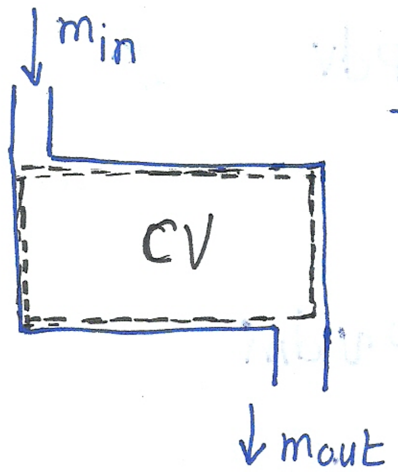
Chapter 6

1st law For a control Volume

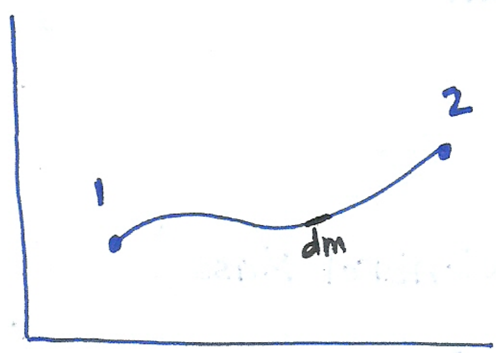
Control Volume "Open System": Mass can cross the boundaries. Must keep track at mass.



For The CV

$$m_2 - m_1 = \Delta m_{cv} = m_{in} - m_{out}$$

Conservation of mass.

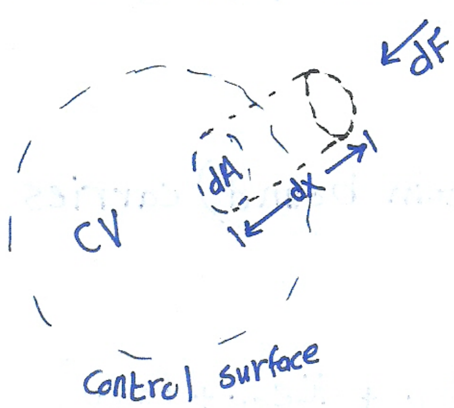


$$\frac{dm_{cv}}{dt} = \frac{dm_{in}}{dt} - \frac{dm_{out}}{dt}$$

$$\frac{dm_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad (kg/s)$$

Rate form

Mass Flow Rate



$dV = dA\,dx$: differential volume entering CV.

$\frac{dV}{dt} = dA \frac{dx}{dt} = dA\,V_n$: Volume flow rate

V_n : velocity normal to dA

mass flow rate : $d\dot{m} = \rho \frac{dV}{dt} = \frac{1}{v} \frac{dV}{dt}$

$$dm^{\circ} = \rho\, dA\, V_n = \frac{1}{v}\, dA\, V_n$$

Flow work

Work done by dF to push volume through CS.

$$\delta W = dF\,dx = P\,dA\,dx = P\,dV$$

Rate of flow work

$$\delta W^{\circ}_{fw} = \frac{\delta w}{dt} = P\frac{dV}{dt} = P v\, dm^{\circ}$$

$$\delta \dot{W}_{FW} = P v\, dm^{\circ} \qquad \dot{w}_{flow} = P\dot{V}$$

1st Law

For a closed system (Control Mass)

$$\left(\frac{dE}{dt}\right)_{CM} = Q^{\circ} - W^{\circ}$$

This needs to be modified for an open system (CV). How? why?

mass crossing the CS (system bound) carries energy with it.

$$E^{\circ}_{cs} = m^{\circ}_{cs}\, e_{cs} = m^{\circ}\,(u_{cv} + ke_{cv} + pe_{cv})$$

"θ"

It helps to account separately for incoming (i) and exiting (e) mass so we have:

$$\dot{E}_{in} = \dot{m}_{in} \overset{``\theta_i"}{e_{in}} \quad , \quad \dot{E}_{out} = \dot{m}_{out} \overset{``\theta_e"}{e_{out}}$$

Thus, for a CV

$$\left(\frac{dE}{dt}\right)_{CV} = \dot{Q} - \dot{W} + \underbrace{\overbrace{\dot{m}_i e_i - \dot{m}_e e_e}^{\text{mass energy}} + \overbrace{\dot{m}_i P_i v_i - \dot{m}_e P_e v_e}^{\text{Flow work}}}_{\text{new terms}}$$

using enthalpy to combine terms

$$e_i = u_i + ke_i + pe_i$$

thus:

$$\dot{m}_i (e_i + P_i v_i) = \dot{m} (\underline{u_i} + ke_i + pe_i + \underline{P_i v_i})$$

$\underline{h_i = u_i + P_i v_i}$ enthalpy of incoming mass.

<u>Then, 1st Law for CV</u>

$$\left(\frac{dE}{dt}\right)_{CV} = \dot{Q} - \dot{W} + \dot{m}_i (h_i + ke_i + pe_i) - \dot{m}_e (h_e + ke_e + pe_e)$$

when kinetic energy, potential energy, and any other forms of energy are negligable:

$$\left(\frac{dU}{dt}\right)_{CV} = \dot{Q} - \dot{W} + \dot{m}_i h_i - \dot{m}_e h_e$$

Types of Processes For CV

1) Steady - state process
2) Transient process

Steady - state Process

- Process occurs continuously and does not change with time.
- Mass and Energy inside CV is always the same amount.

Thus:

$$\left(\frac{dm}{dt}\right)_{cv} = 0 \quad , \quad \left(\frac{dE}{dt}\right)_{cv} = 0$$

Steady flow devices

Assume

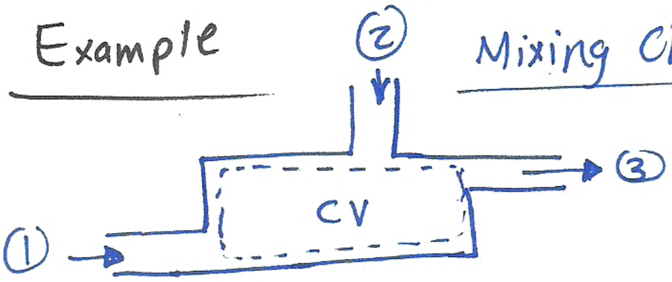
- Heat Exchangers. $w = 0$, $P = \text{constant}$
- Mixing Chambers. $w = 0$, $\boxed{q = 0}$ ← Assume
- Nozzles ⟶ To increase KE. $w = 0$, $q = 0$
- Diffusers ⟶ To Reduse KE. $w = 0$, $q = 0$
- Turbines ⟶ Produce Power. $W_{constant}$, $q = 0$
- Compressors ⟶ To increase pressure. $q \neq 0$
- Throtting Valve ⟶ To reduce pressure. $w = 0$, $q =$
- Pipe and Duct.

Conservation of Mass

$$\cancel{\left(\frac{dm}{dt}\right)_{cv}}^{0} = \dot{m}_i - \dot{m}_e \Rightarrow \dot{m}_i = \dot{m}_e$$

Example

Mixing Champer



- Steady state
- insulated
- inlets : (1) , (2)
- exit : (3)

(1) saturated liquid
$T_1 = 90°C$
$\dot{m}_1 = 5\,kg/s$

(2) superheated vapor
$P_2 = 300\,kPa$
$T_2 = 500°C$

(3) ~~wand~~ saturated vapor
@ $T_3 = 90°C$

What is the needed mass flow rate $\dot{m}_2$ of superheated vapor ?

From Tables

$h_1 = 377\,KJ/kg = h_f\,@90°$
$h_2 = 3486\,KJ/kg$
$h_3 = 2660\,KJ/kg = h_g\,@90°$

Solution

$$\sum \dot{m}_i = \sum \dot{m}_e \Rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3 \longrightarrow \text{Mass Conservation}$$

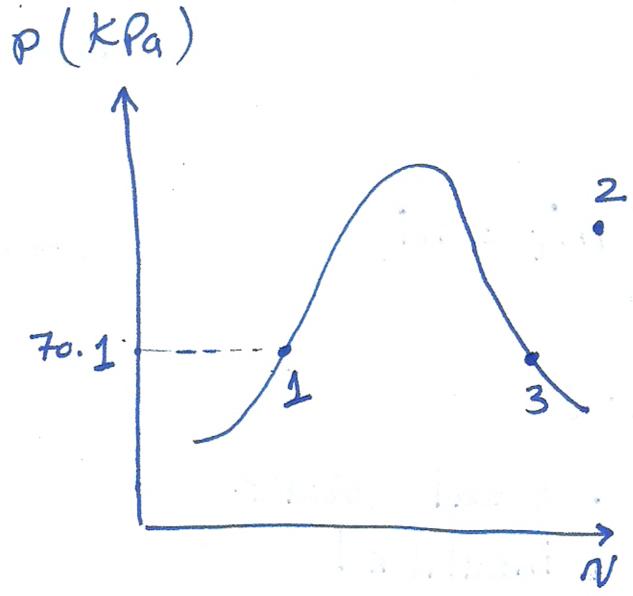
$$\cancel{\left(\frac{du}{dt}\right)_{cv}}^{0} = \cancel{\dot{Q}}^{0} - \cancel{\dot{W}}^{0} + \sum \dot{m}_i h_i - \sum \dot{m}_e h_e$$

$$\dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$$

$$(\dot{m}_1 + \dot{m}_2) h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2 \Rightarrow \dot{m}_1 h_3 + \dot{m}_2 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$$

$$\dot{m}_2 = \frac{\dot{m}_1 (h_3 - h_1)}{h_2 - h_3} = \frac{5\,\frac{kg}{s}\,(2660 - 377)\,\cancel{KJ/kg}}{(3486 - 2660)\,\cancel{KJ/kg}}$$

$$= 13.8\,kg/s$$

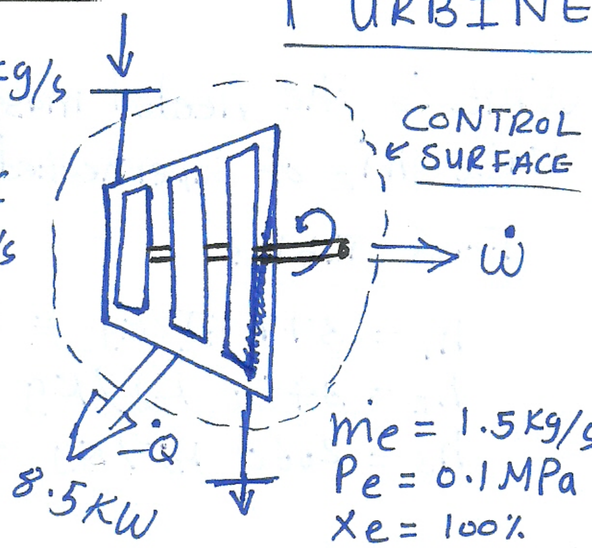


Example

TURBINE

Determine Power Produced by a turbine.

$\dot{m}_i = 1.5\,\text{kg/s}$
$P_i = 2\,\text{MPa}$
$T_i = 350\,^\circ C$
$V_i = 50\,\text{m/s}$
$Z_i = 6\,\text{m}$



$\dot{m}_e = 1.5\,\text{kg/s}$
$P_e = 0.1\,\text{MPa}$
$x_e = 100\%$
$V_e = 100\,\text{m/s}$
$Z_e = 3\,\text{m}$

Tables

inlet: superheated
$h_i = 3137\ \text{KJ/Kg}$
exit: Saturated vapor
$h_e = 2675.5\ \text{KJ/Kg}$

Solution

cons. of mass

$$\dot{m}_i = \dot{m}_e = 1.5\ \text{Kg/s}$$

1st Law

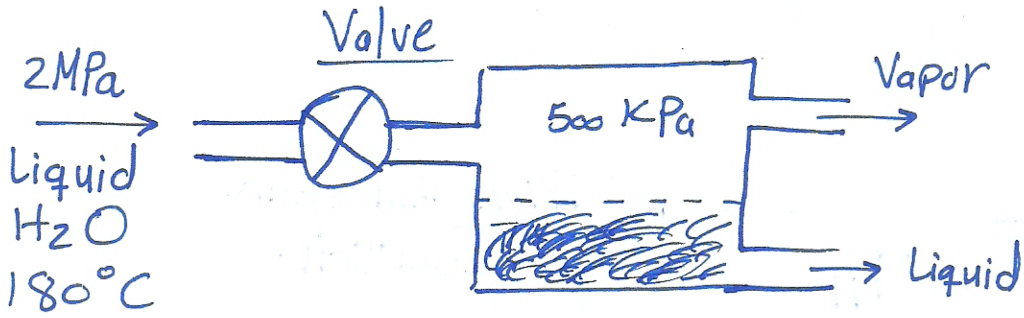
$$\dot{W} = \dot{Q} + \dot{m}\left(h_i - h_e + \tfrac{1}{2}V_i^2 - \tfrac{1}{2}V_e^2 + g z_i - g z_e\right)$$

$$= -8.5\ \text{KW} + 1.5\,\frac{\text{Kg}}{\text{s}}\left(461 + \underbrace{\frac{1}{2}\,\frac{50^2 - 100^2}{1000}}_{-3.7} + \underbrace{\frac{29.4}{1000}}_{0.03}\right)\frac{\text{KJ}}{\text{Kg}}$$

$$\dot{W} = 677\ \text{KW}$$

Example

Find x in chamber



Assume steady-state

Tables

$h_i = 763.05 = h_{i\,@180°}$, inlet: compressed liquid.

