Calculations for e

Calculating r_n and r_c mathematically and subtracting the difference can lead to large errors if not done carefully, since r_n and r_c are typically large values compared to e. Since e is in the denominator of Eqs. (3–64) and (3–65), a large error in e can lead to an inaccurate stress calculation. Furthermore, if you have a complex cross section that the tables do not handle, alternative methods for determining e are needed. For a quick and simple approximation of e, it can be shown that[14]

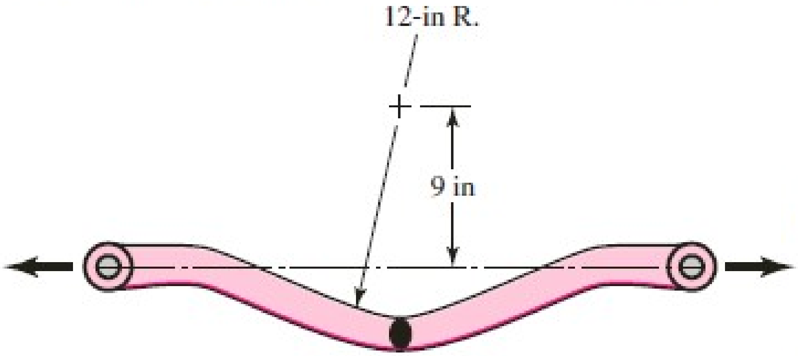
$$e \doteq \frac{I}{r_c A} \tag{3–66}$$

the radius is large compared to the cross section,

$$\sigma \doteq \frac{My}{I}\frac{r_c}{r} \tag{3–67}$$

$$\sigma = \frac{My}{Ae(r_n - y)}$$

3–131 An offset tensile link is shaped to clear an obstruction with a geometry as shown in the figure. The cross section at the critical location is elliptical, with a major axis of 3 in and a minor axis of 1.5 in. For a load of 20 kip, estimate the stresses at the inner and outer surfaces of the critical section.



Problem 3–131

$$M = 20(3+1.5)$$

$$A = \pi ab$$

$$\sigma_i = \frac{F}{A} + \frac{Mc_i r_c}{Ir_i}$$

$$\frac{F}{A} = \frac{20}{A}$$

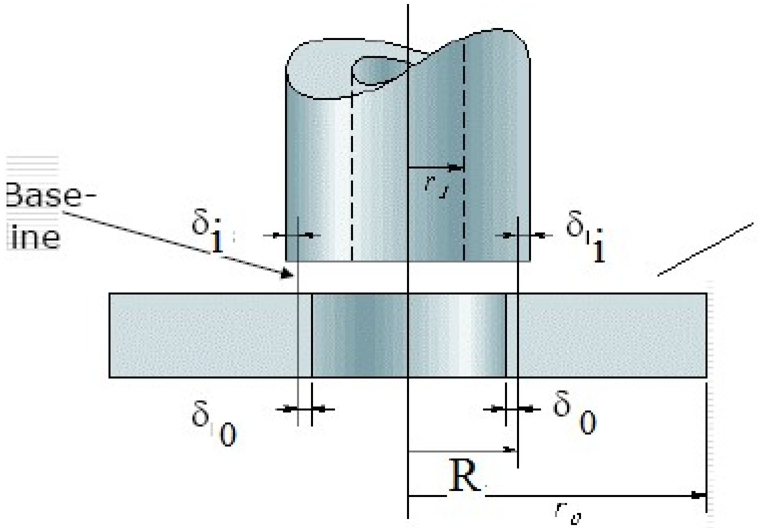
$$\frac{Mc_i r_c}{Ir_i} = \frac{90(1.5)(13.5)}{(1.988)(12)}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o r_c}{Ir_o}$$

