## INSTRUCTOR'S MANUAL TO ACCOMPANY

# FACILITIES PLANNING

**FOURTH EDITION**


**JAMES A. TOMPKINS**
*Tompkins Associates, Inc.*

**John A. White**
*University of Arkansas*

**Yavuz A. Bozer**
*University of Michigan*

**J.M.A. Tanchoco**
*Purdue University*

# PREFACE

The *Instructor's Manual* provides answers to the questions and solutions to the problems at the end of the chapters in the Fourth Edition of *Facilities Planning*. When a question or problem is open-ended, either no answer is provided or guidance is provided relative to the response intended.

Due to human error, it is possible that mistakes exist in the responses provided. Several changes were made to end-of-chapter questions and problems during the production process. Consequently, it is possible that responses are provided in the *Instructor's Manual* in anticipation of changes to the manuscript that did not occur. If you encounter an error in the *Instructor's Manual*, the authors request that you bring it to the attention of Dr. James A. Tompkins, who was coordinating author for this edition of *Facilities Planning*. Correspondence should be sent to Dr. James A. Tompkins, President and CEO, Tompkins Associates, Inc., 8970 Southall Road, Raleigh, NC 27616. If more convenient, you can communicate errors via email using the address: jtompkins@tompkinsinc.com.

This electronic version of the *Instructor's Manual* is available to faculty who adopt *Facilities Planning*. To prevent widespread dissemination of answers and solutions to end-of-chapter questions and problems, the authors and publisher require that you use password-protected access to course websites containing solutions to problems for students. Further, passwords must be changed after each offering of the course. Many instructors do not want students to have access to the *Instructor's Manual*. Hence, it is important for access to be limited.

To prepare the Third Edition of *Facilities Planning*, coordinating authors were assigned for each chapter. Dr. Tompkins had coordinating responsibility for Chapters 1, 4, 5, and 9; Dr. White had coordinating responsibility for Chapters 11, and 12; Dr. Bozer had coordinating responsibility for Chapters 6 and 7; Dr. Tanchoco had coordinating responsibility for Chapters 2, 3, and 8; and Drs. Bozer, White, and Tanchoco shared coordinating responsibility for Chapter 10. Hence, depending on the nature of questions you have regarding material in a chapter, you might wish to contact directly the coordinating author. Contact information for each author is provided following the preface.

To assist you in finding material for a particular chapter, the page numbering in the *Instructor's Manual* incorporates the chapter number. In addition, the manual contains two-levels of bookmarks. The first level corresponds to major divisions of the manual, e.g. parts and chapters. The second level corresponds to the section of each chapter from which the questions originate, which is consistent with the problem groupings at the end of each chapter of the textbook. In this way, we believe you will be able to turn more quickly to the response of interest to you.

We are pleased to acknowledge the assistance of a number of individuals in developing the material for the *Instructor's Manual*. The solutions manual for the fourth edition was compiled by Patrick Brunese (Purdue University). For the previous editions, we express our appreciation to David Ciemnoczolowski (University of Michigan), Ronald Gallagher (Tompkins Associates), E. Harjanto (Purdue University), Mark Rukamathu (University of Arkansas), and S. Sugiarta (Purdue University).

Thank you for adopting *Facilities Planning*. We trust it measures up to your expectations and you view it as an improvement over the previous editions. We have endeavored to respond to all feedback we received from instructors who used the previous editions of the text. And, as in the past, we welcome your suggestions for ways to improve the book.

James A. Tompkins
John A. White
Yavuz A. Bozer
J. M. A. Tanchoco

# Author's Addresses

James A. Tompkins
President and CEO
Tompkins Associates, Inc.
8970 Southall Road
Raleigh, NC 27616
919-876-3667 (B)
919-872-9666 (F)
jtompkins@tompkinsinc.com

John A. White
Chancellor Emeritus and Distinguished
Professor of Industrial Engineering
University of Arkansas
308 Engineering Hall
Fayetteville, AR 72701
479-575-2773 (B)
479-575-8431 (F)
jawhite@uark.edu

Yavuz A. Bozer
Professor
Department of Industrial and
Operations Engineering
University of Michigan
Ann Arbor, MI 48109
734-936-2177 (B)
734-764-3451 (F)
Yavuz.Bozer@umich.edu

J. M. A. Tanchoco
Professor
School of Industrial Engineering
Purdue University
Grissom Hall
West Lafayette, IN 47907
317-494-4379 (B)
317-494-5448 (F)
tanchoco@ecn.purdue.edu

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# Chapter 1

# *Introduction*

**SECTION 1.1**

**1.1a** To plan for a football game, a visiting head football coach must:

- Make sure that his players and coaches understand the game plan
- Prepare his players physically and mentally for the game
- Study opposing team game films
- Understand strengths and weaknesses of the opponent
- Maintain the discipline of his players

To operate the team during a football game, a visiting head coach must:

- Continuously update his game plan based upon what the opposing team does
- Communicate these updates with his coaches and players
- Keep players physically ready to play
- Replace injured players with other quality players if at all possible
- Call the best plays that will score the most points and make victory most likely

**1.1b** To plan for a football game, the home team quarterback must:

- Practice drills and fundamentals
- Understand role in game plan
- Become familiar with pass receivers' skills and abilities
- Assume leadership role of team

The operating activities that a home team quarterback must do are:

- Operate coaches' game plan on the field
- Lead team against opposition
- Use skills learned in practice
- Minimize mistakes
- Relay plans from sidelines to all players in the huddle
- Win the game

**1.1c** To plan for a football game, the manager of refreshment vending must:

- Make sure all food, beverages, condiments, and paper supplies are ordered and stocked
- Hire employees to work the vending locations
- Pay suppliers
- Understand what refreshments field personnel will need during the game

The operating activities that a manager of refreshment vending must do are:

- Serve customers as efficiently and professionally as possible
- Restock vending locations whenever necessary
- Keep fans updated on game activities with televisions at vending locations
- Solve customer problems

**1.1d** To plan for a football game, the ground crew manager must ensure that:

- Lines are painted on the field
- Grass is mowed
- Damaged turf is replaced
- Necessary equipment for the teams is in its proper place
- Locker rooms are prepared for the teams

The operating activities that a ground crew manager must do are:
- Replace damaged turf, if possible, during the game if weather is bad
- Be ready for any grounds problems that may arise during the game

**1.1e** To plan for a football game, the stadium maintenance manager must:
- Examine plumbing fixtures to make sure they are operational
- Clean the stadium and pick up trash from the previous event
- Repair any seats or bleachers that are damaged
- Clean bathrooms
- Repair and replace any necessary audio or video equipment for the stadium

The operating activities that a stadium maintenance manager must do are:
- Respond to any equipment breakdown
- Respond to any plumbing breakdown
- Respond to any power outage

**1.2** Ten components of a football facility are:
- The stadium structure
- Parking lots around the stadium
- Vendor selling areas
- Maintenance components
- Grounds-keeping equipment and personnel
- Security
- Customers
- Athletic team personnel
- Locker rooms
- Vending equipment and supplies

**1.3a** Activities that would be involved in planning the location of an athletic stadium are:
- Marketing analysis: Is there a fan base to support the teams or events that will occupy the facility?
- Determining a suitable plot of land for the facility and its associated parking lots in terms of size and levelness
- Determine by what routes suppliers will supply the facility
- Determine if there are other facilities that will interact with the stadium and how the location of the facility will affect those interactions
- Determine if existing structures will need to be demolished to accommodate the new stadium and how those people or businesses will be compensated
- Determine the stability of the land that will be used to hold this structure

**1.3b** Activities that would be involved in planning the design of an athletic stadium are to determine the facility layout and systems and the material handling systems that are necessary to operate the facility. Items that need to be determined within the facility layout are as follows:

- Number of people it will need to hold
- Equipment is necessary to run this facility.
- How the facility can host multiple types of sports—the design must accommodate all of them
- Whether the stadium be an outdoor stadium or a domed stadium
- Artificial or natural turf
- Materials of the track surface be
- How and where upper-deck access will be
- Vending locations in the facility
- Where administrative offices of the facility will be
- How many restrooms there will be and their locations
- Where the locker rooms will be

Facilities systems that need to be examined are:

- Structural and enclosure elements.
- Power and natural gas requirements
- Lighting requirements
- Heating, ventilation and air conditioning requirements
- Water and sewage needs

Handling systems that need to be examined are:

- Material handling system
- Personnel required to operate the stadium
- Information systems required to operate the stadium
- Equipment needed to support the stadium

**1.3c** Activities that would be involved in facilities planning for an athletic stadium are determining the facility location and design, explained in greater detail in 1.3a and 1.3b, plus the ongoing maintenance and improvement of the facility.

**1.4** Customers in the transportation, communication, and service sectors do have a need for facilities planners. Service facilities such as hospitals, restaurants, athletic facilities, and retail shopping establishments all can and do use facilities planners to optimize the facility layout, handling systems, and facility systems on a continuous basis.

The communications industry uses the world as its facility. Communication networks can be thought of as the handling systems of a communications customer. Due to the rapidly changing technologies found in the communications industry, these networks have to be continuously updated and improved, or new ones have to be created. Also, the equipment and personnel required to run multiple communication networks have to be considered as well as where the communication points within the network will be located. Finally, the facilities required to house a communication hub or hubs must be located and designed, and a facilities planner is the optimal person to do this job.

Just like the communications industry, the transportation industry uses the world as its facility. Facilities planners can play an integral role in determining where airports, train terminals, bus depots, truck depots, and shipping docks are located and in designing them to accommodate the traffic that travels through them. Also, facilities planners can assist in determining the equipment and layout of those facilities as well.

Service industries such as retail shopping establishments, restaurants, hospitals, and athletic facilities all use facilities planners to lay out their buildings. Also, retail establishments and restaurants generally have warehouses in which product is stored before it comes to its point of use. Facilities planners play a large role in the location and design of warehouses of all types.

**SECTION 1.3**

**1.5** Use the following criteria for determining the optimal facilities plan:

- Does the facility provide for the company's future storage requirements?
- Is there cost justification for the facility?
- Is the facility centrally located for suppliers or customers if it is a manufacturing facility; for manufacturing centers if it is a shipping warehouse; for fans if it is an athletic facility, etc.?
- Does it minimize receiving and putaway times while providing enough spaces for those functions?
- Does the product from the manufacturing operations flow smoothly through the facility?
- Are there enough dock doors for shipping and receiving functions?
- Is the material handling equipment proper for the product that is being moved? Is there enough space for the equipment to maneuver around the facility?
- Is the lease cost, property cost, or building cost cheaper than other alternatives?
- Can the manufacturing or warehouse facility accommodate sales forecasts?
- Does the plan minimize the cost of operation in terms of labor and equipment?
- Will scrap be minimized with this plan?
- Can the facility be expanded easily to accommodate growth?
- Is space being utilized to the highest extent?

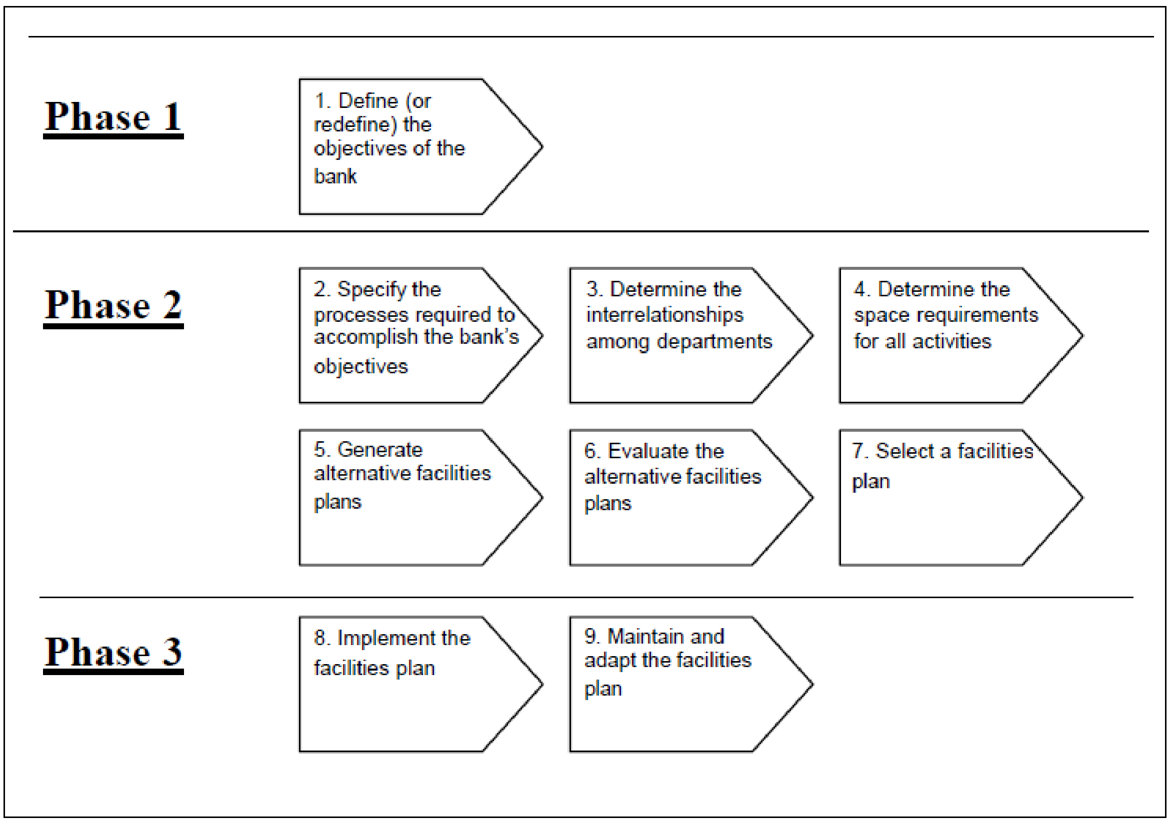
**1.6** This answer will vary from campus to campus, but some things of which the student should be aware are:

- Traffic patterns on campus
- Whether administration offices are easily accessible
- Handicapped accessibility to all buildings
- Whether areas mix high vehicle and pedestrian traffic
- What it would take to remove dilapidated buildings

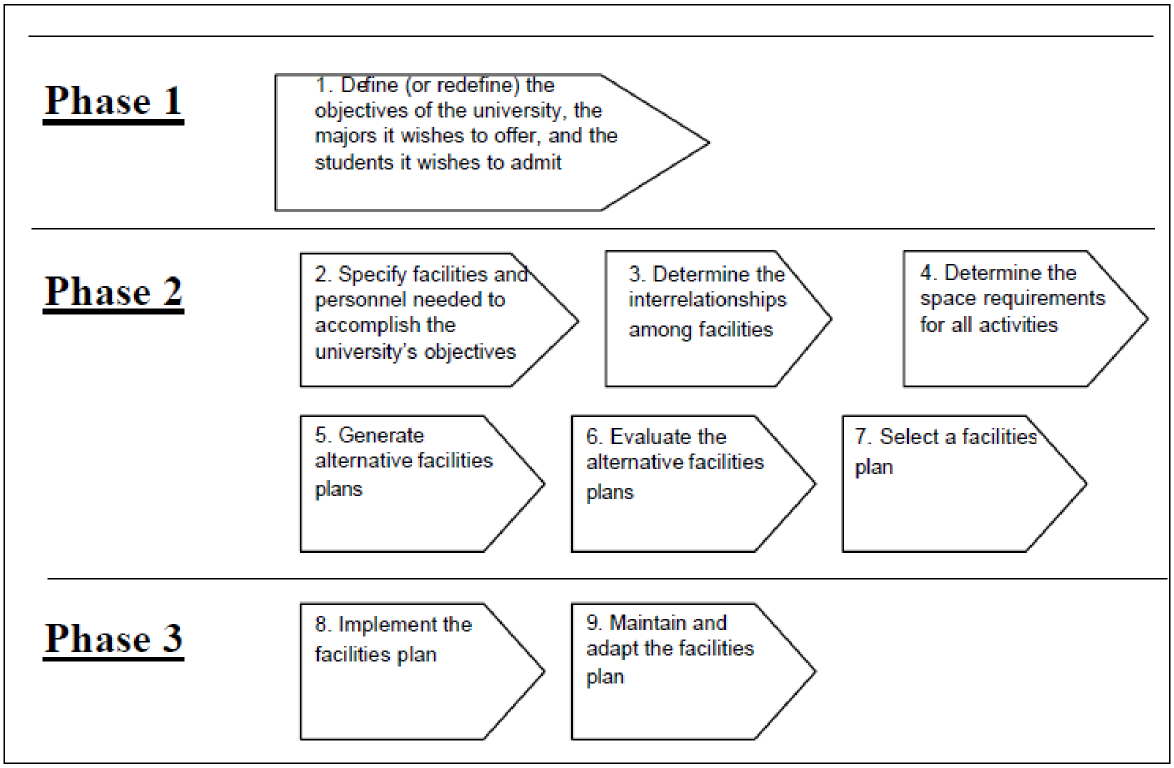
- The locations of certain buildings or functions if they are related to one another, e.g. financial aid should be close to the cashier's office
- Maintenance facilities should be hidden from the main traffic areas
- Whether there are adequate parking facilities for students, faculty, and staff
- Whether there are adequate eating establishments, restrooms, etc.
- Whether adequate computer facilities are accessible to all students
- Whether there are adequate recreational facilities
- If there is room to expand in the future
- If there is room on or near campus for all students to live
- Whether there adequate security on campus

**SECTION 1.4**

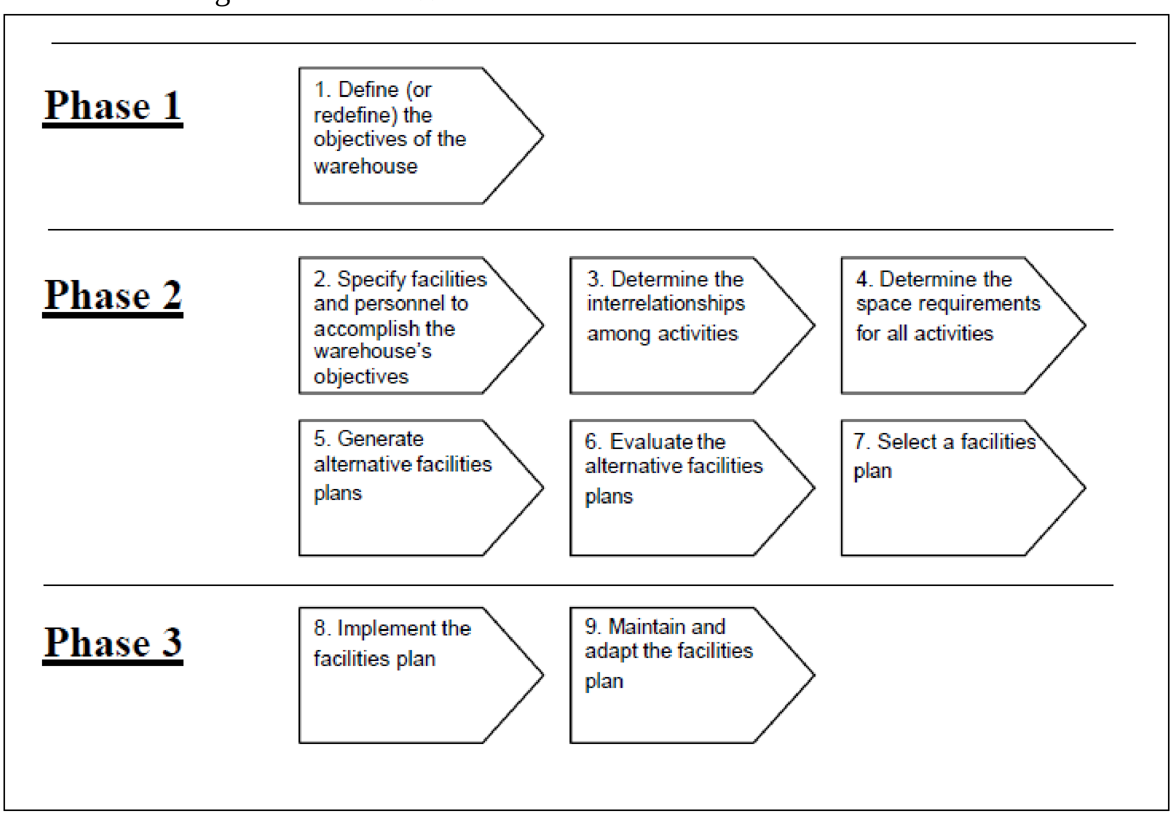
**1.7a** Facilities Planning Process for a Bank



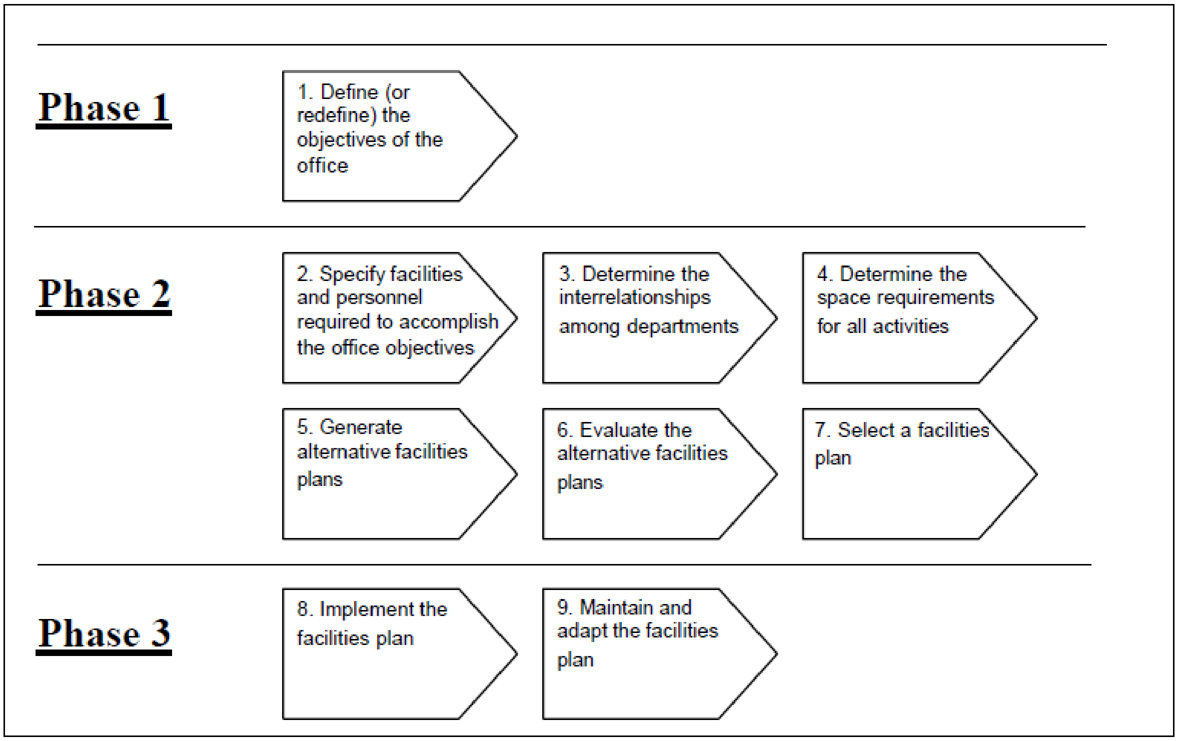
**1.7b** Facilities Planning Process for a University




**1.7c** Facilities Planning Process for a Warehouse



**1.7d** Facilities Planning Process for a Consulting Office




**1.8** The first task that needs to be accomplished is to determine the site of the library on the campus. Questions such as do existing structures need to be demolished, do roads need to be rerouted, do possible sites meet federal, state, and local codes need to be asked. Next, a determination of office space, book storage space, storage rack layout, and student study areas must be made so that the structural design of the facility can be created. Also, the power, gas, heat, ventilation, security, plumbing fixtures, and maintenance issues must be settled. Finally, material handling issues such as how to transport supplies and books, how to reshelve books, how to get from floor to floor, the information systems required to keep track of the locations of all of the books, and the personnel who will be required to maintain the library must be resolved.

**SECTION 1.6**

**1.9** Facilities planning is never completed for an enterprise. If facilities planners believed this, then numerous enterprises would be doomed to failure. New technologies, different enterprises would be doomed to failure. New technologies, different packaging methods, better storage methods, and more advanced material handling equipment make the process of updating and continuously improving facilities a must, or else the company will be left behind with antiquated structures, information systems, and storage and material handling mechanisms.

**1.10** Certain items to look for from this paper are that an architect is more involved with the design of the structure of the facility, the materials that will go into the structure, and the location of items such as plumbing, lighting, HVAC systems, and electrical systems. The facilities planner will be consulted on the above issues, but is more likely to plan the material flow within the facility, how materials will be handled within the facility, how materials will be stores, how materials will be manufactured, the location of storage areas, the location of shipping and receiving docks, the location and types of manufacturing equipment, the types of material handling equipment required in the facility.

**1.11** The IIE description of Industrial Engineering is:
Industrial engineering (IE) is about choices. Other engineering disciplines apply skills to very specific areas. IE gives an opportunity to work in a variety of businesses. The most distinctive aspect of industrial engineering is the flexibility that it offers. Industrial engineers figure out how to do things better. They engineer processes and systems that improve quality and productivity. They work to eliminate waste of time, money, materials, energy, and other commodities. Most important of all, IEs save companies money. Industrial Engineering draws upon specialized knowledge and skills in the mathematical, physical and social sciences together with the principles and methods of engineering analysis and design to specify, predict, and evaluate the results to be obtained from such systems. "Facilities planning" could easily be substituted for the worlds "industrial engineering." Clearly, facilities planning is concerned with the design, improvement, and installation of tangible fixed assets to achieve an activity's objectives. While designing, improving, and installing these assets, the impacts on people, material, information, equipment, and energy are very important. It is also clear that facilities planning draws upon sciences, together with the principles and methods of engineering analysis and design to specify, predict, and evaluate the results on an activity's tangible fixed assets. Therefore, it is clear that the profession most able to perform facilities planning is the profession of industrial engineering.

**1.12** The articles read should relate to the strategic planning process and should draw parallels to the facilities planning process. An awareness of the true meaning of strategy should be demonstrated.

**1.13a** Strategic uses for an airport that must be addressed in facilities planning include:

- Location
- Types of planes that will be flying into the airport
- Type of air traffic control system
- Location of air traffic control tower
- Location of baggage carousels
- Location of ticket counters
- Number of hangars required for servicing airplanes and for safety inspections
- Employee skill levels for servicing airplanes between arrivals and departures and for maintaining airplanes
- Number of employees required to run vending services, ticketing, baggage claim, etc.
- Location of parking lots
- Many other items, including security control

**1.13b** Strategic issues for a community college that must be addressed include:

- Location of classroom facilities
- Location of administrative facilities
- Skill level of faculty in each major
- Majors that will be offered
- Location of roads and sidewalks through campus
- Skill level of students that will be admitted
- Financial aid distribution to students
- Services to be provided to the students and the fees charged to the students
- Level of athletic focus, if any
- Level of maintenance and the location of facilities

**1.13c** Strategic issues for a bank that must be addressed include:

- Types of services provided
- Skill levels of different types of employees
- Security system and personnel
- Interest rates for loans
- Types of loans granted
- Minimum borrower qualifications
- Interest rates for savings and money market accounts
- ATM machine on premises?
- How many drive-thru windows?
- Accounting procedures
- Vault size

**1.13d** Strategic issues for a grocery store chain that must be addressed include:

- Types of products to sell
- Arrangement of shelves within the store
- Locations of items on those shelves
- Parking area required
- Accounting procedures
- Skill level of employees
- Freshness qualifications for open-air food such as produce
- Amount of warehouse space where customers do not shop
- Location of grocery stores throughout the country
- Should stores have the same layout or should they vary?
- Number of shopping carts required
- Number of docks required for receiving purposes
- Unpacking methods
- Predicted sales volume
- Product pricing considerations

**1.13e** Strategic issues for a soft drink bottler and distributor that must be addressed include:

- Location of bottling facilities

- Distribution routes
- Number of trucks required for distribution
- Types of equipment required for bottling
- Number of bottles shipped per day
- Number of SKUs that have to be bottled
- Methods of conveyance through the facility
- Methods of loading and unloading trucks
- Material composition of bottles
- Location and type of liquid components of beverage to be bottled
- Skill level of employees
- Automation level
- Inventory control methods
- Storage requirements

**1.13f** Strategic issues for a library that must be addressed include:

- Location of the library
- Number of books that the library can contain
- Number and location of shelves to hold the books
- Skill level of employees
- Types of magazine and newspaper subscriptions
- Manual or automated system for finding items
- Types of reference materials
- From where will funding come?
- Amount of study area required
- Number of computers needed for public use
- Size of children's section within the library
- Types of annual special events

**1.13g** Strategic issues for an automobile dealership that must be addressed include:

- Parking area required
- Level of knowledge of automobiles that employees must have
- Number of salespeople
- Sales tactics, professionalism of salespeople
- Types of automobiles to sell
- Sales trends in the automobile industry
- Relationship with automobile manufacturer
- Service levels offered at the dealership
- Parts for cars offered at the dealership
- Number of service bays
- Storage area required for service parts
- Elegance of automobile showroom
- Relationships with lending institutions

**1.13h** Strategic issues for a shopping center that must be addressed include:
- Types of shops to be included in the center
- Number of shops and their sizes
- Location of shopping center
- Lease rates for tenants
- Public facilities for customers
- Parking area for customers, should it be decked parking, etc.
- Security
- Shopping center design, how elegant should it be?
- Will it be a strip mall or an enclosed shopping center?
- Finding possible investors in the shopping center

**1.13i** Strategic issues for a public warehousing firm that must be addressed include:
- Size of the warehouse
- Location of the warehouse
- Level of automation within the warehouse
- Number of warehouses
- Types of products that can be stored in the warehouse
- Material handling mechanisms within the warehouse
- Rental rates
- Policies for unclaimed product
- Number of racks required for the storage of the product
- Number of docks required for shipping and receiving of the product
- Will there be crossdocking capability?
- Skill level of employees

**1.13j** Strategic issues for a professional sports franchise that must be addressed include:
- How does the ownership manage the franchise?
- Location of a facility in which to play
- Rent a facility or own one
- Team colors, name, logo
- Advertising methods
- Determining the makeup of the fan base
- Ticket prices
- Revenues from luxury suites
- Training facilities
- Selection criteria for choosing players and coaches
- Salaries for players and coaches
- Revenues from licensed sportswear sales
- Player and coach discipline if they break the rules
- Management’s role in determining the direction of the team

**1.14a** When there are many critical short-term problems that a company has to solve at once, it is generally due to a lack of good strategic planning. Many times, critical problems such as not enough storage space, too few material handlers, improper storage techniques, and too much work-in-process inventory all result from poor strategic facilities planning. Strategic facilities planning is important when there are many “critical” problems to solve because planning itself will generally solve many of the problems.

**1.14b** Everyone needs to have a say in the strategic planning process. However, if critical individuals are too busy to do some sound strategic facilities planning, they will always remain too busy because the problems that result from poor strategic planning will keep these individuals busy attacking short-term problems. Furthermore, there are consulting firms that specialize in this type of work, so not as much time would have to be taken by these individuals who are too busy, but their input would still be required while the consultants are working on the problem.

**1.14c** A good facilities plan will be flexible enough to change with future events. For example, a company that supplies the automobile industry builds a warehouse during a downturn in automotive sales. If good strategic planning has occurred, the company will not build the warehouse based on the storage requirement of the sales volume during the downturn; instead, the company will build on some large percentage of the maximum sales volume they had during an upswing in the economy and on the forecast of automotive sales over some time frame. Otherwise, the supplier could run out of storage space in the new facility and have to add on or build another warehouse before it has the financial capacity to do so.

**1.14d** This is a response of laziness. It takes much work and research to determine what alternatives are available besides the one the company is using. Data collection is needed to determine product throughput, production flow, storage capacities, and inventory control procedures so that the proper facility size and alternative control systems can be determined. Development of labor standards and evaluation of material handling methods is needed to look at different alternatives. If this information is gathered and certain types of vendors are brought in to look at the problem, numerous alternatives for a facility can be generated. Those that apply must meet scientific and financial criteria that are determined at the beginning of the project.

**1.14e** Strategic facilities planning is an ongoing process. Technologies come and go, and a good facilities plan will enable a company to adapt to rapidly changing technologies as well as to discard those that will not help the company achieve its goals. A strategic plan is a plan for the future, not the present. Do not incorporate technologies into a facility for the present product mix—incorporate those technologies that can be used to produce the future product mix. However, a facilities plan cannot take into account technologies that have not yet been created; if a company creates a strategic facility plan that incorporates technologies that have not been created, there is no guarantee that the technology will be available when the company is ready to implement the facilities plan and it will be doomed to failure.

**1.14f** One cannot get an exact dollar figure on the cost of a strategic plan implementation and the savings it will generate. However, examination of past trends and the prices of different types of technologies can give the strategic planner a good estimate of the costs and the savings. The only way to get exact costs is to get quotes from vendors, and the only way to determine the exact savings is to implement the facility plan. This should not stop a company from doing strategic planning, however. Sometimes the plan may be infeasible and may not be implemented, but a company that does not engage in strategic planning will not get a return on an investment because the investment will never be made, or there will never have been a determination on how much money the investment would make or lose.

**1.15** Doing facilities planning for a manufacturing facility is a positive exercise for a company in terms of its competitiveness. The production flow can be examined by simulation, and potential bottlenecks can be smoothed. Also, a facilities plan for a manufacturing facility outlines the skill levels of employees required to operate the equipment on the manufacturing floor, and it discovers the latest and best equipment to make the product. Companies that do not engage in facilities planning usually end up having uneven production flows, improper labor skills, too much or too little labor on the floor, and outdated equipment that must be heavily maintained. These problems as well as many others that could be eliminated by facilities planning cause companies to lose their competitive edge.

**1.16** Using strategic planning to assist with your career allows you to evaluate where you are compared with the goals you set for yourself. If you have fallen short of your goals, it allows you to easily evaluate why and determine how you intend to correct the shortcoming, if it is possible. Furthermore, it can show you if your career is at a dead end and where you need to go to make a change in your career. Strategic planning gives you a path to follow once the plan is in place. However, this plan should be continuously updated just like any other strategic plan or you will lose your competitive edge over others on the same career path.

**1.17** Automation, if planned properly, can have a positive impact upon facilities planning. If there is a large product throughput, automation can reduce costs by reducing labor requirements, improving quality, and perhaps improving product throughput. However, many things could go wrong, and a facilities planner needs to be aware of them. First, the automation may not justify itself. If the throughput required to meet sales is 1,000 units per day and automation equipment is purchased that can produce 20,000 units per day, cost justification probably will not occur. Also, is the automated equipment flexible enough to handle changes in product design or production methods? While the first example showed the automation that could produce 20,000 units per day had too much excess capacity and therefore was not cost-justifiable, when a process is automated, there needs to be extra capacity built into it so that it is not obsolete when the manufacturing requirement increases to meet future sales. Also, the automated process needs to be able to make product of higher quality than if it was made manually or with a cheaper automated process. It does no good to have a machine that can produce 5,000 units per hour to meet production requirements of 4,000 units per hour when only half of the pieces pass a quality inspection. Finally, it is necessary to examine how the automated process will fit within the existing facility and how much rearrangement will have to occur so that production will continue to flow smoothly. Automation is a wonderful way to improve the quality and throughput of a product, but only if it is done properly. Automation that does not do what it is supposed to do ends up giving manufacturing personnel more problems than the process it replaced.

**1.18** Issues that should be addressed during strategic planning for warehousing/distribution include:

- Number of shipping and receiving docks
- How the product will be shipped from the warehouse to the customer
- Product that will be stored
- Number of SKUs
- Size of the product that will be stored
- Storage cube requirement
- Pallet rack, flow rack, bulk storage, conveyor requirements
- Inventory turns per a specific time
- Warehouse staffing levels
- Inventory investment levels
- Material handling procedures and equipment
- Building size
- Will refrigeration be necessary?
- Inventory control methods
- Inventory control equipment
- Building and rack layout
- Power requirements

The primary customer service consideration for a warehouse/distribution strategic plan is that the faster the turnover from receipt of an order to shipment to the customer, the better. Also, proper product storage and product shipment will reduce the amount of damaged product that a customer receives. Finally, the better the inventory control system, the easier it is to determine when there are stockouts and where all of the inventory is, thereby making it easier to fill a customer's order in less time.

Cost implications are that if excess inventory has to be carried due to inaccurate inventory control data, then a larger facility has to be built, more rack has to be installed, more labor has to be used to find the inventory in a larger building, capital is tied up in inventory rather than earning money or being used in a more productive way, and more inventory will have to be thrown away because it is outdated. Facilities planning can reduce or eliminate all of these problems, thereby reducing a company's warehousing costs, which can result in product being shipped at a cheaper price to customers, which will result in more sales and revenues.

**1.19** All of the requirements for success in Supply Chain Synthesis are directly linked to the facilities planning process. A proper facilities plan will enable a company to achieve synthesis much more quickly than a competitor who does not use facilities planning. Strategic facilities issues in manufacturing such as smoothing the production flow, eliminating bottlenecks, using the proper amount of labor that has the appropriate skill level, using the proper equipment that can meet or exceed throughput at a high level of quality will help reduce manufacturing costs, enable manufacturing and marketing to work together to achieve sales goals, reduce lead-times, reduce setup times and production lot sizes, reduce work-in-process inventories, simplify process, balance the production flow, adapt to changing product and technologies, reduce uncertainty, increase quality, and reduce process failures. Also, reducing problems in the manufacturing process will increase the number of happy employees and encourage them to become team players. This will allow every part of the manufacturing process to become integrated and facilitate not only a greater understanding by the employees of the entire manufacturing process and how to achieve synthesis, but also an understanding of the company's goals and directions.

**1.20** The main difference between strategic planning and contingency planning is that strategic planning is a *proactive* process in which problems are being eliminated before they occur, while contingency planning is a *reactive* process in which plans are made to eliminate problems as or after they occur.

**1.21** Facilities planning is not a thing that can be rushed or done halfway because the personnel have "more important things to do." If facilities planning is done that way, the cost of implementation will increase because all alternatives may not have been examined, or errors will have been made in calculations. In order for facilities planning to be done properly, sufficient lead-time in the implementation project must be granted. The amount of lead-time is never the same for two different projects because each project has a different level of complexity.

# Chapter 2

# *Product, Process, and Schedule Design*

**SECTION 2.1**

**2.1** Identify the hospital functions and departments to be incorporated in the facilities plan. Identify the key entities for which flow requirements will be needed, e.g., people, paperwork, operating rooms, emergency rooms, vehicular movement, etc. Identify the various criteria that will be used to evaluate the alternative facilities plans generated, e.g. patient service, cleanliness, logic of travel between departments, ease of expansion/ updating, cost. Determine the time period over which the facilities requirements will be estimated. Estimate space and flow requirements and determine activity relationships. Generate alternative facilities plans.

**2.2** It is important for the various design decisions to be integrated so that all critical issues have been considered before product and process designs are finalized. Using a linear or series approach can result in multiple re-starts of the design process because of "down-stream" consequences of "up-stream" design decisions that are made. Overall optimization is the goal, rather than piecewise optimization.

Knowledgeable representatives from each of the activities or functions need to be involved in the design process.

Several concurrent engineering techniques can be used to improve the design process. Quality Function Deployment is one technique that can prove extremely beneficial. However, all of the approaches described in Section 2.5 should receive serious consideration. Since the text is devoted to facilities planning, every technique presented in the text is a candidate for use in a specific application.

**2.3** Research question. Depending on the comprehensiveness of the collection of periodicals and manuscripts in the university library, it might be more helpful to the students to modify the assignment and encourage them to use the Internet in performing the assigned search.

**SECTION 2.2**

**2.4 - 2.6** Research question. Depending on the comprehensiveness of the collection of periodicals and manuscripts in the university library, it might be more helpful to the students to modify the assignment and encourage them to use the Internet in performing the assigned search.

**2.7** Research question. The Internet will likely be the best source of information needed to answer this question.

**SECTION 2.3**

**2.8** The assigned chart submitted for a cheeseburger will vary depending on the assumptions regarding ingredients, e.g., mayonnaise, mustard, catsup, onions, pickles, lettuce, tomato, multiple beef patties, multiple slices of cheese, intermediate layer of bread. Likewise, the chart submitted for a taco will vary depending on the ingredients included. It is important to verify that the student follows the steps described in Section 2.3.

**2.9** The assembly chart shows only the operations and inspections associated with the assembly of the product. The operation process chart includes all operations and inspections, fabrication and assembly operations and processing times, and purchased materials.

**2.10 - 2.11** The solution depends on the recipe chosen.

**SECTION 2.4**

**2.12** $I_1 = 2000/[(0.92)(0.95)(0.95)(0.97)] = 2{,}483$ enclosures (rounded to nearest integer)

**2.13** Given System: (all $I_k$ values are rounded to the nearest integer)
$I_1 = 1{,}000/[0.93(0.95)(0.97)] = 1{,}167$ units
$I_2 = 1{,}000/[0.93(0.95)] = 1{,}132$ units
$I_3 = 1{,}000/0.93 = 1{,}075$ units

The scrap values and scrap cost at each step are as follows:
Process 3: 1,075 - 1,000 = 75 units → Scrap Cost = \$15(75) = \$1,125
Process 2: 1,132 - 1,075 = 57 units → Scrap Cost = \$10(57) = \$570
Process 1: 1,167 - 1,132 = 35 units → Scrap Cost = \$5(34) = \$170
Total Scrap Cost for given system = \$1,865

System w/Reversed Scrap Rates:
$I_1 = 1{,}167$ units
$I_2 = 1{,}000/[0.97(0.95)] = 1{,}085$ units
$I_3 = 1{,}000/0.97 = 1{,}031$ units

The scrap values and scrap cost at each step are as follows:
Process 3: 1,031 - 1,000 = 31 units → Scrap Cost = \$15(31) = \$465
Process 2: 1,085 - 1,031 = 54 units → Scrap Cost = \$10(54) = \$540
Process 1: 1,167 - 1,085 = 82 units → Scrap Cost = \$5(82) = \$410
Total Scrap Cost for given system = \$1,415

Due to the lower scrap cost the system with reversed scrap rates would be preferred, which is consistent with the claim made in Section 2.4.2.