Cons. of mass

$\dot{m}_i = \dot{m}_e$

1st Law: $\dot{W} = 0$; $\dot{Q} = 0$, $Ke \approx 0$, $Pe \approx 0$

$$\dot{m}_i h_i = \dot{m}_e h_e \Rightarrow h_i = h_e$$

Exit

$h_e = 763.05$ kJ/kg
$P_e = 500$ kPa.

$h_f = \cancel{649.54}$ 640.09 kJ/kg , $h_g = 2748.1$ kJ/kg

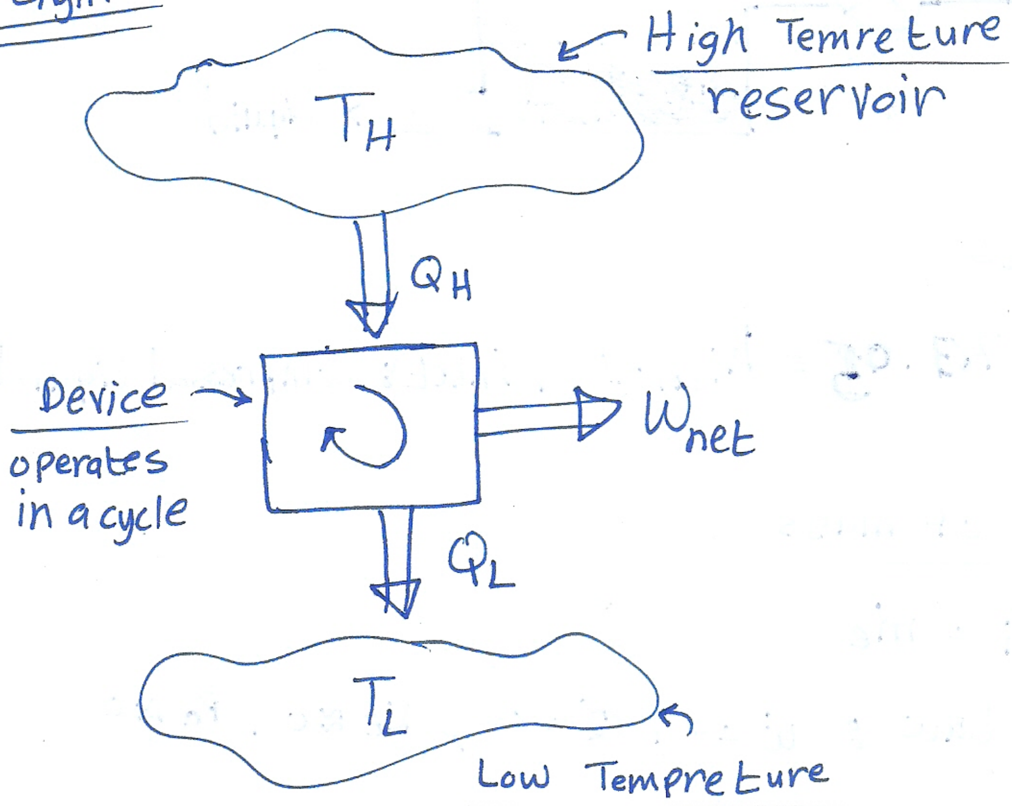
$$h_f < h_e < h_g$$

$$x = \frac{h_e - h_f}{h_{fg}} = 0.0447 = 4.47\%$$

Chaper 7

The 2nd law of thermodynamics

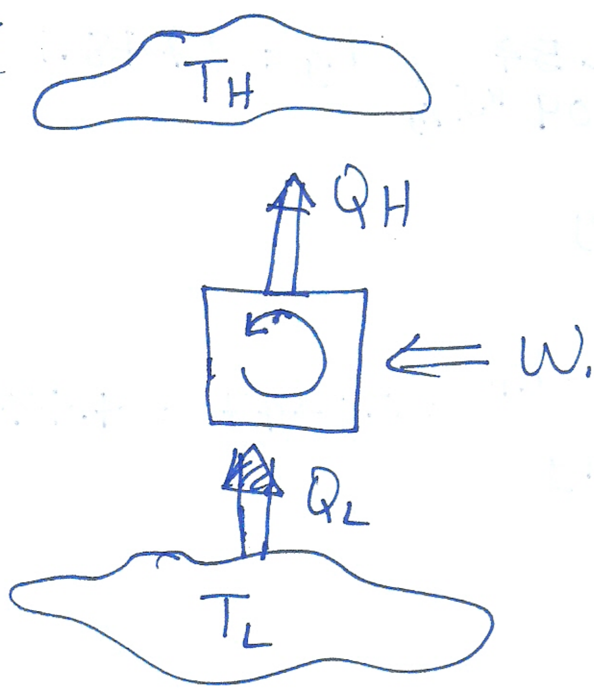
Heat Engine



Heat Engine: A Device that operates in a cycle, recieves energy from a high tempreture ~~resevoir~~ reservoir, delivers net amount of work and rejects left energy to a low tempreture reservoir.

Refrigerator

Heat Pump



Heat Engine

- Operates in a thermodynamic cycle.
- delivers a net amount of work.
- transfers heat from a high-temp reservoir to a low-T reservoir.

Refrigerator or Heat Pump

$\rightarrow$ Remove Q_L $\rightarrow$ Supply Q_H

- operates in a thermodynamics cycle.
- take in some amount of work
- transfers heat from a low-T reservoir to a high-T reservoir.

Thermal Reservoir

A system large enough that it remains at constant temprature while it receives or gives heat.

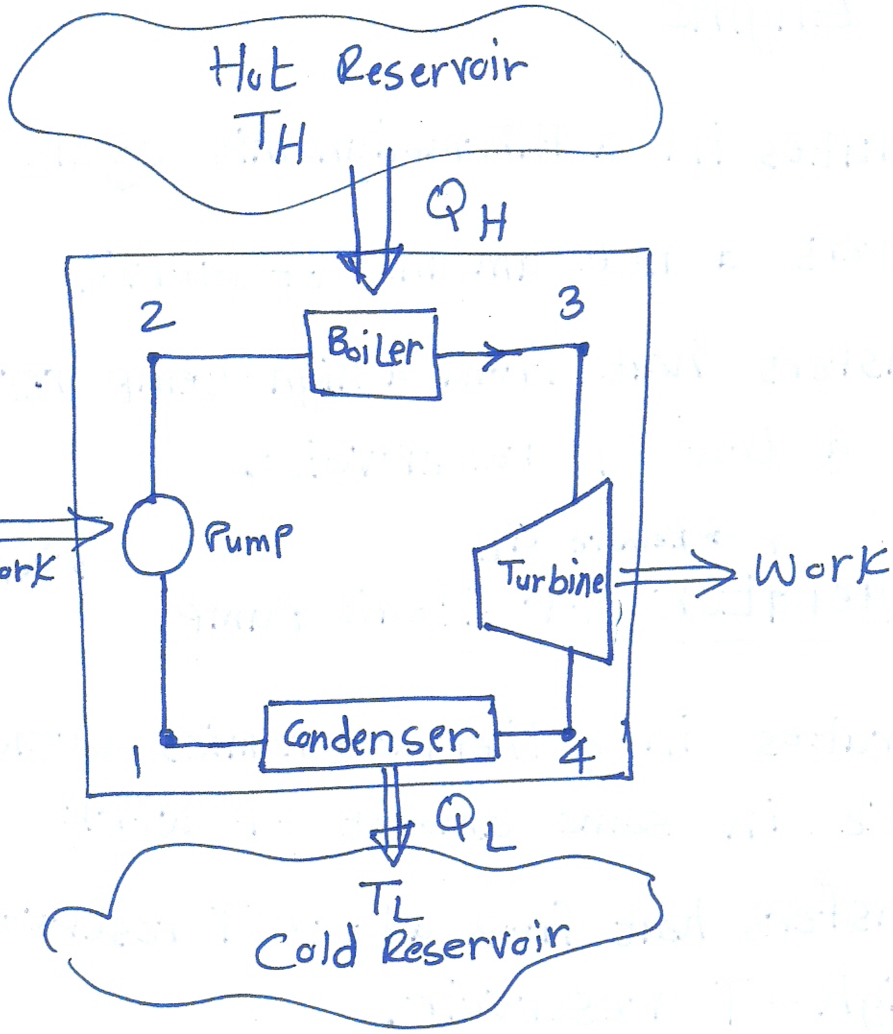
Example: "A Large body of water (ocean, river, lake..)"
"The Atmosphere".

2
Liquid

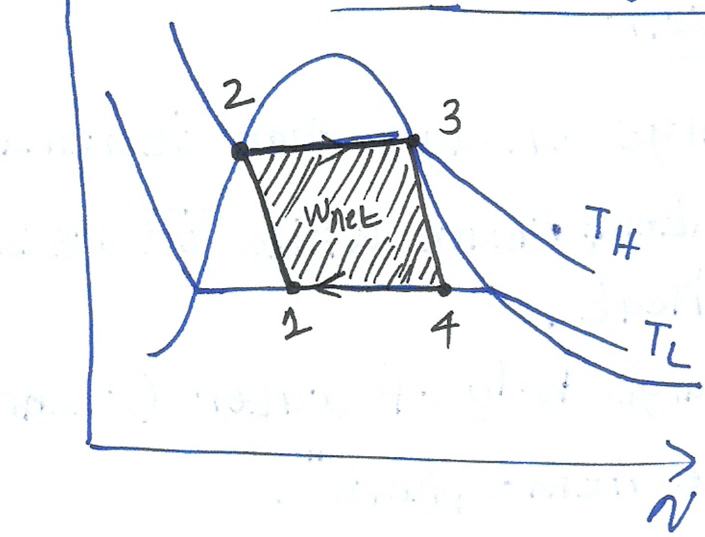
3
High Pressure Steam

4
Low Pressure steam

1
Liquid



Pv - Heat engine evolving water



Efficiency of Heat Engine

"eta" $\eta = \dfrac{W_{net}}{Q_H} = \dfrac{\text{want}}{\text{costs}}$

1^{st} Law : $Q_H - Q_L = W_{net}$, $\Delta U = 0$ (cycle)

$$\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Coefficient of Performance

$$COP_R \doteq \frac{Q_L}{W} = \frac{\text{want}}{\text{costs}}$$

1^{st} Law : $W = Q_H - Q_L$

$$COP_R = \frac{Q_L}{Q_H - Q_L} \longrightarrow \text{For Refrigerator}$$

$$COP_{HP} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \longrightarrow \text{For Heat Pump}$$

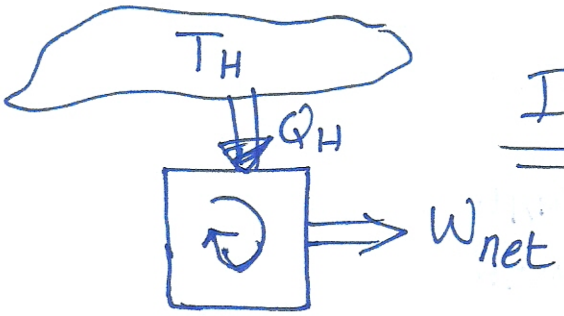
2nd Law statements

- Kelvin-Planck statement (K-P)
- Clausius statement (C)

  - K-P statement refers to HE.
  - C statement refers to Refrig.
  - They are equivalent.