$$\frac{Mc_o r_c}{Ir_o} = \frac{90(1.5)(13.5)}{1.988(15)}$$

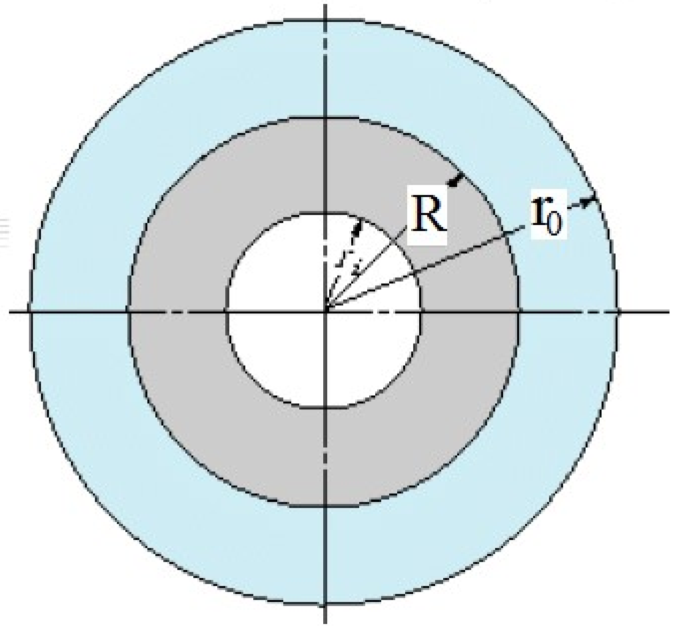
3-18 Press and Shrink Fits

- a contact pressure p exist between the hole and shaft at the transition radius R
- p causes a radial stresses $\sigma_r = -p$ at the contacting surfaces



Tangential stress at inner member

$$\sigma_{it} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2}$$



Tangential stress at the inner surface of the outer member

$$\sigma_{0t} = p\frac{r_0^2 + R^2}{r_0^2 - R^2}$$

$$\delta = |\delta_i| + |\delta_0|$$

Total radial interference

The tangentional strain of the outer member is measured by the change in the circumference

$$\varepsilon_{0t} = \frac{2\pi\left(R+\delta_0\right)-2\pi R}{2\pi R} = \frac{\delta_0}{R}$$

$$\varepsilon_{0t} = \frac{\sigma_{ot}}{E_o} - \frac{\nu_o\sigma_{or}}{E_o}$$

$$\delta_0 = \frac{\sigma_{ot}}{E_o} - \frac{\nu_o\sigma_{or}}{E_o}$$

The change in radius of the inner member is :

$$\delta_i = -\frac{pR}{E_i}\left[\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right] \text{ (inner)}$$

The change in radius of the outer member is :

$$\delta_0 = \frac{pR}{E_o}\left[\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right] \text{ (outer)}$$

The total deformation is :

$$\delta = pR\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]$$

- Once δ is known we can calculate p, or vice versa.
- Typically, δ is very small, approximately 0.001 in. or less.

If the materials are the same:

$$E = E_i = E_o$$

$$\upsilon = \upsilon_i = \upsilon_o$$

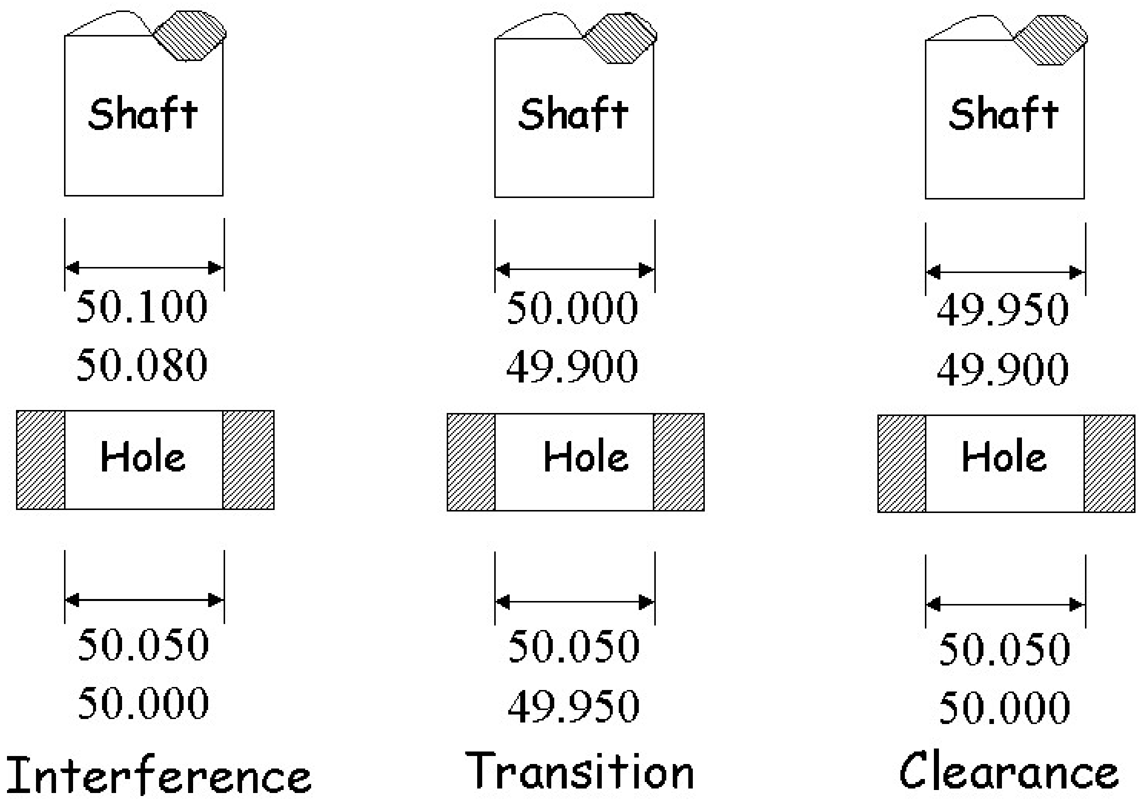
$$p = \frac{\delta E}{R}\left[\frac{(r_o^{\,2} - R^2)(R^2 - r_i^{\,2})}{2R^2\,(r_o^{\,2} - r_i^{\,2})}\right]$$

Types of Fits

Since there will be tolerances on both diameters of the inner and outer members, The max. P and min. P can be determined by using the and max. δ and min δ

$\delta_{min}=d_{min}-D_{max}$

$\delta_{max}=d_{max}-D_{min}$



the designer may choose to specify an interference δ, or the contact pressure *p* itself and then solve for the necessary δ to achieve that *p*.

D: for hole d: for shaft

A solid shaft is to be press fit into a gear hub. Find the maximum stresses in the shaft and the hub. Both are made of carbon steel ($E = 30\times10^6$ psi, $\nu = 0.3$).

- Solid shaft
  - $r_i = 0$ in, $R = 0.5$ in. (nominal)
  - Tolerances: $+2.3\times10^{-3}/+1.8\times10^{-3}$ in.

- Gear hub
  - $R = 0.5$ in. (nominal), $r_o = 1$ in
  - Tolerances: $+0.8\times10^{-3}/0$ in.

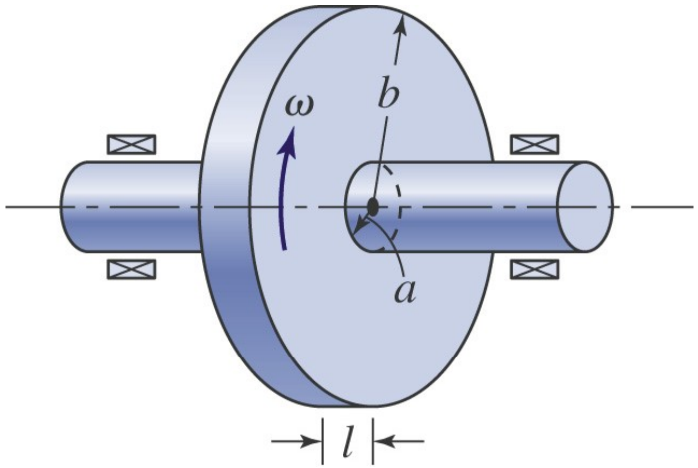
Once p is determined, assume a friction factor *f*: usually $0.15 < f < .20$. The assembly force F to assemble a shrink-fit assembly is given by

$F = 2\pi Rp f L$, where L is the length of the fit.

Holding Torque T is given by:

$T = FR = 2\pi R^2 f p L$

Flywheels



A flywheel is a typically a disc which rotates on a shaft. They are used to smooth out small oscillations and to store energy (kinetic energy of rotation). Examples: cars, hybrids, punch press