**2.14** For simplicity the rework operations are indicated by Rk. All $I_k$ values are rounded to the nearest integer.

Given system: Following the derivation method given in Section 2.4.2.2, we have the expression for $I_1$:

$$I_1 = \frac{O_3}{[[(1-d_3)+d_3(1-d_{R3})][(1-d_2)+d_2(1-d_{R2})][(1-d_1)+d_1(1-d_{R1})]]}$$
$$I_1 = 1{,}000/[[0.93+(0.8)(0.07)][0.95+(0.75)(0.05)][0.97+(0.60)(0.03)]]$$
$$I_1 = 1040 \text{ units}$$

To find the rework cost we need only the required input to each rework process. Thus,

$$I_{R1} = I_1(d_1) = 1{,}040(0.03) = 31 \text{ units}$$
$$I_{R2} = I_1(d_2)[(1-d_1)+d_1(1-d_{R1})] = 1{,}040(0.05)(0.988)$$
$$= 51 \text{ units}$$
$$I_{R3} = I_1(d_3)[(1-d_1)+d_1(1-d_{R1})][(1-d_2)+d_2(1-d_{R2})]$$
$$I_{R3} = 1{,}040(0.07)(0.988)(0.9875) = 71 \text{ units}$$

Therefore, the total rework cost = \$2(31) + \$3(51) + \$4(71) = \$499.

System w/Reversed Scrap Rates: Using the same derivation, we have the following:

$$I_1 = 1{,}000/[[0.97+(0.8)(0.03)][0.95+(0.75)(0.05)][0.93+(0.60)(0.07)]]$$
$$I_1 = 1048 \text{ units}$$

$$I_{R1} = 1{,}048(0.07) = 73 \text{ units}$$
$$I_{R2} = 1{,}048(0.05)(0.972) = 51 \text{ units}$$
$$I_{R3} = 1{,}048(0.03)(0.972)(0.9875) = 30 \text{ units}$$

Therefore, the total rework cost for the system with the reversed scrap rates = \$2(73) + \$3(51) + \$4(30) = \$419.

Based solely on the total rework costs of the two systems, the system with reversed scrap rates is preferred, which is consistent with the result of Problem 2.13. You should note, however, that the system with reversed scrap rates requires more input to the system to meet the demand.

**2.15** All $I_k$ values are rounded to the nearest integer.

$$I_B = Q_B = O_B/(1-d_B) = 3{,}000/0.95 = 3{,}158 \text{ units}$$
$$I_A = Q_A = O_B/[(1-d_B)(1-d_A)] = 3{,}000/[0.95(0.98)] = 3{,}222 \text{ units}$$

$$F = S_A(Q_A)/[(E_A)(H)(R_A)] + S_B(Q_B)/[(E_B)(H)(R_B)] + [30(Q_A/500)]/H$$

$$H = 5 \text{ days/week}(18 \text{ hours/day})(60 \text{ min/hr}) = 5{,}400 \text{ min/week}$$

$$F = 3(3{,}222)/[(0.95)(5{,}400)(0.95)] + 5(3{,}158)/[(0.95)(5{,}400)(0.90)]$$
$$+ [30(3{,}222/500)]/5{,}400$$
$$F = 5.439 \approx 6 \text{ milling machines}$$

**2.16** We solve this problem working in reverse.

$$O_1 = (1 - d_1)I_1$$
$$O_2 = (1 - d_2)(1 - d_1)I_1$$
$$I_4 = d_2(1 - d_1)I_1$$
$$O_4 = (1 - d_4)(d_2)(1 - d_1)I_1$$
$$I_3 = O_2 + O_4 = [(1 - d_2)(1 - d_1) + (1 - d_4)(1 - d_1)d_2]I_1$$
$$O_3 = (1 - d_3)[(1 - d_2)(1 - d_1) + (1 - d_4)(1 - d_1)d_2]I_1$$

Solving for $I_1$ and substituting in the appropriate parameters, we have the following:

$$I_1 = 5{,}000/(0.90)[(0.95)(0.95) + (0.98)(0.95)(0.5)]$$
$$I_1 = 5{,}854 \text{ units (rounded to nearest integer)}$$

**2.17** All $I_k$ values rounded to the nearest integer. For machines 1, 2, and 3:
$H = (16 \text{ hours/day})(60 \text{ mins/hour})(5 \text{ days/week}) = 4{,}800 \text{ mins/week}$
Machine 4 operates for half of the amount of time as machines 1, 2, and 3.

We know that $I_k = Q_k$. So,

$$F_1 = Q_1(S_1)/[(H)(E_1)(R_2)]$$
$$F_1 = (5{,}854)(3)/[(4{,}800)(1)(0.95)] = 3.85 \approx 4 \text{ machines}$$

$$I_2 = (1 - d_1)I_1 = 0.95(5{,}854) = 5{,}561 \text{ units}$$
$$F_2 = (5{,}561)(2)/[(4{,}800)(0.95)(0.90)] = 2.71 \approx 3 \text{ machines}$$

$$I_3 = O_3/(1 - d_3) = 5{,}000/0.90 = 5{,}556 \text{ units}$$
$$F_2 = (5{,}556)(5)/[(4{,}800)(1.02)(0.90)] = 6.3 \approx 7 \text{ machines}$$

$$I_4 = d_2O_1 = 0.05(0.95)(5{,}584) = 278 \text{ units}$$
$$F_4 = (278)(10)/[(2{,}400)(0.90)(0.95)] = 1.35 \approx 2 \text{ machines}$$

Running the rework operation on the same shift as the remainder of the cell would cause the machine fraction to reduce to 0.68, or 1 machine. This may allow for the addition of the rework machine to the cell.

**2.18** All $I_k$ values rounded to nearest integer. Let A1 denote the first step in the production process, and A2 denote the last step.

$$I_{A1} = O_3/[(1 - d_3)(1 - d_2)(1 - d_1)]$$
$$I_{A1} = 10{,}000/[(0.95)(0.95)(0.97)] = 11{,}423 \text{ units}$$

Similarly,

$$I_B = 10{,}000/[(0.95)(0.97)] = 10{,}080 \text{ units}$$
$$I_{A2} = 10{,}000/[0.95] = 10{,}526 \text{ units}$$

$H = (8 \text{ hours/day})(60 \text{ mins/hour})(6 \text{ days/week}) = 2{,}880 \text{ mins/week}$

We know that $I_k = Q_k$. So,

$$F_A = Q_{A1}(S_{A1})/[(H)(E_{A1})(R_{A1})] + Q_{A2}(S_{A2})/[(H)(E_{A2})(R_2)]$$
$$F_A = (11{,}423)(5)/[(2{,}880)(1.08)(0.98)] + (10{,}526)(3)/[(2{,}880)(0.90)(0.95)]$$
$$F_A = 31.57 \approx 32 \text{ machines}$$

$$F_B = Q_B(S_B)/[(H)(E_B)(R_B)]$$
$$F_A = (11{,}080)(5)/[(2{,}880)(1.08)(0.98)] = 12.79 \approx 13 \text{ machines}$$

**2.19** All $I_k$ values are rounded to the nearest integer.

$$I_{XC} = O_X/[1 - d_{XC}] = 100{,}000/0.97 = 103{,}093 \text{ units}$$
$$I_{XB} = O_X/[(1 - d_{XC})(1 - d_{XB})]$$
$$I_{XB} = 100{,}000/[(0.97)(0.96)] = 107{,}388 \text{ units}$$
$$I_{XA} = O_X/[(1 - d_{XC})(1 - d_{XB})(1 - d_{XA})]$$
$$I_{XA} = 100{,}000/[(0.97)(0.96)(0.95)] = 113{,}040 \text{ units}$$

$$I_{YC} = O_Y/[1 - d_{YC}] = 200{,}000/0.97 = 206{,}186 \text{ units}$$
$$I_{YA} = O_Y/[(1 - d_{YC})(1 - d_{YA})]$$
$$I_{YA} = 200{,}000/[(0.97)(0.95)] = 217{,}037 \text{ units}$$
$$I_{YB} = O_Y/[(1 - d_{YC})(1 - d_{YA})(1 - d_{YB})]$$
$$I_{YB} = 200{,}000/[(0.97)(0.95)(0.96)] = 226{,}081 \text{ units}$$

Setup times are identical for machines A, B, and C for a particular product. The setup time for product X, regardless of the machine, is 20 mins; the setup time for product Y is 40 mins., regardless of the machine. A critical piece of information needed to determine the number of machines required is the length of production runs between setups. If a single setup is needed to produce the annual requirement of a product on a machine, then the number of machines required is determined as follows:

$$F_A = \frac{0.15(113{,}040)}{(0.85)(1600)(0.95)} + \frac{0.15(217{,}037)}{(0.85)(1600)(0.95)} + \frac{20 + 40}{(60)(1600)}$$
$$F_A = 13.12 + 16.8 + 0.00065 = 29.92 \approx 30 \text{ machines}$$

$$F_B = \frac{0.25(107{,}388)}{(0.90)(1600)(0.90)} + \frac{0.25(226{,}081)}{(0.90)(1600)(0.90)} + \frac{20 + 40}{(60)(1600)}$$
$$F_B = 20.72 + 17.44 + 0.00065 = 38.16 \approx 39 \text{ machines}$$

$$F_C = \frac{0.15(103{,}093)}{(0.95)(1600)(0.85)} + \frac{0.15(206{,}186)}{(0.95)(1600)(0.85)} + \frac{20 + 40}{(60)(1600)}$$
$$F_C = 7.98 + 23.94 + 0.00065 = 31.92 \approx 32 \text{ machines}$$

If setups occur more frequently, then additional machines might be required due to the lost production time consumed by setups.

**2.20** All $I_k$ values are rounded to the nearest integer.

Part A: $I_1 = O_6/[(1-d_6)(1-d_4)(1-d_2)(1-d_1)]$
$I_1 = 15{,}000/[(0.70)(0.85)(0.90)(0.80)] = 35{,}014$ units

Part B: $I_3 = 3 * I_4/(1-d_3)$
$I_4 = O_6/[(1-d_6)(1-d_3)] = 15{,}000/[(0.70)(0.85)] = 25{,}210$
Then, $I_3 = (3 * 25{,}210)/0.70 = 108{,}043$ units

Part C: $I_5 = O_6/[(1-d_6)(1-d_5)] = 15{,}000/[(0.70)(0.75)] = 28{,}571$ units

**2.21** Using the equations to solve Problem 2.20, we simply change the input values. Let $d_k'$ represent the new scrap percentage. For this solution, we are assuming the 5% reduction is as follows: $d_k' \rightarrow d_k - 0.05$. You could also view the problem statement as indicating that $d_k' \rightarrow 0.95 * d_k$. All $I_k$ values are rounded to the nearest integer.

Part A: $I_1 = 15{,}000/[(0.75)(0.90)(0.95)(0.85)] = 27{,}520$ units
This represents a 21.4% reduction in the input requirement.

Part B: $I_4 = 15{,}000/[(0.75)(0.90)] = 22{,}222$
$I_3 = (3 * 22{,}222)/0.75 = 88{,}888$ units
This represents a 17.7% reduction in the input requirement.

Part C: $I_4 = 15{,}000/[(0.75)(0.80)] = 25{,}000$ units
This represents a 12.5% reduction in the input requirement.

The opinion response will depend on the student. However, the response should look something like the following.

Estimation Perspective: For most tasks that a process designer may have to plan for there are likely to be many alternatives. The designer must be able to identify the issues related to each alternative and be able to generate an accurate estimate of the scrap rate. Even in the case where there may be only a single alternative the ability to accurately estimate the scrap rate is of great significance.

Continuous Improvement Perspective: Being able to reduce the scrap produced by a process can be shown to significantly reduce the input requirement to a process.

**2.22** The value of H is up to the student or instructor. Any value will provide sufficient illustration for the follow-up opinion question. For the purposes of this solution, we will assume H is a variable value and solve symbolically.

Before reduction: $I_4 = Q_4 = 25{,}210$. Thus,
$$F_4 = S_4Q_4/[HE_4R_4] = (4 * 25{,}210)/[(H)(0.95)(0.98)] = 108{,}313.64/H$$

After reduction: $I_4 = Q_4 = 22{,}222$. Thus,
$$F_4 = S_4Q_4/[HE_4R_4] = (4 * 22{,}222)/[(H)(0.95)(0.98)] = 95{,}475.83/H$$

It should be apparent that by reducing the scrap percentage will reduce the number of machines necessary. Assuming one 8 hour shift per day, 5 days per week, reducing the scrap percentage as indicated in Problem 2.21 would result in a reduction of 6 machines. This could be a significant reduction in the floor space required to perform a specific process. In addition, reduction of the input needed for a specific process may also reduce the amount of storage space for raw materials/WIP, further reducing floor space requirements. For the facilities planner, space is at a premium, so every advantage should be taken to either improve processes or select processes that produce less scrap.

**2.23** $\mathrm{I = Q = O/(1-d) = 750/0.80 = 938}$ units (rounded to nearest integer)

$$\mathrm{F = SQ/EHR = [(1/4)(938)]/[(15/20)(7)(0.75)] = 44.67 \approx 45}\text{ machines}$$

Alternatively, you could say that the loss of one hour per shift is a reduction in the reliability. Thus, the denominator would have the following: H = 8, R = 7/8.

**2.24** Let CA, CB, B, F, FA, IN, and M represent component A, component B, blanking, forging, final assembly, inspection, and machining, respectively. Let $O_{SA}$ = Amount of demand for spare parts of Component A = 1,000 units; and $I_{FA}$ = Input needed for final assembly for Component A. All $I_k$ values are rounded to the nearest integer.

$$\mathrm{I_{FA} = 2(O_{FA}/(1-d_{FA})) = 2(5{,}000/0.95) = 10{,}526}\text{ units}$$

$$\mathrm{I_{CA} = (O_S + I_{FA})/[(1-d_M)(1-d_F)(1-d_B)]}$$
$$\mathrm{I_{CA} = (1{,}000 + 10{,}526)/[(0.75)(0.85)(0.90)] = 20{,}089}\text{ units}$$

$$\mathrm{I_{CB} = O_{FA}/[(1-d_{FA})(1-d_{IN})] = 5{,}000/[(0.95)(0.98)] = 5{,}371}\text{ units}$$

**2.25** $I_1 = \lceil O_6/[(1-d_6)(1-d_5)(1-d_4)(1-d_3)(1-d_2)(1-d_1)]\rceil$

Where ⌈xx⌉ stands for the least integer ≥ xx.

**2.26** Let BC, BY, CB, DA, IN, KP, and TC represent the bottom cover, battery cover, circuit board, disassembly, inspection, keypad, and top cover, respectively. In addition, let Mxx represent molding operations for specific components; Pxx represent painting operations for specific components; and INx represent inspection stations in order of occurrence. All $I_k$ values are rounded to the nearest integer.

$$\mathrm{I_{TC} = O/[(1-d_{MTC})(1-d_{PTC})] = 10{,}000/[(0.95)(0.95)] = 11{,}080}\text{ units}$$
$$\mathrm{I_{KP} = O/[(1-d_{MKP})(1-d_{PKP})] = 10{,}000/[(0.98)(0.95)] = 10{,}741}\text{ units}$$

$I_{BY} = O/(1 - d_{MBY}) = 10{,}000/0.92 = 10{,}870$ units
$I_{CB} = O/[(1 - d_{IN1})(1 - d_{IN2})] = 10{,}000/[(0.97)(0.90)] = 11{,}455$ units

To find the number of units of the bottom cover to mold, we'll assume that it only goes through the inspection process once.

$I_{IN2} = (1 - d_{MBC})I_{BC}$
$O_{IN2} = (1 - d_{IN2})I_{IN2} = (1 - d_{IN2})(1 - d_{MBC})I_{BC}$
$O_{DA} = d_{IN2}I_{IN2}(1 - d_{DA}) = (1 - d_{MBC})d_{IN2}(1 - d_{DA})I_{BC}$

Thus, $O = (1 - d_{IN2})(1 - d_{MBC})I_{BC} + (1 - d_{MBC})d_{IN2}(1 - d_{DA})I_{BC}$
$I_{BC} = O/[(1 - d_{IN2})(1 - d_{MBC}) + (1 - d_{MBC})(d_{IN2})(1 - d_{DA})]$
$I_{BC} = 10{,}000/[(0.97)(0.90) + (0.90)(0.03)(0.25)] = 11{,}367$ units

**2.27** Shown below is the probability mass function for number of good castings produced (x), based on Q castings scheduled for production.

| # Good | Number of Castings Scheduled | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Castings | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 12 | 0.10 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 13 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 14 | 0.10 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 |
| 15 | 0.10 | 0.10 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 |
| 16 | 0.10 | 0.10 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 |
| 17 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 |
| 18 | 0.20 | 0.10 | 0.10 | 0.10 | 0.10 | 0.05 | 0.05 | 0.10 | 0.05 | 0.05 | 0.05 |
| 19 | 0.10 | 0.15 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.05 | 0.05 |
| 20 | 0.10 | 0.10 | 0.15 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.05 |
| 21 | 0.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 22 | 0.00 | 0.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 23 | 0.00 | 0.00 | 0.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 | 0.10 | 0.10 | 0.10 |
| 27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 | 0.10 | 0.10 |
| 28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 | 0.05 |
| 29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.05 |
| 30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 |

Shown below is the matrix of net income for each combination of Q and x.

| # Good Castings | Number of Castings Scheduled | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 12 | -$8,000 | -$8,400 | -$8,800 | -$9,200 | -$9,600 | -$10,000 | -$10,400 | -$10,800 | -$11,200 | -$11,600 | -$12,000 |
| 13 | -$8,000 | -$8,400 | -$8,800 | -$9,200 | -$9,600 | -$10,000 | -$10,400 | -$10,800 | -$11,200 | -$11,600 | -$12,000 |
| 14 | -$8,000 | -$8,400 | -$8,800 | -$9,200 | -$9,600 | -$10,000 | -$10,400 | -$10,800 | -$11,200 | -$11,600 | -$12,000 |
| 15 | -$8,000 | -$8,400 | -$8,800 | -$9,200 | -$9,600 | -$10,000 | -$10,400 | -$10,800 | -$11,200 | -$11,600 | -$12,000 |
| 16 | -$8,000 | -$8,400 | -$8,800 | -$9,200 | -$9,600 | -$10,000 | -$10,400 | -$10,800 | -$11,200 | -$11,600 | -$12,000 |
| 17 | -$8,000 | -$8,400 | -$8,800 | -$9,200 | -$9,600 | -$10,000 | -$10,400 | -$10,800 | -$11,200 | -$11,600 | -$12,000 |
| 18 | -$8,000 | -$8,400 | -$8,800 | -$9,200 | -$9,600 | -$10,000 | -$10,400 | -$10,800 | -$11,200 | -$11,600 | -$12,000 |
| 19 | -$8,000 | -$8,400 | -$8,800 | -$9,200 | -$9,600 | -$10,000 | -$10,400 | -$10,800 | -$11,200 | -$11,600 | -$12,000 |
| 20 | $16,000 | $15,600 | $15,200 | $14,800 | $14,400 | $14,000 | $13,600 | $13,200 | $12,800 | $12,400 | $12,000 |
| 21 | $0 | $16,300 | $15,900 | $15,500 | $15,100 | $14,700 | $14,300 | $13,900 | $13,500 | $13,100 | $12,700 |
| 22 | $0 | $0 | $16,600 | $16,200 | $15,800 | $15,400 | $15,000 | $14,600 | $14,200 | $13,800 | $13,400 |
| 23 | $0 | $0 | $0 | $16,200 | $15,800 | $15,400 | $15,000 | $14,600 | $14,200 | $13,800 | $13,400 |
| 24 | $0 | $0 | $0 | $0 | $15,800 | $15,400 | $15,000 | $14,600 | $14,200 | $13,800 | $13,400 |
| 25 | $0 | $0 | $0 | $0 | $0 | $15,400 | $15,000 | $14,600 | $14,200 | $13,800 | $13,400 |
| 26 | $0 | $0 | $0 | $0 | $0 | $0 | $15,000 | $14,600 | $14,200 | $13,800 | $13,400 |
| 27 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $14,600 | $14,200 | $13,800 | $13,400 |
| 28 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $14,200 | $13,800 | $13,400 |
| 29 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $13,800 | $13,400 |
| 30 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $13,400 |

For a given value of Q, multiplying the net income in the column by the probability of its occurrence and summing over all values of x yields the following expected profits for each value of Q.

| Expected Profit<br>Number of Castings Scheduled | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| -$5,600 | -$3,530 | -$190 | $750 | $2,890 | $5,030 | $5,900 | $6,770 | $8,910 | **$11,050** | $10,720 |

Based on the results obtained, scheduling 29 castings for production yields the maximum expected profit of $11,050.

**2.28** Shown below is the probability mass function for number of good castings produced (x), based on Q castings scheduled for production.

| # Good | Number of Castings Scheduled | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Castings | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 5 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 7 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 8 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 9 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 11 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 12 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 13 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 14 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 15 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 16 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 |
| 17 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 |
| 18 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 | 0.00 |
| 19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.00 |
| 20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| 21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| 22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 | 0.05 |
| 23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 | 0.05 |
| 24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 | 0.05 |
| 25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 | 0.05 |
| 26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 | 0.10 |
| 27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 | 0.10 |
| 28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 | 0.15 |
| 29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.15 |
| 30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 |

Shown below is the matrix of net income for each combination of Q and x.

| # Good | Number of Castings Scheduled | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Castings | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 5 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 6 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 7 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 8 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 9 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 10 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 11 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 12 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 13 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 14 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 15 | $8,625 | $8,150 | $7,675 | $7,200 | $6,725 | $6,250 | $5,775 | $5,300 | $4,825 | $4,350 | $3,875 | $3,400 | $2,925 | $2,450 | $1,975 | $1,500 |
| 16 | $0 | $9,200 | $8,725 | $8,250 | $7,775 | $7,300 | $6,825 | $6,350 | $5,875 | $5,400 | $4,925 | $4,450 | $3,975 | $3,500 | $3,025 | $2,550 |
| 17 | $0 | $0 | $9,775 | $9,300 | $8,825 | $8,350 | $7,875 | $7,400 | $6,925 | $6,450 | $5,975 | $5,500 | $5,025 | $4,550 | $4,075 | $3,600 |
| 18 | $0 | $0 | $0 | $10,350 | $9,875 | $9,400 | $8,925 | $8,450 | $7,975 | $7,500 | $7,025 | $6,550 | $6,075 | $5,600 | $5,125 | $4,650 |
| 19 | $0 | $0 | $0 | $0 | $10,925 | $10,450 | $9,975 | $9,500 | $9,025 | $8,550 | $8,075 | $7,600 | $7,125 | $6,650 | $6,175 | $5,700 |
| 20 | $0 | $0 | $0 | $0 | $0 | $11,500 | $11,025 | $10,550 | $10,075 | $9,600 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 21 | $0 | $0 | $0 | $0 | $0 | $0 | $11,025 | $10,550 | $10,075 | $9,600 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 22 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $10,550 | $10,075 | $9,600 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 23 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $10,075 | $9,600 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 24 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $9,600 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 25 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 26 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 27 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $8,175 | $7,700 | $7,225 | $6,750 |
| 28 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $7,700 | $7,225 | $6,750 |
| 29 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $7,225 | $6,750 |
| 30 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $6,750 |

For a given value of Q, multiplying the net income in the column by the probability of its occurrence and summing over all values of x yields the following expected profits for each value of Q.

| Expected Profit<br>Number of Castings Scheduled | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| -$3,975 | -$1,878 | $378 | $2,003 | $3,733 | $4,780 | $5,670 | $6,455 | $7,135 | $7,763 | $8,338 | $8,125 | $7,860 | $7,543 | $7,173 | $6,750 |

Based on the results obtained, scheduling 25 castings for production yields the maximum expected profit of $8,338.

**2.29** Shown below is the probability mass function for number of good castings produced (x), based on Q castings scheduled for production.

| # Good Castings | Number of Castings Scheduled | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 7 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 0.0008 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 9 | 0.0047 | 0.0013 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 0.0208 | 0.0067 | 0.0019 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 11 | 0.0694 | 0.0266 | 0.0091 | 0.0028 | 0.0008 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 12 | 0.1696 | 0.0814 | 0.0332 | 0.0120 | 0.0039 | 0.0012 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 13 | 0.2870 | 0.1837 | 0.0937 | 0.0405 | 0.0154 | 0.0053 | 0.0017 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 14 | 0.3006 | 0.2886 | 0.1963 | 0.1060 | 0.0483 | 0.0193 | 0.0070 | 0.0023 | 0.0007 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 15 | 0.1470 | 0.2822 | 0.2878 | 0.2072 | 0.1181 | 0.0567 | 0.0238 | 0.0090 | 0.0031 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 16 | 0.00 | 0.1293 | 0.2638 | 0.2850 | 0.2166 | 0.1299 | 0.0655 | 0.0288 | 0.0114 | 0.0041 | 0.0014 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 17 | 0.00 | 0.00 | 0.1138 | 0.2458 | 0.2803 | 0.2242 | 0.1413 | 0.0746 | 0.0343 | 0.0141 | 0.0053 | 0.0018 | 0.0006 | 0.0002 | 0.0001 | 0.0000 |
| 18 | 0.00 | 0.00 | 0.00 | 0.1002 | 0.2284 | 0.2740 | 0.2302 | 0.1519 | 0.0839 | 0.0403 | 0.0173 | 0.0067 | 0.0024 | 0.0008 | 0.0003 | 0.0001 |
| 19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0881 | 0.2115 | 0.2665 | 0.2345 | 0.1618 | 0.0932 | 0.0466 | 0.0208 | 0.0084 | 0.0031 | 0.0011 | 0.0004 |
| 20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0776 | 0.1955 | 0.2580 | 0.2374 | 0.1709 | 0.1025 | 0.0533 | 0.0247 | 0.0104 | 0.0040 | 0.0014 |
| 21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0683 | 0.1802 | 0.2487 | 0.2387 | 0.1790 | 0.1117 | 0.0603 | 0.0290 | 0.0126 | 0.0050 |
| 22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0601 | 0.1658 | 0.2387 | 0.2387 | 0.1862 | 0.1207 | 0.0676 | 0.0336 | 0.0151 |
| 23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0529 | 0.1522 | 0.2283 | 0.2375 | 0.1924 | 0.1293 | 0.0750 | 0.0386 |
| 24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0465 | 0.1395 | 0.2177 | 0.2351 | 0.1973 | 0.1374 | 0.0825 |
| 25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0409 | 0.1277 | 0.2069 | 0.2317 | 0.2016 | 0.1451 |
| 26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0360 | 0.1167 | 0.1961 | 0.2274 | 0.2047 |
| 27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0317 | 0.1065 | 0.1853 | 0.2224 |
| 28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0279 | 0.0971 | 0.1747 |
| 29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0245 | 0.0884 |
| 30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0216 |

Net incomes for feasible combinations of Q and x are shown below.

| # Good Castings | Number of Castings Scheduled | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 7 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 8 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 9 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 10 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 11 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 12 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 13 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 14 | -$7,125 | -$7,600 | -$8,075 | -$8,550 | -$9,025 | -$9,500 | -$9,975 | -$10,450 | -$10,925 | -$11,400 | -$11,875 | -$12,350 | -$12,825 | -$13,300 | -$13,775 | -$14,250 |
| 15 | $19,500 | $19,500 | $19,500 | $19,500 | $19,500 | $19,500 | $19,500 | $19,500 | $19,500 | $19,500 | $19,500 | $19,500 | $19,481 | $19,436 | $19,389 | $19,367 |
| 16 | $0 | $20,800 | $20,800 | $20,800 | $20,800 | $20,800 | $20,800 | $20,800 | $20,800 | $20,800 | $20,800 | $20,800 | $20,781 | $20,736 | $20,689 | $20,667 |
| 17 | $0 | $0 | $22,100 | $22,100 | $22,100 | $22,100 | $22,100 | $22,100 | $22,100 | $22,100 | $22,100 | $22,100 | $22,081 | $22,036 | $21,989 | $21,967 |
| 18 | $0 | $0 | $0 | $23,400 | $23,400 | $23,400 | $23,400 | $23,400 | $23,400 | $23,400 | $23,400 | $23,400 | $23,381 | $23,336 | $23,289 | $23,267 |
| 19 | $0 | $0 | $0 | $0 | $24,700 | $24,700 | $24,700 | $24,700 | $24,700 | $24,700 | $24,700 | $24,700 | $24,681 | $24,636 | $24,589 | $24,567 |
| 20 | $0 | $0 | $0 | $0 | $0 | $26,000 | $26,000 | $26,000 | $26,000 | $26,000 | $26,000 | $26,000 | $25,981 | $25,936 | $25,889 | $25,867 |
| 21 | $0 | $0 | $0 | $0 | $0 | $0 | $11,025 | $10,550 | $10,075 | $9,600 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 22 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $10,550 | $10,075 | $9,600 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 23 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $10,075 | $9,600 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 24 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $9,600 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 25 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $9,125 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 26 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $8,650 | $8,175 | $7,700 | $7,225 | $6,750 |
| 27 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $8,175 | $7,700 | $7,225 | $6,750 |
| 28 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $7,700 | $7,225 | $6,750 |
| 29 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $7,225 | $6,750 |
| 30 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $6,750 |

For a given value of Q, multiplying the net income in the column by the probability of its occurrence and summing over all values of x yields the following expected profits for each value of Q. Based on the results obtained, scheduling 21 castings for production yields the maximum expected profit of $22,663.

| Expected Profit<br>Number of Castings Scheduled | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| -$3,212 | $3,721 | $10,915 | $16,362 | $19,904 | $22,170 | **$22,663** | $20,984 | $17,885 | $14,593 | $11,914 | $10,038 | $8,800 | $7,958 | $7,324 | $6,785 |

**2.30a** Shown below is the probability mass function for the number of good custom-designed castings produced (x), based on Q castings scheduled for production.

| # Good Castings | Number of Castings Scheduled | | | |
|---|---|---|---|---|
| | 4 | 5 | 6 | 7 |
| 0 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.0036 | 0.0005 | 0.0001 | 0.0000 |
| 2 | 0.0486 | 0.0081 | 0.0012 | 0.0002 |
| 3 | 0.2916 | 0.0729 | 0.0146 | 0.0026 |
| 4 | 0.6561 | 0.3281 | 0.0984 | 0.0230 |
| 5 | 0.0000 | 0.5905 | 0.3543 | 0.1240 |
| 6 | 0.0000 | 0.0000 | 0.5314 | 0.3720 |
| 7 | 0.0000 | 0.0000 | 0.0000 | 0.4783 |

Net incomes for feasible combinations of Q and x are shown below.

| # Good Castings | Number of Castings Scheduled | | | |
|---|---|---|---|---|
| | 4 | 5 | 6 | 7 |
| 0 | -$60,000 | -$75,000 | -$90,000 | -$105,000 |
| 1 | -$60,000 | -$75,000 | -$90,000 | -$105,000 |
| 2 | -$60,000 | -$75,000 | -$90,000 | -$105,000 |
| 3 | -$60,000 | -$75,000 | -$90,000 | -$105,000 |
| 4 | $60,000 | $45,000 | $30,000 | $15,000 |
| 5 | $0 | $45,000 | $30,000 | $15,000 |
| 6 | $0 | $0 | $30,000 | $15,000 |
| 7 | $0 | $0 | $0 | $15,000 |

For a given value of Q, multiplying the net income in the column by the probability of its occurrence and summing over all values of x yields the following expected profits for each value of Q. As shown, the optimum number to schedule is 5, with an expected net profit of $35,335.

| Expected Profit Number of Castings Scheduled | | | |
|---|---|---|---|
| 4 | 5 | 6 | 7 |
| $18,732 | **$35,225** | $28,098 | $14,673 |

**2.30b** The probability of losing money on the transaction is the probability of the net income being negative when Q equals 5. From above, a negative net cash flow occurs if less than 4 good castings are produced. The probability of producing less than 4 good castings equals 0.0005 + 0.0081 + 0.0729, or 0.0815.

**2.30c** Using Excel it is easy to perform the sensitivity analysis. By varying the cost parameter, it is found that a cost of $22,044.96 yields an expected profit of zero for Q = 5. Likewise, when the cost of producing a casting is reduced to $7,873.19, the optimum number to schedule increases to 6. We could not find a cost that would reduce the optimum production batch to 4 and still have a positive expected profit.

**2.30d** Again, using Excel it is easy to perform sensitivity analyses. For example, if the probability of a good casting is reduced to 0.84248 then the optimum production batch increases to 6; likewise, if the probability of a good casting increases to 0.963781, then the optimum batch production quantity decreases to 4.

**2.31** Shown below is the probability mass function for number of good high precision formed parts (x), based on Q parts scheduled for production.

| # Good Parts | Number Scheduled | | |
|---|---|---|---|
| | 10 | 11 | 12 |
| 0 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0001 | 0.0000 | 0.0000 |
| 4 | 0.0012 | 0.0003 | 0.0001 |
| 5 | 0.0085 | 0.0023 | 0.0006 |
| 6 | 0.0401 | 0.0132 | 0.0040 |
| 7 | 0.1298 | 0.0536 | 0.0193 |
| 8 | 0.2759 | 0.1517 | 0.0683 |
| 9 | 0.3474 | 0.2866 | 0.1720 |
| 10 | 0.1969 | 0.3248 | 0.2924 |
| 11 | 0.00 | 0.1673 | 0.3012 |
| 12 | 0.00 | 0.00 | 0.1422 |
| 13 | 0.00 | 0.00 | 0.00 |

Shown below is the matrix of net income for batch sizes of 10, 11, and 12. Also shown below is the expected profit based on batch sizes of 10, 11, and 12, as well as the probability of losing money. A batch size of 12 yields the smallest expected profit. Based on the probability of losing money, the least attractive alternative is a batch size of 10.

| # Good Castings | Number Scheduled | | |
|---|---|---|---|
| | 10 | 11 | 12 |
| 0 | -$4,750 | -$5,225 | -$5,700 |
| 1 | -$4,750 | -$5,225 | -$5,700 |
| 2 | -$4,750 | -$5,225 | -$5,700 |
| 3 | -$4,750 | -$5,225 | -$5,700 |
| 4 | -$4,750 | -$5,225 | -$5,700 |
| 5 | -$4,750 | -$5,225 | -$5,700 |
| 6 | -$4,750 | -$5,225 | -$5,700 |
| 7 | -$4,750 | -$5,225 | -$5,700 |
| 8 | $150,000 | $85,000 | $20,000 |
| 9 | $250,000 | $185,000 | $120,000 |
| 10 | $350,000 | $285,000 | $220,000 |
| 11 | $0 | $285,000 | $220,000 |
| 12 | $0 | $0 | $220,000 |
| 13 | $0 | $0 | $0 |

| Expected Profit and Probability of Losing Money | | |
|---|---|---|
| 10 | 11 | 12 |
| $196,293 | **$205,834** | $183,746 |
| 0.1798 | 0.0694 | 0.0239 |

**2.32** Shown below is the probability mass function for the number of good wafers (x) resulting from a production batch size of Q.

| # Good Wafers | Number of Wafers Scheduled | | | | | | |
|---|---|---|---|---|---|---|---|
| | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | 0.0039 | 0.0010 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.0469 | 0.0146 | 0.0044 | 0.0013 | 0.0004 | 0.0001 | 0.0000 |
| 2 | 0.2109 | 0.0879 | 0.0330 | 0.0115 | 0.0038 | 0.0012 | 0.0004 |
| 3 | 0.4219 | 0.2637 | 0.1318 | 0.0577 | 0.0231 | 0.0087 | 0.0031 |
| 4 | 0.3164 | 0.3955 | 0.2966 | 0.1730 | 0.0865 | 0.0389 | 0.0162 |
| 5 | 0.0000 | 0.2373 | 0.3560 | 0.3115 | 0.2076 | 0.1168 | 0.0584 |
| 6 | 0.0000 | 0.0000 | 0.1780 | 0.3115 | 0.3115 | 0.2336 | 0.1460 |
| 7 | 0.0000 | 0.0000 | 0.0000 | 0.1335 | 0.2670 | 0.3003 | 0.2503 |
| 8 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1001 | 0.2253 | 0.2816 |
| 9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0751 | 0.1877 |
| 10 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0563 |

Shown below is the matrix of net profit resulting from combinations of Q and x.

| # Good Wafers | Number of Wafers Scheduled | | | | | | |
|---|---|---|---|---|---|---|---|
| | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | -$180,000 | -$200,000 | -$220,000 | -$240,000 | -$260,000 | -$280,000 | -$300,000 |
| 1 | -$180,000 | -$200,000 | -$220,000 | -$240,000 | -$260,000 | -$280,000 | -$300,000 |
| 2 | -$180,000 | -$200,000 | -$220,000 | -$240,000 | -$260,000 | -$280,000 | -$300,000 |
| 3 | $70,000 | $50,000 | $30,000 | $10,000 | -$10,000 | -$30,000 | -$50,000 |
| 4 | $120,000 | $100,000 | $80,000 | $60,000 | $40,000 | $20,000 | $0 |
| 5 | $0 | $150,000 | $130,000 | $110,000 | $90,000 | $70,000 | $50,000 |
| 6 | $0 | $0 | $130,000 | $110,000 | $90,000 | $70,000 | $50,000 |
| 7 | $0 | $0 | $0 | $110,000 | $90,000 | $70,000 | $50,000 |
| 8 | $0 | $0 | $0 | $0 | $90,000 | $70,000 | $50,000 |
| 9 | $0 | $0 | $0 | $0 | $0 | $70,000 | $50,000 |
| 10 | $0 | $0 | $0 | $0 | $0 | $0 | $50,000 |

Shown below are the expected profits and probabilities of losing money for various batch sizes. The optimum batch size is 7, with a 0.0129 probability of losing money.

| Expected Profit<br>Number of Wafers Scheduled and Probability of Losing Money | | | | | | |
|---|---|---|---|---|---|---|
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $20,391 | $67,627 | $88,826 | **$91,073** | $81,888 | $66,718 | $48,734 |
| 0.2617 | 0.1035 | 0.0376 | 0.0129 | 0.0273 | 0.0100 | 0.0035 |

**2.33** Shown below is the probability mass function for the number of good die castings (x) in a production batch of size Q.

| # Good Parts | Number of Parts Scheduled | | | | | |
|---|---|---|---|---|---|---|
| | 25 | 26 | 27 | 28 | 29 | 30 |
| 20 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 21 | 0.0013 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 22 | 0.0118 | 0.0015 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
| 23 | 0.0754 | 0.0131 | 0.0018 | 0.0002 | 0.0000 | 0.0000 |
| 24 | 0.3079 | 0.0801 | 0.0144 | 0.0020 | 0.0002 | 0.0000 |
| 25 | 0.6035 | 0.3138 | 0.0847 | 0.0158 | 0.0023 | 0.0003 |
| 26 | 0.0000 | 0.5914 | 0.3194 | 0.0894 | 0.0173 | 0.0026 |
| 27 | 0.0000 | 0.0000 | 0.5796 | 0.3246 | 0.0941 | 0.0188 |
| 28 | 0.0000 | 0.0000 | 0.0000 | 0.5680 | 0.3294 | 0.0988 |
| 29 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.5566 | 0.3340 |
| 30 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.5455 |

Shown below is the matrix of net profits resulting from various combinations of Q and x.

| # Good Parts | Number of Parts Scheduled | | | | | |
|---|---|---|---|---|---|---|
| | 25 | 26 | 27 | 28 | 29 | 30 |
| 20 | -$37,500 | -$39,000 | -$40,500 | -$42,000 | -$43,500 | -$45,000 |
| 21 | -$37,500 | -$39,000 | -$40,500 | -$42,000 | -$43,500 | -$45,000 |
| 22 | -$37,500 | -$39,000 | -$40,500 | -$42,000 | -$43,500 | -$45,000 |
| 23 | -$37,500 | -$39,000 | -$40,500 | -$42,000 | -$43,500 | -$45,000 |
| 24 | -$37,500 | -$39,000 | -$40,500 | -$42,000 | -$43,500 | -$45,000 |
| 25 | $87,500 | $86,000 | $84,500 | $83,000 | $81,500 | $80,000 |
| 26 | $0 | $86,000 | $84,500 | $83,000 | $81,500 | $80,000 |
| 27 | $0 | $0 | $84,500 | $83,000 | $81,500 | $80,000 |
| 28 | $0 | $0 | $0 | $83,000 | $81,500 | $80,000 |
| 29 | $0 | $0 | $0 | $0 | $81,500 | $80,000 |
| 30 | $0 | $0 | $0 | $0 | $0 | $80,000 |

Shown below are the expected profits and probabilities of losing money for various values of Q, the batch size. From the results obtained, the optimum batch size is 28. The probability of losing money, which is the probability of less than 25 die cast parts being acceptable, equals 0.0022.

| Expected Profit & Probability of Losing Money<br>Number of Parts Scheduled | | | | | |
|---|---|---|---|---|---|
| 25 | 26 | 27 | 28 | 29 | 30 |
| $37,933 | $74,150 | $82,456 | **$82,721** | $81,468 | $79,997 |
| 0.3965 | 0.0948 | 0.0164 | **0.0022** | 0.0003 | 0.0000 |