Kelvin Plank statement

Impossible to make a device which operates in a cycle and delivers a net amount of work, while exchanging energy (heat) with a single reservoir.



Impossible!

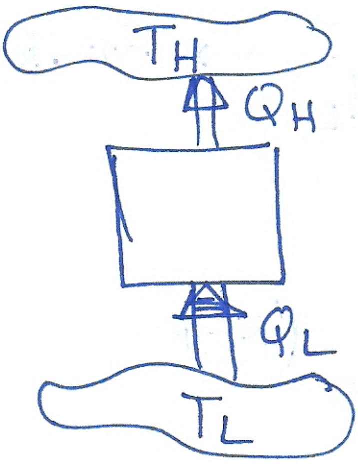
$Q_L \neq 0$

$\eta \neq 1$

$\eta < 1$

CLAUSIUS statement

Impossible to make a device which operates in a cycle and produces no effect other than the transfer of heat from a cold reservoir to a hot reservoir.



Impossible!

$W \neq 0$

$COP \neq \infty$

1st Law

$Q_H = Q_L$ ✓

Note: These devices satisfy the 1st law if:

$W_{net} = Q_H$ For (K-P)

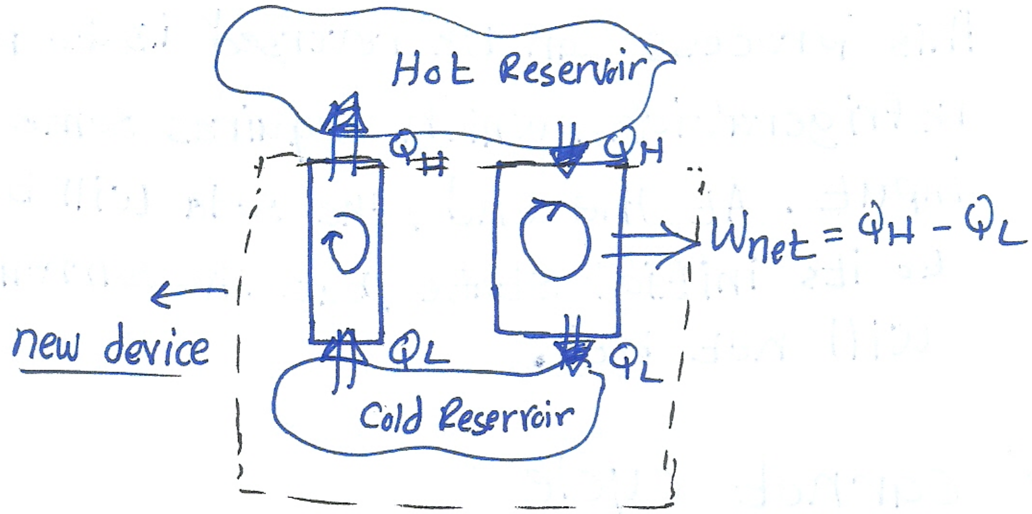
$Q_H = Q_L$ For (C)

However they violate the 2nd Law!

Prove equivalence of two statement

- To Prove that they are equivalent, you would have to prove that if you violate one statement you are in other way violate the second statement.

- The only way to prove this is to assign a device that has both ~~HE~~ HE and Refrig.



The Refrig. violates the second statement (c).

The new device violats (K-P) statement

so the two statement are equivalent.

Perpetual Motion Machines

- of the first Kind: violate the 1^{st} law.
- of the 2^{nd} Kind: violate the 2^{nd} Law

Reversible Process

A process that can be reversed, returning both the System and the surrounding to their original states.

All real process are <u>irreversible</u>.

<u>why?</u> Friction, Finit-Time process, heat transfer through a finit temprature difference, ---

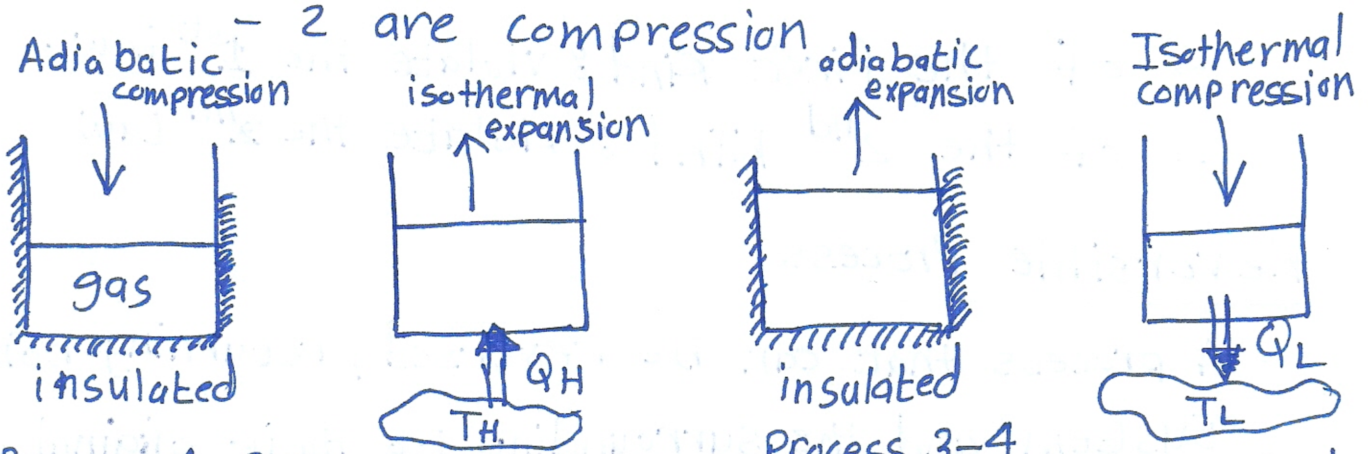
<u>Explaination</u>

Consider a can of soda left in a warm room. Heat is trasfered to the soda. The only way this process can be reversed is to provide refrigeration, which requires some work input. At the end, the soda will be restored to its initial state, but the surroundings will not be.

The carnot cycle

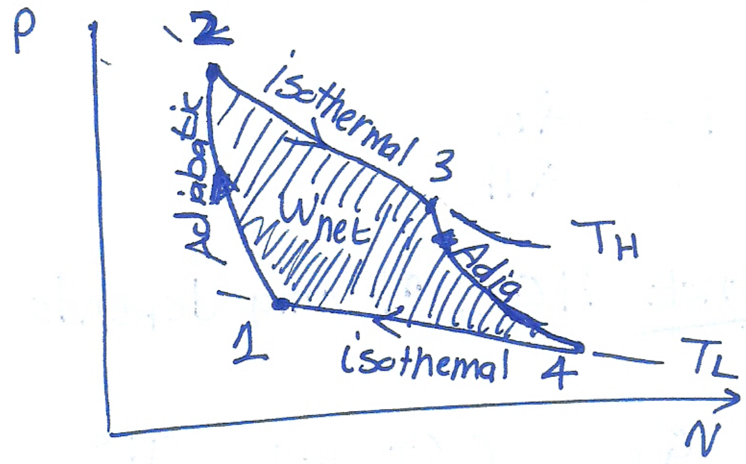
- Reversible
- consist of 4 proesses
  - 2 processes ARe adiabatic.
  - 2 are isothermal

  - 2 are expansion
  - 2 are compression



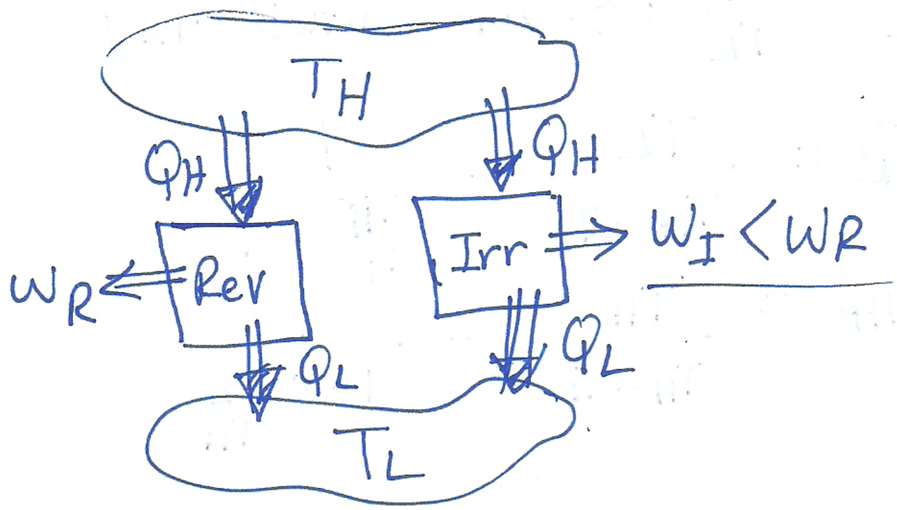
Process 1-2 Process 2-3 Process 3-4 Process 4-1

P–V diagram of the carnot cycle



The carnot Principles

1. For Given T_H and T_L, A reversible (HE) is more efficient than any irreversible (HE).
2. For Given T_H and T_L, All carnot HE have the same efficiency.



Absolute Temp Scale

$$\eta_{HE} = 1 - \frac{Q_L}{Q_H}$$

For a carnot HE, η only depends on T_H and T_L

Thus:

$$\frac{Q_L}{Q_H} = f(T_H, T_L) = \frac{T_L}{T_H}$$

$$\eta_{CHE} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$ Kelvin

Carnot Refrigerator

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

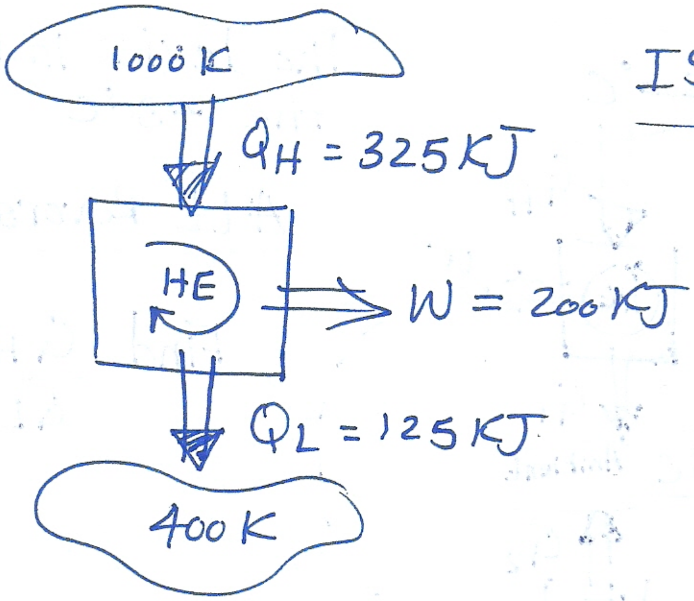
$$\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$$ $\Rightarrow$ For Carnot only

$$COP_R = \frac{T_L}{T_H - T_L} = \frac{1}{\frac{T_H}{T_L} - 1}$$

$$COP_{HP} = \frac{T_H}{T_H - T_L} = \frac{1}{1 - \frac{T_L}{T_H}}$$

Example

IS THIS POSSIBLE ?



Solution

1st law: $Q_H - Q_L = W$

2nd Law: (K-P)

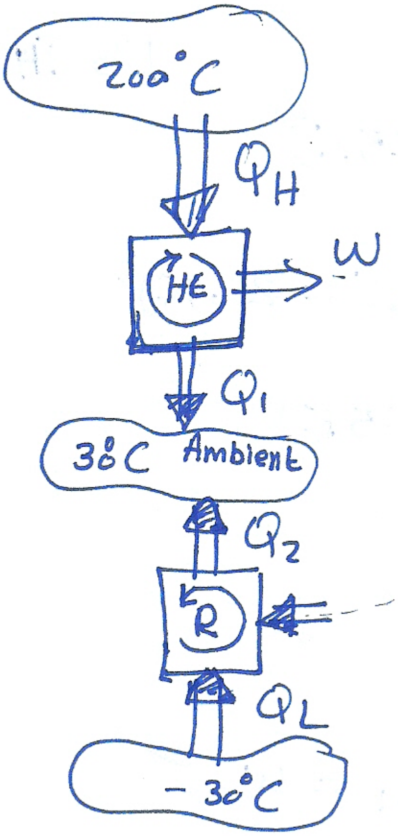
$$\eta_{HE} = \frac{W}{Q_H} = \frac{200}{325} = 0.615$$

$$\eta_{CHE} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = 0.6$$

$\eta_{HE} > \eta_{CHE}$ Impossible !

If $\eta_{HE} = \eta_{CHE}$, also Impossible !

Example



The device is maintain the -30°C

ALL Reversible

Find $\frac{Q_H}{Q_L}$

Solution

Heat Engine

$$\eta = 1 - \frac{T_A}{T_H} = 1 - \frac{30+273}{200+273} = 0.36$$

$$\eta = \frac{W}{Q_H} \Rightarrow W_{HE} = 0.36\, Q_H \quad (1)$$

Refrigerator

$$\cancel{POC}\, COP_R = \frac{1}{\frac{30+273}{-30+273} - 1} = 4.05$$

$$\cancel{POC}\, COP_R = \frac{Q_L}{W} \Rightarrow W = \frac{Q_L}{4.05} \quad (2)$$

$$(1) = (2) \Rightarrow 0.36\, Q_H = \frac{Q_L}{4.05} \Rightarrow \frac{Q_H}{Q_L} = 0.685$$

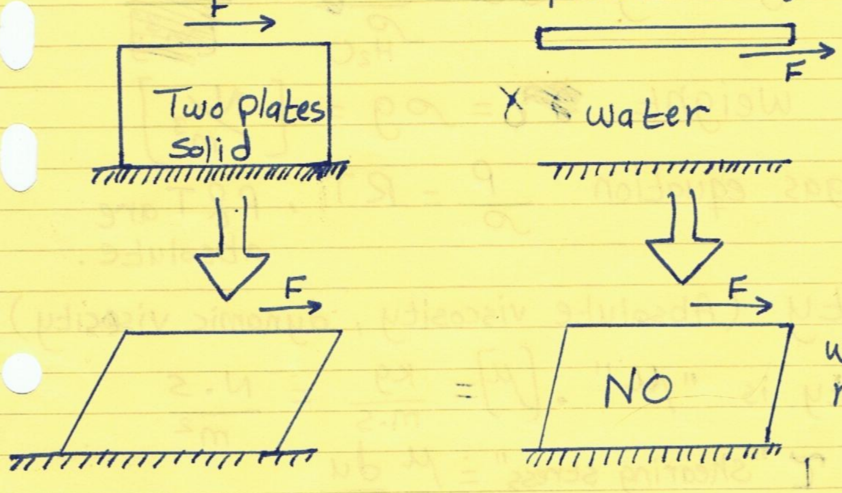
Fluid Mechanics

"flow"

Fluid – A substance that deforms continuously when acted upon by a shearing stress of any size. "gas". "Liquid".

$$\text{Stress} = \frac{\text{force}}{\text{Area}}$$

Shearing stress – The stress component parallel to a given surface.



Fluid will not stop moving.

Solving problems in Fluid Mechanics

Fluids ~~must~~ must satisfy:

1. conservation of mass.
2. Newton's 2nd Law.
3. 1st Law of Thermodynamics.
4. 2nd Law of Thermodynamics.

- Ice is not a fluid. It always remain in its shape.

- Solids has ability to retain its shape by resisting the shearing action. but fluids cannot.

Fluid Properties

Density $\rho = \frac{m}{\forall} = \frac{[kg]}{[m^3]}$

specific volume $v = \frac{1}{\rho}$

Specific gravity $SG = \frac{\rho}{\rho_{H_2O}}$

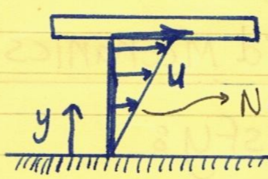
Specific weight $\gamma = \rho g = \left[\frac{N}{m^3}\right]$

Ideal gas equation $\frac{P}{\rho} = RT$, P & T are absolute.

* Viscosity (Absolute viscosity, dynamic viscosity)

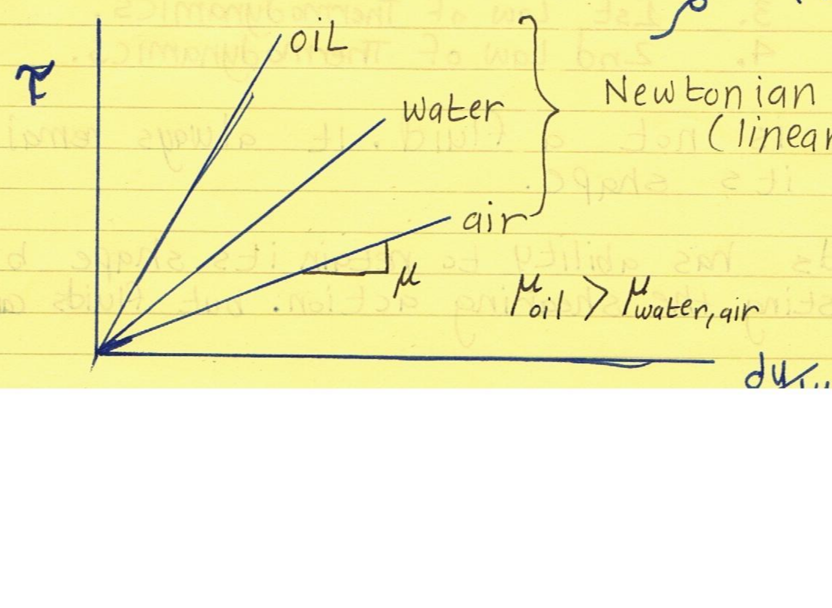
Viscosity is "μ". $[\mu] = \frac{kg}{m.s} = \frac{N.s}{m^2}$

τ "shearing stress" $= \mu \underbrace{\frac{du}{dy}}_{\text{velocity gradient.}}$



No Slip Condition "V is zero at the fixed plate and increasing in y direction"

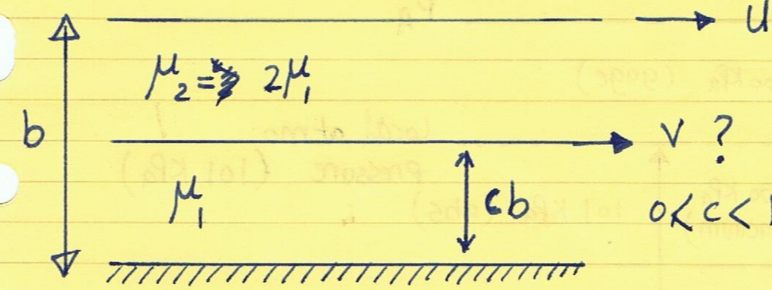
Kinematic viscosity "ν" $= \frac{\mu}{\rho}$,



Newtonian fluids (linear).

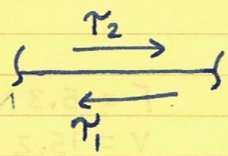
$\mu_{oil} > \mu_{water, air}$

Example Determine V if Top plate is moving.



Solution

Apply force balance on middle plate

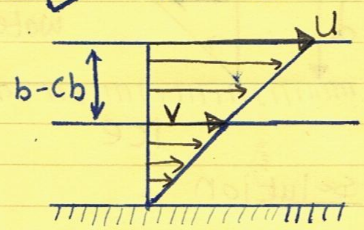


$$\tau = \frac{F}{A} \Rightarrow F = \tau A$$

$$\sum F_H = 0 = \tau_1 A - \tau_2 A \Rightarrow \tau_1 = \tau_2$$

$$\tau_1 = \mu_1 \frac{du}{dy} = \mu_1 \frac{V}{cb}$$

$$\tau_2 = \mu_2 \frac{u - V}{b(1-c)}$$



$$\cancel{\mu_1} \frac{V}{c\cancel{b}} = 2\cancel{\mu_1} \frac{u-V}{\cancel{b}(1-c)} \quad , \quad V = \frac{2c}{1+c} u$$

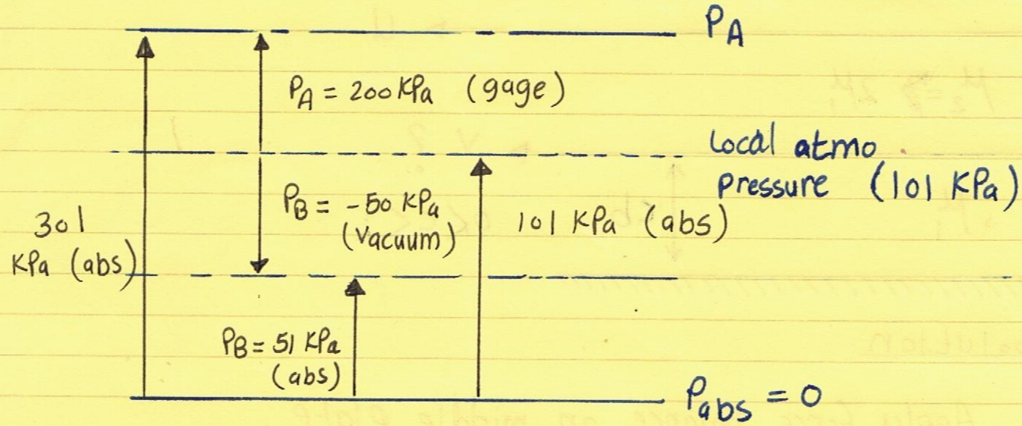
Pressure - A normal force exerted by a fluid per unit area.

Absolute pressure - The actual pressure (refrenced to zero pressure).

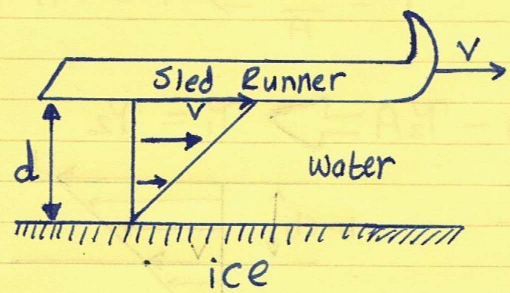
gage pressure - refrenced to $\underbrace{\text{local}}$ atmospheric (can be + or −) pressure.
where you are when you make the measurement.

sometimes called vacuum Pressure

$P_{atm} = 101\ KPa$



Example Find d?



$F = 5.3\ N$
$V = 15.2\ m/s$
$A = 0.08\ m^2$
$\mu = 1.7 \times 10^{-3}\ \frac{Kg}{m \cdot s}\ \frac{N.s}{m^2}$

Solution

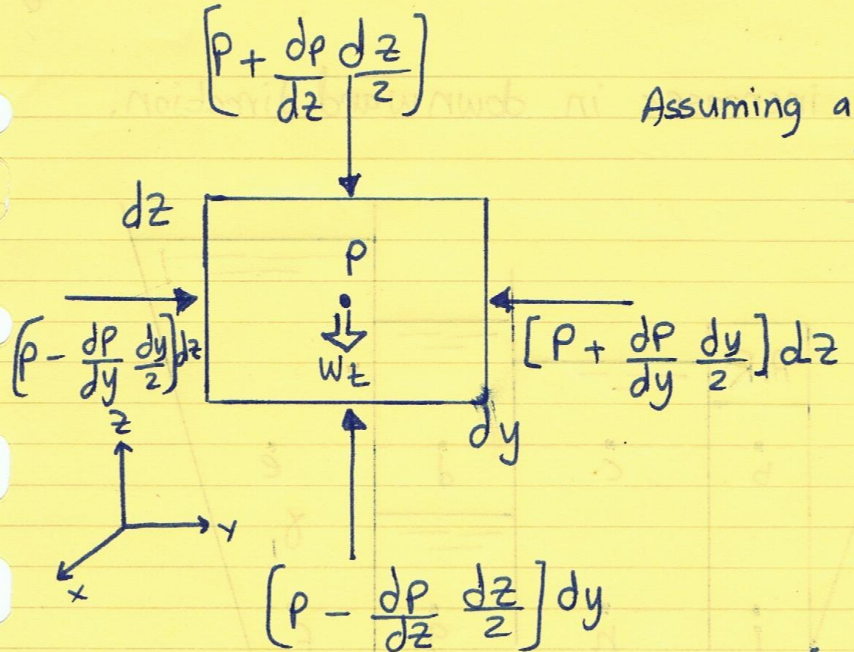
$$\tau = \mu \frac{du}{dy} = \mu \frac{V-0}{d-0}$$

$$\frac{F}{A} = \mu \frac{V}{d} \Rightarrow \frac{5.3\ \cancel{N}}{0.08\ \cancel{m^2}} = 1.7 \times 10^{-3}\ \frac{\cancel{N.s}}{\cancel{m^2}} \left(\frac{15\ m}{d\ \cancel{s}}\right)$$

$$\Rightarrow d = 3.85 \times 10^{-4}\ m$$

* Pressure is the same from any direction at a point.