**2.34a** Shown below is the probability mass function for the number of good castings (x) produced when Q castings are produced based on a probability of 0.85 that an individual casting is good. Also shown below is a matrix of net profits resulting from the combination of Q and x. Finally, the expected profit is shown for various values of Q. Based on the results obtained, the optimum lot size is 65, with an expected profit of $55,925.

| # Good | Number of Castings Scheduled | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Castings | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |
| 35 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 36 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 37 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 38 | 0.0029 | 0.0014 | 0.0007 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 39 | 0.0067 | 0.0035 | 0.0017 | 0.0008 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 40 | 0.0142 | 0.0078 | 0.0041 | 0.0021 | 0.0010 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 41 | 0.0275 | 0.0162 | 0.0091 | 0.0049 | 0.0025 | 0.0012 | 0.0006 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 42 | 0.0483 | 0.0307 | 0.0184 | 0.0105 | 0.0057 | 0.0030 | 0.0015 | 0.0007 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 43 | 0.0764 | 0.0525 | 0.0339 | 0.0207 | 0.0120 | 0.0066 | 0.0035 | 0.0018 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 |
| 44 | 0.1082 | 0.0812 | 0.0568 | 0.0374 | 0.0232 | 0.0137 | 0.0077 | 0.0041 | 0.0021 | 0.0011 | 0.0005 | 0.0002 | 0.0001 |
| 45 | 0.1363 | 0.1124 | 0.0859 | 0.0612 | 0.0409 | 0.0259 | 0.0155 | 0.0089 | 0.0049 | 0.0026 | 0.0013 | 0.0006 | 0.0003 |
| 46 | 0.1511 | 0.1385 | 0.1163 | 0.0904 | 0.0656 | 0.0446 | 0.0287 | 0.0175 | 0.0102 | 0.0057 | 0.0030 | 0.0015 | 0.0008 |
| 47 | 0.1457 | 0.1503 | 0.1403 | 0.1199 | 0.0949 | 0.0700 | 0.0484 | 0.0317 | 0.0196 | 0.0116 | 0.0065 | 0.0035 | 0.0018 |
| 48 | 0.1204 | 0.1419 | 0.1490 | 0.1416 | 0.1232 | 0.0991 | 0.0743 | 0.0523 | 0.0348 | 0.0219 | 0.0131 | 0.0075 | 0.0041 |
| 49 | 0.0836 | 0.1149 | 0.1379 | 0.1474 | 0.1424 | 0.1261 | 0.1031 | 0.0786 | 0.0563 | 0.0380 | 0.0243 | 0.0148 | 0.0086 |
| 50 | 0.0474 | 0.0781 | 0.1094 | 0.1336 | 0.1453 | 0.1429 | 0.1286 | 0.1070 | 0.0829 | 0.0603 | 0.0413 | 0.0269 | 0.0166 |
| 51 | 0.0210 | 0.0434 | 0.0729 | 0.1039 | 0.1292 | 0.1429 | 0.1429 | 0.1307 | 0.1105 | 0.0870 | 0.0643 | 0.0448 | 0.0295 |
| 52 | 0.0069 | 0.0189 | 0.0397 | 0.0679 | 0.0985 | 0.1246 | 0.1401 | 0.1435 | 0.1325 | 0.1138 | 0.0911 | 0.0683 | 0.0483 |
| 53 | 0.0000 | 0.0061 | 0.0170 | 0.0363 | 0.0632 | 0.0932 | 0.1199 | 0.1371 | 0.1417 | 0.1339 | 0.1168 | 0.0049 | 0.0723 |
| 54 | 0.0000 | 0.0013 | 0.0053 | 0.0152 | 0.0332 | 0.0587 | 0.0880 | 0.1151 | 0.1338 | 0.1405 | 0.1349 | 0.1195 | 0.0986 |
| 55 | 0.0000 | 0.0001 | 0.0011 | 0.0047 | 0.0137 | 0.0302 | 0.0544 | 0.0830 | 0.1103 | 0.1303 | 0.1389 | 0.1355 | 0.1219 |
| 56 | 0.0000 | 0.0000 | 0.0001 | 0.0010 | 0.0041 | 0.0122 | 0.0275 | 0.0504 | 0.0781 | 0.1054 | 0.1265 | 0.1371 | 0.1357 |
| 57 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0008 | 0.0037 | 0.0110 | 0.0250 | 0.0456 | 0.0734 | 0.1006 | 0.1227 | 0.1349 |
| 58 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0007 | 0.0032 | 0.0098 | 0.0228 | 0.0430 | 0.0688 | 0.0959 | 0.1186 |
| 59 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0006 | 0.0028 | 0.0087 | 0.0207 | 0.0397 | 0.0645 | 0.0912 |
| 60 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0005 | 0.0025 | 0.0078 | 0.0187 | 0.0365 | 0.0603 |
| 61 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0005 | 0.0022 | 0.0070 | 0.0170 | 0.0336 |
| 62 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0004 | 0.0019 | 0.0062 | 0.0153 |
| 63 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0003 | 0.0017 | 0.0055 |
| 64 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0003 | 0.0015 |
| 65 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0003 |
| 66 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

| # Good Castings | Number of Castings Scheduled | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |
| 35 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 36 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 37 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 38 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 39 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 40 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 41 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 42 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 43 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 44 | -$43,200 | -$44,000 | -$44,800 | -$45,600 | -$46,400 | -$47,200 | -$48,000 | -$48,800 | -$49,600 | -$50,400 | -$51,200 | -$52,000 | -$52,800 |
| 45 | $46,800 | $46,000 | $45,200 | $44,400 | $43,600 | $42,800 | $42,000 | $41,200 | $40,400 | $39,600 | $38,800 | $38,000 | $37,200 |
| 46 | $48,800 | $48,000 | $47,200 | $46,400 | $45,600 | $44,800 | $44,000 | $43,200 | $42,400 | $41,600 | $40,800 | $40,000 | $39,200 |
| 47 | $50,800 | $50,000 | $49,200 | $48,400 | $47,600 | $46,800 | $46,000 | $45,200 | $44,400 | $43,600 | $42,800 | $42,000 | $41,200 |
| 48 | $52,800 | $52,000 | $51,200 | $50,400 | $49,600 | $48,800 | $48,000 | $47,200 | $46,400 | $45,600 | $44,800 | $44,000 | $43,200 |
| 49 | $54,800 | $54,000 | $53,200 | $52,400 | $51,600 | $50,800 | $50,000 | $49,200 | $48,400 | $47,600 | $46,800 | $46,000 | $45,200 |
| 50 | $56,800 | $56,000 | $55,200 | $54,400 | $53,600 | $52,800 | $52,000 | $51,200 | $50,400 | $49,600 | $48,800 | $48,000 | $47,200 |
| 51 | $58,800 | $58,000 | $57,200 | $56,400 | $55,600 | $54,800 | $54,000 | $53,200 | $52,400 | $51,600 | $50,800 | $50,000 | $49,200 |
| 52 | $60,800 | $60,000 | $59,200 | $58,400 | $57,600 | $56,800 | $56,000 | $55,200 | $54,400 | $53,600 | $52,800 | $52,000 | $51,200 |
| 53 | $0 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 | $56,400 | $55,600 | $54,800 | $54,000 | $53,200 |
| 54 | $0 | $64,000 | $63,200 | $62,400 | $61,600 | $60,800 | $60,000 | $59,200 | $58,400 | $57,600 | $56,800 | $56,000 | $55,200 |
| 55 | $0 | $66,000 | $65,200 | $64,400 | $63,600 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 56 | $0 | $0 | $65,200 | $64,400 | $63,600 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 57 | $0 | $0 | $0 | $64,400 | $63,600 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 58 | $0 | $0 | $0 | $0 | $63,600 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 59 | $0 | $0 | $0 | $0 | $0 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 60 | $0 | $0 | $0 | $0 | $0 | $0 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 61 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 62 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 63 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $59,600 | $58,800 | $58,000 | $57,200 |
| 64 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $58,800 | $58,000 | $57,200 |
| 65 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $58,000 | $57,200 |
| 66 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $57,200 |

| Expected Profit<br>Number of Castings Scheduled | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |
| $24,084 | $32,877 | $39,656 | $44,687 | $48,305 | $50,873 | $52,699 | $54,001 | $54,914 | $55,514 | $55,841 | $55,925 | $55,793 |

**2.34b** Shown below is the probability mass function for the number of good castings (x) produced when Q castings are produced based on a probability of 0.98 that an individual casting is good. Also shown below is a matrix of net profits resulting from the combination of Q and x. Finally, the expected profit is shown for various values of Q. Based on the results obtained, the optimum lot size is 56, with an expected profit of $64,315.

| # Good Castings | Number of Castings Scheduled | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |
| 45 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 46 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 47 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 48 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 49 | 0.0038 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 50 | 0.0184 | 0.0041 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 51 | 0.0708 | 0.0195 | 0.0044 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 52 | 0.2002 | 0.0734 | 0.0206 | 0.0047 | 0.0009 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 53 | 0.0000 | 0.2036 | 0.0760 | 0.0217 | 0.0050 | 0.0010 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 54 | 0.0000 | 0.3695 | 0.2069 | 0.0786 | 0.0228 | 0.0054 | 0.0011 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 55 | 0.0000 | 0.3292 | 0.3687 | 0.2101 | 0.0813 | 0.0240 | 0.0058 | 0.0012 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 56 | 0.0000 | 0.0000 | 0.3236 | 0.3678 | 0.2133 | 0.0839 | 0.0252 | 0.0061 | 0.0013 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
| 57 | 0.0000 | 0.0000 | 0.0000 | 0.3161 | 0.3667 | 0.2164 | 0.0865 | 0.0264 | 0.0065 | 0.0014 | 0.0003 | 0.0000 | 0.0000 |
| 58 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.3098 | 0.3656 | 0.2194 | 0.0892 | 0.0277 | 0.0070 | 0.0015 | 0.0003 | 0.0000 |
| 59 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.3036 | 0.3644 | 0.2223 | 0.0919 | 0.0289 | 0.0074 | 0.0016 | 0.0003 |
| 60 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2976 | 0.3630 | 0.2151 | 0.0945 | 0.0302 | 0.0079 | 0.0017 |
| 61 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2916 | 0.3616 | 0.2278 | 0.0972 | 0.0316 | 0.0083 |
| 62 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2858 | 0.3601 | 0.2304 | 0.0999 | 0.0330 |
| 63 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2801 | 0.3585 | 0.2330 | 0.1025 |
| 64 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2745 | 0.3568 | 0.2355 |
| 65 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2690 | 0.3550 |
| 66 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2636 |

| # Good Castings | Number of Castings Scheduled | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |
| 45 | $46,800 | $46,000 | $45,200 | $44,400 | $43,600 | $42,800 | $42,000 | $41,200 | $40,400 | $39,600 | $38,800 | $38,000 | $37,200 |
| 46 | $48,800 | $48,000 | $47,200 | $46,400 | $45,600 | $44,800 | $44,000 | $43,200 | $42,400 | $41,600 | $40,800 | $40,000 | $39,200 |
| 47 | $50,800 | $50,000 | $49,200 | $48,400 | $47,600 | $46,800 | $46,000 | $45,200 | $44,400 | $43,600 | $42,800 | $42,000 | $41,200 |
| 48 | $52,800 | $52,000 | $51,200 | $50,400 | $49,600 | $48,800 | $48,000 | $47,200 | $46,400 | $45,600 | $44,800 | $44,000 | $43,200 |
| 49 | $54,800 | $54,000 | $53,200 | $52,400 | $51,600 | $50,800 | $50,000 | $49,200 | $48,400 | $47,600 | $46,800 | $46,000 | $45,200 |
| 50 | $56,800 | $56,000 | $55,200 | $54,400 | $53,600 | $52,800 | $52,000 | $51,200 | $50,400 | $49,600 | $48,800 | $48,000 | $47,200 |
| 51 | $58,800 | $58,000 | $57,200 | $56,400 | $55,600 | $54,800 | $54,000 | $53,200 | $52,400 | $51,600 | $50,800 | $50,000 | $49,200 |
| 52 | $60,800 | $60,000 | $59,200 | $58,400 | $57,600 | $56,800 | $56,000 | $55,200 | $54,400 | $53,600 | $52,800 | $52,000 | $51,200 |
| 53 | $0 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 | $56,400 | $55,600 | $54,800 | $54,000 | $53,200 |
| 54 | $0 | $64,000 | $63,200 | $62,400 | $61,600 | $60,800 | $60,000 | $59,200 | $58,400 | $57,600 | $56,800 | $56,000 | $55,200 |
| 55 | $0 | $66,000 | $65,200 | $64,400 | $63,600 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 56 | $0 | $0 | $65,200 | $64,400 | $63,600 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 57 | $0 | $0 | $0 | $64,400 | $63,600 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 58 | $0 | $0 | $0 | $0 | $63,600 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 59 | $0 | $0 | $0 | $0 | $0 | $62,800 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 60 | $0 | $0 | $0 | $0 | $0 | $0 | $62,000 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 61 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $61,200 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 62 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |
| 63 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $59,600 | $58,800 | $58,000 | $57,200 |
| 64 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $58,800 | $58,000 | $57,200 |
| 65 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $58,000 | $57,200 |
| 66 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $57,200 |

| Expected Profit<br>Number of Castings Scheduled | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |
| $17,627 | $63,800 | **$64,315** | $64,120 | $63,528 | $62,784 | $61,997 | $61,199 | $60,400 | $59,600 | $58,800 | $58,000 | $57,200 |

**2.35** In Example 2.7, a = 2 min, b = 1 min, t = 6 min, n' = 2.67, $C_o$ = $15/hr, and $C_m$ = $50/hr. Without other constraints the optimum number of machines to assign to an operator was shown to equal 2. Since 2 < n' < 3, the economic choice was between 2 and 3 machines. Hence, two groups of 2 would be less costly, on a cost per part produced basis, than one group of 4 machines. Here, 11 machines are required to meet the production requirements. How should they be assigned? One of the alternatives being considered is to assign 2 machines to each of 4 operators and then assign 3 machines to one operator; the alternative assignment being considered is to assign 2 machines to each of 5 operators and then assign 1 machine to one operator.

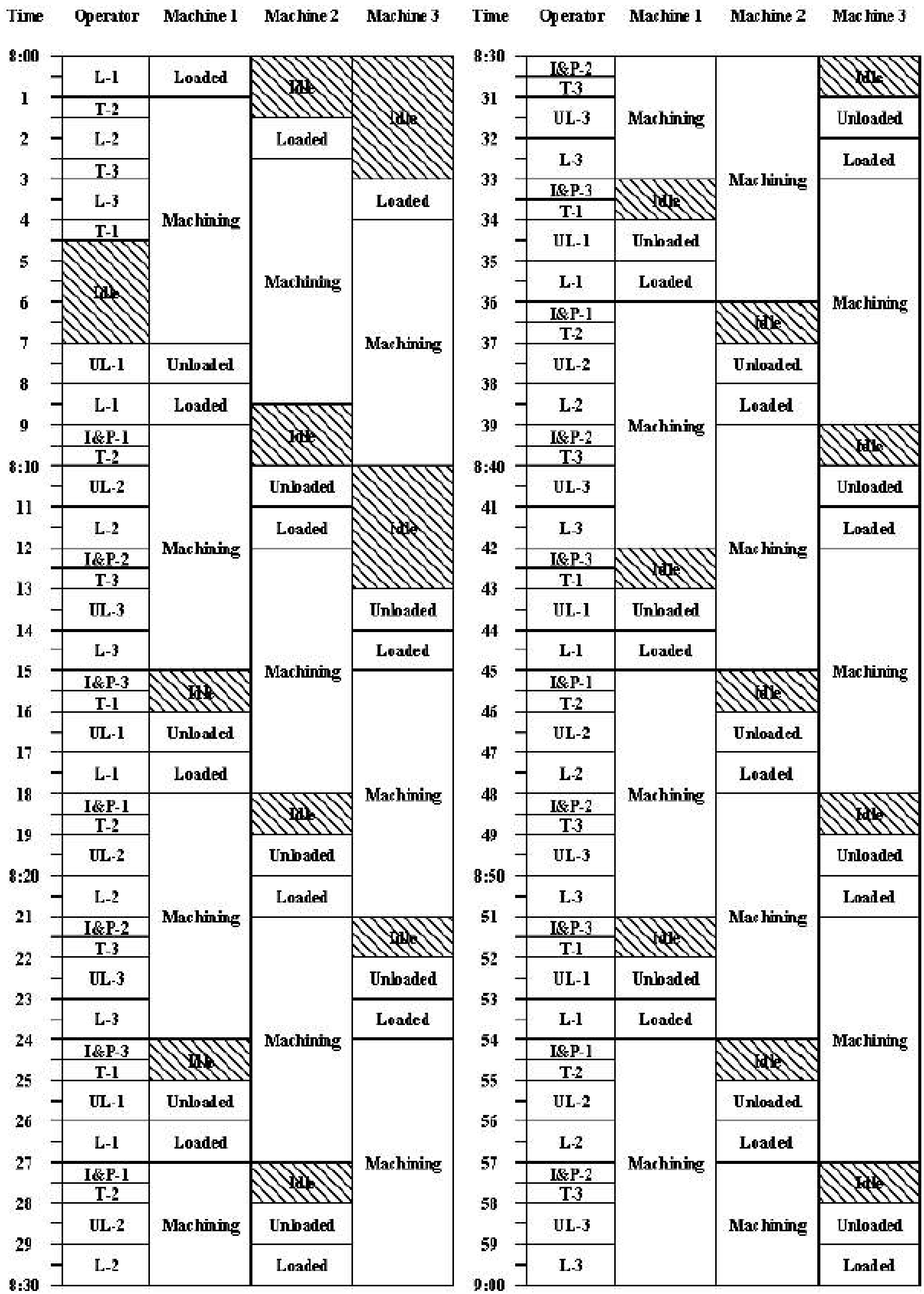
To calculate the cost per unit produced for each scenario, it is useful to evaluate each alternative using a length of time equal to the least common multiple of the cycle times for each machine-operator assignment in the scenario. For example, with the {2, 2, 2, 2, 3} scenario, $T_c$ = 8 min for m = 2 and $T_c$ = 9 min for m = 3; therefore, a time period equal to 72 minutes will be used. During a period of 72 mins each 2-machine combination will perform 9 cycles and produce 18 parts; likewise, over the same time period, the 3-machine assignment will perform 8 cycles and produce 24 parts. Hence, over a 72 min. period, a total of 4(18) + 24, or 96 parts will be produced. The total cost per unit produced over a 72 min. period equals [(5 op)($15/hr-op) + (11 mach)($50/hr-mach)](72 min/60 min/hr.)(1/96) parts), or $7.81/part.

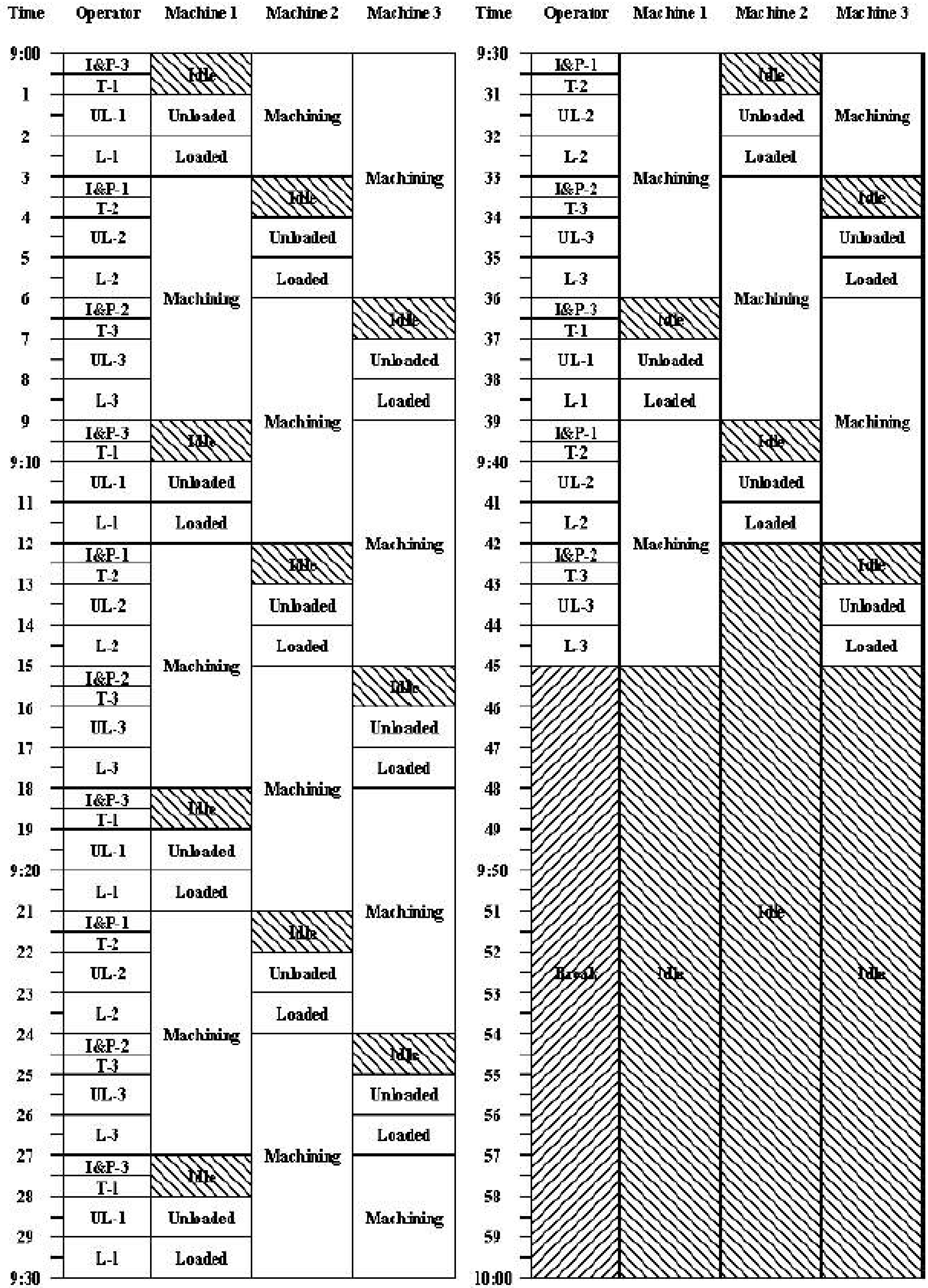
For the {2, 2, 2, 2, 2, 1} scenario, $T_c$ = 8 min for both m = 1 and m = 2. During an 8 min. time period a total of 11 parts are produced. The total cost per unit produced over an 8 min. period equals [6(15) + 11(50)](8/60)(1/11), or $7.76/part. Hence, the least cost alternative, in terms of cost per unit produced, is the {2, 2, 2, 2, 2, 1} scenario.

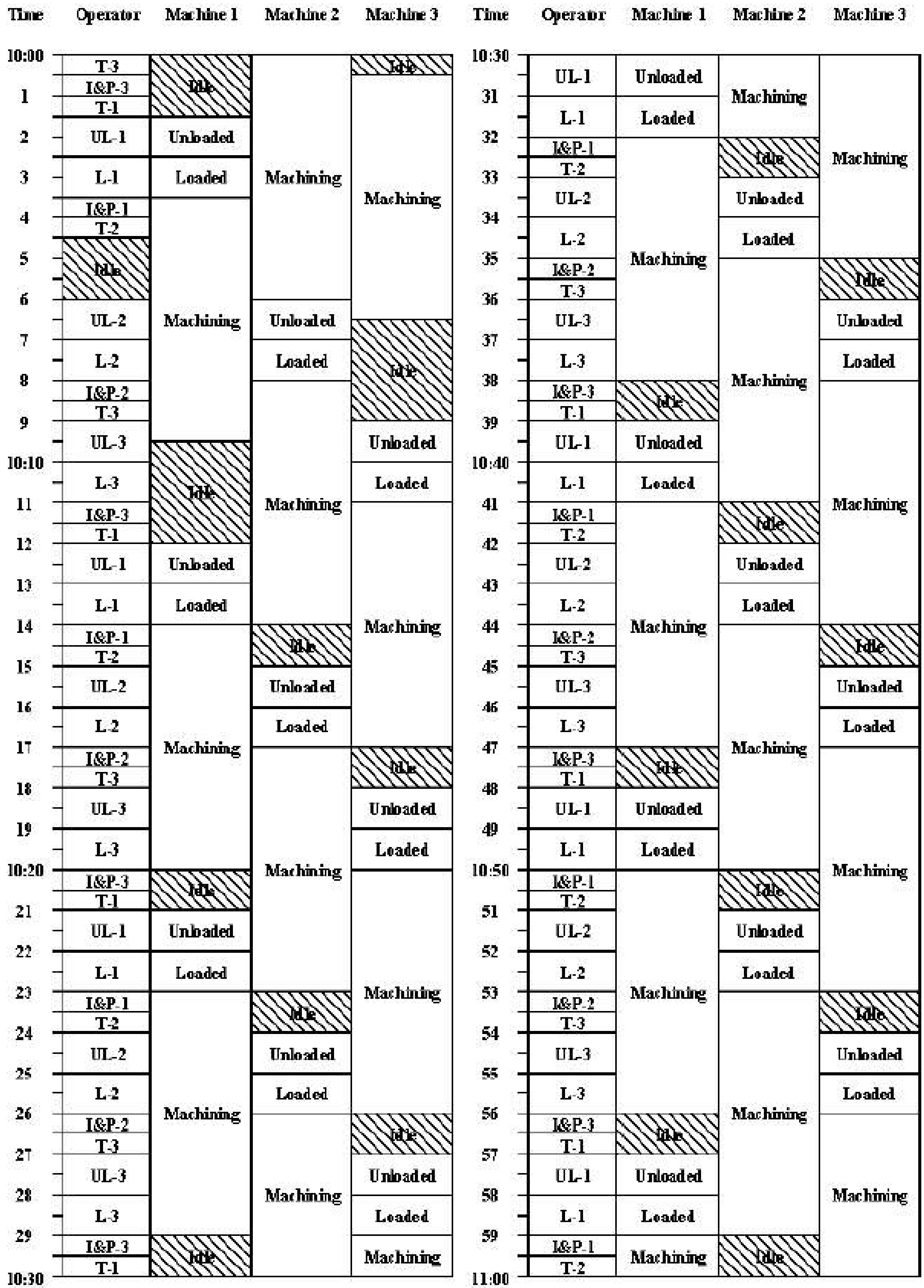
Are there other scenarios that are less costly than the two considered? No! From Example 2.7, n* = 2. Hence, any scenario involving multiple assignments of single machines will be more costly than assignments of 2 machines per operator. Likewise, from the analysis performed above, any scenario involving a 3-machine assignment will be more expensive than one with a 2-machine and a 1-machine assignment. Further, any scenario having a 4-machine assignment will be more costly than one that substitutes two 2-machine assignments for the 4-machine assignment. From the analysis performed above, a 5-machine assignment will be more expensive than a {2, 2, 1} assignment. By similar analyses, there are no other scenarios that need to be considered for the assignment of 11 machines.

**2.36** For the optimum assignment in Example 2. 5 to remain unchanged, M < 1. Recall, in Example 2.5, n' = 2.67, $C_o$ = $15/hr, $C_m$ = $50/hr, and , = $C_o/C_m$ = 15/Cm. Therefore, for M < 1, $(\epsilon + n)(n') < (\epsilon + n + 1)(n)$, or $(\epsilon + 2)(2.67) < (\epsilon + 3)(2)$, or $(15 + 2C_m)(2.67) < (15 + 3C_m)(2)$, or $(15)(2.67 - 2) < (6 - 5.33)C_m$, or $C_m >$ $15/hr. Hence, for a machine cost of $15 or more per machine-hour, the optimum assignment will be 2 machines per operator.

**2.37** During 7 hours of work for the operator between 8:00 a.m. and 4:00 p.m., 136 units were produced by the 3 machines. In steady state conditions, the repeating cycle is 9 minutes. Hence, in steady state conditions a total of 140 units are produced. Transient conditions due to start-up and shut-down for breaks, lunch, and beginning/ending of the shift diminish the production by only 4 units. If replacement labor is provided to keep the machines working during the entire 8-hour shift and 3 shifts operate per day, then steady state production will result in 160 units being produced per 8-hour shift. This situation is illustrated in the following multiple activity charts.


Time
Operator
Machine 1
Machine 2
Machine 3
8:00
8:10
8:20
8:30
8:40
8:50
9:00
L-1
T-2
L-2
T-3
L-3
T-1
I&P-1
UL-1
I&P-2
UL-2
I&P-3
UL-3
Idle
Loaded
Unloaded
Machining


Time
Operator
Machine 1
Machine 2
Machine 3
9:00
9:10
9:20
9:30
9:40
9:50
10:00
I&P-3
T-1
UL-1
L-1
I&P-1
T-2
UL-2
L-2
I&P-2
T-3
UL-3
L-3
Idle
Unloaded
Loaded
Machining
Break


Time
Operator
Machine 1
Machine 2
Machine 3
10:00
1
2
3
4
5
6
7
8
9
10:10
11
12
13
14
15
16
17
18
19
10:20
21
22
23
24
25
26
27
28
29
10:30
T-3
I&P-3
T-1
UL-1
L-1
I&P-1
T-2
Idle
UL-2
L-2
I&P-2
T-3
UL-3
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I&P-3
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I&P-1
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Machining
Idle
Unloaded
Loaded
Machining
Idle
Unloaded
Loaded
Machining
Time
Operator
Machine 1
Machine 2
Machine 3
10:30
31
32
33
34
35
36
37
38
39
10:40
41
42
43
44
45
46
47
48
49
10:50
51
52
53
54
55
56
57
58
59
11:00
UL-1
L-1
I&P-1
T-2
UL-2
L-2
I&P-2
T-3
UL-3
L-3
I&P-3
T-1
UL-1
L-1
I&P-1
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I&P-2
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UL-1
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I&P-1
T-2
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Loaded
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Idle
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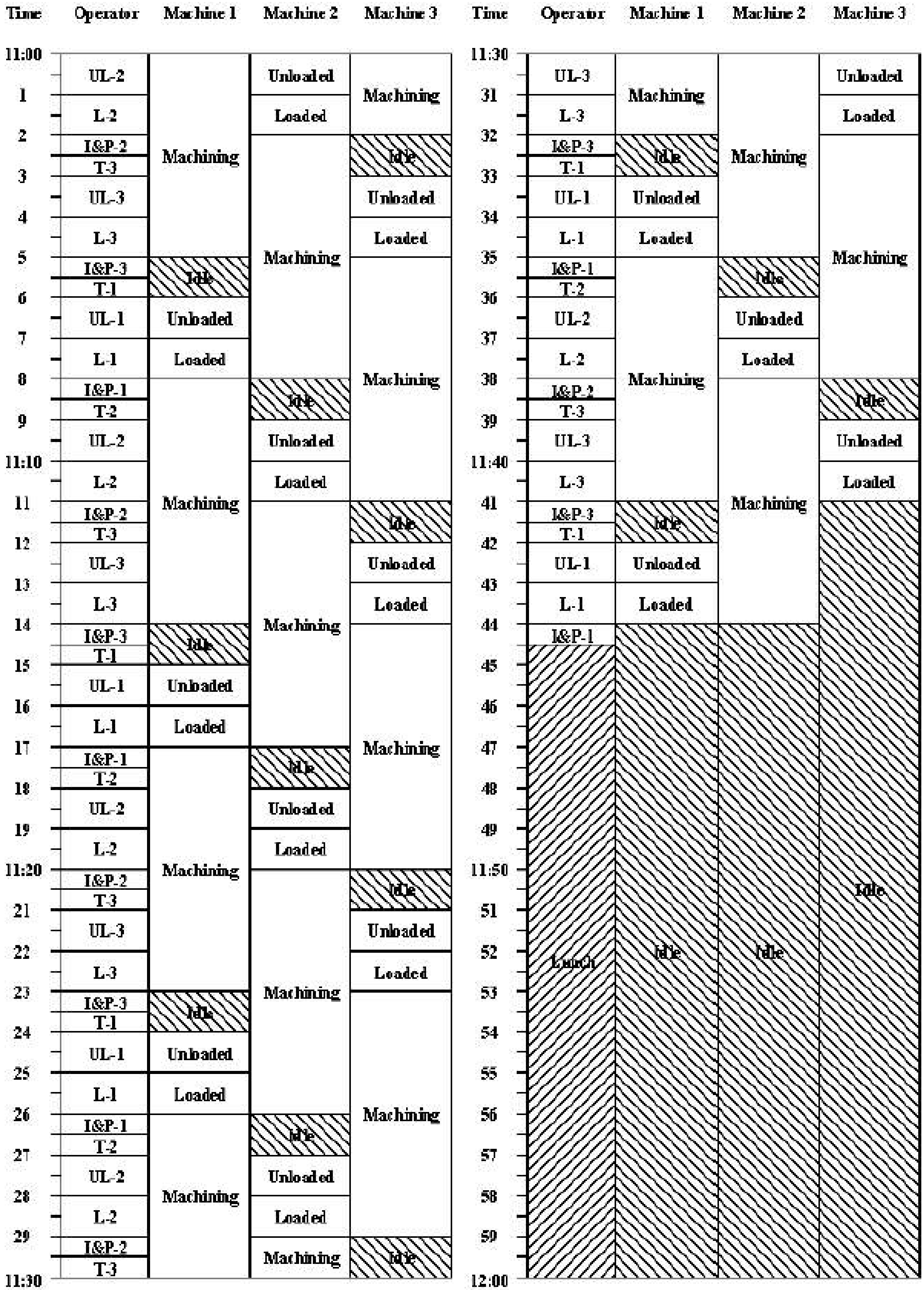
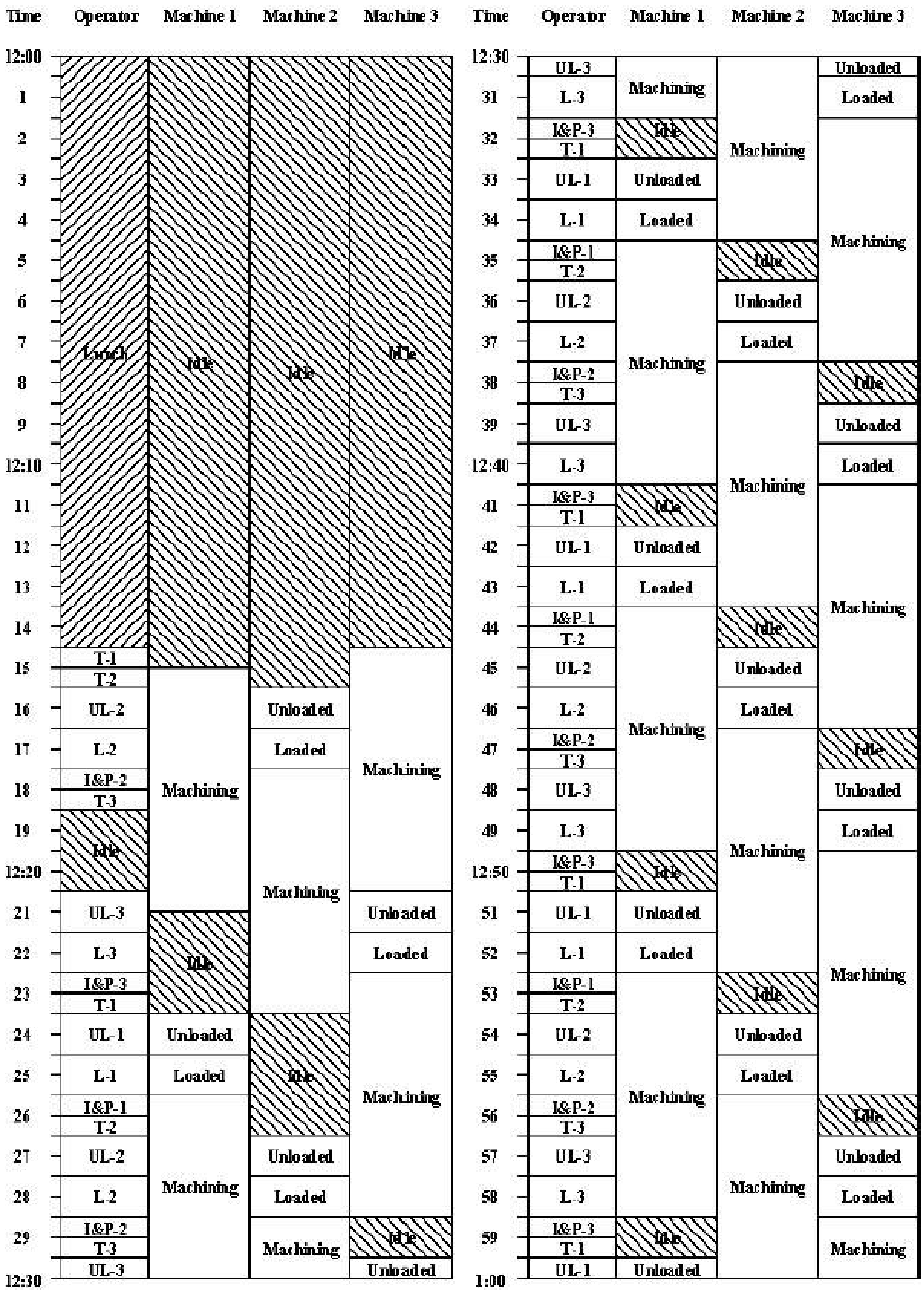
Loaded
Machining


Time
Operator
Machine 1
Machine 2
Machine 3
Time
Operator
Machine 1
Machine 2
Machine 3
11:00
11:10
11:20
11:30
11:40
11:50
12:00
Lunch
Idle


Time
Operator
Machine 1
Machine 2
Machine 3
12:00
1
2
3
4
5
6
7
8
9
12:10
11
12
13
14
15
16
17
18
19
12:20
21
22
23
24
25
26
27
28
29
12:30
Lunch
Idle
Idle
Idle
T-1
T-2
UL-2
L-2
I&P-2
T-3
Idle
UL-3
L-3
I&P-3
T-1
UL-1
L-1
I&P-1
T-2
UL-2
L-2
I&P-2
T-3
UL-3
Machining
Idle
Unloaded
Loaded
Machining
Unloaded
Loaded
Machining
Idle
Unloaded
Loaded
Machining
Machining
Unloaded
Loaded
Machining
Idle
Unloaded
Time
Operator
Machine 1
Machine 2
Machine 3
12:30
31
32
33
34
35
36
37
38
39
12:40
41
42
43
44
45
46
47
48
49
12:50
51
52
53
54
55
56
57
58
59
1:00
UL-3
L-3
I&P-3
T-1
UL-1
L-1
I&P-1
T-2
UL-2
L-2
I&P-2
T-3
UL-3
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UL-2
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I&P-2
T-3
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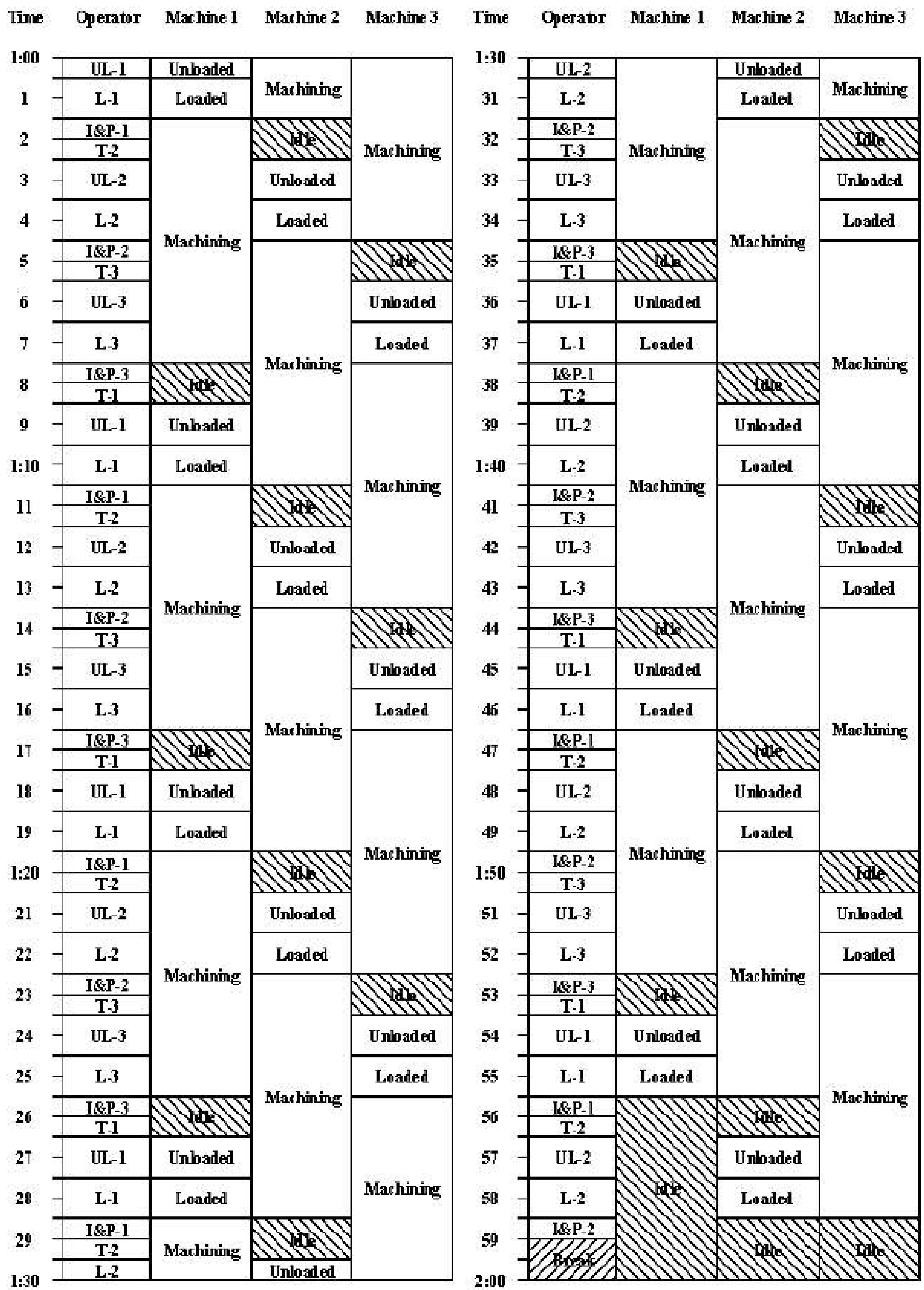
Time
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Machine 1
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1:20
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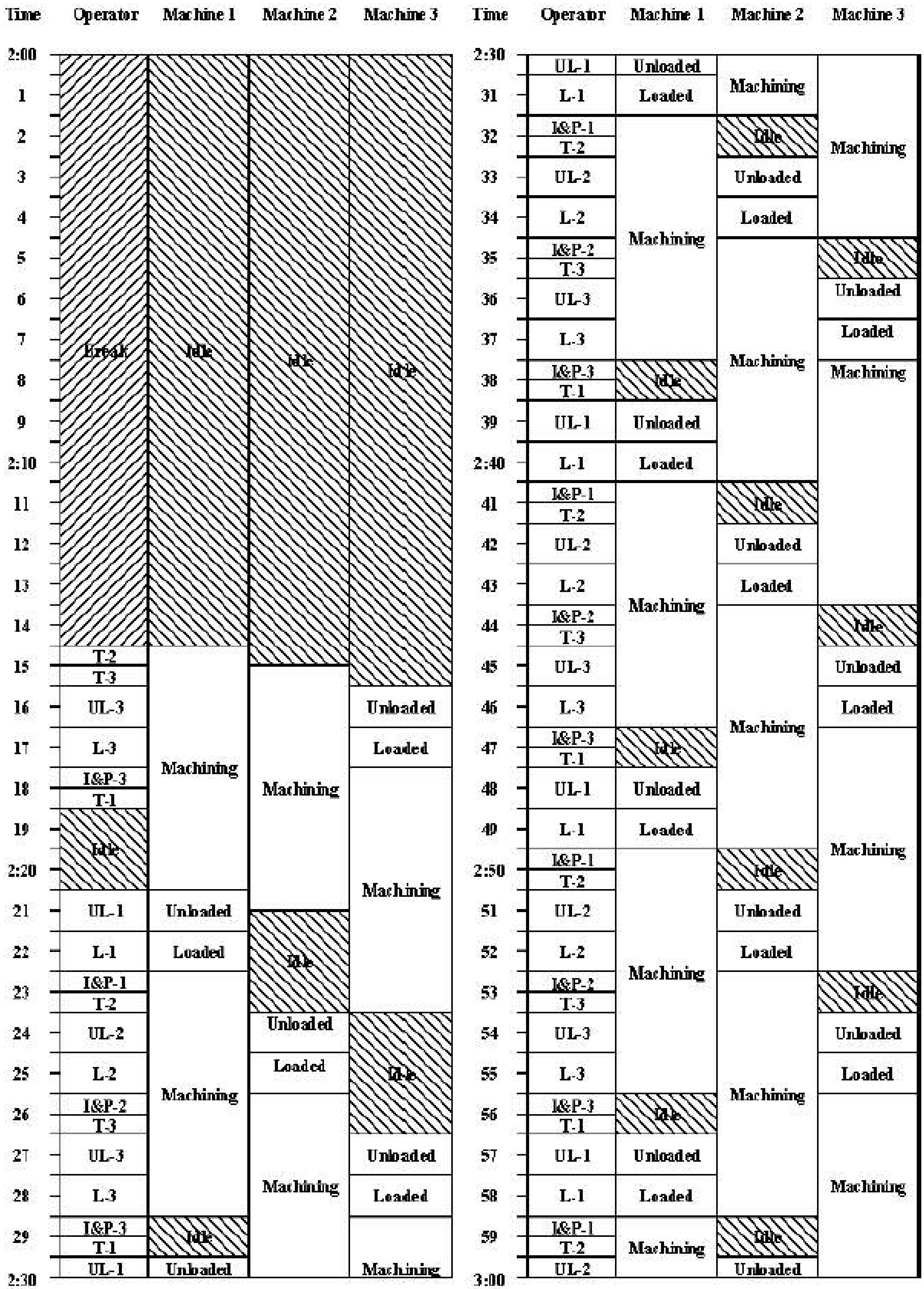
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Machine 3
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**2.38** The problem statement was overly simplified, assuming sufficient demand exists to keep 5 machines busy and sales prices of the products are such that the calculation of cost per unit produced can be performed by summing the units produced for both products. Also, it is assumed that a machine is dedicated to producing either product 1 or product 2 and cannot be assigned to produce a combination of the two products due to changeover times.