Pascal's Law $\boxed{P = P_y = P_z = P_s}$



Assuming a unit depth
$dx = 1$

$\xrightarrow{+} \sum F_Y = ma \Rightarrow \left[\cancel{p} - \frac{dp}{dy}\frac{dy}{2}\right]dz - \left[\cancel{p} + \frac{dp}{dy}\frac{dy}{2}\right]dz = ma$

$a = 0$ since it is a static fluid.

$$\frac{dp}{dy}\,dy\,dz = 0 \Rightarrow \frac{dp}{dy} = 0$$

in x-direction $\Rightarrow \frac{dp}{dx} = 0$

* So pressure does not vary on horizontal planes as long as it is the <u>same fluid</u> and <u>continuous</u>.

$+\uparrow \sum F_z = 0$

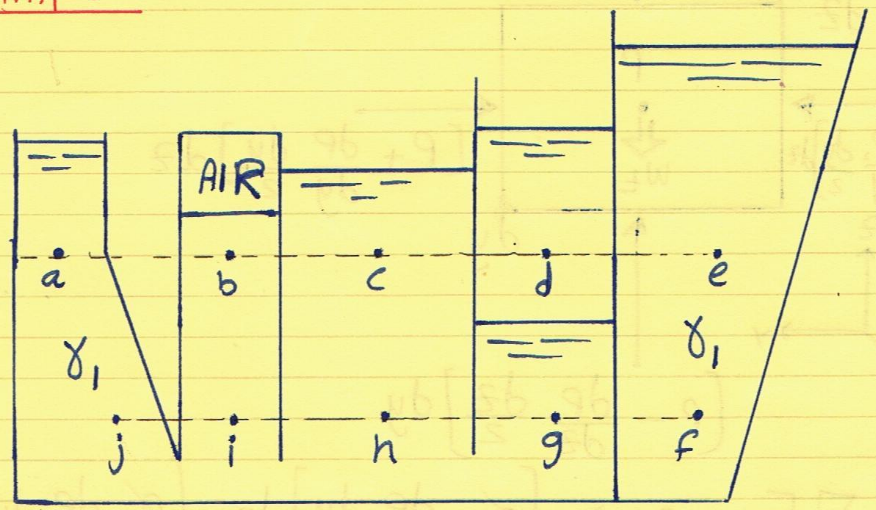
$$\left[\cancel{p} - \frac{dp}{dz}\frac{\cancel{dz}}{2}\right]\cancel{dy} - \left[\cancel{p} + \frac{dp}{dz}\frac{\cancel{dz}}{2}\right]\cancel{dy} - \underbrace{W_t}_{\gamma(dy)(dz)(1)} = 0$$

$$-\frac{dp}{dz} - \gamma = 0$$

$$\boxed{\frac{dp}{dz} = -\gamma}$$

* pressure increases in downward direction.

Example



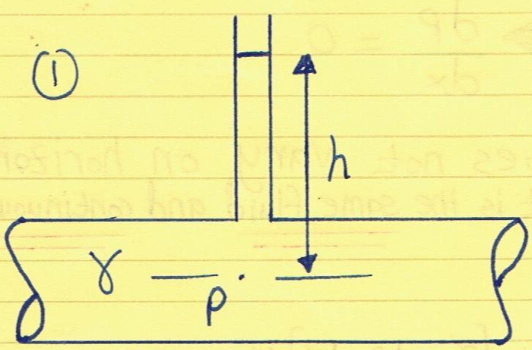
$P_a = P_b = P_c \neq P_d \neq P_e$

$P_j = P_i = P_h = P_g \neq P_f$

Pressure Measurement

equal $\left\{ \begin{array}{l} 101.3 \text{ KPa} \\ 14.7 \text{ psia} \\ 760 \text{ mmHg} \\ 30 \text{ in Hg} \\ 10.3 \text{ m } H_2O \end{array} \right.$

(1)



piezometric tube

if $P = 202.6$ KPa $= 20.6$ m H_2O

20.6 m height !!

It is not practical to measure the pressure using piezometric tube.

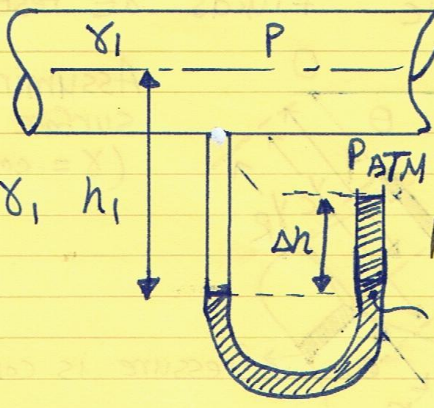
Assuming incompressible, $\gamma = \underline{cons}$

$$\frac{dP}{dz} = -\gamma \qquad \underline{h = z}$$

$$\Delta P = -\gamma \Delta h$$

$$P_1 - P_2 = -\gamma(h_2 - h_1)$$

up down



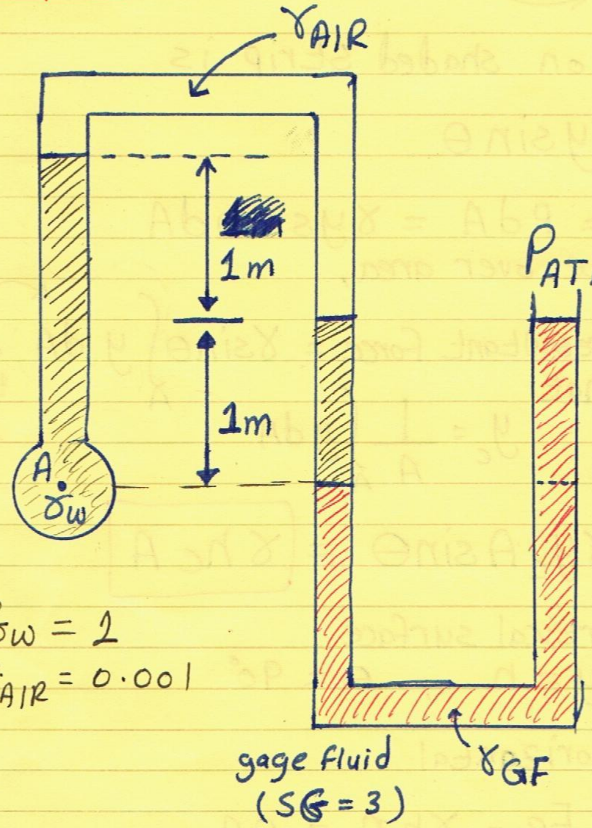
$P_{ATM} = 0$ gage

note $\gamma_2 > \gamma_1$

$$P + h_1\gamma_1 - \Delta h\gamma_2 = 0$$

$$P = \gamma_2\Delta h - \gamma_1 h_1$$

Example



$SG_w = 1$

$SG_{AIR} = 0.001$

$$P_A - 2\gamma_w + 1\gamma_{AIR} + 1\gamma_w - 1\gamma_{GF} = 0$$

$$P_A = \gamma_w - \gamma_{AIR} + \gamma_{GF}$$

$$P_A = (SG_w - \cancel{SG_{AIR}} + SG_{GF})\gamma_w$$

neglict

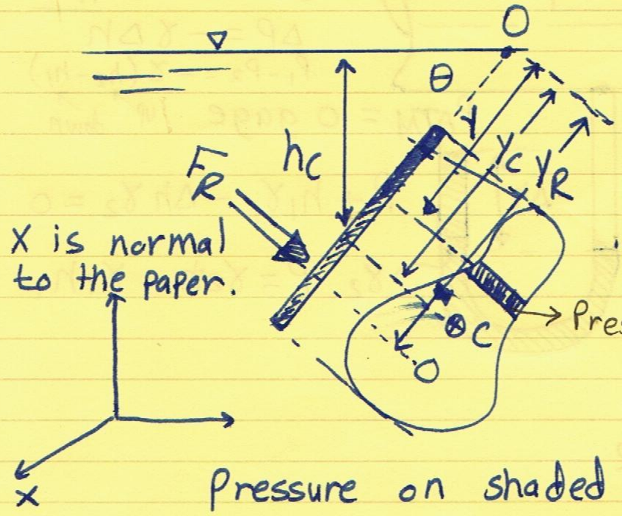
$$P_A = (1+3)\left(1000\,\frac{kg}{m^3}\right)(9.8)$$

$$P_A = (9.8)(4)(1000) = 39.2\ \text{KPa}$$

Forces on Submerged surface

Fluid static – fluids at rest.

Assuming plane surface and incompressible (γ = constant).



Pressure on shaded strip is

$$P = \gamma y \sin\theta$$

so

$$dF = P\,dA = \gamma y \sin\theta\, dA$$

integrate over area,

$$F_R = \text{resultant force} = \gamma \sin\theta \int_A y\, dA$$

1st momen of area w/ to x-axis

recall that

$$y_c = \frac{1}{A}\int_A y\, dA$$

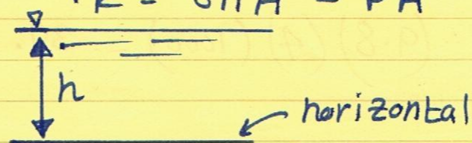
$$F_R = \gamma y_c A \sin\theta = \boxed{\gamma h_c A}$$

for vertical surface

$$y = h \quad , \quad \theta = 90^\circ$$

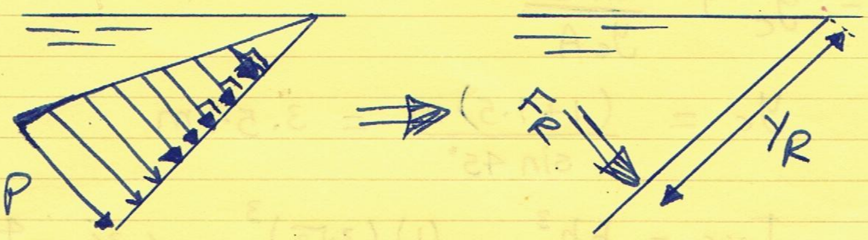
for horizontal

$$F_R = \gamma h A = PA$$

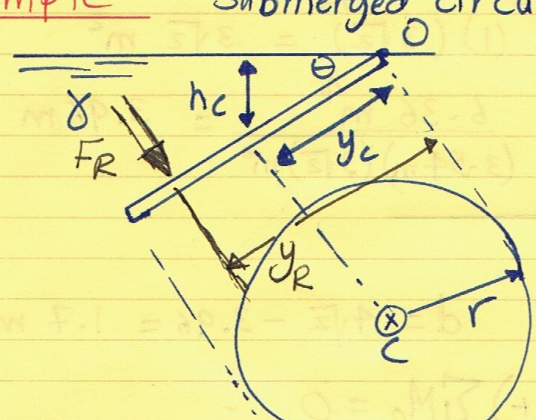


The force F_R acts at,

$$y_R = y_c + \frac{I_{xc}}{y_c A} \text{, From statics.}$$



Example submerged circular area.



(a) F_R (b) y_R

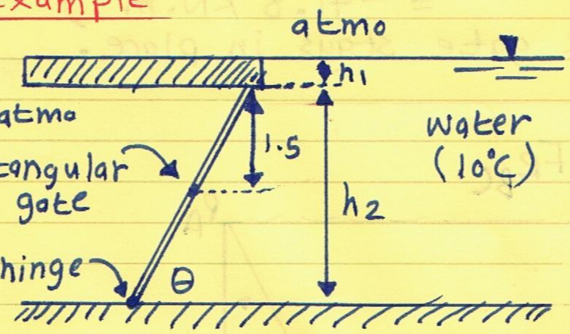
(a) $F_R = \gamma h_c A$

(b) $y_R = y_c + \dfrac{I_{xc}}{y_c A}$

$= r + \dfrac{\frac{\pi r^4}{4}}{r(\pi r^2)}$

$y_R = r + \dfrac{r}{4}$

Example



$\theta = 45^\circ$
$h_1 = 1\,m$
$h_2 = 3\,m$
2 m into paper.
W_t Gate = 90 KN

Will the gate stay in place or fall ?

If the water moment force is greater than gate weight moment then gate will stay in place.

$$F_R = \gamma h_c A = \left(9810 \ \frac{N}{m^3}\right)\left(1+1.5 \ m\right)(1)\left(\frac{3}{\sin 45^\circ}\right) m^2$$

$$= 104{,}051 \ N = 104.1 \ KN$$

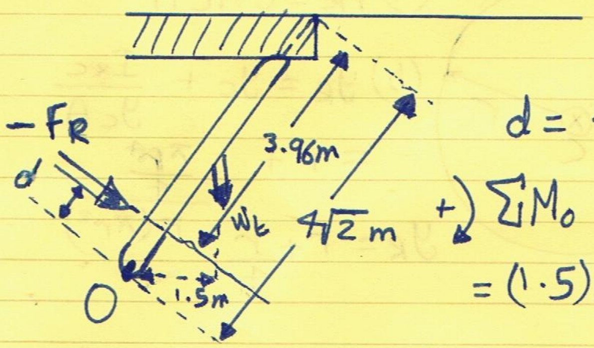
$$y_R = y_c + \frac{I_{xc}}{y_c A}$$

$$y_c = \frac{(1+1.5) \ m}{\sin 45^\circ} = 3.54 m$$

$$I_{xc} = \frac{bh^3}{12} = \frac{(1)(3\sqrt{2})^3}{12} = 6.36 \ m^4$$

$$A = bh = (1)(3\sqrt{2}) = 3\sqrt{2} \ m^2$$

$$y_R = 3.54 \ m + \frac{6.36 \ m^4}{(3.54 m)(3\sqrt{2}) m^2} = 3.96 m$$



$$d = 4\sqrt{2} - 3.96 = 1.7 \ m$$

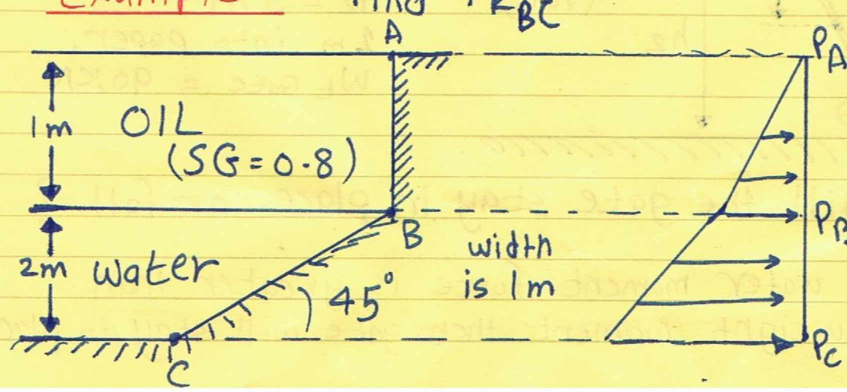
$$+\circlearrowright \sum M_o = 0$$

$$= (1.5)(90) - (104.1)(1.7) =$$

$$= -41.8 \ KN.m$$

* The gate stays in place.

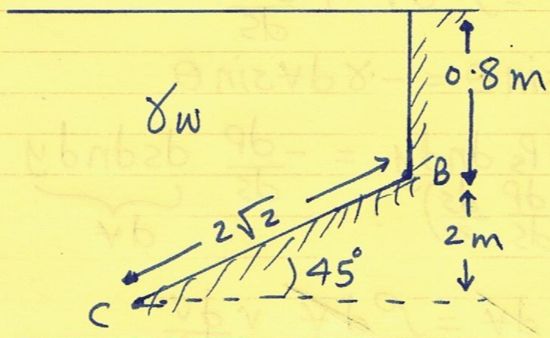
Example Find $F_{R_{BC}}$



You can solve it using statics or you can replace Oil with ~~an~~ equivalent layer of water which we will do.

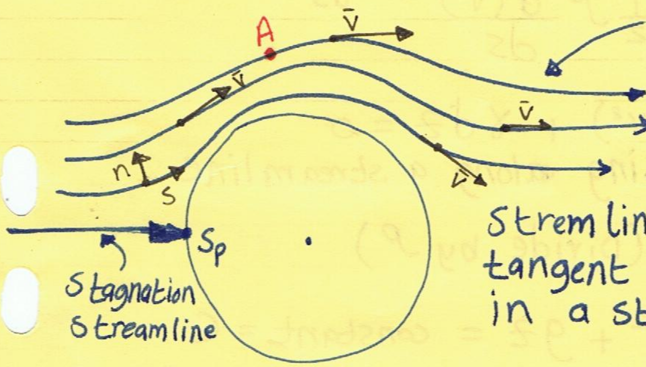
$$p_B = \gamma_{oil}(1) = \gamma_{water}\, h_w$$

$$h_w = SG_{oil}\,(1) = 0.8\ m$$



$$F_{R_{BC}} = \gamma_w\, h_c\, A$$
$$= (9810)(0.8+1)(2\sqrt{2})(1)$$
$$= 499.45\ KN$$

Bernoulli Equation



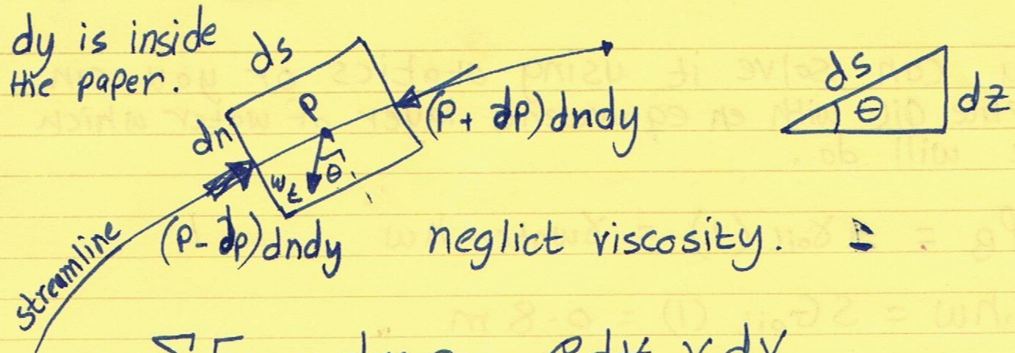
"A" does not change with time unless the body is moving.

Stremlines are lines that are tangent to the velocity vector in a steady flow field.

Two component of acceleration

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V$$

$$a_n = \frac{V^2}{R}$$ raduis of curvature



neglict viscosity.

$$\Sigma F_s = dm\, a_s = \rho\, d\forall\, V \frac{dV}{ds}$$

$$dW_s = -dw \sin\theta = -\gamma\, d\forall \sin\theta$$

$$dF_{ps} = -2\, dp_s\, dn\, dy = -\frac{dp}{ds} \underbrace{ds\, dn\, dy}_{d\forall}$$

$$\left(dp_s = \frac{dp}{ds}\frac{ds}{2}\right)$$

so,

$$\left(-\gamma \sin\theta - \frac{dp}{ds}\right) \cancel{d\forall} = \rho\, \cancel{d\forall}\, \underbrace{V \frac{dV}{ds}}_{\frac{1}{2}\frac{d}{ds}(V^2)}$$

$$-\gamma \frac{dz}{ds} - \frac{dp}{ds} = \frac{1}{2}\rho \frac{d(V^2)}{ds}$$

$$dp + \frac{1}{2}\rho\, d(V^2) + \gamma\, dz = 0$$

using along a streamline.

integrate (Divide by ρ)

$$\int \frac{dp}{\rho} + \frac{1}{2}V^2 + gz = \text{constant} = C$$

note, C has the same value anywhere along the streamline.

Assume incompressible fluid ($\rho = $ const)

Bernoulli Equation $\boxed{\frac{p}{\rho} + \frac{V^2}{2} + gz = C}$ $\left[\frac{m^2}{s^2}\right]$

Assumption (Limitations)

(1) steady flow.
(2) neglict viscosity.
(3) incompressible.
(4) along streamline.
(5) neglict heat transfer.
(6) no shaft work.

— multiply eq by ρ

$$\boxed{P + \frac{1}{2}\rho V^2 + \gamma z = \text{constant}}$$

[~~KPa~~] [Pa]

[N/m²]

P : static pressure.

$\frac{1}{2}\rho V^2$: dynamic pressure.

γz : hydrostatic pressure.

$P + \frac{1}{2}\rho V^2$: stagnation pressure.

* a stagnation point in a flowfield is where $V = 0$.

— divide by g

$$\boxed{\frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{const}}$$

[m]

Head form of Bernoulli

$\frac{P}{\gamma}$: pressure head.

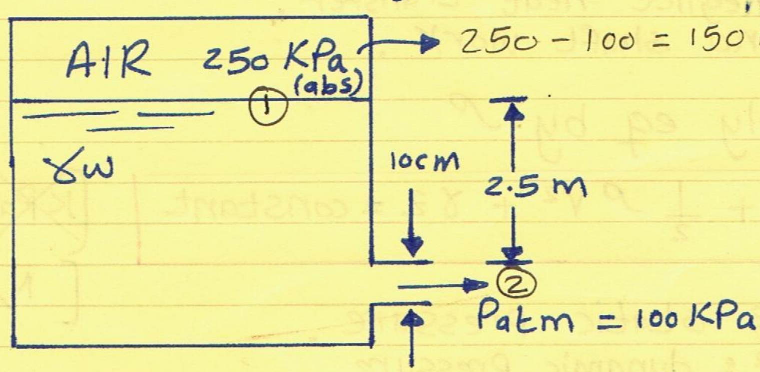
$\frac{V^2}{2g}$: velocity head.

z : elevation head.

* When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (Not just along a streamline)

Example (12 - 30p)

$\dot{V}_w$? Neglecting frictional effects



→ 250 − 100 = 150 KPa (gage)

$$\frac{P_1}{\gamma_w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_w} + \frac{V_2^2}{2g} + Z_2$$

$P_2 = 0$, $Z_2 = 0$, $V_1 = 0$

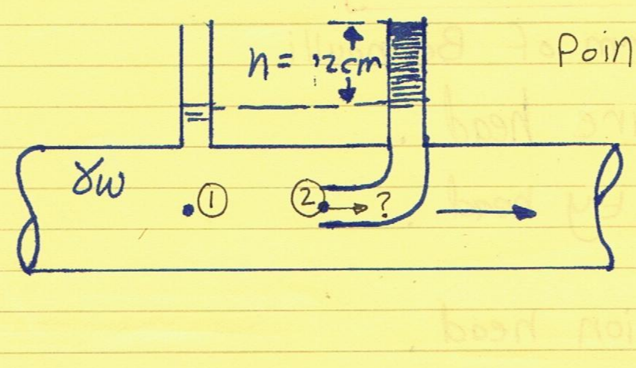
$$\frac{150 \times 10^3}{9810} + 2.5 = \frac{V_2^2}{2 \times 9.81}$$

$(0.1)^2 \pi/4$

$V_2 = 18.68$ m/s , $\dot{V} = A_2 V_2 = 0.147\ m^3/s$

Example

V ?