From Example 2.7, we know that two machines producing product 1 should be assigned an operator to minimize the cost per unit produced. For product 2, the optimum number of machines to assign an operator is obtained as follows for producing product 2: $n_2' = (2.5 + 8)/(2.5 + 1.5) = 2.625$; $\epsilon = 0.3$; and $\Phi = (2.3/3.3)(2.625/2) = 0.9148 < 1$; therefore, $n_2^* = 2$. Hence, it appears that 2 machines should be assigned to produce product 1 and 2 machines should be assigned to produce product 2; however, that leaves 1 machine unassigned. From the solution to Problem 2.35, we know that {2, 1} is less costly than {3} for product 1.

The alternatives to be evaluated are as follows: assign 2 machines producing product 1 to an operator and 3 machines producing product 2 to an operator; assign 2 machines producing product 1 to an operator, 2 machines producing product 2 to an operator, and 1 machine producing product 2 to an operator; assign 2 machines producing product 1 to an operator, 1 machine producing product 1 to an operator, and 2 machines producing product 2 to an operator.

*Assign 2 machines producing product 1 to an operator and 3 machines producing product 2 to an operator*. From Example 2.7, for product 1 the repeating cycle is 8 minutes. The repeating cycle is 3(2.5 + 1.5), or 12 min., for product 2. Hence, in 24 minutes, the 2 machines producing product 1 perform 3 repeating cycles and produce 6 parts, and 6 parts are produced by the 3 machines making product 2 while performing 2 repeating cycles. The total hourly cost of 2 operators and 5 machines is $280. During a period of 24 minutes, the cost will be $112.00; hence, the cost per unit to produce 12 parts in 8 minutes is $9.33/part.

*Assign 2 machines producing product 1 to an operator, 2 machines producing product 2 to an operator, and 1 machine producing product 2 to an operator*. The repeating cycle for product 2 is 10.5 minutes. Hence, in 168 minutes there will be 21 repeating cycles for machines producing product 1 and 16 repeating cycles of machines producing product 2. The total hourly cost of 3 operators and 5 machines is $295. During a period of 168 minutes, the cost will be $826; hence, the cost per unit to produce 90 parts (42 of product 1 and 48 of product 2) in 168 minutes is $9.18/part.

*Assign 2 machines producing product 1 to an operator, 1 machine producing product 1 to an operator, and 2 machines producing product 2 to an operator*. As in the previous case, the repeating cycles are 8 and 10.5 minutes. Hence, over a 168 minute time frame, there will be 21 repeating cycles of the 3 machines producing product 1 and 16 repeating cycles of the 2 machines producing product 2. The cost per unit to produce 63 units of product 1 and 32 units of product 2 is ($295)(168)/(60)(95), or $8.69/part. Since this is the least cost option, it would be recommended.

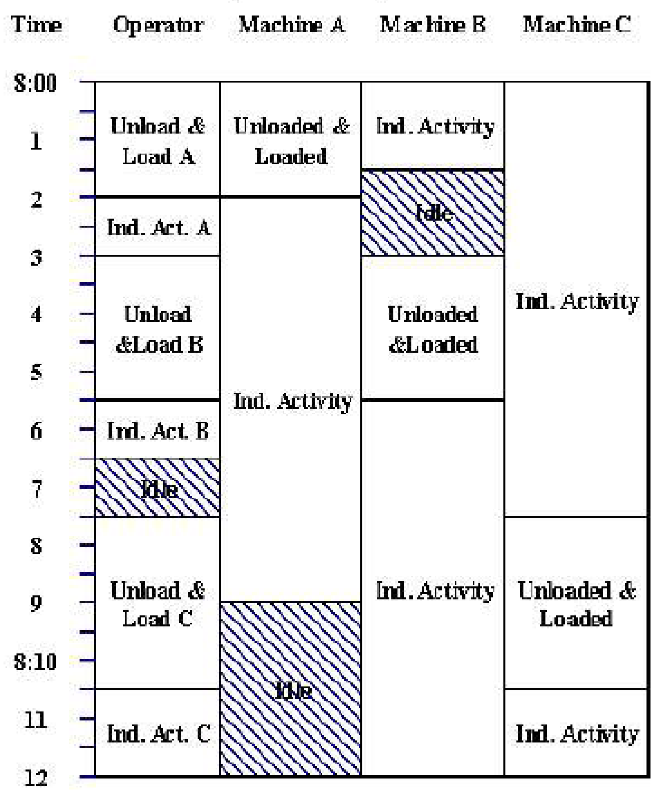
As noted, a simplified approach was used to arrive at a preference in the assignment of the 5 machines to the 2 products. With more information regarding sales prices, demands, changeover times, etc., a more informed decision could be made. The underlying objective in presenting the machine-assignment problem was to provide students with experience in using simple mathematical models in making decisions regarding the assignment of machines to operators.

**2.26a** a = 6 + 4 = 10; b = 6; and t = 30. n' = (10 + 30)/(10 + 6) = 2.5. No more than 2 mixers can be assigned without idle mixer time.

**2.26b** $C_o$ = $12/hr; $C_m$ = $25/hr; and $\epsilon = C_o/C_m = 0.48$.
Therefore, $\Phi = [(\epsilon + n)(n')]/[(\epsilon + n + 1)(n)] = [(2.5)(2.48)]/[(3.48)(2)]$, or
$\Phi = 0.89 < 1$. Hence, 2 mixers should be assigned to an operator.

**2.40** The multiple activity chart is provided on the following page. The length of the repeating cycle is given by the maximum of the following values: $(a_A + b_A + a_B + b_B + a_C + b_C)$; $(a_A + t_A)$; $(a_B + t_B)$; $(a_C + t_C)$, or max (11, 9, 10.5, 12), or 12. The repeating cycle is determined by machine C. As shown, the operator will have 1 minute of idle time during a repeating cycle, machine A will have 3 minutes of idle time, machine B will have 1.5 minutes of idle time, and machine C will have no idle time during a repeating cycle.

Multiple Activity Chart



**2.41a** a = 5 min; b = 1 min; t = 20 min; $C_o$ = \$12/hr, and $C_m$ = \$30/hr.
n' = (5 + 20)/(5 + 1) = 4.167; Therefore, 4 is the maximum number of machines that can be assigned an operator without creating machine idle time during a repeating cycle.

**2.41b** $\epsilon = C_o/C_m = 0.4$. Therefore,
$\Phi = [(\epsilon + n)(n')]/[(\epsilon + n + 1)(n)] = [(4.4)(4.167)]/[(5.4)(4)]$, or $\Phi = 0.8488 < 1$.
Hence, 4 mixers should be assigned to an operator.

**2.41c** $TC(m = 4) = [(12 + 4(30))(5 + 20)]/[(60)(4)] = \$13.75/\text{unit}$

**2.41d** For n* = 4, either $\Phi < 1$ when n = 4 or $\Phi > 1$ for n = 3.
<u>Φ < 1 case</u>: $\Phi = [(0.4 + 4)(n')]/[(0.4 + 5)(4)] < 1$ or
$\Phi = [(4.4)(a + 20)]/[(a + 1)(5.4)(4)] \Rightarrow (4.4a + 88) < (21.6a + 21.6)$.
Hence, 17.2a > 66.4, or a > 3.86 min.

<u>Φ > 1 case</u>: $\Phi = [(3.4)(a + 20)]/[(a + 1)(4.4)(3)] < 1$. Thus, (3.4a + 68) > (13.2a + 13.2). Hence, 9.8a < 54.8, or a < 5.59 min. Hence, the optimum assignment of 4 machines occurs when 3.86 min < a < 5.59 min.

**2.41e** Consider the alternatives: {5, 5, 5}, {4, 4, 4, 3}, and {4, 5, 6}.
<u>{5, 5, 5} case</u>: $T_c$ = 30 min.
TC{5, 5, 5} = [3(\$12) + 15(\$30)](30/60)(1/15) = \$16.20/unit.

<u>{4, 4, 4, 3} case</u>: $T_c$ = 25 min.
TC{4, 4, 4, 3} = [4(\$12) + 15(\$30)](25/60)(1/15) = \$13.83/unit.

<u>{4, 5, 6} case</u>: $T_c(4)$ = 25 min., $T_c(5)$ = 30 min., $T_c(6)$ = 36 min. In 2,700 minutes, 1,332 units will be produced: 4(108), or 432, by the 4- machine assignment; 5(90), or 450, by the 5-machine assignment; and 6(75), or 450, by the 6-machine assignment.
TC{4, 5, 6} = [3(\$12) + 15(\$30)](2700/60)(1/1332) = \$16.42/unit.

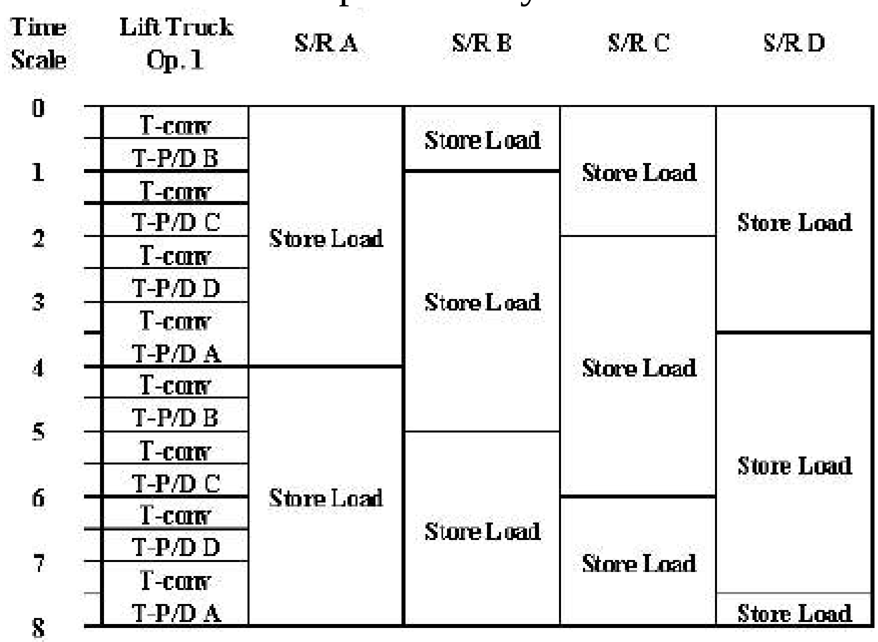
The least costly assignment of 15 machines is {4, 4, 4, 3}. We do not need to consider {3, 3, 3, 3, 3} since it has the same repeating cycle as {4, 4, 4, 3} and requires an additional operator. Likewise, there is no reason to consider an alternative involving a {3, 5} combination since we know from part a) that {4, 4} is less costly.

**2.42** a = 4 min; b = 5 min + 3 min = 8 min; and t = 40 min. n' = (4 + 40)/(4 + 8) or n' = 3.67 and 3 is the maximum number of automatic palletizers on operator can be assigned without creating idle time for the palletizers.

**2.43** The problem statement does not mention retrievals by the S/R; hence, no travel between the P/D station and the outbound conveyor is required. It is assumed that the travel between the P/D station of one S/R aisle and the inbound conveyor for another S/R aisle also requires 0.5 minute.

For the problem, there is no concurrent activity. As long as a load is at the P/D station, the S/R machine will retrieve it automatically. Hence, for the problem, b = 0.5 min to travel to the P/D station + 0.5 min to travel from the P/D station = 1.0 and t = 4 min to store a load and return to the end-of-the-aisle. (In Chapter 10, we define this as a single command cycle for the S/R machine.) With b = 1 and t = 4, n’ = 4. Hence, as shown below, one lift truck operator can service 4 S/R machines; the operator and the S/R machines will be busy 100% of the time. For 5 S/R machines, 2 lift truck operators will be required.

Multiple Activity Chart

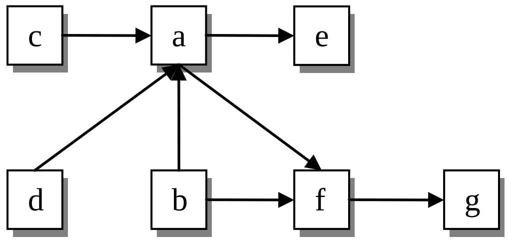



**2.44** a = 0.25 min; b = 0; and t = 1.0. Therefore, n’ = 1.25/0.25 = 5.0. An operator can tend 5 carousels without creating idle time for the conveyors; the operator will also be 100% occupied.

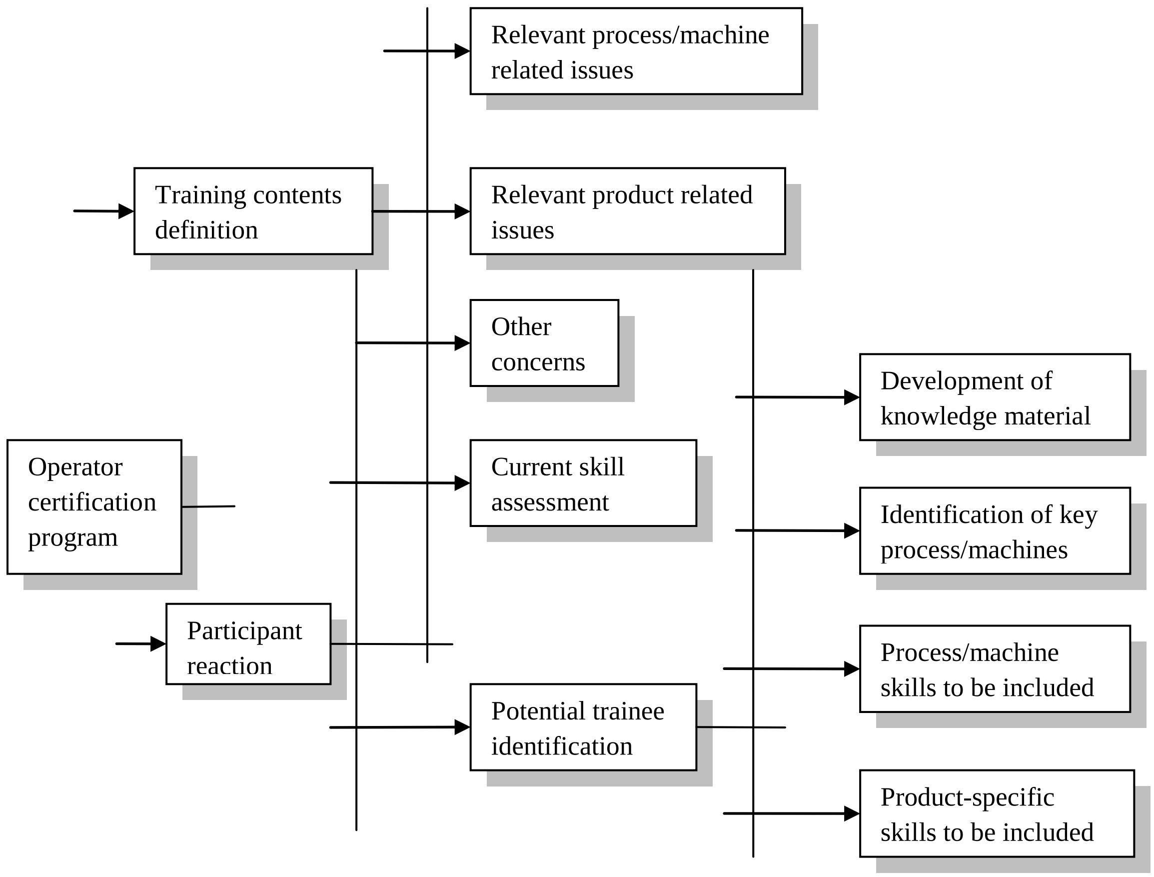
**SECTION 2.5**

**2.45** a. Board for part shortages
b. Board for back-orders
c. Feedback from material handlers when part has low physical inventory
d. Feedback from operator handlers when part has low physical inventory

**2.46**




**2.47**




**2.48 - 2-50** The answers to these questions depend on choices made and course specifics. See Section 2.5 for details on each of the 7 M&P tools.

**2.51** Follow the instructions provided in Section 2.5 for each of the 7 M&P tools.

# Chapter 3

# *Flow Systems, Activity Relationships, and Space Requirements*

## SECTION 3.1

**3.1** Top Management: Alternative issues and strategies to consider in the analysis such as layout classification, storage-handling strategies, organizational structure, and environmental policy.
Product Designer: components and their dimensions and quantities per unit.
Process Designer: machines and their specifications and product routings.
Schedule Designer: lot sizes, production control method (e.g., kanbans), and production volume.

Modern Manufacturing Approaches: kanbans, standardized containers, total productive maintenance, short set-ups, small lots, manufacturing cells, quality at the source, delivery to points of use, visual management, total quality management, multi-functional employees, decentralized storage, and team-based environments.

## SECTION 3.2

**3.2** A material management system for a bank refers to the flow process *into* the bank. The subjects of this system might include, but are not limited to, coin, currency, checks, other monetary instruments, deposit and withdrawal forms, loan documents, customers, suppliers, employees, banking supplies, and equipment required to operate the bank. The material flow system for a bank refers to the movement of materials, supplies, equipment, and personnel *within* the bank. The physical distribution system at a bank describes the flow of money and information, including loan and other documents, *out of* the bank.

**3.3** The impact includes delivery of small lots, the elimination of paperwork due to electronic data interchange (EDI), receiving material in decentralized storage areas, by-passing incoming inspection since suppliers have been certified, reduced movement of material, and simpler material handling equipment alternatives. The overall impact to the logistics systems includes shorter lead times, lower cost, and better quality.

**3.4** Logistics encompasses the arrival and departure of parts and products, including the quantities of each. Material handling and storage are extremely important for the facilities planner. In fact, some define manufacturing as production processes located strategically in a logistics system.

## SECTION 3.3

**3.5** The basic layout of the hospital will depend on "who moves" and "who remains fixed" in location. Actually, all subjects are candidates for movement, except for the most expensive diagnostic equipment; it cannot always be taken to the patient. However, advances in medical technology are such that more and more the technology is being brought to the patient via distance medicine.

If it is desired that patients not be transported around the hospital to the various x-ray, cat-scan, and MRI facilities, then either remote sensing methods must be provided or the diagnostic equipment must be duplicated. Likewise, if the movement of medical personnel is to be minimized, then patients can be assigned to rooms/beds that are designated for particular doctors and nurses; the down-side of such an assignment is the likelihood of empty beds existing, since not all doctors will always have the maximum number of patients in the hospital at all times.

Hospitals are often designed using group technology layouts, where the grouping is based on the medical services provided the patients.

**3.6** Backtracking results in excessive flow or travel, longer lead-times, and complications in scheduling. Backtracking may be avoided by duplicating machines, redefining the process plans to complete the machining in consecutive steps, specifying the use of another machine not requiring backtracking, or redesigning the product to eliminate the processing requiring backtracking.

**3.7** <u>Pros</u>: simpler material handling equipment, shorter travel distances, less complex flow

<u>Cons</u>: duplication of equipment, more discipline to monitor inventory and quality, more complex logistics to distinguish deliveries and shipments at different docks

Considerations should include evaluation of aisle requirements, outside accessibility for trucks, requirements for receiving inspection, delivery lot sizes, delivery/shipment unit load sizes, frequency of deliveries/shipments, and required paperwork and communications.

**3.8** Research Question: answer depends on the restaurant visited.

**3.9** The answer may depend on the region in which the store is located; however, some basic principles will likely apply. The ordering of the answers is up to the student.

Customer perspective:

- Aisles must be wide enough to accommodate customer traffic in both directions, based on the width of the shopping carts.
- “Like” items should be relatively close to each other.

Employee perspective:

- The location of the stockroom entrance/exit, is the stockroom entrance/exit located on a main aisle.
- The angle at which the aisles are located relative to the stockroom.
- The width of the aisles must be sufficient to allow for whatever means of material movement is used.

**3.10** Benefits of U-shaped flow in manufacturing cells include, but are not limited to, enhanced visibility, improved communication, improved teamwork, simplified flow, reduced travel distance, improved quality, reduced WIP, reduced space, reduced handling, and improved control over input/output to the cell.

**3.11** The space requirements in terms of the shape of the department will be different. The means of material handling will also change. In addition, switching from a straight line flow to a u-shaped flow will change the tasks of the workers on the line. The flow strategy may also change, since a conveyor line is often associated with push or continuous systems and u-shaped cells are most often associated with pull or just-in-time production systems.

**3.12** Research Question: answer depends on the restaurant visited.

**3.13** Research Question: answer depends on the individual.

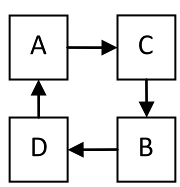
**3.14a** 100' + 75' + 75' + 25' + 25' = 300'

**3.14b** 175' + 75' + 150' + 25' + 25' = 450'

**3.14c** 300' + 25' + 25' + 75' + 75' + 100' + 300' = 900'

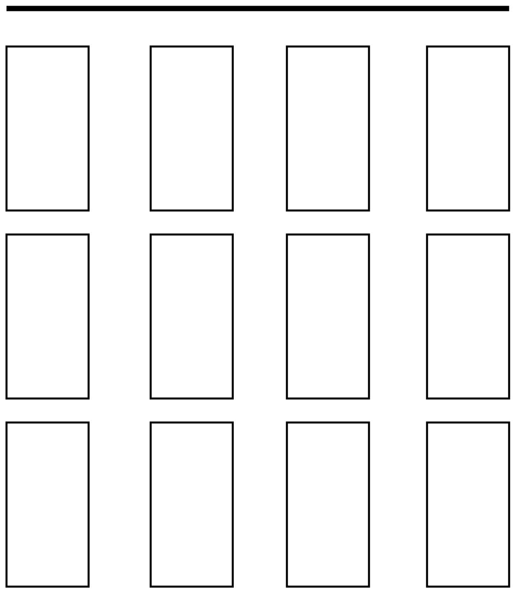
**3.14d** 175' + 100' + 175' + 150' + 50' = 650'

**3.15**

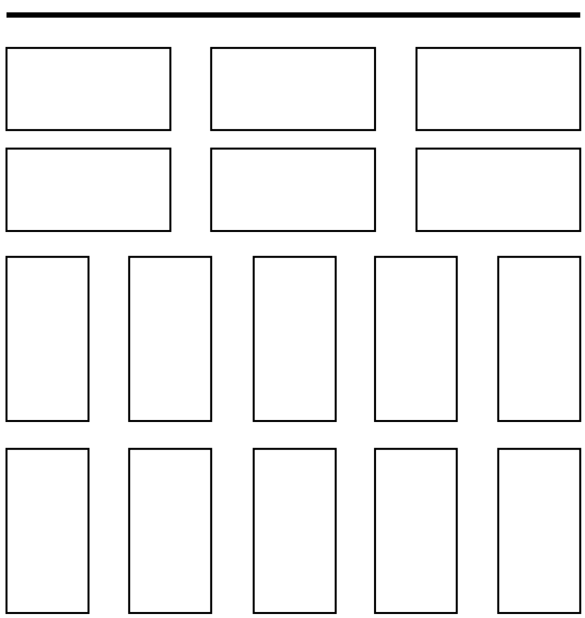



**3.16** When streets are not at right angles, turning of vehicles can be more problematic. In such instances, wider turning lanes might be required, particularly for delivery trucks.

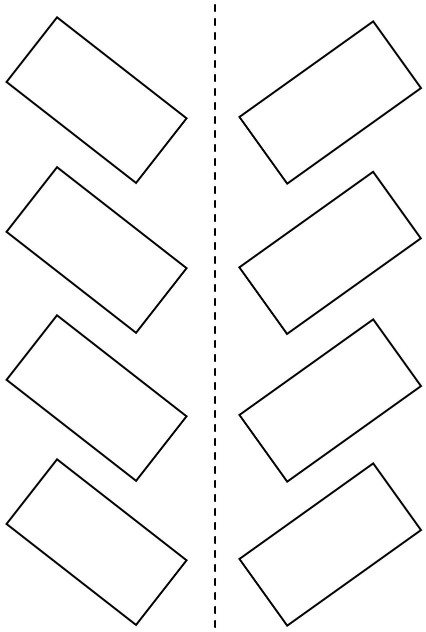
**3.17** Parallel parking advantages: space utilization and flow in one direction; okay if last-in, first-out arrival and departure is used Parallel parking disadvantages: excessive vehicular movement required; poor if first-in, first-out arrival and departure is used



Perpendicular parking advantages: none come to mind
Perpendicular parking disadvantages: flow in two directions, excessive movement, and requires more open spaces



Diagonal parking advantages: requires less vehicular movement
Diagonal parking disadvantages: requires more space



**SECTION 3.4**

**3.18** A group technology layout is a candidate for consideration when a medium volume of a medium variety of products are to be produced. It is used when families of products can be grouped according to physical characteristics or production sequence information.

**3.19** A fixed position layout would be recommended when the product is very large, awkward, or expensive to move. It is also a candidate when low or sporadic volume of low variety production occurs.

**3.20** Group technology layout using manufacturing cells is popular in JIT facilities because cellular layout encourages small lots, kanbans, simple material handling systems, short setup times, cross trained personnel, teamwork, and quality at the source.

**3.21** Process layouts can result in complex flows, excessive handling, large inventories, and long production lead-times.

**3.22**

| | Machine # | | | | |
|---|---|---|---|---|---|
| Part # | 4 | 1 | 3 | 2 | Sum |
| 4 | 1 | 1 | | | 2 |
| 2 | | 1 | | | 1 |
| 1 | | | 1 | 1 | 2 |
| 3 | | | 1 | | 1 |
| Sum | 1 | 2 | 2 | 1 | |

**3.23**

| | Machine # | | | | | |
|---|---|---|---|---|---|---|
| Part # | 5 | 4 | 2 | 1 | 3 | Sum |
| 3 | 1 | 1 | 1 | | | 3 |
| 6 | 1 | 1 | | | | 2 |
| 5 | | | 1 | | | 1 |
| 4 | | | | 1 | 1 | 2 |
| 1 | | | | 1 | 1 | 2 |
| 2 | | | | 1 | | 1 |
| Sum | 2 | 2 | 2 | 3 | 2 | |

**3.24**

| | Machine # | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Part # | 7 | 4 | 5 | 8 | 6 | 2 | 1 | 3 | Sum |
| 10 | 1 | 1 | 1 | | | | | | 3 |
| 5 | 1 | | 1 | 1 | | | | | 3 |
| 4 | 1 | | 1 | 1 | | | | | 3 |
| 8 | | 1 | 1 | | 1 | | | | 3 |
| 2 | | 1 | 1 | | | | 1 | | 3 |
| 7 | | | | | 1 | 1 | | 1 | 3 |
| 6 | | | | | 1 | | 1 | 1 | 3 |
| 1 | | | | | | 1 | 1 | 1 | 3 |
| 9 | | | | | | 1 | 1 | 1 | 3 |
| 3 | | | | | 1 | 1 | | 1 | 3 |
| Sum | 3 | 3 | 5 | 2 | 4 | 4 | 4 | 5 | |

| | Machine # | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Part # | 7 | 4 | 5 | 8 | 6a | 1a | 6b | 2 | 1b | 3 |
| 10 | 1 | 1 | 1 | | | | | | | |
| 5 | 1 | | 1 | 1 | | | | | | |
| 4 | 1 | | 1 | 1 | | | | | | |
| 8 | | 1 | 1 | | 1 | | | | | |
| 2 | | 1 | 1 | | | 1 | | | | |
| 7 | | | | | | | 1 | 1 | | 1 |
| 6 | | | | | | | 1 | | 1 | 1 |
| 1 | | | | | | | | 1 | 1 | 1 |
| 9 | | | | | | | | 1 | 1 | 1 |
| 3 | | | | | | | 1 | 1 | | 1 |

**3.25**

| | Machine # | | | | | | |
|---|---|---|---|---|---|---|---|
| Part # | 6 | 3 | 5 | 1 | 4 | 2 | Sum |
| 8 | 1 | 1 | | | | | 2 |
| 6 | 1 | | 1 | | | | 2 |
| 1 | 1 | | 1 | | | | 2 |
| 3 | 1 | 1 | | | | | 2 |
| 5 | | | | 1 | | | 2 |
| 2 | | | | 1 | 1 | | 2 |
| 4 | | | | 1 | | 1 | 2 |
| 7 | | | | | 1 | 1 | 2 |
| Sum | 4 | 2 | 2 | 3 | 2 | 2 | |

**3.26**

| Part # | Machine # 6 | 3 | 5 | 4 | 2 | 1 | Sum |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | | | | 3 |
| 8 | 1 | 1 | | | | | 2 |
| 3 | 1 | 1 | | | | | 2 |
| 6 | 1 | | 1 | | | | 2 |
| 5 | | 1 | | | | 1 | 2 |
| 7 | | | | 1 | 1 | | 2 |
| 4 | | | | | 1 | 1 | 2 |
| 2 | | | | 1 | | 1 | 2 |
| Sum | 4 | 4 | 2 | 2 | 2 | 3 | |

| Part # | Machine # 6 | 3a | 5 | 3b | 4 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | | | | |
| 8 | 1 | 1 | | | | | |
| 3 | 1 | 1 | | | | | |
| 6 | 1 | | 1 | | | | |
| 5 | | | | 1 | | | 1 |
| 7 | | | | | 1 | 1 | |
| 4 | | | | | | 1 | 1 |
| 2 | | | | | 1 | | 1 |

Alternatively,

| Part # | Machine # 6 | 3 | 5 | 4 | 2 | 1 | Sum |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | | | | 3 |
| 8 | 1 | 1 | | | | | 2 |
| 3 | 1 | 1 | | | | | 2 |
| 6 | 1 | | 1 | | | | 2 |
| 5 | | 1 | | | | 1 | 2 |
| 7 | | | | 1 | 1 | | 2 |
| 4 | | | | | 1 | 1 | 2 |
| 2 | | | | 1 | | 1 | 2 |
| Sum | 4 | 4 | 2 | 2 | 2 | 3 | |

| Part # | Machine # 6 | 3a | 5 | 3b | 4 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | | | | |
| 8 | 1 | 1 | | | | | |
| 3 | 1 | 1 | | | | | |
| 6 | 1 | | 1 | | | | |
| 5 | | 1 | | 1 | | | |
| 7 | | | | | 1 | 1 | |
| 4 | | | | | | 1 | 1 |
| 2 | | | | | 1 | | 1 |

**3.27**

| Part # | Machine # 1 | 2 | 3 | 4 | Sum |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | 1 | 3 |
| 5 | 1 | 1 | | | 2 |
| 2 | 1 | | 1 | | 2 |
| 4 | | | 1 | | 1 |
| 3 | | | | 1 | 1 |
| Sum | 3 | 2 | 2 | 2 | |

| Part # | Machine # 1 | 2 | 3 | 4a | 4b |
|---|---|---|---|---|---|
| 1 | 1 | 1 | | 1 | |
| 5 | 1 | 1 | | | |
| 2 | 1 | | 1 | | |
| 4 | | | 1 | | |
| 3 | | | | | 1 |

Alternatively,

| | Machine # | | | | |
|---|---|---|---|---|---|
| Part # | 4 | 2 | 1 | 3 | Sum |
| 1 | 1 | 1 | 1 | | 3 |
| 5 | | 1 | 1 | | 2 |
| 2 | 1 | | | | 2 |
| 4 | | | 1 | 1 | 1 |
| 3 | | | | 1 | 1 |
| Sum | 3 | 2 | 2 | 2 | |

| | Machine # | | | | |
|---|---|---|---|---|---|
| Part # | 4 | 2 | 1a | 1b | 3 |
| 1 | 1 | 1 | 1 | | |
| 5 | | 1 | 1 | | |
| 2 | 1 | | | | |
| 4 | | | | 1 | 1 |
| 3 | | | | | 1 |

**3.28** For the 1st presented solution of Problem 3.27:

| | Total Units | |
|---|---|---|
| Machine # | Cell 1 | Cell 2 |
| 1 | 4000 | |
| 2 | 2000 | |
| 3 | 4000 | |
| 4 | | 1000 |

There is sufficient capacity remaining in machine 4 in cell 2 to allow us to produce part 1 in that cell for the operation that requires machine 4. Therefore, as long as material movement was not an issue (and scheduling) we can use the minimum amount of machines to meet the production requirements by combining all machines into one cell.

For the 2nd presented solution of Problem 3.27:
In this case the conflicts were resolved by adding machine 1 to both cells.

| | Total Units | |
|---|---|---|
| Machine # | Cell 1 | Cell 2 |
| 1 | 2000 | 2000 |
| 2 | 2000 | |
| 3 | | 4000 |
| 4 | 2000 | |

We could eliminate the duplicate of machine 1 in the second cell for the same reason as in the 1st case.

**3.29** Benefits: *reduction of* inventories, space, machine breakdowns, rework, paperwork, warranty claims, storage and handling equipment, employee turnover and absenteeism, production lead-times, cost, and stockouts; *simplification of* communication, handling, and production scheduling; and *improvement of* productivity, flexibility, inventory turnover, quality, customer satisfaction, and employee morale.

**3.30** Research Question: answer depends on problem statement.

**3.31** A kanban is a signal, typically a card, that indicates to the supplying workstation that the consuming workstation requests more parts. There are two types of kanbans: a production card that is used to authorize production of more parts and a withdrawal card that is used to authorize delivery of more parts or components. The benefits of kanbans include simplified production control, reduced inventories, increased visibility, improved quality, reduced material handling, and reduced lead-times. Kanbans are a reminder that each employee has a customer, i.e., someone who depends on the employee doing her or his job well; in this case, kanbans come from internal "customers" and serve a similar function to a customer's order for material. Kanbans are important in cellular manufacturing because of the limited space and need for better production control to prevent excessive work-in-process (WIP).

**3.32** Research Question: answer depends on article chosen.

**SECTION 3.5**

**3.33**

| Component | Quantity/Week* | Equivalent Loads/Week | Routing |
|---|---|---|---|
| 1. Medical Record | 50 | 50 | R-M |
| 2. Certificate | 35 | 35 | P-M |
| 3. Blood Sample | 30 | 60 | M-L |
| 4. Blood Sample Report | 30 | 30 | L-M |
| 5. Box of Medical Records | 1 | 10 | M-R |

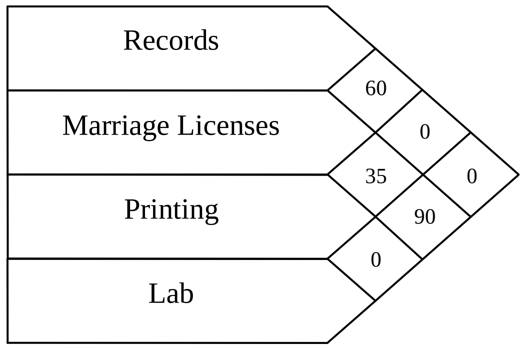
* Based on five working days per week

From-To Chart:

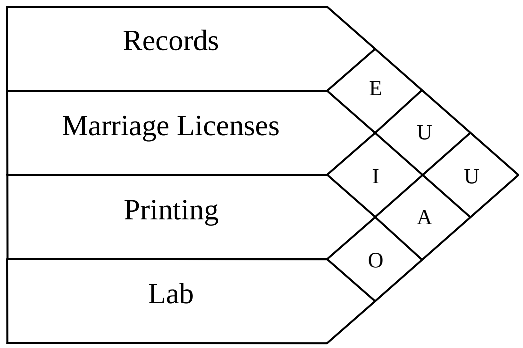
| | Records | Marriage Licenses | Printing | Lab |
|---|---|---|---|---|
| Records | – | 50 | 0 | 0 |
| Marriage Licences | 10 | – | 0 | 60 |
| Printing | 0 | 35 | – | 0 |
| Lab | 0 | 30 | 0 | – |

"Flow between," expressed in equivalent loads/day, is depicted below. As noted, 90 equivalent loads move weekly between Marriage Licenses and the Lab (60 + 30). Given the differences in numerical values a closeness rating was assigned, as shown in the activity relationship chart.

Flow Between Chart:




Activity Relationship Chart:




**3.34** Functions:

- Process improvement and quality activities
- Training for improved cross-functionality
- Production coordination
- Performance measures tracking
- Material handling

The new roles impact resources (less people, less inventories, less space) and change the structure of traditional activity relationships (storage at points of use, training and continuous improvement).

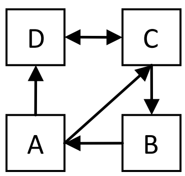
**3.35** Research Question: answer depends on assumptions made.

**SECTION 3.6**

**3.36** Research Question: answer depends on the components in the kitchen and the eating habits of the person for whom the kitchen is designed.