Point 2 is a stagnation Point $V_2 = 0$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

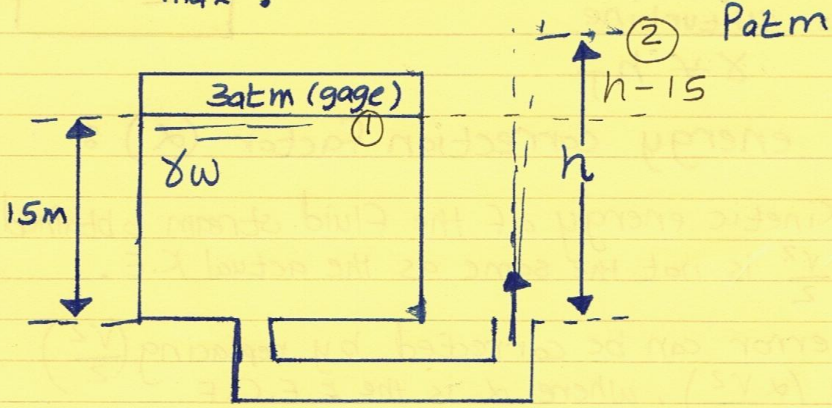
$Z_1 = Z_2$, $V_2 = 0$

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} \Rightarrow \frac{V_1^2}{2} = \frac{\rho g h}{\rho}$$

$$V_1 = \sqrt{2gh} = 1.53 \text{ m/s}$$

Example (12-33P)

h_{max} ? 2 atm → 101.3 KPa



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$P_2 = 0$, $Z_1 = 0$, $V_1 = 0$, $V_2 = 0$

$$\frac{3\,(101.3) \times 10^3}{9810} = h - 15$$

$h = 46$ m

Energy Equation (Extended Bernoulli)

$$\boxed{\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_T + h_L}$$

Units : [m] . The form we will use.

for a pump $\dot{W}_p = \gamma \dot{\forall} h_p$ (watt) (out)
for a turbine $\dot{W}_T = \gamma \dot{\forall} h_T$ (watt) (in)

$$\eta_p = \frac{\dot{W}_p}{\dot{W}_{\text{into pump}}} = \frac{\gamma \dot{\forall} h_p}{\dot{W}_{\text{into pump}}}$$

$$\boxed{h_L > 0}$$

$$\eta_T = \frac{\dot{W}_{\text{turbine}}}{\gamma \dot{\forall} h_T}$$

Kinetic energy correction factor (α) :

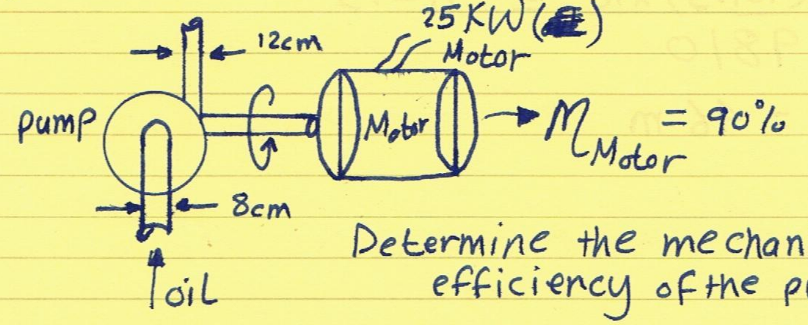
The Kinetic energy of the fluid stream obtained from $\frac{V^2}{2}$ is not the same as the actual K.E.

This error can be corrected by replacing $\left(\frac{V^2}{2}\right)$ by $\left(\alpha \frac{V^2}{2}\right)$, where α is the K.E.C.F

α is often ignored ($\alpha = 1$) since most fluid flows as turbulent ($\alpha \approx 1$).

Example (12-50p)

$\Delta P_{oil} = 250\,kPa$, $\rho_{oil} = 860\,kg/m^3$, $\dot{\forall} = 0.1\,m^3/s$, $\alpha = 1.05$



Determine the mechanical efficiency of the pump.

① inlet ② outlet

$$\frac{P_1}{\gamma} + \alpha\frac{V_1^2}{2g} + z_1 + h_P = \frac{P_2}{\gamma} + \alpha\frac{V_2^2}{2g} + z_2 + h_T + h_L$$

$z_1 \approx z_2$, $h_L \approx 0$, $h_T = 0$, $P_2 - P_1 = 250\,kPa$

$$h_P = \frac{P_2 - P_1}{\gamma} + \frac{\alpha}{2g}\left(V_2^2 - V_1^2\right)$$

$$h_P = \frac{250\times10^3}{(9.81)(860)} + \frac{1.05}{2(9.81)}\left(V_2^2 - V_1^2\right)$$

$V_1 = \frac{\dot{\forall}}{A_1} = \frac{0.1}{\frac{\pi}{4}(0.08)^2} = 19.894$ m/s

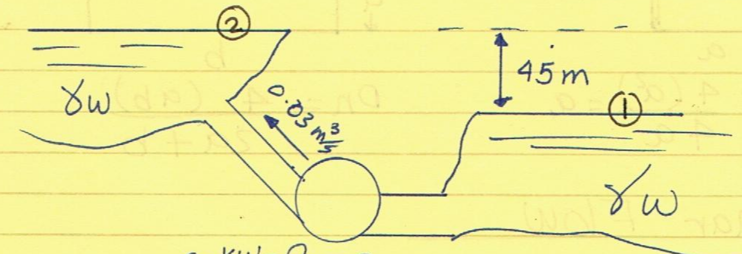
$V_2 = \frac{0.1}{\frac{\pi}{4}(0.12)^2} = 8.842$ m/s

$h_P = 12.63$ m

$$\eta_P = \frac{\dot{W}_P}{\dot{W}_{into}} = \frac{\gamma\dot{\forall}h_P}{\dot{W}_{in}} = \frac{\gamma\dot{\forall}h_P}{\eta_M\,(25\,kW)}$$

$$= \frac{(9.81)(860)(0.1)(12.63)[W]}{(0.9)(25)\times10^3[W]} = 0.474 = 47.4\%$$

Example (12-56P)



20KW-Pump

$$\cancel{\frac{P_1}{\gamma}}^{0} + \cancel{\alpha\frac{V_1^2}{2g}}^{0} + \cancel{z_1}^{0} + h_P = \cancel{\frac{P_2}{\gamma}}^{0} + \cancel{\alpha\frac{V_2^2}{2g}}^{0} + z_2 + \cancel{h_T}^{0} + h_L$$

$$h_P = z_2 + h_L$$

$$\Rightarrow h_L = h_P - z_2 = \frac{\dot{W}_P}{\gamma\dot{\forall}} - z_2 = \frac{25\times10^3}{(9810)(0.03)} - 45m$$

$$= 22.9\,m\ ,\quad \dot{W}_L = \gamma\dot{\forall}h_L = 6756.5\ W$$

Internal Flow CH 14

The Reynolds Number – an important dimiensionless number in fluid.

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

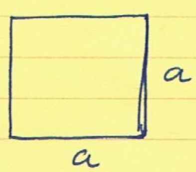
If $Re < 2300$ flow is laminar
If $Re > 4000$ flow is turbulent

If noncirculer ducts

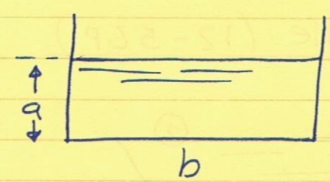
$$Re = \frac{V D_h}{\nu}$$

Where D_h = hydraulic diameter $= \frac{4A}{P}$

A = cross-sectional area
P = wetted perimeter

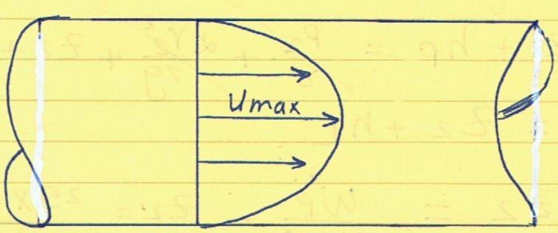


$$D_h = \frac{4(a^2)}{4a} = a$$



$$D_h = \frac{4\ (ab)}{2a + b}$$

Laminar Flow



$U_{max} = 2V_{avg}$
max velocity

velocity profile is parabolic

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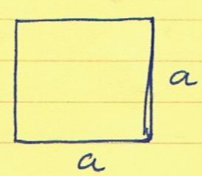
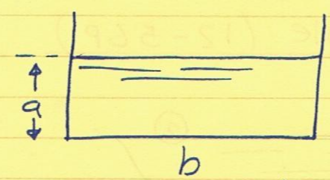
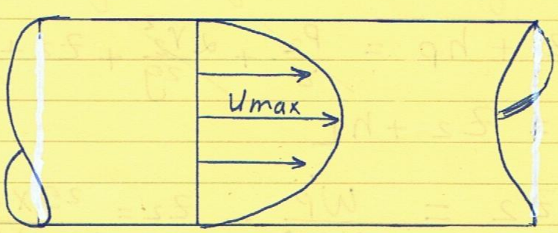


$$D_h = \frac{4(a^2)}{4a} = a$$



$$D_h = \frac{4\ (ab)}{2a + b}$$

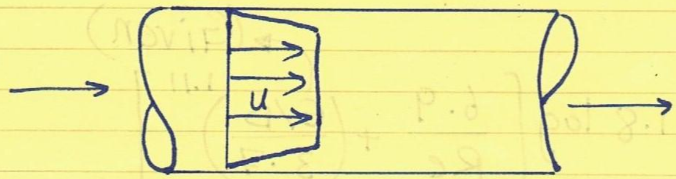
Laminar Flow



$U_{max} = 2V_{avg}$
max velocity

velocity profile is parabolic

Turbulent flow



velocity profile is almost uniform over central region of flow but very steep near walls.

$$\frac{V}{U_{max}} \approx \frac{4}{5}$$

Head loses

$$h_L = \frac{\Delta P}{\gamma} = f \, \frac{l}{D} \, \frac{V^2}{2g} \quad [m]$$

l: length of a pipe

D: Diameter of the pipe

f: friction factor

$$F_{\text{shear stress}} = \tau A$$

$$f = \frac{64}{Re} \quad \text{for laminar flow}$$

(Given)

$$\underbrace{\frac{1}{\sqrt{f}} \approx -1.8 \log\left[\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]}_{\text{for turbulent flow}}$$

Example

OIL (SG = 0.85), $\nu = 6 \times 10^{-4}\ m^2/s$

$D = 15$ cm dia pipe, $\dot{V} = 0.02\ m^3/s$

Find head loses in 100 m length of pipe.

1) $Re = \frac{VD}{\nu} = \frac{\frac{\dot{V}}{A} \cdot D}{\nu} = \frac{\frac{0.02}{\frac{\pi}{4}(0.15)^2}(0.15)}{6 \times 10^{-4}}$

$= 283 < 2300$

$\therefore$ flow is laminar

2) $f = \frac{64}{Re} = \frac{64}{283} = 0.226$

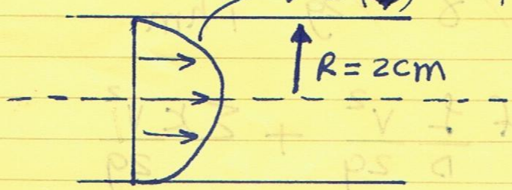
3) $h_L = \frac{\Delta P}{\gamma} = f\left(\frac{l}{D}\right)\left(\frac{V^2}{2g}\right)$

$= (0.226)\left(\frac{100}{0.15}\right)\left(\frac{1.13^2}{(2)(9.81)}\right)$

$= 9.83$ m

Example (P 14-32)

$$u(r) = 4\left(1 - \frac{r^2}{R^2}\right)$$



Find the average & maximum velocities.

$$u(r) = U_{max}\left(1 - \frac{r^2}{R}\right) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$\underline{\underline{U_{max} = 4}}$ $\qquad V_{avg} = \frac{U_{max}}{2} = \underline{\underline{2}}$

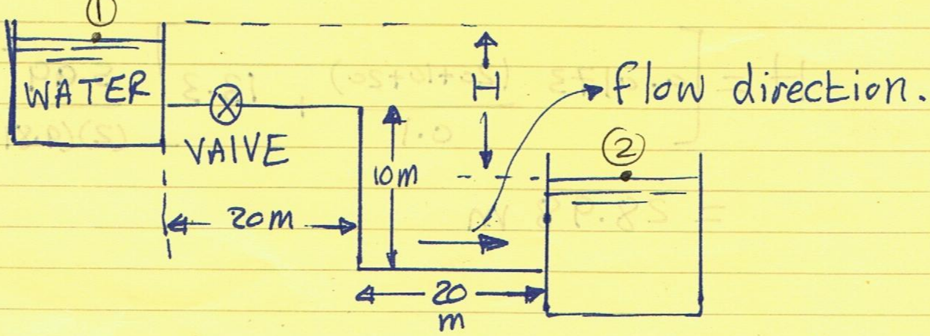
Minor loses

h_m = minor loses (elbows, tees, valves, etc)

$h_m = \Sigma K \frac{V^2}{2g}$, K: minor loses coeffecient.