**3.37**

| Dept | A | B | C | D |
|---|---|---|---|---|
| A | – | 0 | 32 | 35 |
| B | 20 | – | 0 | 0 |
| C | 15 | 20 | – | 32 |
| D | 0 | 28 | 35 | – |




**3.38** Using the sum of the distances it takes for each product to complete production routings given in Problem 3.37, and assuming that it only takes one trip to transfer between each department, we have the following:

(a)

| Product | Distance |
|---|---|
| 1 | 350 |
| 2 | 250 |
| 3 | 250 |
| 4 | 200 |
| Total | 1050 |

(b)

| Product | Distance |
|---|---|
| 1 | 221.71 |
| 2 | 150 |
| 3 | 191.42 |
| 4 | 170.71 |
| Total | 733.84 |

**3.39** Research Question: answer depends on student and professor.

**3.40** Research Question: answer depends on services provided by and the organization of the bank.

**3.41** Research Question: answer depends on student's perspective and the degree to which the student interacts with the various activity areas listed.

**3.42** Research Question: answer depends on student's perspective of the professor and the various roles played by the professor in question.

**3.43** Typically, decreasing lot sizes increases the number of setups that must be performed. Therefore, although decreasing lot sizes lead to a decrease in the amount of storage space needed for materials, the increased number of setups cause an increase in the amount of workstation area as extra space is needed for tools, fixtures, etc.

This trade-off may be alleviated by having tool-cribs located such that they serve several workstations instead of having tooling available at all stations (this may have its own negative impact as it may increase setup times).

**3.44** Research Question: answer depends on the local fast-food restaurant chosen for study.

# Chapter 4

# *Personnel Requirements*

## SECTION 4.1

**4.1** A well-fed employee is not necessarily a happy employee. Likewise, a happy employee is not necessarily a productive one. These two statements relate to pervasive philosophies held by some companies, and they are not universally accepted. Because of particular company philosophies, facilities planners must adapt their plans to conform.

## SECTION 4.2

**4.2** Generally, if the location of the facility is in a cold environment, the employee may bring a coat to work. Also, the employee may bring a lunch, a change of clothes if working in a dirty or hot environment, and some personal toiletries. These items can all fit into a locker.

**4.3** When parking spaces are assigned, there must be a 1:1 ratio between the number of spaces and the number of employees that work in the facility, and the parking space utilization will be low depending on whether or not some personnel are not at the facility during the day. If there is a random parking philosophy at the company, then anywhere between a 1:1.25 to a 1:3 ratio of parking spaces to employees is required, depending on whether or not the facility is serviced by public transportation. Consequently, only 33%-80% of the space required for an assigned space parking lot is necessary for a random space parking lot.

**4.4** Standard cars (8’6”), 90° (because a perpendicular alignment has the best utilization of space), W2: Module width (from Table 4.1) = 66’0”

First, determine how many modules the depth of the parking lot can hold:

(370’/66’) = 5.606 modules (round down to 5.5, because each module has two rows in it.).

Next, determine the number of cars that can be parked per module:

(400’/8’6”) = 47 cars/row * 2 rows/module = 94 cars/module

Finally, multiply the number of cars/module by the number of modules in the parking lot:

94 cars/module * 5.5 modules/parking lot = 517 cars/parking lot

Note: If a 9’ standard car width is used, then 484 cars can be parked in the lot. If a 9’6” standard car width is used, then 462 cars can be parked in the parking lot.

**4.5** Answers vary depending on the specific parking lot analyzed. The user should try various parking angles and pay attention to best placement of aisles and cross-aisles relative to entry and exit points in the lot.

**4.6** The primary advantage of parking decks over surface lots is that parking decks can accommodate more cars per square foot of land, which is significant when land is expensive or unavailable. Also, parking decks provide shelter against the elements (rain, snow, UV rays from the sun) for the users and their vehicles. However, parking decks are more expensive than comparable surface lots and take longer to construct. They may require elevators, adding to the expense. Many drivers have difficulty navigating parking decks and may take longer to find a parking spot and exit later. Finally, a parking deck leaves fewer options for future expansion than a surface lot that could be more easily built over.

**SECTION 4.3**

**4.7** There must be two restrooms, one for males and one for females. There must be a minimum of three water closets and three lavatories in each restroom (from Table 4.2). In the men's restroom, one urinal can be substituted for a water closet. Consequently, only l6 $ft^2$ of space is required for the urinal. For each lavatory, 6 $ft^2$ of space should be designated. For each water closet, 15 $ft^2$ of space should be included. Also, 15 $ft^2$ of space should be allocated for the entrance. Since there are fewer than 100 females employed in the facility, only one bed or cot should be provided. For each bed, 60 $ft^2$ of space should be provided. The summation of space required for each gender's restroom is as follows:

| | | |
|---|---|---|
| Men: | Urinal (1 @ 6 ft2) | = 6 |
| | Water Closet (3 @ 15 ft2) | = 30 |
| | Lavatories (3 @ 6 ft2) | = 18 |
| | Entrance (1 @ 15 ft2) | = 15 |
| | Subtotal | = 69 |
| | 40% allowance | = 28 |
| | **Total** | **= 97 $ft^2$** |
| Women: | Water Closet (3 @ 15 ft2) | = 45 |
| | Lavatories (3 @ 6 ft2) | = 18 |
| | Bed or Cot (1 @ 60 ft2) | = 60 |
| | Entrance (1 @ 15 ft2) | = 15 |
| | Subtotal | = 138 |
| | 40% allowance | = 55 |
| | **Total** | **= 193 $ft^2$** |

**4.8** Since an employee typically spends the first third of his lunch break preparing to eat and obtaining his meal, for a one-hour lunch break, dining shifts may begin every 40 minutes (rounded up to 45 minutes because shifts must begin and end on 15 minute increments). Consequently, 2 lunch shifts can be included in the 11:00 A.M. to 1:00 P.M. time frame. The following table shows the shift timing for each lunch break.

| Beginning of Lunch Break | Time Sat Down in Chair | End of Lunch Break |
|---|---|---|
| 11:00 A.M. | 11:20 A.M. | 12:00 Noon |
| 11:45 A.M. | 12:05 P.M. | 12:45 P.M. |

**4.9** If 800 employees eat during one shift at an industrial facility that has a cafeteria with 36 $in^2$ tables, then the following space requirements are necessary for a vending machine and cafeteria food service:

| | |
|---|---|
| Vending Machine Area (800 people @ 1 $ft^2$ per person) | = 800 |
| Cafeteria Area (800 people @ 13.5 $ft^2$ per person) | = 10,800 |
| **Total Area Requirement** | **= 11,600 ft2** |

If 400 employees eat during two shifts at a commercial facility that has a cafeteria with 42 $in^2$ tables, then the following space requirements are necessary for a vending machine and cafeteria food service:

| | |
|---|---|
| Vending Machine Area (400 people @ 1 $ft^2$ per person) | = 400 |
| Cafeteria Area (400 people @ 17.5 $ft^2$ per person) | = 7,000 |
| **Total Area Requirement** | **= 7,400 ft2** |

If 200 employees eat during four shifts at an industrial facility that has a cafeteria with rows of 10 foot long rectangular tables, then the following space requirements are necessary for a vending machine and cafeteria food service:

| | |
|---|---|
| Vending Machine Area (200 people @ 1 $ft^2$ per person) | = 200 |
| Cafeteria Area (200 people @ 12 $ft^2$ per person) | = 2,400 |
| **Total Area Requirement** | **= 2,600 ft2** |

**4.10** If 800 employees eat during one shift at an industrial facility that has a cafeteria with 36 $in^2$ tables, and lunch breaks are one hour, then the following space requirements are necessary for a serving line and cafeteria food service:

| | |
|---|---|
| Serving Line Area (6 lines @ 300 $ft^2$ per line) | = 1,800 |
| Cafeteria Area (800 people @ 13.5 $ft^2$ per person) | = 10,800 |
| **Total Area Requirement** | **= 12,600 $ft^2$** |

The reason why six serving lines are required with a one-hour lunch break for 800 employees is that a serving line can serve 7 employees per minute. In a 20-minute time frame, one line can serve 140 employees. If five lines are utilized only 700 employees will be served in twenty minutes. Consequently, size lines are required.

If 400 employees eat during two one-hour shifts at a commercial facility that has a cafeteria with 42 in2 tables, then the following space requirements are necessary for a serving line and cafeteria foot service:

| | |
|---|---|
| Serving Line Area (3 lines @ 300 $ft^2$ per line) | = 900 |
| Cafeteria Area (400 people @ 17.5 $ft^2$ per person) | = 7,000 |
| **Total Area Requirement** | **= 7,900 $ft^2$** |

If 200 employees eat during four one-hour shifts at an industrial facility that has a cafeteria with rows of 10 foot long rectangular tables, then the following space requirements are necessary for a serving line and cafeteria food service:

| | |
|---|---|
| Serving Line Area (2 lines @ 300 $ft^2$ per line) | = 600 |
| Cafeteria Area (200 people @ 12 $ft^2$ per person) | = 2,400 |
| **Total Area Requirement** | **= 3,000 $ft^2$** |

**4.11** If 800 employees eat during one shift at an industrial facility that has a cafeteria with 36 $in^2$ tables, and lunch breaks are one hour, then the following space requirements are necessary for a full kitchen and cafeteria food service:

| | |
|---|---|
| Serving Line Area (6 lines @ 300 $ft^2$ per line) | = 1,800 |
| Kitchen Area for 800 people (from Table 4.5) | = 2,400 |
| Cafeteria Area (800 people @ 13.5 $ft^2$ per person) | = 10,800 |
| **Total Area Requirement** | **= 15,000 $ft^2$** |

If 400 employees eat during two one-hour shifts at a commercial facility that has a cafeteria with 42 $in^2$ tables, then the following space requirements are necessary for a full kitchen and cafeteria food service:

| | |
|---|---|
| Serving Line Area (3 lines @ 300 $ft^2$ per line) | = 900 |
| Kitchen Area for 800 people (from Table 4.5) | = 2,400 |
| Cafeteria Area (400 people @ 17.5 $ft^2$ per person) | = 7,000 |
| **Total Area Requirement** | **= 10,300 $ft^2$** |

If 200 employees eat during four one-hour shifts at an industrial facility that has a cafeteria with rows of 10 foot long rectangular tables, then the following space requirements are necessary for a full kitchen and cafeteria food service:

| | |
|---|---|
| Serving Line Area (2 lines @ 300 $ft^2$ per line) | = 600 |
| Kitchen Area for 800 people (from Table 4.5) | = 2,400 |
| Cafeteria Area (200 people @ 12 $ft^2$ per person) | = 2,400 |
| **Total Area Requirement** | **= 5,400 $ft^2$** |

## SECTION 4.5

**4.12** The space requirement in the health services are for the employment of two nurses and a part-time physician are as follows

| | |
|---|---|
| Waiting Room (1 @ 100 $ft^2$) | = 100 |
| First Aid Room (2 Nurses @ 250 $ft^2$ per nurse) | = 500 |
| Examination Room (1 Physician @ 150 $ft^2$ per physician) | = 150 |
| **Total Health Services Area Requirement** | **= 750 $ft^2$** |

**4.13** Answers will vary depending on the campus.

**4.14** In office facilities, the restrooms must be the same size for the same amount of people. However, due to the fact that office personnel tend to have longer lunch breaks, the food services are does not need to be as large as it would for a production facility, because office personnel tend to go out to lunch for more than production personnel. Also, office workers tend not to get injured as much as production personnel, so the health services area does not need to be as large in an office facility as they do in a production facility. Finally, production workers that work in hot environments need more locker space and shower stalls than those people in an office environment.

**4.15** It is important to consult personnel such as the human resources department industrial relations department, or personnel department on the types of personnel requirements necessary for the facility. However, these people may not understand the cost effectiveness of their decisions, so the facilities planner should only gain insight from them, not use them to make final facilities decisions.

**4.16** In a multi-level facility, there have to be more personnel service locations than in a single-level facility because a multi-level facility is much more decentralized. There have to be more restrooms because each level has to have one for men and women. Also, there should be more vending facilities in a multi-level facility than a single-level facility because it is more difficult for people to walk stairs to vending locations on different levels. if it is a multi-level production facility, health services need to be located on several, if not all floors. Furthermore, on top of health service, food service and restrooms that must comply with ADA regulations in a multi-level facility, ADA compliance also must occur so that all disabled employees can reach all levels of the facility.

**SECTION 4.6**

**4.17a** The classroom building in which this class is taught must have tables at which disabled students can sit. Also, if the classroom is above the ground floor, then ADA regulations state that the student must be able to access the room by other means than stairs. The elevator control panel must be at a height that is accessible to disabled students. All doors leading to the classroom must be wide enough to accommodate a wheelchair. The fire alarm and electric switches must be at a height where someone in a wheelchair can reach it. The room between aisles of decks in the classroom must be at least three feet. For any sight impaired students, Braille instructions should be placed on the door so the student can determine where the classroom is.

**4.17b** The upper decks of the football stadium should have ramped access to them and all decks should have areas in which people in wheel chairs can enjoy the game without someone standing in front of them impeding their view. Also, vending locations should be low enough so that people in wheelchairs can reach the service shelf and cash register without having to strain themselves. Finally, all restroom facilities should have doors that are wide enough for disables individuals and Braille instructions for sight-impaired individuals.

**4.17c** The school's student center should have access to all levels for disabled personnel. Sight-impaired individuals should have Braille instructions and Braille labels throughout the center, as well as seeing-eye dog access. Wheelchair-bound individuals should have their campus mailbox at a level where they can reach it. Any activities within the student center, such as bowling alleys, cafeteria, game rooms, movie theaters, and student organization offices should have wheelchair access. The entrances and exits to the center should have ramp access, if necessary. All aisles within the facility should be wide enough to contain two wheelchairs side by side.

**SECTION 4.7**

**4.18** Individual employees can give valuable input to verify and refine office requirements. This can be accomplished through personal interviews when the number of employees is small. When the number of employees is large, survey can be used to get feedback on things to consider for employees to be more productive. It is important for employees to feel that they are part of the process of making the entire organization more efficient.

**4.19** Advantages of open office structure:

- Improved communications
- Improved supervision
- Better access to common files and equipment
- Easier to illuminate, heat, cool, and ventilate
- Lower maintenance cost
- Reduce space requirements due to space flexibility

Disadvantages of open office structure:

- Lack of privacy
- Lack of status recognition
- Difficulty in controlling noise
- Easy access for interruptions and interference

**4.20** Campus office concept gives higher level of personal satisfaction. Every employee is provided with the same employee services and facilities, which put everyone in an equal "status". No one feels being treated unfairly. When moving to a larger cluster/new office, everything remains the same, where as in the traditional office concept; the employee may have to readjust to the new environment.

**4.21** Additional features:

- Computers at every workstation
- Vending Machines

- Child care services

**4.22** List of criteria to evaluate office plans can be looked at from different perspective:

a. Employee
   - Size
   - Noise
   - Proximity of facilities
   - Proximity of colleague
   - Supporting features such as a computer, internet, phone line, etc

b. Manager
   - Visibility for supervision
   - Productivity

c. Company
   - Maintenance cost
   - Impact to business operation’
   - Proximity to supporting facility such as postal service

# Chapter 5

# *Material Handling*

**SECTION 5.3**

**5.1a** Work principle: The measure of work is material flow (volume, weight, or count per unit of time) multiplied by the distance moved. The work principle promotes minimizing work whenever possible. Materials movement should be in as practically large amount as possible and material-handling devices should be fully utilized by carrying loads at its capacity.

**5.1b** Ergonomic principle: Ergonomics is the science that seeks to adapt work or working conditions to suit the abilities of the worker. Human capabilities and limitations must be incorporated into material handling equipment and procedures designed for effective interaction with the people using the system. As an example, for humans picking in a warehouse, do not store any items above the top of the head. The optimum picking zone in order to minimize stooping and stretching is from the hips to the shoulder.

**5.2a** Unit Load principle: A unit load is one that can be stored or moved as a single entity at one time, such as pallet, container, etc, regardless of the number of individual items that make up the load. In this principle, product should be handled in as large a unit load as practical. As an example, instead of moving individual products, a number of products can be palletized and move as a single load. However, the size of the load must be considered as excessive unit load can reduce the visibility of the forklift driver or may not be practical in utilizing truck space. Moving as large a unit load as practical must be exercised in the context of just-in-time delivery. There is no point in moving materials if it will not be used for a long duration.

**5.2b** Space Utilization principle: Space in material handling is three dimensional and therefore is counted as cubic space and effective utilization of all cubic space is the essence of this principle. If products are stored in a container, the product should fill up the container. In storage application, the storage dimension should be designed to fit a unit load with some tolerance for storage and retrieval.

**5.3a** An illustrative example is given below.

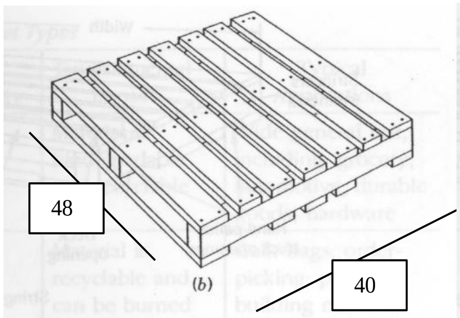
Environmental principle: Environmental consciousness stems from a desire not to waste natural resources and to predict and eliminate the possible negative effects of our daily action to the environment. Practical application to this principle is to replace combustion engine forklift to electric forklift.

**5.3b** An illustrative example is given below.

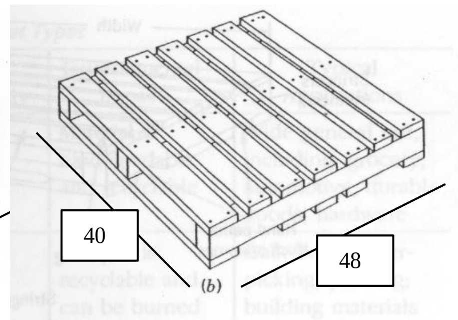
Standardization principle: Standardization means less variety and customization in the methods and equipment employed. With unit load standardization, a common material handling device can be employed to move different products. Higher space utilization can also be attributed to unit load standardization. Storage space is the same across all products; therefore there is no need for differentiating the storage area. By doing this, space wasted from excess storage space dedicated to each product can be eliminated due to the pooling effect.

## SECTION 5.5

**5.4** The first dimension of pallet size correspond to the length of the stringer board and the second dimension correspond to the length of the deck board.

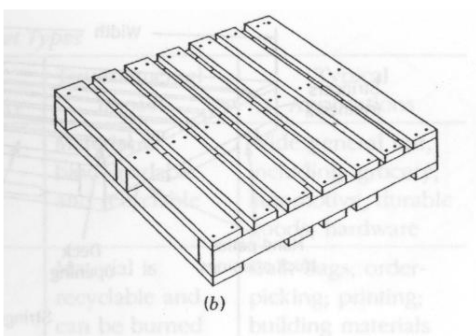



40” x 48”

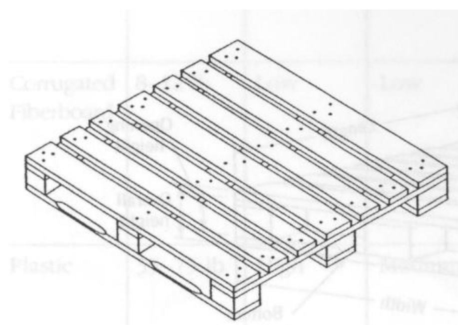



48” x 40”

**5.5** Two-way pallet - fork entry can be only on 2 opposite sides of the pallet and is parallel to the stringer board. Four-way pallet - fork entry can be on any side of the pallet.



A two-way pallet



A four-way pallet

**5.6** Research question. The answer depends on the factory visited.

**5.7** Research question. The answer depends on the factory visited.

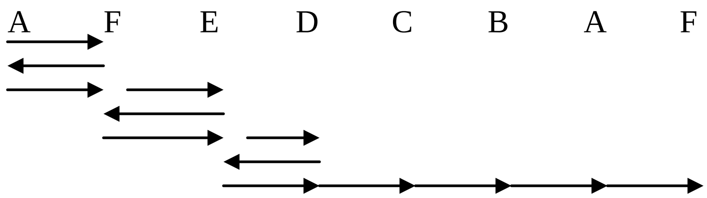
**5.8** Research question. Answer will depend on the case developed.

**5.9** A-F-E-D-C-B-A-F

There are several different answers depending on the assumption. Two different cases will be presented in these examples: The vehicle is assumed to stop at point F.

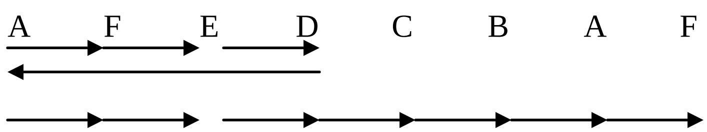
<u>Case 1:</u>

Unit load size = 50, so assuming for every 100 loads the vehicle will make the following trip:
A-F-A-F-E-F-E-D-E-D-C-B-A-F




Number of trips = 13
There are 13 trips for every 100 loads. Since we have 2000 pieces, then the total number of trip is 2000/100 * 13 = 260 trips

Case 2:
Unit load size 50, so assuming for every 100 loads the vehicle will make the following trip:
A-F-E-D-A-F-E-D-C-B-A-F




Number of trips = 11
There are 11 trips for every 100 loads. Since we have 2000 pieces, then the total number of trips is 2000/100 * 11 = 220 trips

These two examples are just two possible answers available. There are many other possibilities depending on the assumption.

**SECTION 5.6 AND APPENDIX**

**5.10** An illustrative example is given below.

Bridge Crane: A bridge crane can be classified into two basic types: top running and under running, both with single or multiple girders design. The girders which functions as a bridge beam may support one or multiple trolley and hoist. The bridge travels along the runway on wheeled carriages called end trucks, which are mounted to each bridge end. The hoist body is either mounted on top of the bridge or suspended from it. The hoist moves along the bridge on a trolley. Bridge crane operates on three axes of movement and a swivel that rotates the load around the vertical axis is regularly featured. An operator standing on the floor using a pendant, remote radio, or infrared control unit can do the controlling. The operator may also be in a self-contained control cab in larger and faster cranes. Finally, automation is the last control option.

Jib Crane: The main configuration of a jib crane is a horizontal beam that pivots along a vertical axis. A trolley and hoist may be perched from the beam. Jib cranes have three degrees of freedom - vertical, radial and rotary. Jib cranes may be attached to a vertical wall member, or mounted to a mast attached to the floor, and some may be constructed to move along a wall. Jib crane is economical; uses little floor space, and pivots in a pie-shaped area. However, it cannot reach into corners and it doesn't lift objects outside the circular area. Another disadvantage is the lack of portability. Typical application includes localized activity like in a machine shop.

Gantry Crane: A gantry crane is a bridge crane using a horizontal beam that is supported at each end. There are two versions of gantry crane: single leg and double leg both with provisions for the legs to travel along rails. Gantry cranes are used when overhead runway are not practical. It can be built on wheels for portability, is economical, and is able to lift heavy loads. Its major disadvantage is the limited reach since it cannot expand much beyond the inside span.

**5.11** Attributes for comparing sorting conveyor:

- Maximum number of sorting capacity per minute
- Load range
- Load size
- Minimum distance between spurs
- Diverter impact on load
- Safety
- Initial cost
- Maintenance cost.

**5.12** Attributes for comparing automated guided vehicles systems:

- Weight capacity
- Safety
- Travel speed
- Throughput capacity
- Vehicle Cost
- System cost per vehicle
- Operating costs

**5.13** Attributes for comparing unit load storage system:

- Cost per position
- Potential storage density
- Load Access: the ease to load/unload the materials stored
- Throughput capacity - the rate of which materials can flow through
- Inventory and location control - visibility permitting for easier inventory control/counting
- FIFO maintenance - storage arrangement so that the first material that is stored will also be the first one that is retrieved

- Ability to house variable load sizes
- Ease of installation

**5.14** Attributes for comparing unit load retrieval technologies:
- Vehicle cost
- Lift height capacity
- Aisle width - aisle width required for vehicle to operate
- Weight capacity
- Lift speed - the speed in the vertical axis
- Travel speed - speed of the vehicle

**5.15** Attributes for comparing small part storage alternatives:
- Gross system cost - initial cost/purchased $ft^3$
- Net system cost - initial cost/available $ft^3$
- Floor space requirements - $ft^3$ of inventory housed per $ft^2$ of floor space
- Human factors - ease of retrieval
- Maintenance requirements
- Items security
- Flexibility - ease of reconfigure
- Pick rate - order lines per person-hour
- Key

**5.16** Attributes for comparing automated data collection systems:
- Real time - data collected will be straight recorded in the database
- Hands free
- Eyes free
- Cost

**5.17** Attributes for comparing bar code readers:
- Range - the range where the bar code can still be read
- Depth of field - orthogonal distance to read the bar code
- Scan rate
- Resolution
- Price

**5.18** Attributes for comparing bar code printers:
- Print Quality
- Throughput ($in^2$ / sec)
- Typical price range
- Relative ownership cost

**5.19a** A pallet truck is used to lift and transport pallet loads of material for distance preclude walking. Platform truck is a version of industrial truck. A platform truck uses a platform for supporting the load. It does not have lifting capabilities and used for transporting. The main difference is that a platform truck needs another device to load and unload the material while a pallet truck doesn't need one.

**5.19b** A reach truck is mostly used on double deep rack. A reach truck has the capability to extend its forks for easier storage/retrieval on deep racks. A turret truck is mostly used on narrow aisle. A turret truck has a pivot point on the fork and a constant mask for more maneuverability along narrow aisles.

**5.20a** Drive-in rack extends the reduction of aisle space that begun with the double deep rack. Drive-in rack allows a lift truck to drive in to the rack several positions and store or retrieve a unit load. A drive-thru rack is a drive-in rack that is accessible from both sides of the rack. It is staged for a flow-thru fashion where load is loaded at one end and retrieve at the other end. Both racks have the same design considerations.

**5.20b** Push back rack is a carrier with rail guide for each pallet load. Push back rack provides a last-in-first-out storage system. A mobile rack is a single deep selective rack on wheels/tracks that permits the entire row of racks to move on adjacent rows.

**5.21** An illustration is given below.

Bridge Crane vs. Stacker Crane: Bridge Crane is a bridge that spans a work area. The bridge is mounted on tracks. The bridge crane and hoist can provide 3 dimensional coverage of the department. A stacker crane is similar to a bridge crane, but instead of using a hoist, a mast is supported by the bridge. The mast is equipped with forks or a platform, which are used to lift unit loads.

# Chapter 6

# *Layout Planning Models and Design Algorithms*

**SECTION 6.1-6.2**

**6.1** Several important factors such as:

- Product type,
- Manufacturing,
- Marketing Distribution,
- Management Distribution,
- Human resource plans will be impacted by and will impact on the facility layout.

**6.2** The material handling decisions can have a significant impact on the effectiveness of a layout. For example:

- Centralized vs. decentralized storage of WIP, tooling, and supplies.
- Fixed path vs. variable path handling.
- Single vs. bi-directional material handling equipments.
- The handling unit (unit load) planned for the system.
- The degree of automation used in handling.
- The type of level of inventory control, physical control, and computer control of materials

**6.3a** Fixed Product Layout: It should be used when the product is too large or cumbersome to move through the various processing steps, e.g. shipbuilding industry, aspects of the aircraft industry, and the construction industry.

**6.3b** Product Layout: It is used when high-volume production conditions exist. The product volume must be sufficient to achieve satisfactory utilization of the machine.

**6.3c** Group Layout: It is used when production volumes for individual products are not sufficient to justify product layouts, but by grouping products into logical product families, a product layout can be justified for the family.

**6.3d** Process Layout: It is used when there exist many low-volume, dissimilar products to be planned.

**6.4**

| Manufacturing Situation | | Primary Layout Design |
|---|---|---|
| a. | Soda Bottler | Product Layout |
| b. | Printing Shop | Group Layout |
| c. | Meat-Process Plant | Fixed Product Layout |
| d. | Furniture Manufacturing Plant | Group Layout |
| e. | Computer Chip Maker | Product Layout |
| f. | Shipyard | Fixed Product Layout |
| g. | Refinery Plant | Product Layout |
| h. | College Campus | Group Layout |

**SECTION 6.3**

**6.5** The construction type of facility layout procedure involves developing a new layout from scratch. The improvement procedure generates layout alternatives based on an existing layout.

**6.6** All three layout procedures provide a systematic step-by-step methodology in designing a facility layout. All of them take the activity interrelationship, space requirements, and etc. into account. All of them are based on the construction type of facility layout procedure and improved upon the initial layout with consideration of other constraints.

**6.7**

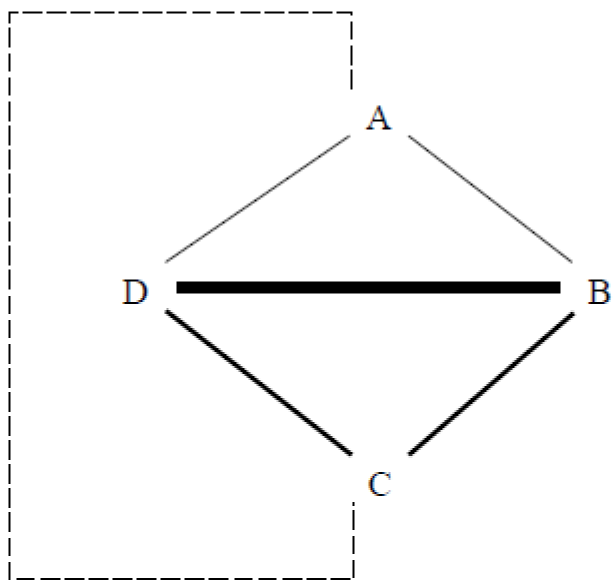
Flow-Between Diagram

| | A | B | C | D |
|---|---|---|---|---|
| A | – | 375 | 125 | 365 |
| B | | – | 400 | 620 |
| C | | | – | 400 |
| D | | | | – |

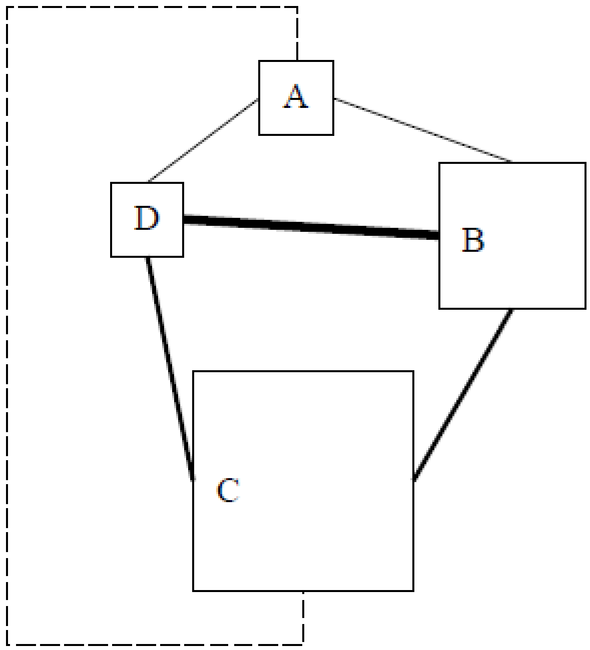
Activity Relationship Chart

| | A | B | C | D |
|---|---|---|---|---|
| A | – | I | O | I |
| B | | – | E | A |
| C | | | – | E |
| D | | | | – |

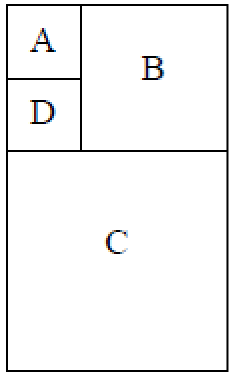
Relationship Diagram



Space Relationship Diagram




Block Layout




**6.8a** From-To Chart

| Dept. | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | – | 13,320 | 1,800 | 0 | 0 | 0 |
| B | 0 | – | 11,400 | 6,600 | 4,920 | 5,400 |
| C | 0 | 6,600 | – | 2,400 | 4,200 | 3,600 |
| D | 0 | 3,000 | 1,200 | – | 5,040 | 960 |
| E | 0 | 5,400 | 7,800 | 1,200 | – | 5,160 |
| F | 0 | 0 | 0 | 0 | 0 | – |

**6.8b** SLP Approach:

Since we do not have the activity relationship chart, we base our closeness rating on the material flow information.

Flow-Between Chart

| Dept. | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | – | 13,320 | 1,800 | 0 | 0 | 0 |
| B | | – | 18,000 | 9,600 | 10,320 | 5,400 |
| C | | | – | 3,600 | 12,000 | 3,600 |
| D | | | | – | 6,240 | 960 |
| E | | | | | – | 5,160 |
| F | | | | | | – |

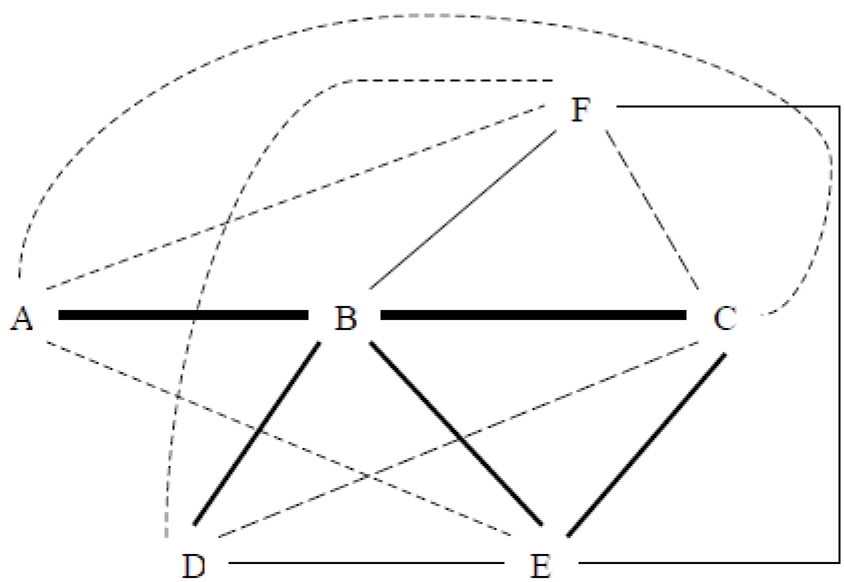
Rank flow values

| | | | |
|---|---|---|---|
| 1. | B-C | 18,000 | A |
| 2. | A-B | 13,320 | A |
| 3. | C-E | 12,000 | E |
| 4. | B-E | 10,320 | E |
| 5. | B-D | 9,600 | E |
| 6. | D-E | 6,240 | I |
| 7. | B-F | 5,400 | I |
| 8. | E-F | 5,160 | I |
| 9. | C-D | 3,600 | O |
| 10. | C-F | 3,600 | O |
| 11. | A-C | 1,800 | U |
| 12. | D-F | 960 | U |
| 13. | A-D | 0 | U |
| 14. | A-E | 0 | U |
| 15. | A-F | 0 | U |

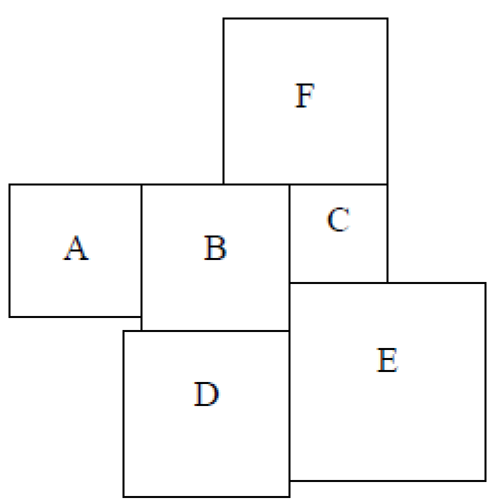
Activity Relationship Chart

| Dept. | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | – | A | U | U | U | U |
| B | | – | A | E | E | I |
| C | | | – | O | E | O |
| D | | | | – | I | U |
| E | | | | | – | I |
| F | | | | | | – |

Relationship Diagram




Block Layout



**6.9a** From-To Chart

| Dept. | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | – | 500 | 440 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | | – | 500 | 500 | 940 | 440 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | | | – | 1,000 | 0 | 1,140 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D | | | | – | 640 | 1,140 | 0 | 1,140 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| E | | | | | – | 700 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| F | | | | | | – | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G | | | | | | | – | 0 | 0 | 0 | 0 | 0 | 630 | 550 | 0 |
| H | | | | | | | | – | 250 | 400 | 300 | 300 | 0 | 0 | 0 |
| I | | | | | | | | | – | 150 | 0 | 0 | 0 | 0 | 0 |
| J | | | | | | | | | | – | 0 | 0 | 0 | 0 | 0 |
| K | | | | | | | | | | | – | 150 | 0 | 0 | 0 |
| L | | | | | | | | | | | | – | 0 | 0 | 0 |
| M | | | | | | | | | | | | | – | 280 | 0 |
| N | | | | | | | | | | | | | | – | 1,450 |
| O | | | | | | | | | | | | | | | – |

**6.9b** SLP Approach

Rank flow values

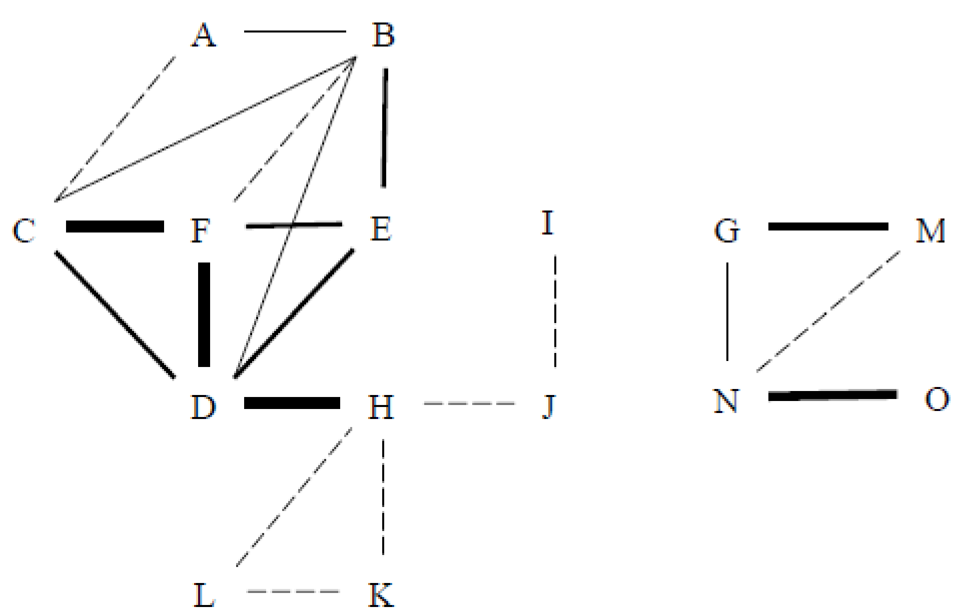
| | | | | | |
|---|---|---|---|---|---|
| N-O | 1,450 | A | B-C | 500 | I |
| C-F | 1,140 | A | B-D | 500 | I |
| D-F | 1,140 | A | A-C | 440 | O |
| D-H | 1,140 | A | B-F | 440 | O |
| C-D | 1,000 | E | H-J | 400 | O |
| B-E | 940 | E | H-K | 300 | O |
| E-F | 700 | E | H-L | 300 | O |
| D-E | 640 | E | M-N | 280 | O |
| G-M | 630 | E | H-I | 250 | O |
| G-N | 550 | I | I-J | 150 | O |
| A-B | 500 | I | K-L | 150 | O |

* This is an incomplete list since we know that any flow value equaling zero in 6.9a will receive a relationship value of U.

Activity Relationship Chart

| Dept. | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | – | I | O | U | U | U | U | U | U | U | U | U | U | U | U |
| B | | – | I | I | E | O | U | U | U | U | U | U | U | U | U |
| C | | | – | E | U | A | U | U | U | U | U | U | U | U | U |
| D | | | | – | E | A | U | A | U | U | U | U | U | U | U |
| E | | | | | – | E | U | U | U | U | U | U | U | U | U |
| F | | | | | | – | U | U | U | U | U | U | U | U | U |
| G | | | | | | | – | U | U | U | U | U | E | I | U |
| H | | | | | | | | – | O | O | O | O | U | U | U |
| I | | | | | | | | | – | O | U | U | U | U | U |
| J | | | | | | | | | | – | U | U | U | U | U |
| K | | | | | | | | | | | – | O | U | U | U |
| L | | | | | | | | | | | | – | U | U | U |
| M | | | | | | | | | | | | | – | O | U |
| N | | | | | | | | | | | | | | – | A |
| O | | | | | | | | | | | | | | | – |

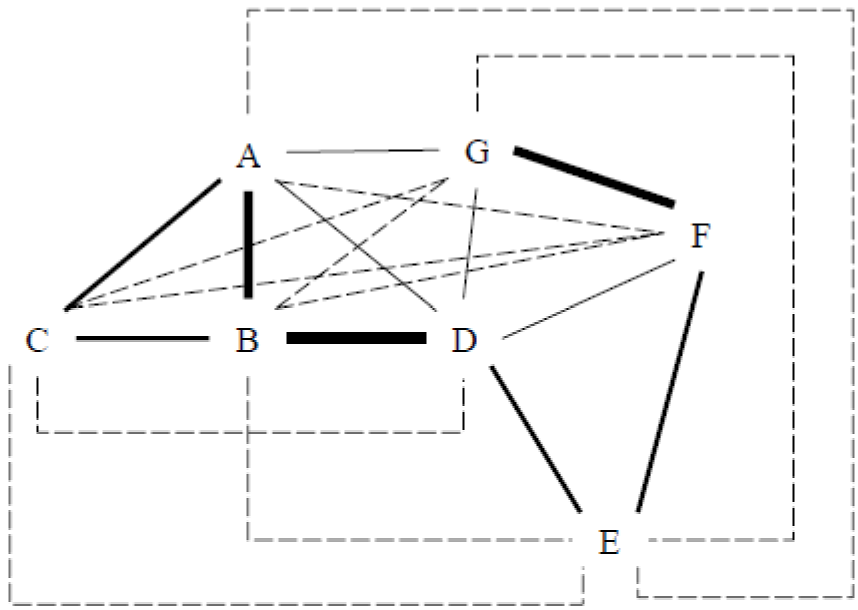
Relationship Diagram



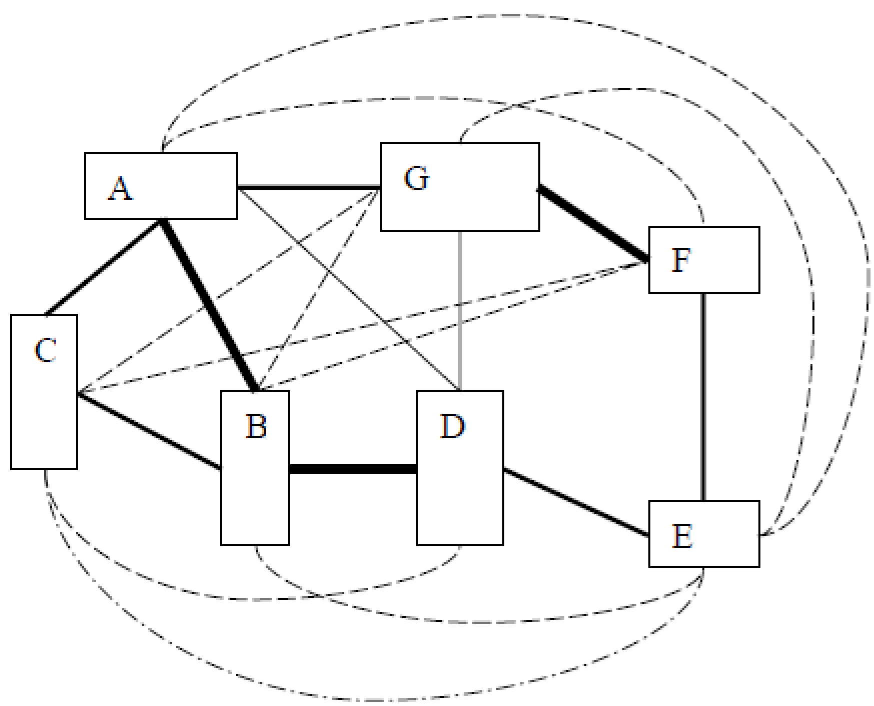

Block Layout

| | | | | |
|---|---|---|---|---|
| A | B | E | J | I |
| C | F | D | H | K |
| O | N | G | M | L |

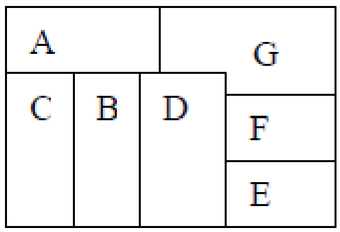
**6.10a** Relationship Diagram



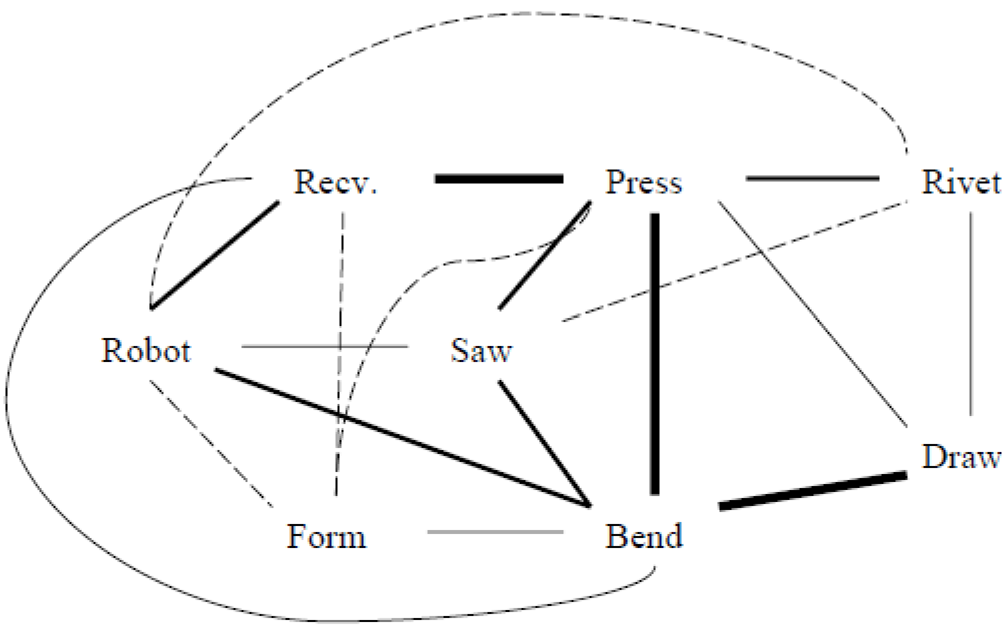

Space Relationship Diagram



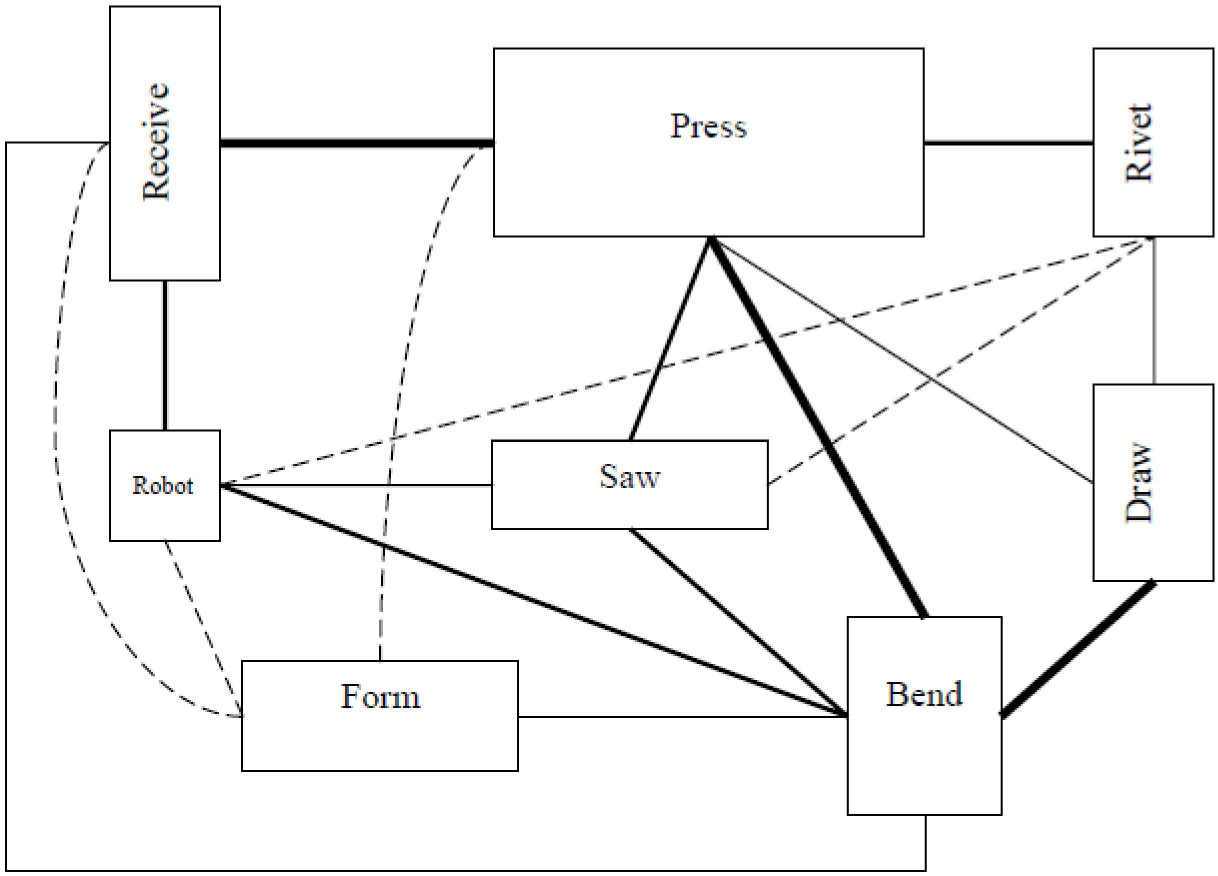
**6.10b** Block Layout using SLP



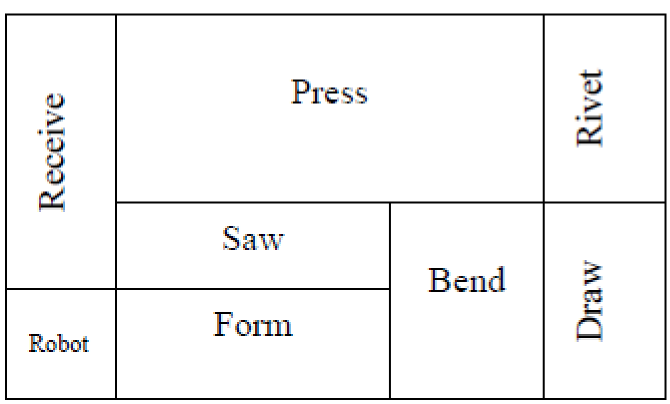

**6.11** Relationship Diagram



Space Relationship Diagram




Block Layout



**SECTION 6.4**

**6.12** $\sum_{(i,j)\in F} f_{ij} = 20$, $\sum_{(i,j)\in \bar{F}} f_{ij} = -10$,
$\sum_{(i,j)\in F} f_{ij} x_{ij} = 14$, $\sum_{(i,j)\in \bar{F}} f_{ij} \left(1 - x_{ij}\right) = -4$;

by equation (6.4) we have $z = (14 - (-4))/(20 - (-10)) = 18/30 = 0.60$.

**6.13** A to PD #3 B to PD #2 C to PD #5
Cost = 5(120) + 30(180) + 10(90) + 25(210) + 25(210) + 5(120)
+ 5(150) + 20(60) + 5(270) = 21,300

A to PD #2 B to PD #3 C to PD #5
Cost = 5(210) + 30(270) + 10(240) + 25(120) + 25(120) + 5(210)
+ 5(60) + 20(150) + 5(60) = 22,200

A to PD #5 B to PD #2 C to PD #3
Cost = 5(210) + 30(60) + 10(210) + 25(90) + 25(120) + 5(240)
+ 5(270) + 20(60) + 5(180) = 14,850

A to PD #3 B to PD #5 C to PD #2
Cost = 5(240) + 30(180) + 10(210) + 25(120) + 25(90) + 5(210)
+ 5(150) + 20(270) + 5(60) = 21,450

A to PD #2 B to PD #5 C to PD #3
Cost = 5(90) + 30(270) + 10(120) + 25(210) + 25(240) + 5(120)
+ 5(60) + 20(270) + 5(210) = 28,350

A to PD #5 B to PD #3 C to PD #2
Cost = 5(120) + 30(60) + 10(120) + 25(240) + 25(210) + 5(90)
+ 5(270) + 20(150) + 5(60) = 19,950

Final Assignment: A to PD #5 B to PD #2 C to PD #3

**6.14** Flow-Between Chart

| Machine | A | B | C | D |
|---|---|---|---|---|
| A | – | 200 | 3,300 | 700 |
| B | | – | 200 | 2,800 |
| C | | | – | 1,200 |
| D | | | | – |

Arrangements/Costs:
A B C D: 30(200 + 200 + 1200) + 60(3300 + 2800) + 90(700) = 477,000
B A C D: 30(200 + 3300 + 1200) + 60(200 + 700) + 90(2800) = 477,000
C B A D: 30(200 + 200 + 700) + 60(3300 + 2800) + 90(1200) = 507,000

D B C A: 30(2800 + 200 + 3300) + 60(1200 + 200) + 90(700) = 336,000

Switch A and D:
B D C A: 30(2800 + 1200 + 3300) + 60(200 + 700) + 90(200) = 291,000
C B D A: 30(200 + 2800 + 700) + 60(1200 + 200) + 90(3300) = 492,000
A B C D: 30(200 + 200 + 1200) + 60(3300 + 2800) + 90(700) = 477,000

Switch B and D:
C D B A: 30(1200 + 2800 + 200) + 60(200 + 700) + 90(3300) = 477,000
A D C B: 30(700 + 2800 + 200) + 60(3300 + 2800) + 90(200) = 495,000

Final Arrangement: B D C A

**6.15** Flow-Between chart is identical to the one presented in the solution to Problem 6.14.

Arrangements/Costs:
A B C D: 200(30) + 3300(45) + 700(75) + 200(15) + 2800(45) + 1200(30)
= 372,000
B A C D: 200(30) + 3300(15) + 700(45) + 200(45) + 2800(75) + 1200(30)
= 342,000
C B A D: 200(30) + 3300(75) + 700(15) + 200(45) + 2800(45) + 1200(90)
= 507,000
D B C A: 200(60) + 3300(45) + 700(105) + 200(15) + 2800(45) + 1200(60)
= 435,000

Switch A and B:
A B C D: 200(30) + 3300(45) + 700(45) + 200(15) + 2800(75) + 1200(30)
= 372,000
C A B D: 200(30) + 3300(45) + 700(45) + 200(75) + 2800(15) + 1200(90)
= 351,000
D A C B: 200(60) + 3300(15) + 700(45) + 200(45) + 2800(105) + 1200(60)
= 468,000

Final Arrangement: B A C D

**6.16** Flow-Between Chart

| Machine | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | – | 15 | 90 | 70 | 0 | 80 |
| B | | – | 75 | 65 | 85 | 0 |
| C | | | – | 85 | 45 | 80 |
| D | | | | – | 90 | 70 |
| E | | | | | – | 10 |
| F | | | | | | – |

ABC-DEF: 30(15 + 0 + 75 + 45 + 90 + 10) + 60(90 + 80 + 85 + 70)
= 26,550
ABC- DFE: 10(45) + 20(80 + 70) + 30(15 + 90 + 10) + 40(80) + 50(90) + 60(90 + 85)
= 25,100
ABC-EDF: 10(65) + 20(85 + 85 + 70) + 30(15 + 75 + 85 + 90) + 40(70) + 50(45 + 10)
+ 60(90 + 80) = 29,150
ABC-EFD: 10(0) + 20(85 +80 + 70) + 30(15 + 75 + 10 +65) +40(80) + 50(90 + 45)
+ 60(90 + 70) = 29,200
ABC-FDE: 10(65 + 45) + 20(70 + 70 + 85) + 30(15 + 50 + 75) + 40(85) + 50(10)
+ 60(90 + 80) = 23,900
ABC-FED: 10(85 + 85) + 20(10 + 65) + 30(15 + 90 + 75) + 40(45) + 50(70 + 70)
+ 60(90 + 80) = 27,600
ACB-DEF: 10(45) + 20(90 + 85 + 85) + 30(90 +75 + 10) + 40(80) + 50(15 + 65)
+ 60(80 + 70) = 27,100
ACB-DFE: 20(90 + 85 + 80 + 70) + 30(75 + 45 + 10) + 50(15 + 65 + 90)
= 18,900
ACB-EDF: 10(45 + 65) + 20(90 + 85 + 70) + 30(90 + 75) + 40(70 + 85 + 80)
+ 50(15 + 10) + 60(80) = 26,400
ACB-EFD: 10(45 + 65) + 20(90 + 80 + 70) + 30(10 + 75) + 40(80 + 85 + 85)
+ 50(15 + 90) + 60(70) = 27,900
ACB-FDE: 20(90 + 70 + 45 + 70) + 30(75 + 45 + 65 + 90) + 50(15 + 10)
= 15,000
ACB-FED: 10(45 + 65) + 20(90 + 80 + 85) + 30(10 + 75 + 90) +40(65) + 50(15)
+ 60(70 + 70) = 23,200
BAC-DEF: 10(65) + 20(85 + 90 + 80) +30(90 + 15 + 10) + 40(70) + 50(75)
+ 60(85 + 70) = 25,050
BAC-DFE: 10(65 + 45) + 20(70 + 80 + 90) + 30(15 + 10) + 40(70 + 85 + 80)
+ 50(90 + 75) + 60(85) = 29,400
BAC-EDF: 20(90 + 80 + 85 +70) + 30(15 + 65 + 90) + 50 (75 + 45 + 10)
= 18,100
BAC-EFD: 20(90 + 80 + 85 +70) + 30(10 + 15) + 50 (75 + 45 + 65 + 90)
= 20,700
BAC-FDE: 10(65 + 45) + 20(70 + 70 + 90) +30(15 + 90) + 40(80 + 85 + 85)
+ 50(10 + 75) + 60(80) = 27,900
BAC-FED: 20(85 + 90 + 70) + 30(10 + 15 + 45 + 90) +40(80) + 50(75 + 65)
+ 60(80 + 70) = 28,900

Final Arrangement: ACB-FDE

**6.17** Flow-Between Chart

| Machine | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 0 | 450 | 500 | 600 |
| B | | – | 700 | 0 | 250 |
| C | | | – | 0 | 850 |
| D | | | | – | 200 |
| E | | | | | – |

Distance Matrix (w/ initial assignments based on diagram)

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 (A) | – | 20 | 70 | 120 | 140 |
| 2 (B) | | – | 50 | 100 | 120 |
| 3 (C) | | | – | 50 | 70 |
| 4 (D) | | | | – | 20 |
| 5 (E) | | | | | – |

Arrangements/Costs:

ABCDE: 450(70) + 500(120) + 600(140) + 50(700) + 250(120) + 850(70) + 200(20)
= 304,000
BACDE: 450(50) + 500(100) + 600(120) + 700(70(120) + 250(140) + 850(70) + 200(20)
= 292,000
CBADE: 450(70) + 500(50) + 600(70) + 700(20) + 250(120) + 850(140) + 200(20)
= 265,500
DBCAE: 450(50) + 500(120) + 600(20) + 700(50) + 250(120) + 850(70) + 200(140)
= 247,000
EBCDA: 450(50) + 500(20) + 600(140) + 700(50) + 250(20) + 850(70) + 200(120)
= 240,000
ACBDE: 450(20) + 500(120) + 600(140) + 700(50) + 250(70) + 850(120) + 200(20)
= 311,500
ADCBE: 450(70) + 500(20) + 600(140) + 700(50) + 250(20) + 850(120) + 200(120)
= 291,500
AECDB: 450(70) + 500(120) + 600(20) + 700(70) + 250(120) + 850(50) + 200(100)
= 245,000
ABDCE: 450(120) + 500(70) + 600(140) + 700(100) + 250(120) + 850(20) + 200(70)
= 304,000
ABEDC: 450(140) + 500(120) + 600(70) + 700(120) + 250(50) + 850(70) + 200(500)
= 331,000
ABCED: 450(70) + 500(140) + 600(120) + 700(50) + 250(100) + 850(50) + 200(20)
= 280,000

Final Arrangement: E B C D A

**6.18** Flow-Between Chart

| Dept. | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 350 | 250 | 250 | 0 |
| B | | – | 350 | 0 | 400 |
| C | | | – | 0 | 50 |
| D | | | | – | 450 |
| E | | | | | – |

1. Determine the departments with largest weight: D and E; weight = 450
2. Select the third department to enter: B; weight = 400

| | D | E | Total |
|---|---|---|---|
| A | 250 | 0 | 250 |
| B | 0 | 400 | 450 |
| C | 0 | 50 | 50 |

3. Select the next department to enter: C; in face D-E-B; weight = 750

| | D | E | B | Total |
|---|---|---|---|---|
| A | 250 | 0 | 350 | 600 |
| C | 0 | 400 | 350 | 750 |

4. Decide which face to locate department A; in face B-C-D; weight = 850

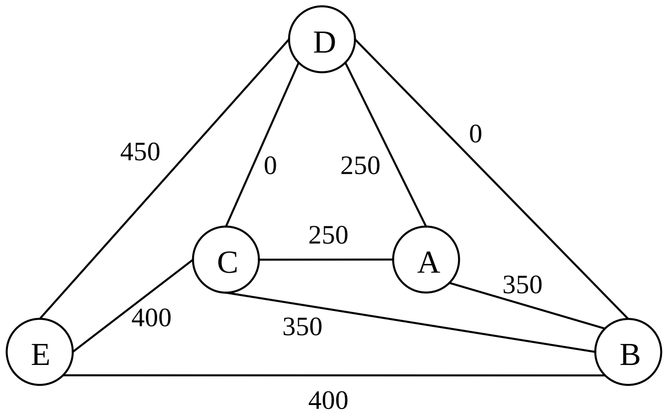
| | C | D | E | Total |
|---|---|---|---|---|
| A | 250 | 250 | 0 | 500 |

| | B | C | E | Total |
|---|---|---|---|---|
| A | 350 | 250 | 0 | 600 |

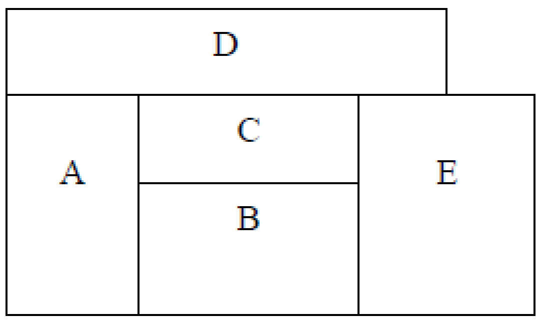
| | B | C | D | Total |
|---|---|---|---|---|
| A | 350 | 250 | 250 | 850 |

| | B | D | E | Total |
|---|---|---|---|---|
| A | 350 | 250 | 0 | 600 |

Adjacency Graph

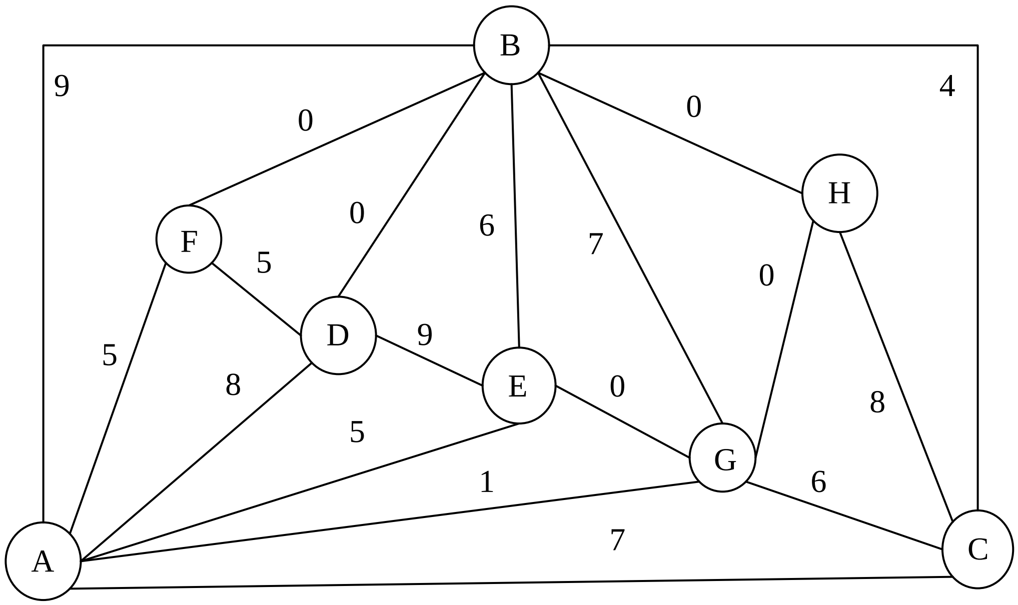



Block Layout

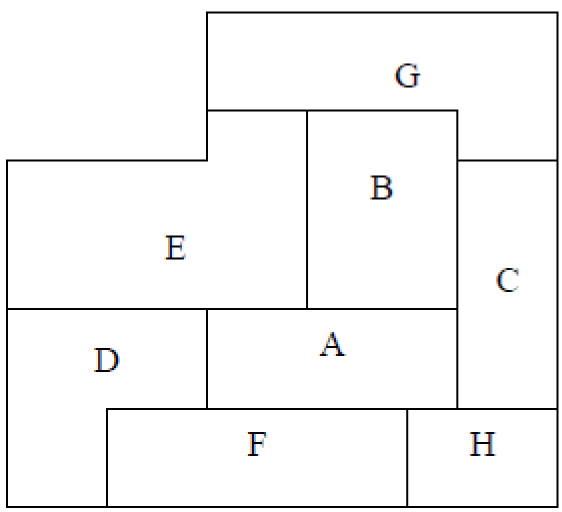



**6.19**
1. Departments with largest weight: A and B; weight = 9
2. Select the third department to enter: C; weight = 11
   Note: Both C and E have weights of 11. C is arbitrary chosen
3. Select the next department to enter: G; weight = 14
4. Select the next department to enter: E; in Face A-B-G; weight = 11
5. Select the next department to enter: D; in Face A-B-E; weight = 16
6. Select the next department to enter: F; in Face A-B-D; weight = 10
7. The last department to enter is H; in Face B-C-G; weight = 8

Adjacency Graph

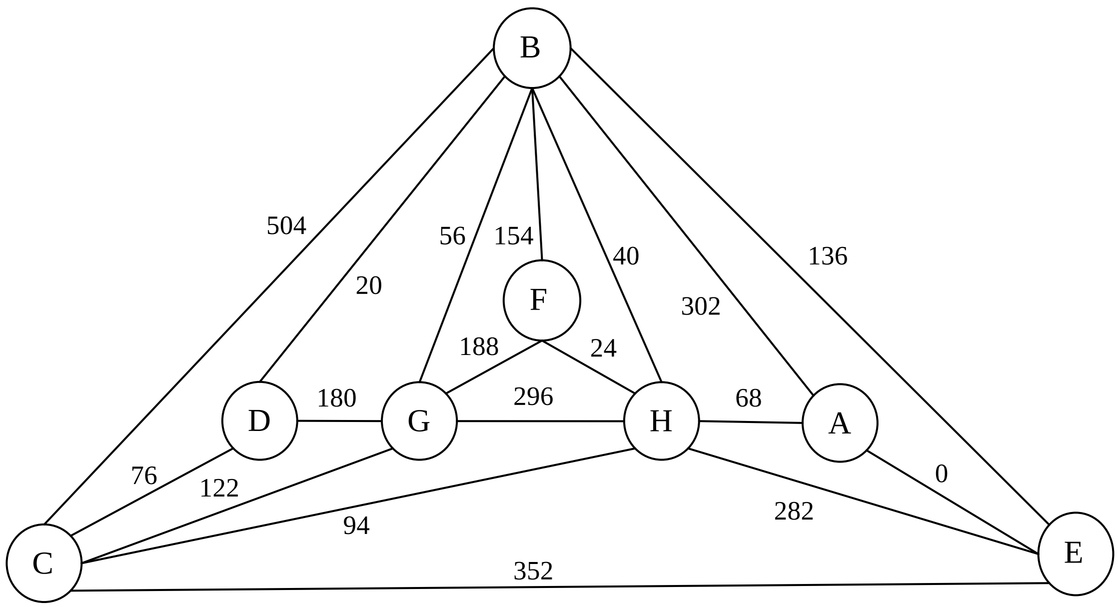



Block Layout

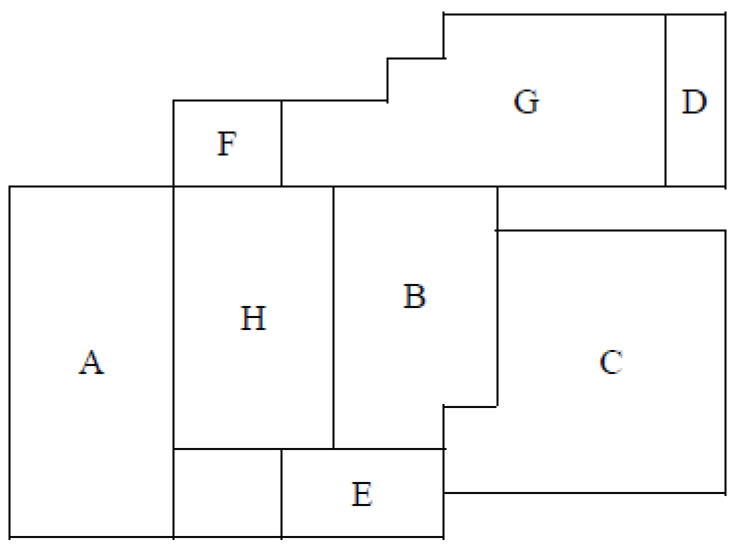



**6.20**
1. Departments with largest weight: B and C; weight = 504
2. Select the third department to enter: E; weight = 488
3. Select the next department to enter: H; weight = 416
4. Select the next department to enter: G; in Face B-C-H; weight = 474
5. Select the next department to enter: A; in Face B-E-H; weight = 370
6. Select the next department to enter: F; in Face B-G-H; weight = 366
7. The last department to enter is D; in Face B-C-G; weight = 276

Adjacency Graph




Block Layout



**6.21a** Flow-Between Chart

| Dept. | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 0 | 5 | 55 | 35 |
| B | | – | 45 | 35 | 55 |
| C | | | – | 60 | 35 |
| D | | | | – | 10 |
| E | | | | | – |

1. Department pair with largest weight: C and D; weight = 60
2. Select the third department to enter: B; weight = 80

| | C | D | Total |
|---|---|---|---|
| A | 5 | 55 | 60 |
| B | 45 | 35 | 80 |
| E | 35 | 10 | 45 |

3. Select the next department to enter: E; weight = 100

| | B | C | D | Total |
|---|---|---|---|---|
| A | 0 | 5 | 55 | 60 |
| E | 55 | 35 | 10 | 100 |

4. Department A enters the face C-D-E; weight = 95

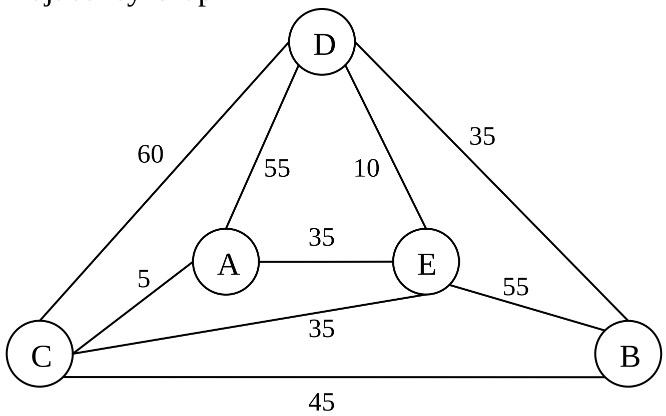
| | C | D | E | Total |
|---|---|---|---|---|
| A | 5 | 55 | 35 | 95 |

| | B | C | E | Total |
|---|---|---|---|---|
| A | 0 | 5 | 35 | 40 |

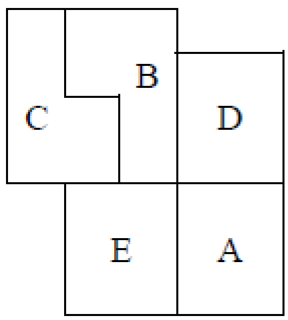
| | B | D | E | Total |
|---|---|---|---|---|
| A | 0 | 55 | 35 | 90 |

| | B | C | D | Total |
|---|---|---|---|---|
| A | 0 | 5 | 55 | 60 |

Adjacency Graph



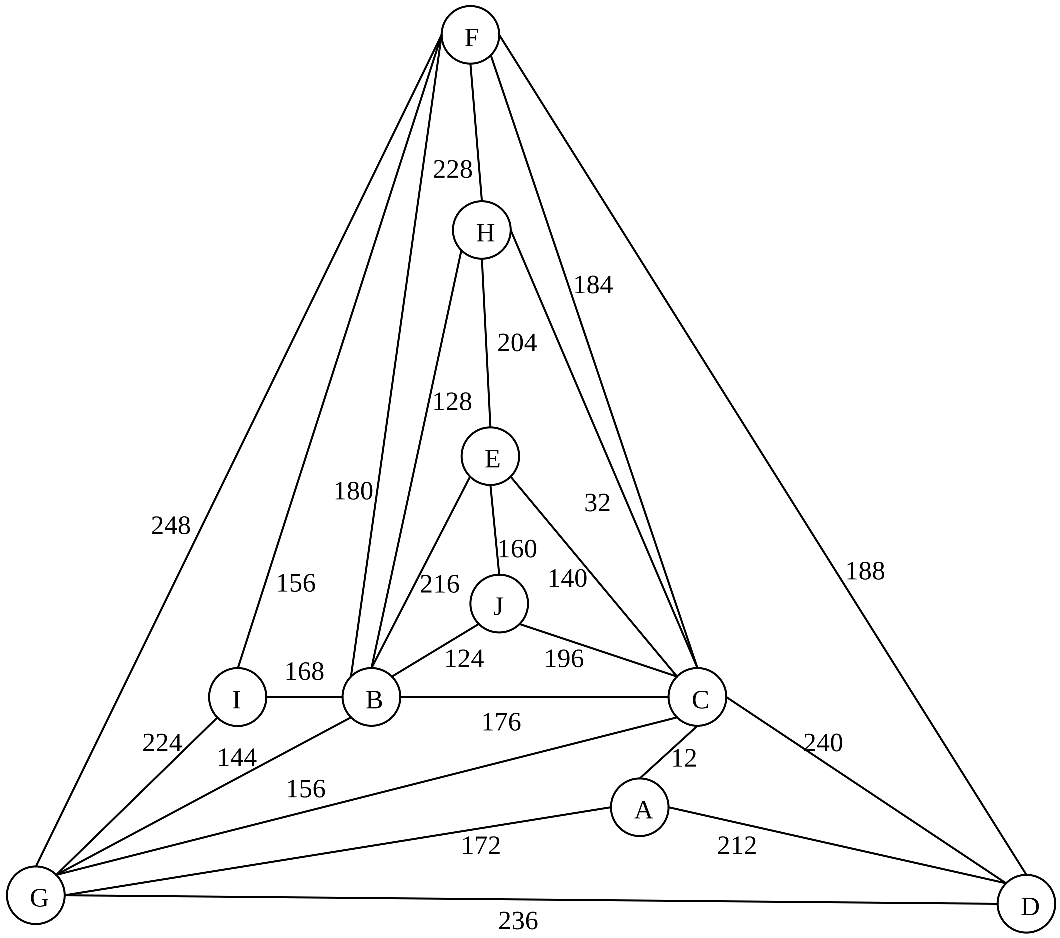

**6.21b** Block Layout



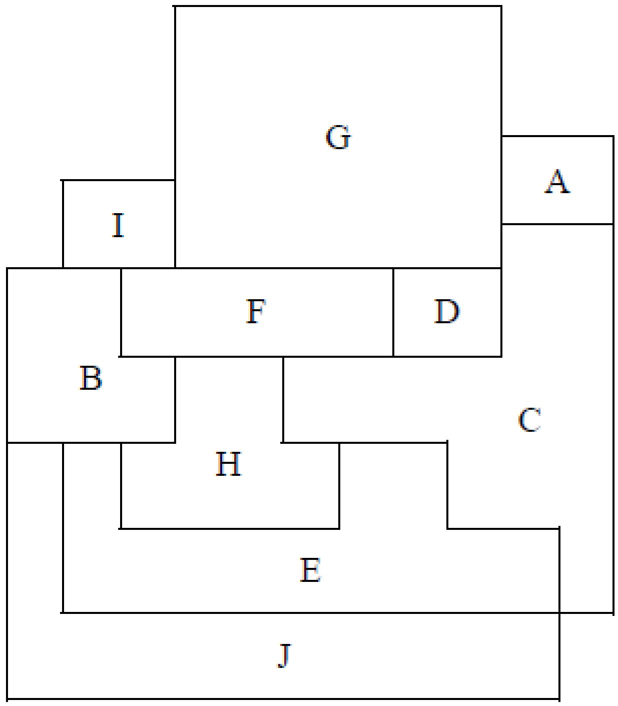

**6.22a** The Flow-Between matrix can be calculated by summing the "ij" and "ji" elements of the given flow matrix for all (i,j) pairs.

1. Departments with largest weight: F and G; weight = 248
2. Select the third department to enter: D; weight = 424
3. Select the fourth department to enter: C; weight = 580
4. Select the next department to enter: B; in Face C-F-G; weight = 500
5. Select the next department to enter: H; in Face B-C-F; weight = 576
6. Select the next department to enter: E; in Face B-C-H; weight = 560
7. Select the next department to enter: I; in Face B-F-G; weight = 548
8. Select the next department to enter: J; in Face B-C-E; weight = 480
9. The last department to enter is A; in Face C-D-G; weight = 396

Adjacency Graph



**6.22b** Block Layout




**6.23a** $E = 1.0$, and $K = 28$.

**6.23b** $E = 1.0$, and $K = 42$.

**6.23c** $E = 0.5$, and $K = 42$.

**6.23d** The two measures are not consistent! See (b) vs (a), and (b) vs (c).

**6.24a** BC, BD, and CD.

**6.24b** 26 units.

**6.24c** The layout obtained by exchanging departments C and D is optimal since $d_{ij} = 1$ for all $f_{ij} > 0$.

**6.25** The layout cost, as computed by CRAFT, will be equal to zero units since the centroids of all the departments overlap. In general, the centroid-to-centroid distance measure can give unrealistic results when the centroid of a department lies outside the department. Note that this may happen with L-shaped departments, which are legal in CRAFT.

**6.26a** If all departments are the same size, the estimated layout cost for exchanging departments will always be equal to the actual layout cost after the departments have been exchanged.

**6.26b** The initial layout cost is 93 units, and the estimated layout cost of exchanging B and C is 75 units; however, the actual layout cost after B and C have been exchanged is 111 units.

**6.27** The following estimated layout costs are computed for the initial layout:

| Exchange: | AB | AC | AE | BC | CD | CE | DE |
|---|---|---|---|---|---|---|---|
| Est. Cost: | 48 | 53.5 | 57.5 | 49.5 | 48 | 54 | 66 |

The lowest estimated layout cost is 48 units with departments D and E exchanged. Since the initial layout cost is 66 units, CRAFT exchanges departments D and E and obtains the layout shown below. The actual cost of this new layout is 48 units.

| | | | | | |
|---|---|---|---|---|---|
| A | A | A | B | B | B |
| A | A | A | C | C | C |
| A | A | A | C | C | C |
| E | E | E | D | D | D |
| E | E | E | D | D | D |

Next, CRAFT repeats the same procedure as above to compute the following estimated layout costs:

| Exchange: | AB | AC | AE | BC | CD | CE | DE |
|---|---|---|---|---|---|---|---|
| Est. Cost: | 48 | 53.5 | 57.5 | 49.5 | 48 | 54 | 66 |

Since none of the estimated layout costs is lower than 48 units (the current layout cost), CRAFT stops and the above layout is the final layout.

**6.28a** The areas of the departments are $A = 48$ , $B = 40$ , $C = 85$, $D = 40$, and $E = 62$ unit squares. Note that the area of department *B* is equal to the area of department *D*. Therefore, CRAFT would consider exchanging department pairs *AB*, *AE*, *BC*, *BD*, *CE*, and *DE*.

**6.28b** Let $d_{ij}$ be the distance between departments *i* and *j* in the current layout; let $d_{ij}^{AE}$ be the estimated distance between departments *i* and *j* if departments A and E are exchanged. Given the flow and cost data, we have:

| Pair | $f_{ij}$ | $c_{ij}$ | $d_{ij}$ | $d_{ij}^{AE}$ |
|---|---|---|---|---|
| AC | 5 | 1 | 20 | 8 |
| AE | 5 | 1 | 11 | 11 |
| BC | 6 | 1 | 12 | 12 |
| BD | 2 | 4 | 22 | 22 |
| CD | 3 | 3 | 10 | 10 |
| DE | 7 | 1 | 7 | 18 |

Thus, the estimated cost of exchanging departments *A* and *E* is
$\sum_{f_{ij}>0} f_{ij}\, c_{ij}\, d_{ij}^{AE} = 559$ units.

**6.29a** The areas (in unit squares) of the departments are *A* = 4, *B* = 8, *C* = 6 , *D* = 6 , *E* = 8 , and *F* = 4 . Note that department pairs *A* and *F*, *C* and *D*, and *B* and *E* have the same area of 4, 6, and 8, respectively. Therefore, department pairs *AD*, *AE*, *BF*, and *CF* will *not* be considered for exchange by CRAFT.

**6.29b** The cost of the initial layout is $\sum_{f_{ij}>0} f_{ij}\, d_{ij} = 1040$ units. (All the $c_{ij}$ values are equal to 1.0).

**6.29c** Using the procedure shown in Problem 6.28 (b), we first compute the estimated distance $d_{ij}^{AE}$ for all $f_{ij} > 0$. The estimated layout cost assuming that departments *E* and *F* are exchanged is $\sum_{f_{ij}>0} f_{ij}\, d_{ij}^{AE} = 1090$ units. (Again, all the $c_{ij}$ values are equal to 1.0).