$\therefore h_{\text{All loses}} = h_L + h_m$

Example



Pipe is $\underbrace{\text{wrought iron}}_{\text{to get } \epsilon}$, $D = 10cm$, $\dot{\forall} = 0.04\ m^3/s$, $\nu = 1\times10^{-6}\ m^2/s$

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1}{2g}} + z_1 + \cancel{h_p} = \cancel{\frac{p_2}{\gamma}} + \cancel{\frac{V_2}{2g}} + \cancel{z_2} + \cancel{h_T} + h_L + h_m$$

$$H = h_L + h_m = f\,\frac{L}{D}\,\frac{V^2}{2g} + \Sigma K\,\frac{V^2}{2g}$$

$$V = \frac{\dot{\forall}}{A} = \frac{0.04}{\frac{\pi}{4}\,(0.1)^2} = 5.09 \text{ m/s}$$

$$Re = \frac{VD}{\nu} = \frac{(5.09)(0.1)}{1\times10^{-6}} = 5.09\times10^{5}$$

$Re > 4000$ flow is turbulent.

$$\frac{1}{\sqrt{f}} = -1.8\,\log\left[\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.1}\right]$$

$f = 0.0173$.

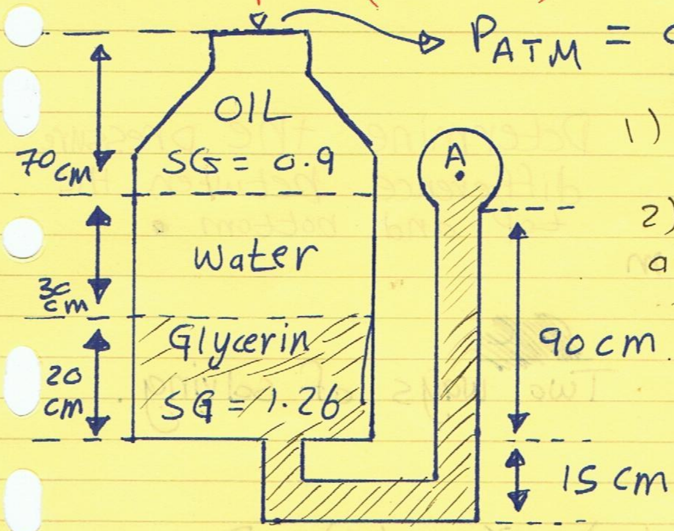
$$\Sigma K = \underbrace{0.5}_{\text{square edge entry}} + \underbrace{1}_{\text{Pipe exit}} + \underbrace{10}_{\text{Globe valve}} + \underbrace{(2)(0.9)}_{90^\circ\text{ smooth bend, THREADED.}}$$

$\Sigma K = 13.3$

$$H = \left[0.0173\,\frac{(20+10+20)}{0.1} + 13.3\right]\frac{5.09^2}{(2)(9.81)}$$

$$= 28.98 \text{ m}$$

Example (P 2-67)



1) Find P_A

2) Find the height of a mercury that would create the same pressure at A.

$\gamma_w = 9810 \ N/m^3$

$\rho_M = 13600 \ kg/m^3$

1) $P_A + \gamma_G (0.9 - 0.2) - \gamma_w (0.3) - \gamma_{OIL} (0.7) = 0$

$$P_A = \gamma_w \left(0.3 + SG_{OIL}(0.7) - SG_G (0.7)\right)$$

$$= 470.88 \ Pa = 0.47 \ kPa$$

2) $P_A = \gamma_M h \Rightarrow h = \frac{P_A}{\gamma_M}$

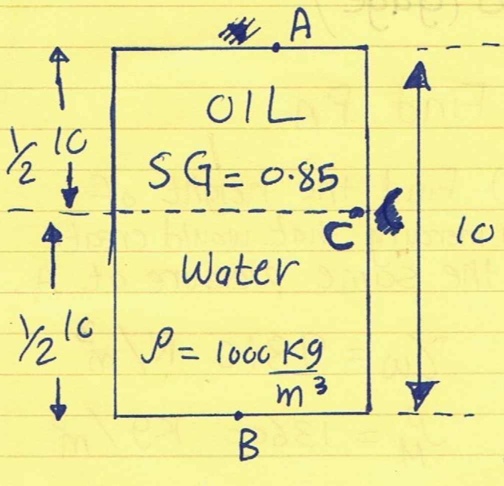
$$h = \frac{P_A}{(\rho_M)(g)} = \frac{470.88}{(13600)(9.81)} = 0.352 \ cm$$

NOTE

$$\underset{\rho}{\frac{kg}{m^3}} \cdot \underset{g}{\frac{m}{s^2}} = \underbrace{\underset{F}{\frac{kg \cdot m}{s^2}} \cdot \underset{A}{\frac{1}{m^2}}}_{\frac{F}{A}} = \underline{Pa}$$

NOT kPa

Example (P 2-81)



Determine the pressure difference between the top and bottom.

Two ways of solving.

1) $P_B - \gamma_w(5) - \gamma_{oil}(5) = P_A$

$P_B - P_A = 5\gamma_w(1 + SG_{oil})$

$\Delta P = 5(1000)(9.81)(1 + 0.85)$

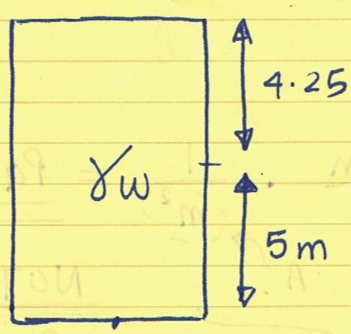
$= 90.7425$ KPa

2) @ C

$\gamma_{oil}(h_o) = \gamma_w(h_w)$

$\Rightarrow SG_{oil}(h) = h_w$

$\Rightarrow h_w = (0.85)(5) = 4.25$



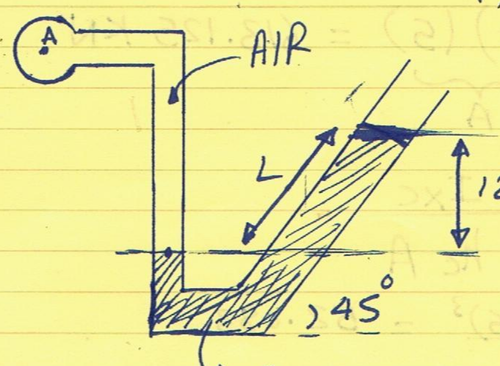
$\Delta P = \gamma_w(5 + 4.25)$

$= 90.7425$ KPa.

Example (P 2-85)

1) Find P_A (gage)

2) Find L.



$\rho = 810\ Kg/m^3 = 0.81\ Kg/L$

NOTE : Pressure does not vary on horizental plane.

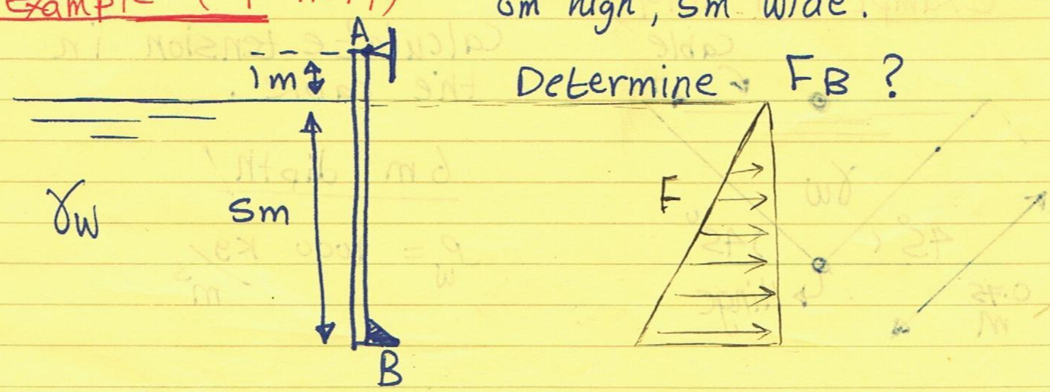
1) $P_A = \gamma h = (810)(9.81)(0.12)$

$= 953.532$ Pa

2) $\sin 45 = \frac{12\,cm}{L} \Rightarrow L = 16.97$ cm

Example (P 11.14) 6m high ; 5m wide.

Determine F_B ?



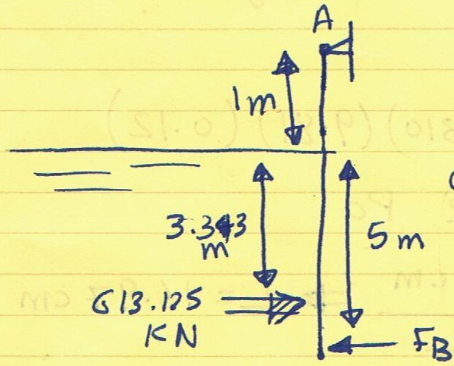
$F_R = \gamma_w h_c A \Rightarrow$ For submerged only.

$$= (9810)\left(\frac{5}{2}\right)\underbrace{(5)(5)}_{A} = 613.125 \text{ KN}$$

$$y_R = h_R = h_c + \frac{I_{xc}}{h_c A}$$

$$I_{xc} = \frac{bh^3}{12} = \frac{\cancel{6}5(5)^3}{12} = 52.083$$

$$y_R = h_R = \frac{5}{2} + \frac{I_{xc}}{\frac{5}{2}(5)^2} = 3.33 \text{ m}$$

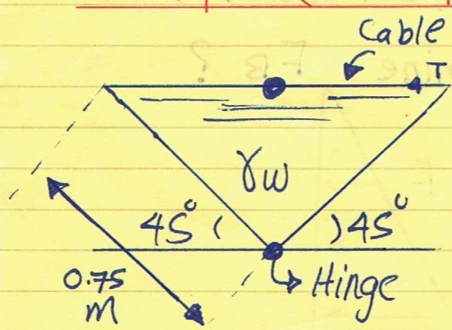


$$\Sigma M_A = 0$$

$$0 = 613.125\,(3.33+1) - F_B\,(5+1)$$

$$\underline{F_B = 442.47 \text{ KN}}$$

Example (P 11.19)

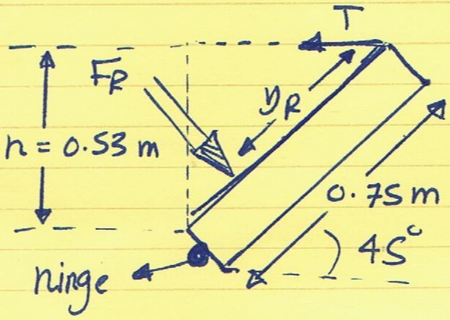


Calculate tension in the cable.

6 m dipth!

$\rho_w = 1000 \text{ kg/m}^3$

Take half of the figure



$$F_R = \gamma_w h_c A$$

$$F_R = (9810)\left(\frac{0.53}{2}\right)\underbrace{(0.75)(6)}_{A}$$

$$F_R = 11698.425 \text{ N}$$

$$h = (0.75)\sin 45 = 0.53 \text{ m}$$

$$y_R = y_c + \frac{I_{xc}}{y_c A} \rightarrow \frac{bh^3}{12} \text{ [Given]}$$

$$= \frac{0.75}{2} + \frac{\cancel{(6)}(0.75)^{\cancel{3}}}{6\cancel{(12)}\cancel{(0.75)}_{2}\cancel{(0.75)}_{1}\cancel{(6)}}$$

$$y_R = 0.375 + \frac{0.75}{6} = 0.5 \text{ m}$$

$$\Sigma M_{\text{hinge}} = 0 \Rightarrow F_R(0.75 - 0.5) = T(h)$$

$$T = \frac{(11698.425)(0.25)}{0.53} = 5518.125 \text{ N}$$