**6.29d** In general, given the same problem data, we would expect MULTIPLE to obtain a lower layout cost than CRAFT since MULTIPLE considers a larger set of (2-way) department exchanges at each iteration; i.e., MULTIPLE considers a larger number of possible department exchanges and therefore explores a larger "neighborhood" of the current solution. However, both MULTIPLE and CRAFT are "path dependent" heuristics. That is, the final solution obtained by either algorithm depends on the starting point and the particular set of departments exchanged in arriving at the final solution. Therefore, there is no guarantee that MULTIPLE will always outperform CRAFT because the two algorithms may follow a different path and CRAFT may ultimately stop at a lower-cost solution. If the layout or departmental areas are such that all departments are either adjacent or equal in area, both CRAFT and MULTIPLE will consider the same (2- way) exchanges at each iteration. Therefore, both algorithms (using two-way exchanges only) will follow the same path and terminate at the same solution.

**6.30a** Principal weaknesses of CRAFT: 1. no control over department shapes; 2. only adjacent or equal-area departments will be exchanged; 3. inter-departmental distances are measured between department centroids (which can sometimes lead to unrealistic distances); and 4. when evaluating possible exchanges, CRAFT uses estimated distances obtained by swapping the department centroids. Principal strengths of CRAFT: 1. CRAFT can capture the details of the initial layout, such as fixed departments, unusable space, and obstacles; 2. CRAFT can generate many alternative layouts in a short period of time.

**6.30b** True. Both CRAFT and NEWCRAFT are "path dependent" heuristics. Since the two programs will not necessarily follow the same path, they will most likely terminate with different layouts, and there is no guarantee that the cost of a layout generated by NEWCRAFT will be less than that obtained with CRAFT.

**6.31**

Flow-Between Chart

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| A | – | 9 | 0 | 3 | 0 | 10 | 0 | 0 |
| B | | – | 0 | 9 | 5 | 0 | 0 | 0 |
| C | | | – | 0 | 4 | 0 | 4 | 0 |
| D | | | | – | 1 | 4 | 2 | 7 |
| E | | | | | – | 0 | 0 | 0 |
| F | | | | | | – | 0 | 0 |
| G | | | | | | | – | 20 |
| H | | | | | | | | – |

Relattionship Chart

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| A | – | I | U | U | U | I | U | U |
| B | | – | U | I | O | U | U | U |
| C | | | – | U | U | U | U | U |
| D | | | | – | U | U | A | O |
| E | | | | | – | U | U | U |
| F | | | | | | – | U | U |
| G | | | | | | | – | A |
| H | | | | | | | | – |

**6.32a** The efficiency rating = $\frac{A+E+A}{A+E+I+X+A} = \frac{25}{37} = 0.6757.$

**6.32b** The distance matrix for the given layout is computed as follows:

| Dept. | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | – | 3 | 6 | 5 | 9 |
| 2 | | – | 3 | 8 | 6 |
| 3 | | | – | 5 | 3 |
| 4 | | | | – | 4 |
| 5 | | | | | – |

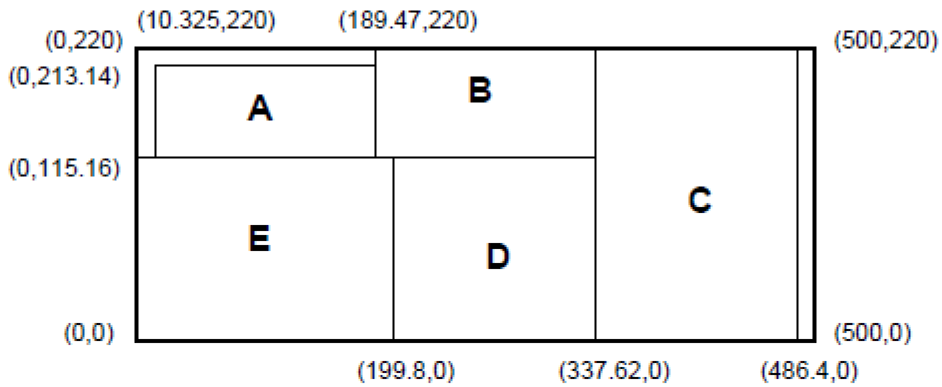
Thus, the REL-DIST score = $A \times 3 + E \times 5 + I \times 6 + X \times 3 + A \times 4$ = 30 + 25 + 12 − 30 + 40 = 77 .

**6.32c** False. BLOCPLAN uses three bands; if two non-adjacent or unequal-area departments are exchanged, it simply recomputes the width of the two bands affected by the exchange. (If the two departments are in the same band, the band width remains the same.)

**6.32d** Advantages: A rectangular shape is often the preferred shape for a department; also, with rectangular departments it is straightforward to measure and control department shapes. Limitations: If fixed departments or obstacles are present, it may be difficult or impossible to maintain rectangular department shapes; also, in some cases an L-shape may be acceptable or preferable for some departments.

**6.33** To avoid unrealistic department shapes, we set $R_i$ equal to two for all the departments. We obtain the results shown in the following Table by using the linear MIP model given by equations (6.21) through (6.37):

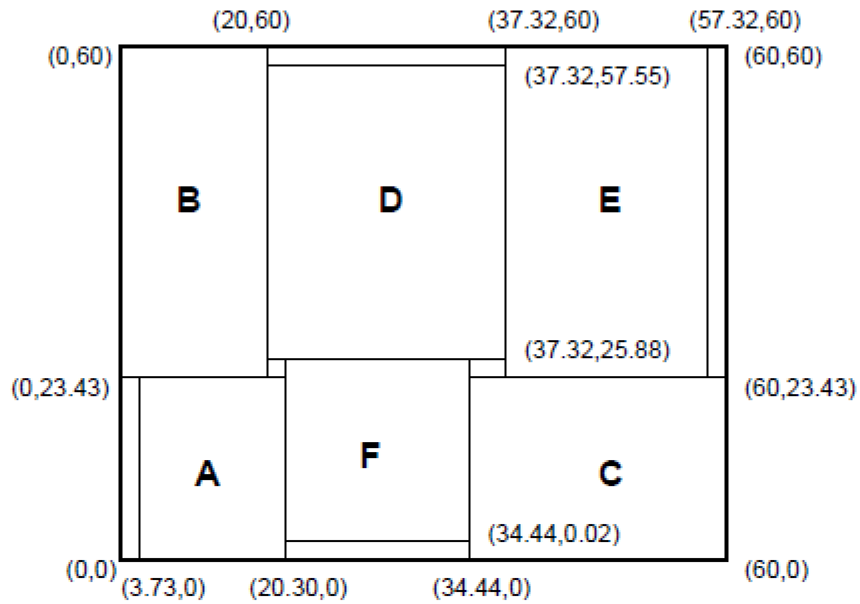
| Dept | $x_i''$ (feet) | $x_i'$ (feet) | $y_i''$ (feet) | $y_i'$ (feet) | Area (req.) | Area (model) | Error (%) |
|---|---|---|---|---|---|---|---|
| A | 189.47 | 10.33 | 213.14 | 115.16 | 19200.00 | 17553.00 | 8.58 |
| B | 337.62 | 189.47 | 220.00 | 115.16 | 16000.00 | 15532.00 | 2.92 |
| C | 486.40 | 337.62 | 220.00 | 0.00 | 34000.00 | 32732.00 | 3.73 |
| D | 337.62 | 199.80 | 115.16 | 0.00 | 16000.00 | 15871.00 | 0.00 |
| E | 199.80 | 0.00 | 115.16 | 0.00 | 24800.00 | 23009.00 | 7.22 |




The objective value we obtained from the model is 14,929.74 units.

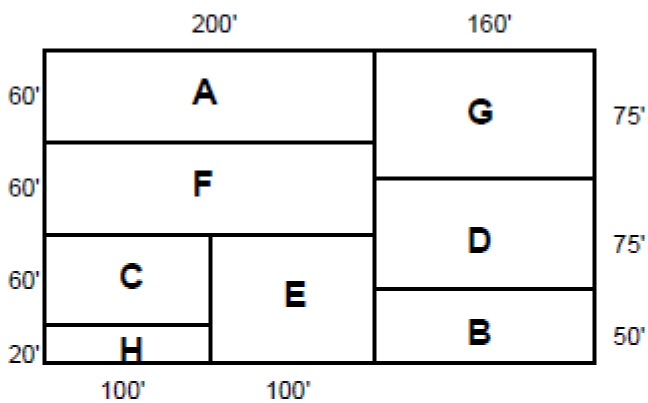
**6.34** To avoid unrealistic department shapes, we set equal to two for all the departments. We obtain the results shown in the following Table by using the linear MIP model given by equations (6.21) through (6.37):

| Dept | $x_i''$ (feet) | $x_i'$ (feet) | $y_i''$ (feet) | $y_i'$ (feet) | Area (req.) | Area (model) | Error (%) |
|---|---|---|---|---|---|---|---|
| A | 20.30 | 3.73 | 23.43 | 0.00 | 400.00 | 388.21 | 2.94 |
| B | 20.00 | 0.00 | 60.00 | 23.43 | 800.00 | 731.38 | 8.58 |
| C | 60.00 | 34.44 | 23.43 | 0.00 | 600.00 | 598.85 | 0.19 |
| D | 37.32 | 20.00 | 57.55 | 25.88 | 600.00 | 548.51 | 8.58 |
| E | 57.32 | 37.32 | 60.00 | 23.43 | 800.00 | 731.38 | 8.58 |
| F | 34.44 | 20.3 | 25.88 | 0.02 | 400 | 365.71 | 8.57 |

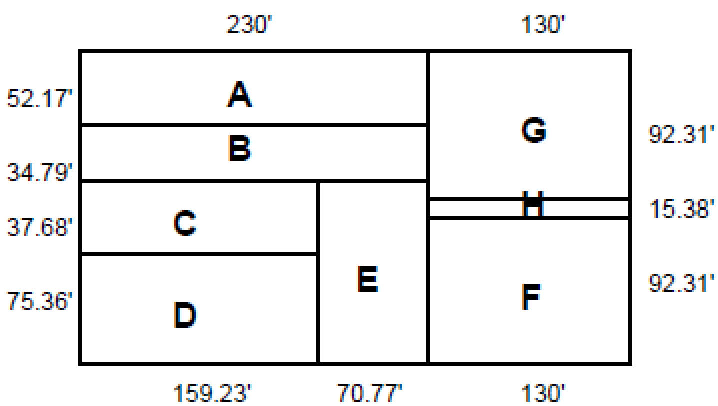



The objective value we obtained from the model is 2,130.70 units.

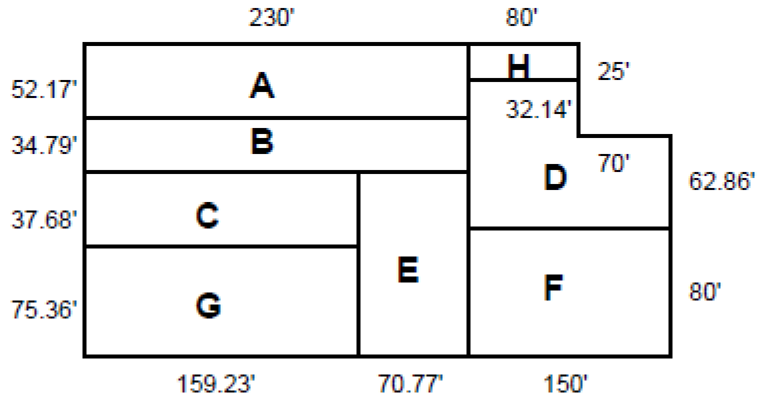
**6.35** Using LOGIC to exchange departments B and F we obtain the following layout:




**6.36** Using LOGIC to exchange departments D and H we obtain the following layout:



**6.37** Using LOGIC to exchange departments G and H we obtain the following layout:




**6.38a-b**

(a)

| | | | |
|---|---|---|---|
| A | B | B | B |
| A | A | C | C |
| A | A | C | C |
| A | A | D | D |

(b)

| | | | |
|---|---|---|---|
| A | D | D | C |
| A | A | C | C |
| A | A | C | B |
| A | A | B | B |

**6.38c** A spacefilling curve "maps" or translates a two-dimensional problem (i.e., the layout problem) into a single dimension (i.e., the layout vector or the fill sequence). Therefore, any two departments can be exchanged (or more general exchange routines such as those implemented within simulated annealing can be used) and the resulting layout can be constructed rapidly. However, department shapes are affected by the spacefilling curve; sometimes it may be difficult to ensure "good" department shapes and massaging will be necessary. Spacefilling curves should not visit fixed departments or obstacles. Otherwise, a fixed department may "float" (see MICRO-CRAFT) or a department may be placed over an obstacle. Note that all the empty space (if any) will automatically appear at the end of the spacefilling curve. If one wishes to "insert" empty space at locations other than the end of the curve, then one must use dummy departments and place them in the appropriate position in the layout vector.

**6.39a** Assuming a 1 x 1 grid for simplicity and using the data given for Problem 6.28, we first enter the initial layout and MULTIPLE computes its cost:

```
2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 4 4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 4 4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 4 4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 4 4 4 4 4 4 4 4 4 4

                INITIAL LAYOUT

COST OF INITIAL LAYOUT: 541.26 UNITS

ITER. NO. 1 - SWAP DEPARTMENTS 4 AND 5
TO OBTAIN FOLLOWING LAYOUT WITH COST 433.69

2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
1 1 1 1 1 1 1 1 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 1 1 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 1 1 4 4 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 4 4 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5

ITER. NO. 2 - SWAP DEPARTMENTS 2 AND 3
TO OBTAIN FOLLOWING LAYOUT WITH COST 399.59
```

```
3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 1 1 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 1 1 4 4 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 4 4 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5

NO TWO-WAY EXCHANGES LEAD TO A FURTHER REDUCTION
IN THE LAYOUT COST.  THEREFORE, THE ABOVE LAYOUT
IS TWO-OPT.

LAYOUT COST 399.59 UNITS; LAYOUT SEQUENCE: 13245
```

**6.39b** Using a conforming spacefilling curve may result in irregular department shapes, especially if some departments in the initial layout have irregular shapes. In part (a), for example, department 4 (D) attains a "long-and-narrow" L-shape primarily due to the shape of department 5 (E) in the initial layout.

**6.40** Assuming a 1 x 1 grid for simplicity and using the data given for Problem 6.29, we first enter the initial layout and MULTIPLE computes its cost:

```
1 1 3 3 5 5
1 1 3 3 5 5
2 2 3 3 5 5        INITIAL LAYOUT
2 2 4 4 5 5
2 2 4 4 6 6
2 2 4 4 6 6

COST OF INITIAL LAYOUT: 104.00 UNITS
```

Note that department labels (A through F) have been replaced by department numbers (1 through 6).

```
ITER. NO. 1 - SWAP DEPARTMENTS 1 AND 3
TO OBTAIN FOLLOWING LAYOUT WITH COST 94.00

3 3 3 1 5 5
3 3 3 1 5 5
2 2 1 1 5 5
2 2 4 4 5 5
2 2 4 4 6 6
2 2 4 4 6 6

NO TWO-WAY EXCHANGES LEAD TO A FURTHER REDUCTION IN
THE LAYOUT COST.  THEREFORE, THE ABOVE LAYOUT IS
TWO-OPT.

LAYOUT COST  94.00 UNITS; LAYOUT SEQUENCE: 231465
```

# Chapter 7

## *Warehouse Operations*

**SECTION 7.4-7.5**

**7.1** A project to evaluate existing dock area, including receiving and shipping, will allow the determination of those aspects that can potentially prove economically advantageous. The following is a partial list of such aspects that warrant attention.

- Eliminate the receiving area. For some materials, e.g., large and bulky ones, drop shipping (having the vendor ship to the customer directly) can save the time and labor associated with receiving and shipping.
- Reduce or eliminate staging in the receiving and shipping area. Determine the location assignment and product identification prior to receiving the product so that materials are stored as soon as they arrive, reducing the need for large staging area.
- Employ the fastest and most productive receiving process possible, i.e., crossdocking or shipping directly from the receiving dock. Palletized materials with a single SKU per pallet, floor-stacked loose cases, and backordered merchandise are excellent candidates for crossdocking.
- Bypass receiving staging and put materials away directly to primary picking locations, essentially replenishing those locations from receiving. In direct putaway systems, the staging and inspection activities are eliminated, saving the time, space, and labor associated with those operations.
- Minimize the floor space required for staging by providing storage locations for receiving staging.
- Prepare shipping from the time the materials are received, thus reducing the area for shipping.
- Ship materials in larger quantities and preferably in unit loads, i.e., pallets with one SKU per pallet.
- Ship directly from storage and without the need for staging, having prepared the shipping information prior to picking.
- Reduce the number of docks, if possible. For example, can receiving and shipping be modified so that less frequent visits to docks are necessary? This will save more space in and around the receiving and shipping areas.
- Modify docks to 90-degree docks. Finger docks should be eliminated, if possible. Otherwise, can the largest angle finger docks be used? This will allow shrinking of maneuvering and staging areas inside the receiving and shipping areas.
- Eliminate dock levelers by requiring uniform-height carriers for loading and unloading, thus reducing or eliminating dock maintenance costs.
- Remove dock shelters in favor of a more streamlined dock, if dock shelters exist. Again, maintenance costs will be reduced or eliminated.

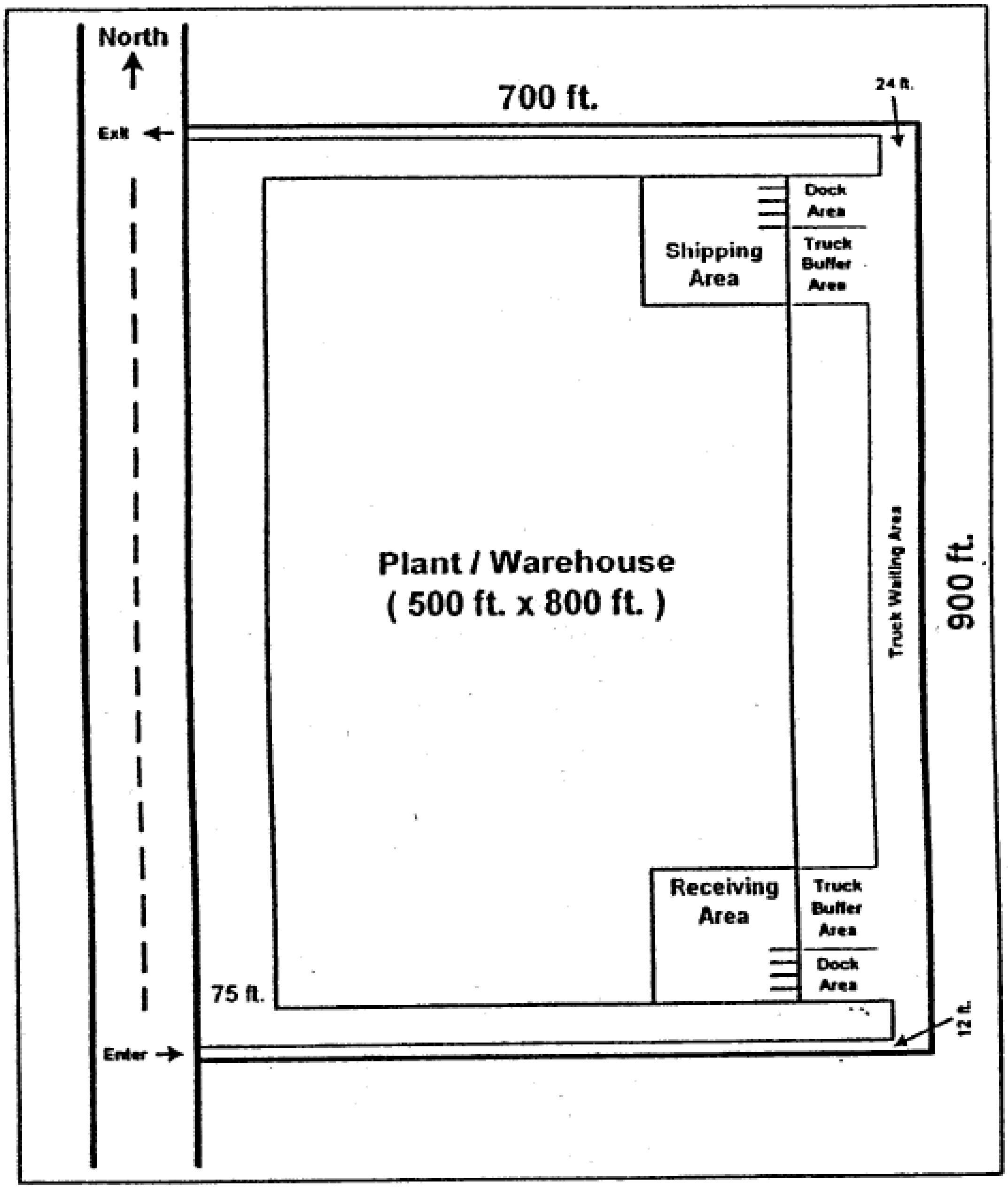
**7.2** This problem should not be assigned if the students have not been exposed to either Monte Carlo simulation or queueing theory. In Chapter 10, we provide queueing models to address problems of this type. The instructor might want to assign this problem before covering the material in Chapter 10 to motivate the students to learn the material on queueing theory.

Based on the material in Chapter 10, we provide the following solution to the dock design problem. Let $\lambda$ denote the arrival rate of trucks and $\mu$ denote the service rate at the docks. Since arrivals occur at a Poisson rate and service times are exponentially distributed, the problem is of the class (M|M|c):(GD|∞|∞). The problem reduces to determining the smallest value of c (number of docks) such that the average time trucks spend in the system (W) is less than 50 minutes.

The relationship between W and the expected number of trucks in the system (L) is given by $L = \lambda W$. Hence, since it is desired that $W < 50/60$ hr., then it is desired that $L/\lambda < 50/60$. Since $\lambda = 20/8$, or 2.5 trucks per hour, then the problem reduces to determining the smallest value of c such that $L < 2.0833$.

From figure 10.63, we can determine the value of c. The utilization factor, $\rho$, is given by $\rho = \lambda/(c\mu)$. With $\mu = 60/40 = 1.5$ trucks per hour, we find that for c equal to 3, $\rho = 2.5/[3(1.5)] = 0.5556$; from Figure 10.28, $L < 2.0833$. For $c = 2$, $\rho = 0.8333$ and, form Figure 10.63, $L > 2.0833$. Therefore, 3 docks are required to ensure that the average amount of time spent waiting and being loaded or unloaded is less than 50 minutes.

**7.3**




**7.4** The impact on space requirement of this proposal is that 90-degree docks require greater apron depth but less bay width, both impacting outside space requirements, as well as requiring greater outside turning area. Finger docks, however, require greater inside maneuvering area. Because inside space costs considerably more to construct and maintain than outside space, finger docks should be used as little as possible. If outside space is sparse, then finger docks should be used, although keeping the angle closer to 90 degrees.

If 45-degree finger docks are utilized, the bay width is increased from the width of the truck to about 45 feet. Furthe rmore, the dock width will increase from about the width of the truck to about 40 percent more than the width of the truck. That is, assuming that the old dock width is about 10 feet, then the new dock width would have to be around 14 feet to accommodate a 45-degree finger dock.

**7.5** Permanent, adjustable dockboards are fastened to the dock and are not moved form position to position. Therefore, the dock board may be longer and wider than portable dockboards. The extra length results in a smaller incline between the dock and the carrier. This allows easier and safer handling of hand carts, reduced power drain on electrically powered trucks, and less of a problem with fork and undercarriage fouling on the dockboard. The greater width allows for safer and more efficient carrier loading and unloading. Permanent, adjustable dockboards also eliminate the safety, pilferage, and alignment problems associated with dockboards. For these reasons, permanent adjustable dockboards should always be given serious consideration, despite their high installation costs.

**7.6** A survey of the receiving areas on a typical campus shows that most dock and carrier height differences are handled by portable dockboards. In many areas, however, the problem is solved by walking up or down a step to accommodate the difference in carrier and dock height.

Because surveys indicate that the campus has scattered receiving docks with minimal carrier visits, it is not justified to invest in expensive dock shelters. The bookstore docking area, ho wever, may benefit from a dock shelter because more frequent stops are made at this location in delivering and shipping more valuable products.

**7.7** Increases in the length of trucks imply that more space will be needed to maneuver trucks around shipping and receiving docks. Furthermore, certain access roads about the plant/warehouse may need to be modified to better accommodate longer trucks/trailers.

Increases in both width and height of trucks and trailers imply that docks may have to be modified to accept taller and, in many cases, wider trucks and trailers. Some of the changes can be handled by using permanent adjustable dockboards whose dynamic ranges can accept taller or wider trucks. Redesigning the dock area as well as the inside of the shipping and receiving areas may nevertheless be required should the width and height of the new vehicles fall outside of the current equipment's dynamic ranges.

**SECTIONS 7.6-7.7**

**7.8** About 55% of all operating costs in a typical warehouse can be attributed to order picking.

**7.9**

- Strict order picking, i.e., each batch is an order, or single-order-pick (SOP),
- Batch picking two or more orders, or sort-while-pick (SWP),
- Batch picking two or more (partial) orders with progressive assembly across zones, and

- Batch picking two or more orders with downstream sorting, or pick-and-sort (PAS)

**7.10**

- Traveling to, from, and between pick locations,
- Searching for pick locations,
- Documenting picking transactions,
- Reaching (and bending) to access pick locations, and
- Extracting items from storage locations

**7.11**

- Simple ranking of SKUs based on the ratio of pick frequency (the number of times a SKU is requested) to shipped cube (the product of unit demand and unit cube) establishes a baseline for an intelligent stock assignment planning. SKUs with high rankings should be assigned to the most accessible locations.
- A distribution illustrating SKUs ranked by popularity and the portion of total picking activity they represent is the ABC or Pareto plot used in intelligent stock assignment planning. More popular SKUs with the highest picking activity are distributed closer to the input/output point.
- Correlated stock assignment planning ranks SKUs by popularity as well as considering the correlations between various items in storage. A small picking zone dedicated to high- density, high-throughput order picking is used for SKUs with high correlations.

**7.12**

- Popularity of items or SKUs
- Similarity of items or SKUs
- Size of items or SKUs
- Characteristics of SKUs, such as perishability, and
- Space utilization of SKUs, i.e., maximizing space used in the layout to store SKUs.

**7.13** Travel time per line item picked … sorting

**7.14** By reducing the amount of stock in the forward picking area, forward picking costs (b) decrease and the cost to replenish the forward picking area (c) increases.

**7.15** Activity, Quantity, and Size

**7.16** Pick frequency (the number of times the item is requested) and shipped cube (the product unit demand and unit cube).

**7.17** Travel times … productivity

**7.18**

- Introduction of *Just-in-time* operating program
- Introduction of *cycle time* reduction operation program

- Introduction of *quick response* operating program, and
- Introduction *of micro-marketing* and *megabrand* strategies

**7.19** Wrong SKU is picked, or wrong quantity of the SKU is picked.

**7.20** The same or nearby locations

**7.21** Visiting a typical warehouse may bring about the following discoveries:

- Both dedicated and randomized storage schemes are used.
- The main mode of transporting materials within the warehouse is counterbalanced lift trucks, also known as fork lift truck.
- In some areas of the warehouse pallet jacks are also used.
- During order picking materials are transported using lift trucks, pallet jacks, or order picking carts.
- Although the warehouse is rather new, the condition of some of the equipments is not good because not all equipment was purchased new when the warehouse was constructed.
- The bar code symbols used are the familiar "three-of-nine" code, though some parts of the warehouse have abandoned the use of bar codes.
- Hand-held optical bar code readers are used in areas where the technology is still being exploited.
- Receiving and shipping are on the same side of the warehouse, though in opposite corners.
- Both receiving and shipping areas employ three 90-degree docks with adjustable permanent dock levelers, but without any shelters.
- The pallets used are all 42 in. x 42 in. double-faced, non-reversible type.
- The warehouse is mostly clear of clutter, but the item pick area could benefit from a better empty case (carton) removal system.
- Most of the warehouse, e.g., 75%, is dedicated to storing the materials in pallet racks, giving ample space to aisles for order picking using counterbalanced lift trucks and walk-behind order picking carts.
- Receiving and shipping areas consist of about 10-15% of the warehouse, while the remaining 10-15% of the warehouse is used for other functions, such as office space.
- The aisles represent 25-30% of the total floor space of the warehouse.
- More than 35% of the pallet rack openings are empty.
- Since most of the loads are partial loads, stored product represents less than 20-25% of the rack face.
- The storage space (cube) utilization factor within a storage opening is about 15-20%.
- The overall cube space utilized in the warehouse is less than 15%.

**7.22a** Total cube demand for item A
$= (100 \text{ units}) \times (1 \text{ ft}^3 / \text{unit A})$
$= 100 \text{ ft}^3$

Total cube demand for item B
$= (50 \text{ units}) \times (0.5 \text{ ft}^3 / \text{unit B})$
$= 25 \text{ ft}^3$

Total cube demand for item C
$= (200 \text{ units}) \times (2 \text{ ft}^3 / \text{unit C})$
$= 400 \text{ ft}^3$

**7.22b** (400 ft3 / Year) / (20 $\text{ft}^3$ / Replenishments) = 20 Replenishments / Year.

**7.22c** Item A: [(100 units / 50 weeks) x (1 $\text{ft}^3$ / unit A)] = 2 $\text{ft}^3$ / week.
(2 / 10.5 = 19.0% of forward pick area.)
Item B: [(50 units / 50 weeks) x (0.5 $\text{ft}^3$ / unit B)] = 0.5 $\text{ft}^3$ / week.
(0.5 / 10.5 = 4.8% of forward pick area.)
Item C: [(200 units / 50 weeks) x (2 $\text{ft}^3$ / unit C)] = 8 $\text{ft}^3$ / week.
(8 / 10.5 = 76.2 % of forward pick area.)

**7.22d** Item A: ( 25 Requests / 50 weeks ) = 0.5 requests / week = Rank 1
Item C: ( 16 Requests / 50 weeks ) = 0.32 requests / week = Rank 2
Item B: ( 5 Requests / 50 weeks ) = 0.1 requests / week = Rank 3.

**7.23**

- When product volume is low and/or product is to be stored for short periods of time, i.e., work in process, storing the product directly on the floor is practiced frequently. For example, liquid products in barrels (55 gallon drums) are often stored directly on the floor.
- When product volume is high and/or product mix is low, e.g., large production runs of palletized and uniform products in cases (cartons), storing the product in block storage or block stacking is quite advantageous. In particular, tight-block stacking wastes the least amount of storage space and thus has a high storage space utilization.
- Pallet racks are mostly used when product mix is high and accessibility to a particular product must be quick for faster order-processing.
- Pallet flow racks provide good storage space (cube) utilization and allow products to be processed in first-in, first-out (FIFO) fashion. They are very good for dated products with high product mix, but lower volume.
- Drive-in racks allow large pallet counts (product volume) of high product mixes to be stored and still obtain good storage space (cube) utilization. Drive-in racks are best used along the walls of plants or warehouses.
- Drive-through racks can be used the same as drive-in racks, but they also allow accessing the product from both sides of the rack. They are thus used for areas in the middle of the plant or the warehouse.

- Placing cases directly on the flow rack is used when the warehouse operation is geared toward full or partial broken case (carton) order picking. It also allows for faster replenishment, but it does lower the storage space (cube) utilization (factor).
- Storing in cantilever racks provides long, unobstructed storage spans. They are mostly helpful in sorting self-supporting long stock, such as bar stocks, pipes, and lumber.
- Portable racks are mostly good for loads (open stock), such as pipes, that need to be protected against crushing and other damage. Portable racks result in flexibility and good space utilization of bulk storage, and are also useful for crushable materials and allow access to materials on all levels.
- Smaller SKUs and/or SKUS that require various protections from damage and loss are usually stored in bin shelving. Orders that require picking items, rather than cartons or pallets, use bin racks or shelving systems.

**7.24** A detailed approach to storage space planning for the 1200 different SKUs this warehouse would have to receive, store, and ship is not possible with the data given. However, the information from the shipping and receiving analysis chart and the storage analysis chart can be used to determine the space requirement of the 1200 SKUs.

As part of the storage space planning, certain inventory parameters would have to be determined. For example, determining the *safety stock* and knowing the *order quantity* are crucial in determining the *average quantity to be stored* for each of the 1200 SKUs. The average quantity to be stored will then help in choosing among dedicated storage, randomized storage, and class-based dedicated storage.

The main difference between dedicated storage and randomized storage is it implication for what happens when a storage location becomes empty or available. In randomized storage, the closest-available-slot is designated as the storage location; retrievals are performed on a first-in, first-out (FIFO) basis, which provides a uniform stock rotation policy. The closest-available-slot storage policy can yield results that are similar to the "pure" randomized storage policy if the storage level remains fairly constant and the level of utilization is high.

Dedicated storage locations (and class-based dedicated storage locations) remain active even after stock has been removed from that location and the location is empty. This is partly due to the fact that the number of openings assigned to an SKU must accommodate its maximum inventory level. The planned quantity of unit loads to be stored for dedicated storage is thus equal to the sum of the openings required for each SKU. With randomized storage, however, the planned quantity of unit loads to be stored in the system is the number of openings required to store all SKUs. Since typically all SKUs will not be at their maximum inventory levels at the same time, randomized storage will generally require fewer openings than dedicated storage.

As part of the storage space planning, consideration must be given to throughput as well. For example, when using dedicated storage, SKUs should be assigned to storage locations based on the ratio of their activity to the number of openings or slots assigned to the SKU. The SKU having the highest ranking is assigned to the preferred opening, with the lowest-ranking SKU assigned to the least-preferred openings. Because fast movers are up front and slow movers are in back, throughput is maximized.

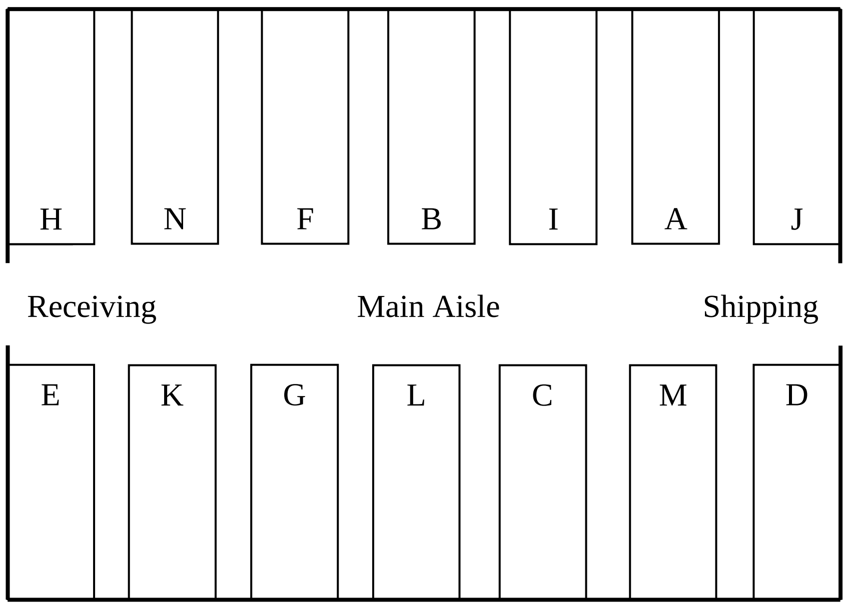
Finally, in ranking SKUs, it is important to define activity as the number of storage/retrieval actions per unit time, not the quantity of materials moved. Also, it is important to think of "part families" as well. "Items that are ordered together should be stored together" is a maxim of activity-based storage.

**7.25** The receiving/shipping ratios (the ratio of the trips to receive and the trips to ship a material) for each of the products A-N are calculated and given in the following table. A receiving/shipping ratio of 1.0 indicates the same number of trips are required to receive as to ship. Therefore, for products B, G, I, and L, the travel distance will be the same no matter where along the main aisle the products are stored. A receiving/shipping ratio less than 1.0 indicates that fewer trips are required to receive the product than to ship it. Therefore, products having ratios less than 1.0 should be located closer to shipping. Products A, C, D, J, and M should be closer to shipping.

| Product | Monthly Throughput | Qty per Receipt | Trips to Receive | Avg. Customer Order Size | Trips to Ship | Receiving/ Shipping Ratio |
|---|---|---|---|---|---|---|
| A | High | 300 pallets | 30 | 2.0 pallets | 150 | 0.20 |
| B | Low | 200 cartons | 50 | 4.0 pallets | 50 | 1.00 |
| C | Low | 10 pallets | 10 | 0.2 pallet | 50 | 0.20 |
| D | High | 400 pallets | 400 | 0.5 pallet | 800 | 0.50 |
| E | High | 6,000 cartons | 1,000 | 10.0 cartons | 600 | 1.67 |
| F | Low | 40 cartons | 40 | 2.0 pallets | 20 | 2.00 |
| G | High | 200 cartons | 200 | 1.0 pallet | 200 | 1.00 |
| H | High | 9,000 cartons | 2,250 | 5.0 cartons | 1,800 | 1.25 |
| I | Low | 50 pallets | 50 | 1.0 pallet | 50 | 1.00 |
| J | High | 500 pallets | 500 | 0.7 pallet | 715 | 0.69 |
| K | Low | 80 pallets | 80 | 2.0 pallets | 40 | 2.00 |
| L | High | 400 pallets | 400 | 1.0 pallet | 400 | 1.00 |
| M | High | 7,000 cartons | 1,167 | 3.0 cartons | 2,334 | 0.50 |
| N | Low | 700 cartons | 140 | 7.0 cartons | 100 | 1.40 |

Similarly, a receiving/shipping ratio greater than 1.0 indicates that fewer trips are required to ship than to receive. Therefore, products having ratios greater than 1.0 (i.e., E, F, H, K, and N) should be located closer to receiving.

The layout shown is typical of an aisle-based system. Products are distributed according to throughput, quantity receipt, and the ratio of receiving to shipping trips. In Chapter 10, we provide guidelines for minimizing travel distances.




**7.26** Minimum Total Cube Requirement:

[(4 ft x 4 ft x 4.5 ft)/load]x[300 loads/product]x[30 products] = 648,000 $ft^3$

Max Usable Height Requirement:

[(4.5 ft/load)x(4 loads)] = 18 ft

Minimum Total Floor Space Requirement:

[648,000 cubic ft]/[18 ft] = 36,000 $ft^2$

Approximate Dimensions of Block Stacking Area:

$[36,000\ ft^2]^{0.5}$ = 190 ft (width or depth of stacking area)

Minimum Aisle Width and Maximum No. of Lanes per Product:

Minimum aisle width = 13 ft. Maximum no. of lanes per product = 2

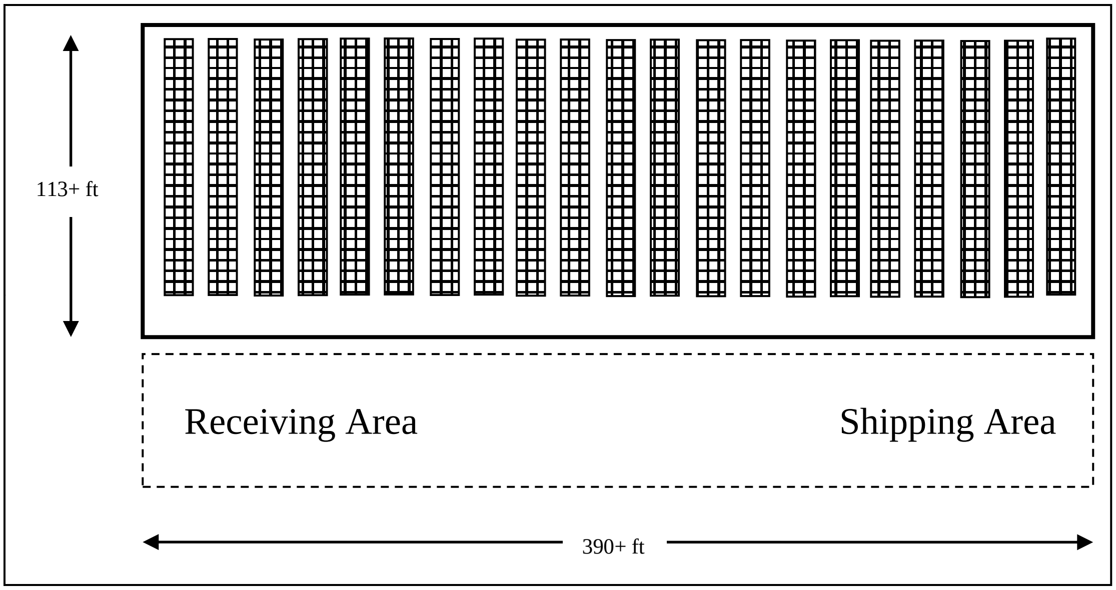
Practical Dimensions of Block Stacking Area:

[30 products]x[3 load widths/product]x[4 ft/load width] = 360 ft (width)
{[300 loads]/[4 loads high]/[3 loads wide]}x[4 ft/load depth] = 100 ft (depth)
Add [30 products]x[13 ft –3 x 4 ft] = 30 ft to 260 ft

Therefore, the practical dimensions for the block stacking storage area are 390 ft wide by 300 ft deep.

Layout for the Warehouse: A rectangular block stacking area of 390 feet wide by 100 feet deep, minimum dimensions. Each product will be stacked in three lanes, each containing 25 stacks of four loads high, for a total of [ 25 + 25 + 25 ] x 4 = 300 loads per product. The 30 products will be stacked in 90 side-by-side lanes (30 aisles), with one foot separating each product aisle.

This scheme tries to reduce the pick face of the stacks, which increases congestion and the time it takes to empty a (deep) lane. Addressing those issues requires a more rectangular block stacking area at the expense of cube utilization. A typical layout is depicted below.




**7.27** Cube utilization increases as products are more densely stored in a warehouse. Thus, cube utilization and product accessibility are inversely proportional. For example, in block stacking storage scheme, cube utilization increases as lane depth increases. This makes accessing (deep) product(s) more difficult.

**7.28** The slightest advantage that layout (a) has over layouts (b) and (c) of Figure 7.36 is that certain conditions may exist to make the overall travel distance somewhat less. In general, however, layouts (b) and (c) are preferred over (a) because they allow more stock accessibility. Layout (c) is preferred over layout (b) because it allows the most accessibility of all layouts. The disadvantage of layout (c) is that it uses more aisle space and its overall storage space utilization (factor) will be slightly less. The overall preference is thus (c) over (b), which are both preferred over (a).

# Chapter 8

# *Manufacturing Systems*

## SECTION 8.1

**8.1** To some extent, the automatic factory is still valid today. There are three components in an automatic factory – manufacturing, material handling, and the information system. In terms of manufacturing, some decisive factors to justify automation are:

- Volume of production. Economics of scale can be achieved by mass production and the financial benefit can compensate the high capital cost of an automatic factory.
- Expensive machinery. Some industries, such as semiconductor, require expensive machinery. By automating, these machines can be fully utilized to reduce production cost.
- Variability reduction. Manual machining, while still within tolerance, often produce parts with high variability. This variability can be reduced significantly by automation.

From a material handling perspective, automation is desirable to reduce cost in time due to savings in labor cost. In addition to cost saving, some product may require careful handling; therefore automation is an alternative to prevent product damage. In addition, the declining costs of computing and data storage continue to fuel the desire to invest in automation.

**8.2** The semiconductor industry would be a good target for the automatic factory. Machinery for semiconductor production cost dearly and should be fully utilized. Product value is also very high; material-handling automation is needed to avert damage.

Another sector would be continuous flow manufacturing such as chemical products.

**8.3** Advantages of automatic warehouse:

- High throughput
- Reduced labor cost
- Elimination of human error
- Reduced material damage
- Greater security

Disadvantages of automatic warehouse:

- High initial capital cost
- Downtime or reliability of equipment
- Software related problems
- Backup is needed to cover the risk of complete reliance on automation if a disaster should strike.
- User interface and training
- Obsolescence
- Lack of flexibility
- Risk of having all eggs in one basket if a disaster should strike the warehouse

**8.4** A cross docking facility can be totally automated. A list that is required for a fully automated cross docking facility with respect to the material handling aspect:

- Software for warehouse management. Software must be provided for a fully automated cross- docking facility since there is no man-labor available to make the arrangement to unload/load.
- Automatic material transport equipment for moving the materials. For example: conveyors, racks that are designed to accommodate cross docking facility, i.e. multi-lane directional conveyor lines from receiving dock to the shipping dock.
- Industrial vehicles for transporting from the dock to storage or storage to dock. For example: automatic crane/hoist to load/unload material to/from trucks/containers. This device will allow automated retrieval of loads from truck.

**8.5** A container shipping yard can be totally automated. A list that is required for a fully automated container shipping yard with respect to the material handling aspect:

- Software for "warehouse" management. Software must be provided for a fully automated container shipping yard since there is no man-labor available to make the arrangement to unload/load.
- Industrial vehicles for transporting from the dock to storage or storage to dock. For example: AGV crane/hoist to load/unload material to/from staging area/yard. This device will allow automated retrieval of loads from dock.

## SECTION 8.2

**8.6** Criteria for comparing the different variations of the straight-line flow pattern for fixed automation systems:

- Actual shape of the line, e.g. straight-line flow, U-flow, circular flow, etc.
- Material handling device used
- Spacing between workstations (buffer size)

**8.7** Dial-indexing machine, single-stage multimachine systems. The student may also come up with other types of systems.

## SECTION 8.3

**8.8** Criteria for comparing transfer line and FMS:

- Volume of production
- Number of products manufactured
- Production of families of work parts
- Reduced flexibility in processing orders because of long production line
- Manufacturing lead time
- Machine utilization
- Direct and indirect labor
- Management control

**8.9** Alternatives for moving part as shown:

- Spurs can be removed so that the system will have an on-track upload/offload. An on-track unload/offload system will allow queueing on tracks. If the system has many AGVs and the process of uploading/offloading takes a significant amount of time, then having spurs is more favorable since it allows an off-track upload/offload system.
- Instead of using AGVs, conveyors can be used for transporting materials. When using conveyors, a spine layout can also be implemented. When using a spine layout, there is no more loop around the system.

**SECTION 8.4**

**8.10** Criteria for comparing FMS and SSMS:

- Part transportation requirement
- Number of setup
- Tool allocation and management
- Flexibility
- Capital cost

**8.11** Tool management problem in:

- <u>FMS</u>: In a FMS environment, tool sharing among machines is common. This trend is perpetuated by limited tool magazine size and more importantly; keeping a complete set of tools in a magazine may not be economically feasible since tools are generally expensive. Tools can be categorized into two types: resident, which reside in a machine permanently and transient, which is shared among machines and kept in central tool storage.

  Determining how many resident and transient tools is the problem. This also translates to how to allocate the transient tools among machines and how many transport devices is needed. Given the machining schedule, usually based on order priority, the required tool sequence can be known.

  Simulation or integer programming can be used to find the optimal solution. In practice, keeping active inventory of all tools in the cell with their size, type, number and location, improving tool forecasts and warnings of tools changes, reducing delays in the system, and improving tool information reliability are the core of a tool management system.
- <u>SSMS</u>: SSMS essentially faced the same tools management problems as in FMS. Tools are still separated into resident tools and transient tools; however the proportion of resident tools is considerably higher than in a FMS setting. This can be attributed to the nature of SSMS where part only visit a machine once; thus more resident tools are needed. This arrangement is more costly; in return it offers more versatility, more machine utilization, easier part scheduling and higher throughput.

**8.12** Answer will depend on the student, but some relationships are as follows:

- The cell design must accommodate the working envelope of the material handling device due to the automation requirements of the FMS.

- The handling device needs to be flexible in the sense that it must be able to handle all of the part types produced within the flexible cell.
- The WIP storage must also conform to the limitations of the cell and material handling system design.

**SECTION 8.5**

**8.13** By having a device that serves multiple functions, in this case, storage and fixturing, the second rule of thumb is automatically satisfied. The storage/fixturing device would be designed such that the part is held so that it can be processed. In addition, the manual handling that would normally be involved moving a part from a storage device (e.g. bin) to a fixturing device is eliminated. Instead the storage/fixturing device would simply be loaded into the machine. The common device would also eliminate the need for extra storage devices and/or fixturing. Therefore, each of the rules-of-thumb are satisfied in at least one way.

**8.14**

- The waiting time can be reduced since there is less handling between the time the part is taken from storage to the time it is processed. Therefore, a worker may save time to perform the task, thus opening the worker to handling more machines.
- It may be that the method of transportation is simpler due to the fact that the storage/fixturing device is designed such that it can be transported without special equipment. In addition, the operator will not have to spend time arranging components.
- By utilizing a storage/fixturing device the operator is not tied up at a machine readying the machine for processing.
- By utilizing a storage/fixturing device the amount of manual operations needed to prepare a part for processing is reduced.

Therefore, at least 4 of the 7 wastes are reduced.

**SECTION 8.6**

**8.15** Research question: Response will be based on the paper chosen for review.

**8.16** Research question: Response will be based on the paper chosen for review.

**8.17** JIT and lean manufacturing have essentially the same objectives. Both strive for eliminating or minimize waste, produce only what is demanded, minimize the use of time and space resources, and manufacture in the shortest cycle time possible. Different companies have different name for JIT, at Hewlett-Packard it is called Stockless Production, Material as Needed at Harley-Davidson, Continuous Flow Manufacturing at IBM.

**8.18** It is not applicable to all types of manufacturing systems. Mass production is still the best process to use for high volume, repetitive products. JIT may also be difficult to implement to very low volume or unique products such as in a job shop environment unless there is flexibility in reordering the machine.

**8.19** JIT can be applied to continuous production systems since JIT requires a pull-based production system. Like discrete-part production systems, a continuous production system starts with a batch that is processed; however, the batch is processed in a continuous manner from process to process. So, the batch size can be determined by the actual demand, and therefore, be thought of as a pull system. By limiting the size of the batch the WIP is naturally limited in the system as well. However, it should be noted that most continuous production systems are used in very large scale production, so reducing batch sizes may reduce the utilization of the system.

**8.20** Research question. Response will be based on the paper chosen for review.

**8.21** Research question. Response will be based on the paper chosen for review.

**8.22** In a straight line-balancing problem, the set of assignable tasks is limited to that task whose predecessors have been assigned. In a U line balancing problem, the set of assignable task is enlarge by those tasks whose successors have been assigned, therefore U line balancing problem is more complex since now task grouping not only move in forward or backward direction as in a straight line, but it can also move in both directions at the same time.

In practice, rebalancing of the line is done quite often following demand changes. Rebalancing involves adding or removing machine from on the line or changing the standard time bases on new layout configuration; it also involves determining the number of operators required and assigning the machines that each operator tends.

**8.23** Quality problems can be communicated and addressed as they occur, thus leading to quicker resolution. The workers should be cross-trained so they can fix each others mistakes or aide in the quality resolution process.

**8.24** To evenly balance the processing time among the workers the following assignments would be made:

| Worker | Processes Assigned |
|---|---|
| 1 | 1, 2, 3, 9 |
| 2 | 5, 7 |
| 3 | 4, 6 |
| 4 | 8, 10 |

Each worker would have 60 units of processing time. There would be cases of overlap, where the one worker would obstruct the path of another worker.

**8.25** In this case each worker would be assigned 80 units of processing time. The assignments are as follows:

| Worker | Processes Assigned |
|---|---|
| 1 | 1, 2, 3, 10 |
| 2 | 4, 5 |
| 3 | 6, 7, 8, 9 |

In this case each worker is not obstructed by any other worker; therefore, it would be a preferable arrangement compared with that of the solution to Problem 8.24.

**8.26** The final placement order is 2-3-1-4 with a total cost of 912.

## SECTION 8.7

**8.27** Research question.

**8.28** Research question.

# Chapter 9

# *Facilities Systems*

## SECTION 9.2

**9.1** The statement, "The grid structure should defer to the function of the facility" is true. Some reasons why this is so are:

- In a warehouse facility, the grid spacing should be dictated by the rack dimensions and access aisles between racks
- In a parking garage, the grid element should be a multiple of the car bay width and length and the circulation needs
- As long as the structural integrity is not compromised, the dimensions of the building grid are secondary to the optimal layout of the facility
- Specific structural members other than 20' and 40' lengths can be obtained without much of a penalty cost

## SECTION 9.3

**9.2** The thermal performance of a facility can be improved by making better use of solar gain. This will greatly reduce the building's dependence on artificial atmospheric systems. Using double-paned windows with a reflective glass member can reduce the solar transmission through windows from 90% to 25%.

A good roof design will also help thermal performance. A membrane layer to prevent water penetration, an insulation layer to assist with thermal comfort, and a vapor check to stop vapor migration are all requirements of a good roof. The floor should have integral water-proofing and an applied membrane to seal the floor against water migration. The primary purpose of an enclosure system for a manufacturing facility is to keep out undesirables.

## SECTION 9.4

**9.3** Allocate the mechanical equipment room space:
Total Gross Area = 120 ft x 180 ft = 21,600 $ft^2$
Assume the system will be roof-mounted

Determine the air-handling requirements:
Air supply = 21,600 $ft^2$ x 1.0 $ft^3$/min-$ft^2$ = 21,600 $ft^3$/min (CFM)
Air exhaust = 21,600 $ft^2$ x 0.3 $ft^2$/min-$ft^2$ = 6,480 $ft^3$/min (CFM)

Determine Louver Supply and Exhaust sizes:
Louver Supply = 21,600 CFM / 1,000 ft/min = 21.6 $ft^2$
Louver Exhaust = 6,480 CFM / 2,000 ft/min = 3.24 $ft^2$

Determine Main Duct Sizing:
2 main ducts, each covering a half of the building's area, times the air supply rate:
0.5 x 21,600 $ft^2$ x 1.0 $ft^3$/min-$ft^2$ = 10,800 $ft^3$/min (CFM)
10,800 CFM / 1,800 ft/min = 6 $ft^2$

**9.4** $Q_F$ = (120,000 ft$^2$)[0.81 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_F$ = 5.83 x 10$^6$ Btu/hr
$Q_R$ = (120,000 ft$^2$)[0.20 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_R$ = 1.44 x 10$^6$ Btu/hr
$Q_D$ = (6) (24 ft$^2$)[1.13 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_D$ = 0.0098 x 10$^6$ Btu/hr
$Q_W$ = [(2)(8,000 ft$^2$)+(2)(6,000 ft$^2$)-(6)(24 ft$^2$)][0.39 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_W$ = 0.65 x 10$^6$ Btu/hr
$Q_I$ = [(120,000 ft$^2$)+(28,000 ft$^2$)][0.02 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_I$ = 0.18 x 10$^6$ Btu/hr

Using equation 9.1, the total heat loss may be calculated as follows:

$Q_H$ = 5.83 x 10$^6$ Btu/hr + 1.44 x 10$^6$ Btu/hr + 0.0098 x 10$^6$ Btu/hr + 0.65 x 10$^6$ Btu/hr + 0.18 x 10$^6$ Btu/hr
$Q_H$ = 8.11 x 10$^6$ Btu/hr

**9.5** $Q_F$ = (120,000 ft$^2$)[0.81 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_F$ = 5.83 x 10$^6$ Btu/hr
$Q_R$ = (120,000 ft$^2$)[0.20 Btu/(hr)(ft$^2$)(°F)](72°F – 12°F)
$Q_R$ = 1.44 x 10$^6$ Btu/hr
$Q_D$ = (6) (24 ft$^2$)[1.13 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_D$ = 0.0098 x 10$^6$ Btu/hr
$Q_G$ = (20)(32 ft2)[0.63 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_G$ = 0.024 x 10$^6$ Btu/hr
$Q_W$ = [(2)(8,000 ft$^2$)+(2)(6,000 ft$^2$)-(6)(24 ft$^2$)-(20)(32 ft$^2$)] [0.39 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_W$ = 0.64 x 10$^6$ Btu/hr
$Q_I$ = [(120,000 ft$^2$)+(28,000 ft$^2$)][0.02 Btu/(hr)(ft$^2$)(°F)](72°F – 12°F)
$Q_I$ = 0.18 x 10$^6$ Btu/hr

Using Equation 9.1, the total heat loss may be calculated as follows:

$Q_H$ = 5.83 x 10$^6$ Btu/hr + 1.44 x 10$^6$ Btu/hr + 0.0098 x 10$^6$ Btu/hr + 0.024 x 10$^6$ + 0.64 x 10$^6$ Btu/hr + 0.18 x 10$^6$ Btu/hr
$Q_H$ = 8.12 x 10$^6$ Btu/hr

**9.6** $Q_F$ = (120,000 ft$^2$)[0.81 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_F$ = 5.83 x 10$^6$ Btu/hr
$Q_R$ = (120,000 ft$^2$)[0.13 Btu/(hr)(ft$^2$)(°F)](72°F – 12°F)
$Q_R$ = 0.94 x 10$^6$ Btu/hr
$Q_D$ = (6)(24 ft$^2$)[1.13 Btu/(hr)(ft$^2$)(°F)](72°F – 12°F)
$Q_D$ = 0.0098 x 10$^6$ Btu/hr
$Q_W$ = [(2)(8,000 ft$^2$)+(2)(6,000 ft$^2$)-(6)(24 ft$^2$)][0.39 Btu/(hr)(ft$^2$)(°F)](72°F - 12°F)
$Q_W$ = 0.65 x 10$^6$ Btu/hr
$Q_I$ = [(120,000 ft$^2$)+(28,000 ft$^2$)][0.02 Btu/(hr)(ft$^2$)(°F)](72°F – 12°F)

$Q_I = 0.18 \times 10^6$ Btu/hr

Using Equation 9.1, the total heat loss maybe calculated as follows:

$Q_H = 5.83 \times 10^6$ Btu/hr + $0.94 \times 10^6$ Btu/hr + $0.0098 \times 10^6$ Btu/hr + $0.65 \times 10^6$ Btu/hr + $0.18 \times 10^6$ Btu/hr
$Q_H = 7.61 \times 10^6$ Btu/hr

**9.7** $Q_F = (120{,}000\ ft^2)[0.55\ Btu/(hr)(ft^2)(^oF)](72^oF - 12^oF)$
$Q_F = 3.96 \times 106$ Btu/hr
$Q_R = (120{,}000\ ft^2)[0.20\ Btu/(hr)(ft^2)(^oF)](72^oF – 12^oF)$
$Q_R = 1.44 \times 10^6$ Btu/hr
$Q_D = (6)(24\ ft^2)[1.13\ Btu/(hr)(ft^2)(^oF)](72^oF – 12^oF)$
$Q_D = 0.0098 \times 10^6$ Btu/hr
$Q_W = [(2)(8{,}000\ ft^2)+(2)(6{,}000\ ft^2)-(6)(24\ ft^2)][0.39\ Btu/(hr)(ft^2)(^oF)](72^oF - 12^oF)$
$Q_W = 0.65 \times 10^6$ Btu/hr
$Q_I = [(120{,}000\ ft^2)+(28{,}000\ ft^2)][0.02\ Btu/(hr)(ft^2)(^oF)](72^oF – 12^oF)$
$Q_I = 0.18 \times 10^6$ Btu/hr

Using Equation 9.1, the total heat loss maybe calculated as follows:

$Q_H = 3.96 \times 10^6$ Btu/hr + $1.44 \times 10^6$ Btu/hr + $0.0098 \times 10^6$ Btu/hr + $0.65 \times 10^6$ Btu/hr + $0.18 \times 10^6$ Btu/hr
$Q_H = 6.24 \times 10^6$ Btu/hr

**9.8** $Q_F = (120{,}000\ ft^2)[0.81\ Btu/(hr)(ft^2)(^oF)](98^oF - 78^oF)$
$Q_F = 19.4 \times 10^5$ Btu/hr
$Q_R = (120{,}000\ ft^2)[0.20\ Btu/(hr)(ft^2)(^oF)](98^oF – 78^oF)$
$Q_R = 4.80 \times 10^5$ Btu/hr
$Q_D = (6)(24\ ft^2)[1.13\ Btu/(hr)(ft^2)(^oF)](98^oF – 78^oF)$
$Q_D = 0.033 \times 10^5$ Btu/hr
$Q_W = [(2)(8{,}000\ ft^2)+(2)(6{,}000\ ft^2)-(6)(24\ ft^2)][0.39\ Btu/(hr)(ft^2)(^oF)](98^oF - 78^oF)$
$Q_W = 2.17 \times 10^5$ Btu/hr
$Q_I = [(120{,}000\ ft^2)+(28{,}000\ ft^2)][0.02\ Btu/(hr)(ft^2)(^oF)](98^oF – 78^oF)$
$Q_I = 0.59 \times 10^5$ Btu/hr
$Q_S = \{[(3)(24\ ft^2)][110\ Btu/(hr)(ft^2)(^oF)] + [(3)(24\ ft^2)][55\ Btu/(hr)(ft^2)(^oF)]\}(98^oF – 78^oF)$
$Q_S = 2.37 \times 10^5$ Btu/hr
$Q_L = (700)(60\ watt)[4.25\ Btu/(hr)(watt)]$
$Q_L = 1.79 \times 10^5$ Btu/hr
$Q_P = (50)(1{,}500\ Btu/hr)+(60)(700\ Btu/hr)$
$Q_P = 1.17 \times 10^5$ Btu/hr

Using Equation 9.3, the total cooling load may be calculated as follows:

$Q_C = 19.4 \times 10^5$ Btu/hr + $4.80 \times 10^5$ Btu/hr + $0.033 \times 10^5$ Btu/hr + $2.17 \times 10^5$ Btu/hr

$+ 0.59 \times 10^5$ Btu/hr + $2.37 \times 10^5$ Btu/hr + $1.79 \times 10^5$ Btu/hr + $1.17 \times 10^5$ Btu/hr
$Q_C = 32.32 \times 10^5$ Btu/hr

**9.9** $Q_F$ = (120,000 ft$^2$)[0.81 Btu/(hr)(ft$^2$)(°F)](98°F - 78°F)
$Q_F = 19.4 \times 10^5$ Btu/hr
$Q_R$ = (120,000 ft$^2$)[0.20 Btu/(hr)(ft$^2$)(°F)](98°F – 78°F)
$Q_R = 4.80 \times 10^5$ Btu/hr
$Q_D$ = (6)(24 ft$^2$)[1.13 Btu/(hr)(ft$^2$)(°F)](98°F – 78°F)
$Q_D = 0.033 \times 10^5$ Btu/hr
$Q_G$ = (10)(32 ft$^2$)[0.63 Btu/(hr)(ft$^2$)(°F)](98°F – 78°F)
$Q_G = 0.040 \times 10^5$ Btu/hr
$Q_W$ = [(2)(8,000 ft$^2$)+(2)(6,000 ft$^2$)-(6)(24 ft$^2$)-(10)(32 ft$^2$)]
[0.39 Btu/(hr)(ft$^2$)(°F)](98°F - 78°F)
$Q_W = 2.15 \times 10^5$ Btu/hr
$Q_V$ = [(120,000 ft$^2$)+(28,000 ft$^2$)][0.02 Btu/(hr)(ft$^2$)(°F)](98°F – 78°F)
$Q_V = 0.59 \times 10^5$ Btu/hr
$Q_S$ = {[(3)(24 ft$^2$)+(5)(32 ft$^2$)][110 Btu/(hr)(ft$^2$)(°F)] + [(3)(24 ft$^2$)
+(5)(32 ft$^2$)][55 Btu/(hr)(ft$^2$)(°F)]}(98°F – 78°F)
$Q_S = 7.65 \times 10^5$ Btu/hr
$Q_L$ = (700)(60 watt)[4.25 Btu/(hr)(watt)]
$Q_L = 1.79 \times 10^5$ Btu/hr
$Q_P$ = (50)(1,500 Btu/hr)+(60)(700 Btu/hr)
$Q_P = 1.17 \times 10^5$ Btu/hr

Using Equation 9.3, the total cooling load may be calculated as follows:

$Q_C = 19.4 \times 10^5$ Btu/hr + $4.80 \times 10^5$ Btu/hr + $0.033 \times 10^5$ Btu/hr + $0.040 \times 10^5$ Btu/hr
+ $2.15 \times 10^5$ Btu/hr + $0.59 \times 10^5$ Btu/hr + $7.65 \times 10^5$ Btu/hr + $1.79 \times 10^5$ Btu/hr
+ $1.17 \times 10^5$ Btu/hr
$Q_C = 37.62 \times 10^5$ Btu/hr

**9.10** $Q_F$ = (120,000 ft$^2$)[0.81 Btu/(hr)(ft$^2$)(°F)](98°F - 78°F)
$Q_F = 19.4 \times 10^5$ Btu/hr
$Q_R$ = (120,000 ft$^2$)[0.20 Btu/(hr)(ft$^2$)(°F)](98°F – 78°F)
$Q_R = 4.80 \times 10^5$ Btu/hr
$Q_D$ = (6)(24 ft$^2$)[1.13 Btu/(hr)(ft$^2$)(°F)](98°F – 78°F)
$Q_D = 0.033 \times 10^5$ Btu/hr
$Q_G$ = (10)(32 ft$^2$)[0.63 Btu/(hr)(ft$^2$)(°F)](98°F – 78°F)
$Q_G = 0.040 \times 10^5$ Btu/hr
$Q_W$ = [(2)(8,000 ft$^2$)+(2)(6,000 ft$^2$)-(6)(24 ft$^2$)-(10)(32 ft$^2$)]
[0.39 Btu/(hr)(ft$^2$)(°F)]*(98°F - 78°F)
$Q_W = 2.15 \times 10^5$ Btu/hr
$Q_V$ = [(120,000 ft$^2$)+(28,000 ft$^2$)][0.02 Btu/(hr)(ft$^2$)(°F)](98°F – 78°F)
$Q_V = 0.59 \times 10^5$ Btu/hr
$Q_S$ = {[(3)(24 ft$^2$)+(5)(32 ft$^2$)][110 Btu/(hr)(ft$^2$)(°F)] + [(3)(24 ft$^2$) +(5)(32 ft$^2$)]
[55 Btu/(hr)(ft$^2$)(°F)]}(0.3)(98°F – 78°F)

$Q_S = 2.30 \times 10^5$ Btu/hr
$Q_L = (700)(60 \text{ watt})[4.25 \text{ Btu/(hr)(watt)}]$
$Q_L = 1.79 \times 10^5$ Btu/hr
$Q_P = (50)(1{,}500 \text{ Btu/hr})+(60)(700 \text{ Btu/hr})$
$Q_P = 1.17 \times 10^5$ Btu/hr

Using Equation 9.3, the total cooling load may be calculated as follows:

$Q_C = 19.4 \times 10^5$ Btu/hr + $4.80 \times 10^5$ Btu/hr + $0.033 \times 10^5$ Btu/hr + $0.040 \times 10^5$ Btu/hr
+ $2.15 \times 10^5$ Btu/hr + $0.59 \times 10^5$ Btu/hr + $2.30 \times 10^5$ Btu/hr + $1.79 \times 10^5$ Btu/hr
+ $1.17 \times 10^5$ Btu/hr
$Q_C = 37.27 \times 10^5$ Btu/hr

If blinds are used, the cooling load required would be $Q_C = 35.26 \times 10^5$ Btu/hr

## SECTION 9.5

**9.11** Step 1: Determine the Level of Illumination
From Table 9.5, for a regular office environment, the minimum level of illumination is 100 foot-candles.

Step 2: Determine the Room Cavity Ratio
Using Equation 9.8: RCR = [(5)(10 – 3)(250 + 150)]/[(250)(150)] = 0.373

Step 3: Determine the Ceiling Cavity Ratio
Since the lights are ceiling mounted, the reflective property of the ceiling will not be impacted by the luminaries' mounting height.

Step 4: Determine the Wall Reflection (WR) and the Effective Ceiling Reflectance (ECR)
From Table 9.6, since the walls and ceiling are painted white, the WR and the base ceiling reflectance (BCR) are 80%. Since the lights are ceiling mounted, BCR = ECR = 80%.

Step 5: Determine the Coefficient of Utilization (CU)
From Table 9.8, since the ECR and the WR are 80%, one can extrapolate from the table for an RCR value of 0.373. The CU is approximately 0.68 and will be the value utilized.

Step 6: Determine the Light Loss Factor (LLF)
From Table 9.10, for fluorescent lamps in prismatic lens fixtures and six months between cleanings, the LLF = 0.92.

Step 7: Calculate the Number of the Laps and Luminaries
From Table 9.9, Lamp output at 70% of rated life for an 85-watt lamp is 5,400 lumens. Using Equation 9.10: Number of Lamps = [(100)(250)(150)]/[(0.68)(0.92)(5,400)] = 1,110 Lamps. If there are 2 lamps per luminary, then 555 luminaries are required for the facility.

Step 8: Determine the Location of the Luminaries
From Table 9.8, Column 2, the spacing between banks of luminaries is equal to 1.2 times the mounting height, or 12 feet between banks. In a 250'X150' office, 20 banks of luminaries spaced 12 feet apart can be mounted along the length of the facility. Since each fluorescent lamp is 4' long, 37 luminaries per bank can be placed along the width of the facility for a total of 740 luminaries, exceeding the required amount of 555 luminaries determined in step 7.

**9.12** The changes in the facility would be that the minimum level of illumination would be 150 foot-candles. This would make the number of lamps and luminaries change as well, as shown in the following equation:

Number of Lamps = [(150)(250)(150)]/[(0.68)(0.92)(5,400)] = 1,665 Lamps
Number of Luminaries = 1,665/2 = 833 luminaries

The spacing between banks of luminaries must be reduced from 12 feet to 10.5 feet to get the required 833 luminaries within the facility. This will allow 851 luminaries to be placed within the facility.

**9.13** From equation 9.8, the RCR changes from 0.373 to 0.64. This result changes the coefficient of utilization in Table 9.8 slightly to 0.67. This changes the number of lamps, as shown in the following equation:

Number of Lamps = [(100)(250)(150)]/[(0.67)(0.92)(5,400)] = 1,127 Lamps
Number of Luminaries = 1,127/2 = 564 luminaries

The spacing between banks of luminaries can remain 12 feet to get the required 564 luminaries within the facility, and must not exceed the 12-foot spacing. This result yields 750 luminaries that can be placed within the facility.

**9.14** The Ceiling Cavity Ratio (CCR) would have to be determined because the luminaries are no longer mounted on recessed into the ceiling. From Equation 9.8:

CCR = [(5)(0.373)]/(7) = 0.266

Because the CCR has to be considered in this problem the ECR is no longer equivalent to the BCR. The ECR is determined by examining Table 9.7 and by interpolation is found to be 0.78. Since the ECR changed, this affects the coefficient of utilization slightly. The CU decreases to 0.67 by extrapolation using Table 9.8. As in Problem 9.3, the number of luminaries required is 564, and 12-foot spacing between banks with 37 luminaries per bank will allow 750 luminaries to be placed within the facility.

**9.15** If the walls and ceiling were painted a medium color, then the WR and ECR would change to 50% instead of 80%. This changes the value of the CU to 0.62, using extrapolation from Table 9.8. This changes the number of lamps required to:

Number of Lamps = [(100)(250)(150)]/[(0.62)(0.92)(5,400)] = 1,217 Lamps
Number of Luminaries = 1,217/2 = 609

The number of luminaries required is 609, and 12-foot spacing between banks with 37 luminaries per banks will allow 750 luminaries to be placed within the facility.

**9.16** If the fixtures were cleaned once every 36 months instead of once every six months, the LLF value changes from 0.92 to 0.78. This changes the number of lamps to:

Number of Lamps = [(100)(250)(150)]/[(0.62)(0.78)(5,400)] = 1,436 Lamps
Number of Luminaries = 1,436/2 = 718

The number of luminaries required is 718, and 12-foot spacing between banks with 37 luminaries per banks will allow 750 luminaries to be placed within the facility.

**SECTION 9.6**

**9.17** The required fire protection in a building, which is governed by the Uniform Building Code (UBC). There are 20 classifications or groups of buildings governed by the UBC.

**9.18a** Note: The correct figure to use as a reference for this problem is Figure 9.12.
Maximum Population = [120 ft x 200 ft]/[20 $ft^2$/person] = 1200 people

**9.18b** Minimum Number of Exits (from Table 9.12) = 4

**9.18c** Minimum Distance Between Exits = $[((200)^2 + (120)^2)^{0.5}]/2$ = 117 feet

**9.18d** The drawing should have two exits on each 200’ side of the building.

**9.19** The major benefit of using a sprinkler system is that it is a very effective means of extinguishing or controlling a fire in its early stages and it can provide protection when the building is not occupied.

**9.20** 1. Safety of regular building occupants
2. Safety of firefighters
3. Salvage of the building
4. The goods and equipment in the building

**9.21** The main purpose of face sprinklers in an in-rack sprinkler system is to oppose vertical development of fire on the external face of the rack.

**SECTION 9.7**

**9.22** The purpose of a trap in a sewage system is to prevent odor from passing back into the facility.

**9.23** Vent stacks can be used to improve water-seal integrity and to provide assurance that smells are effectively removed from the facility and vented to the atmosphere.

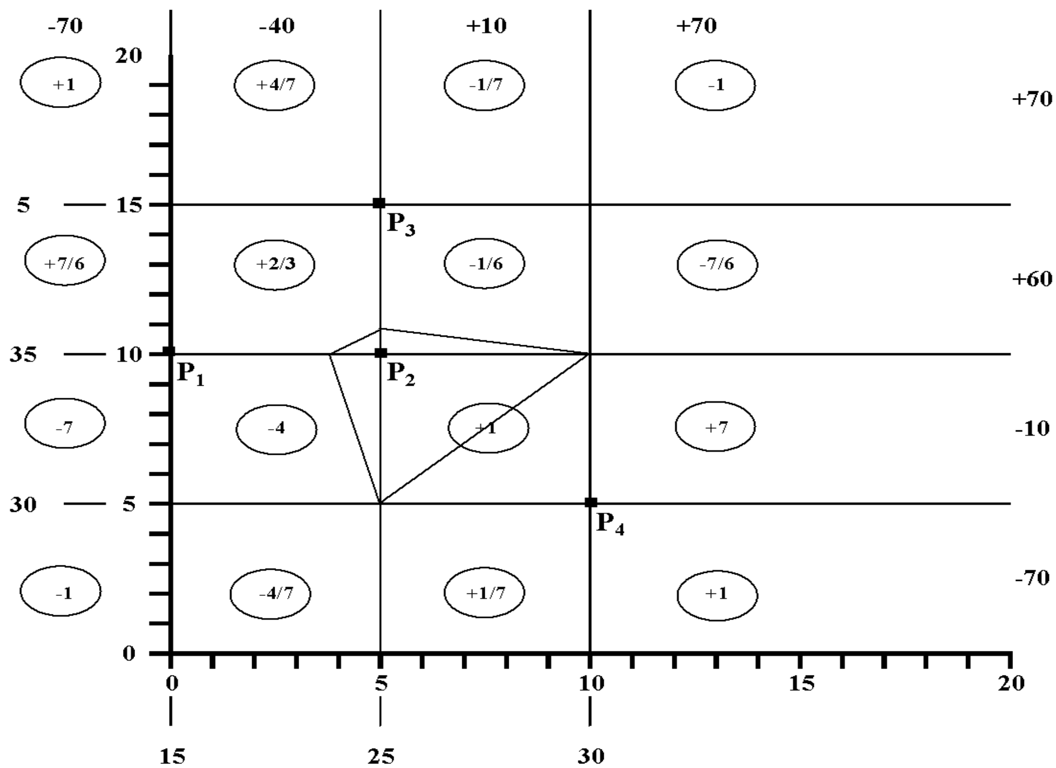
# Chapter 10

# *Quantitative Facilities Planning Models*

## SECTION 10.2

**10.1** Using the half-sum method: half of the sum of the weights equals 35

| **x-coord** | $w_i$ | $\Sigma w_i$ | **y-coord** | $w_i$ | $\Sigma w_i$ |
|---|---|---|---|---|---|
| 0 | 15 | 15 | 5 | 30 | 30 |
| 5 | 25 | 40>35 | 10 | 35 | 65>35 |
| 10 | 30 | 70 | 15 | 5 | 70 |




($\mathbf{x^*}$,$\mathbf{y^*}$) = (5,10), which is the location of existing facility 2

f(10,10) = 15(|10-0|+|10-10|) + 20(|10-5|+|10-10|) + 5(|10-5|+|10-15|)
+ 30(|10-10|+|10-5|)

f(10,10) = 450

**10.2** Using the half-sum method: half of the sum of the weights equals 15

| **x-coord** | $\mathbf{w_i}$ | $\mathbf{\Sigma w_i}$ | **y-coord** | $\mathbf{w_i}$ | $\mathbf{\Sigma w_i}$ |
|---|---|---|---|---|---|
| -5 | 8 | 8 | -3 | 3 | 3 |
| -3 | 3 | 11 | -1 | 8 | 11 |
| 1 | 4+3=7 | 18>15 | 0 | 4 | 15=15 |
| 2 | 7 | 25 | 5 | 7 | 22 |
| 3 | 5 | 30 | 6 | 3 | 25 |
| | | | 8 | 5 | 30 |

**x*** = 1, **y*** = [0,5]. Any point on the defined line segment is an optimum location.

**10.3a** Using the half-sum method; half of the sum of the weights equals 1725

| Hospital $i$ | Coordinate $a_i$ | Weight $w_i$ | |
|---|---|---|---|
| 3 | 8 | 300 | 300 |
| 1 | 10 | 450 | 750 < 1725 |
| 2 | 14 | 1200 | 1950 > 1725 |
| 4 | 32 | 1500 | 3450 |
| | x* = 14 | | |

| Hospital $i$ | Coordinate $b_i$ | Weight $w_i$ | |
|---|---|---|---|
| 3 | 4 | 300 | 300 < 1725 |
| 4 | 6 | 1500 | 1800 > 1725 |
| 2 | 12 | 1200 | 3000 |
| 1 | 20 | 450 | 3450 |
| | y* = 6 | | |

**10.3b** Using the majority algorithm (or Chinese algorithm), locate at x = 26 and y = 15.

**10.4** Using Excel's SOLVER tool,

| Machine | Coordinates | | Squared Difference | | Weight |
|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $(x - a_i)^2$ | $(y - b_i)^2$ | $w_i$ |
| 1 | 0 | 10 | 36.86 | 3.19 | 15 |
| 2 | 5 | 10 | 1.15 | 3.19 | 20 |
| 3 | 5 | 15 | 1.15 | 46.05 | 5 |
| 4 | 10 | 5 | 15.43 | 10.33 | 30 |

| Decision Variables | | Obj Fn | |
|---|---|---|---|
| $x$ | $y$ | f(X*) | |
| 5.00000 | 10.00000 | 312 | SOLVER solution |

Since SOLVER converged on an existing facility location, we check the necessary and sufficient conditions for the optimum solution to the Weber problem to be an existing facility location, say $(a_r,b_r)$:

$$\left(\sum_{i\neq r}\frac{w_i(a_r-a_i)}{d_i(a_r,b_r)}\right)^2+\left(\sum_{i\neq r}\frac{w_i(b_r-b_i)}{d_i(a_r,b_r)}\right)^2\leq w_r^2$$

$$\left(\frac{15(5-0)}{5}+\frac{5(5-5)}{5}+\frac{30(5-10)}{7.0711}\right)^2+\left(\frac{15(10-10)}{5}+\frac{5(10-15)}{5}+\frac{30(10-5)}{7.0711}\right)^2=301.4675<20^2=400$$

The necessary and sufficient condition is satisfied, therefore (x*,y*) = (5,10).

**10.5** Solving mathematically,
x* = [15(0)+25(5)+30(10)]/(15+20+5+30) = 6.07143
y* = [15(10)+20(10+5(15)+30(5)]/70 = 8.21429
Solving with Excel's SOLVER tool,

| Machine | Coordinates | | Squared Difference | | Weight |
|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $(x - a_i)^2$ | $(y - b_i)^2$ | $w_i$ |
| 1 | 0 | 10 | 36.86 | 3.19 | 15 |
| 2 | 5 | 10 | 1.15 | 3.19 | 20 |
| 3 | 5 | 15 | 1.15 | 46.05 | 5 |
| 4 | 10 | 5 | 15.43 | 10.33 | 30 |

| Decision Variables | | Obj Fn | |
|---|---|---|---|
| $x$ | $y$ | f(X*) | |
| 6.07143 | 8.21428 | 1,696 | SOLVER solution |

**10.6** Using the half-sum method: half of the sum of the weights equals 3.

| x-coord | $w_i$ | $\Sigma w_i$ | y-coord | $w_i$ | $\Sigma w_i$ |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 1 | 1 |
| 20 | 1 | 2 | 20 | 1 | 2 |
| 30 | 1 | 3=3 | 40 | 1 | 3=3 |
| 60 | 1 | 4 | 70 | 1 | 4 |
| 70 | 1 | 5 | 80 | 1 | 5 |
| 90 | 1 | 6 | 90 | 1 | 6 |

**x*** = [30,60], **y*** = [40,70]. Any point in the defined square is an optimum location.

**10.7a** Using the half-sum method: half the sum is 5.

| Station(s) $i$ | Coordinate $a_i$ | Weight $w_i$ | |
|---|---|---|---|
| 1 & 2 | 4 | 2 | 2 |
| 3 & 4 | 6 | 2 | 4 < 5 |
| 5 & 9 | 8 | 2 | 6 > 5 |
| 7 & 10 | 10 | 2 | 8 |
| 6 & 8 | 12 | 2 | 10 |
| | x* = 8 | | |

**10.7b** Using the half-sum method: half of the sum is 10.

| Station(s) $i$ | Coordinate $a_i$ | Weight $w_i$ | |
|---|---|---|---|
| 1 & 2 | 4 | 2 | 2 |
| 3 & 4 | 6 | 2 | 4 < 5 |
| 5 & 9 | 8 | 2 | 6 > 5 |
| 7 & 10 | 10 | 2 | 8 |
| 6 & 8 | 12 | 2 | 10 |
| | x* = 10 | | |

**10.7c**

| Work Station ($i$) | Coordinates $a_i$ | $b_i$ | Addends $g_i$ | $a_i+b_i-g_i$ | $a_i+b_i+g_i$ | $-a_i+b_i-g_i$ | $-a_i+b_i+g_i$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 10 | 6 | 8 | 20 | 0 | 12 |
| 2 | 4 | 10 | 2 | 12 | 16 | 4 | 8 |
| 3 | 6 | 10 | 6 | 10 | 22 | 0 | 10 |
| 4 | 6 | 10 | 2 | 14 | 18 | 2 | 6 |
| 5 | 8 | 10 | 2 | 16 | 20 | 2 | 4 |
| 6 | 12 | 10 | 2 | 20 | 24 | 0 | 0 |
| 7 | 10 | 10 | 4 | 16 | 24 | -6 | 4 |
| 8 | 12 | 10 | 4 | 18 | 26 | -4 | 2 |
| 9 | 8 | 10 | 4 | 14 | 22 | -6 | 6 |
| 10 | 10 | 10 | 6 | 14 | 26 | -6 | 6 |
| | | | Min: | 8 | | -6 | |
| | | | Max: | | 26 | | 12 |

$c_1 = 8$, $c_2 = 26$, $c_3 = -6$, $c_4 = 12$, $c_5 = 18$, $x_1^* = 7$, $x_2^* = 7$, $y_1^* = 10$, $y_2^* = 10$, $x^* = 7$.

**10.7d** Using the majority theorem, locate at $x^* = 10$, $y^* = 10$. (Note: the vertical line from (6,10) to (6,12) is not supposed to be present. This was an error that was created in production that was not caught during the editing process.)

**10.7e** The same answer occurs for d) and e): $x^* = y^* = 10$.

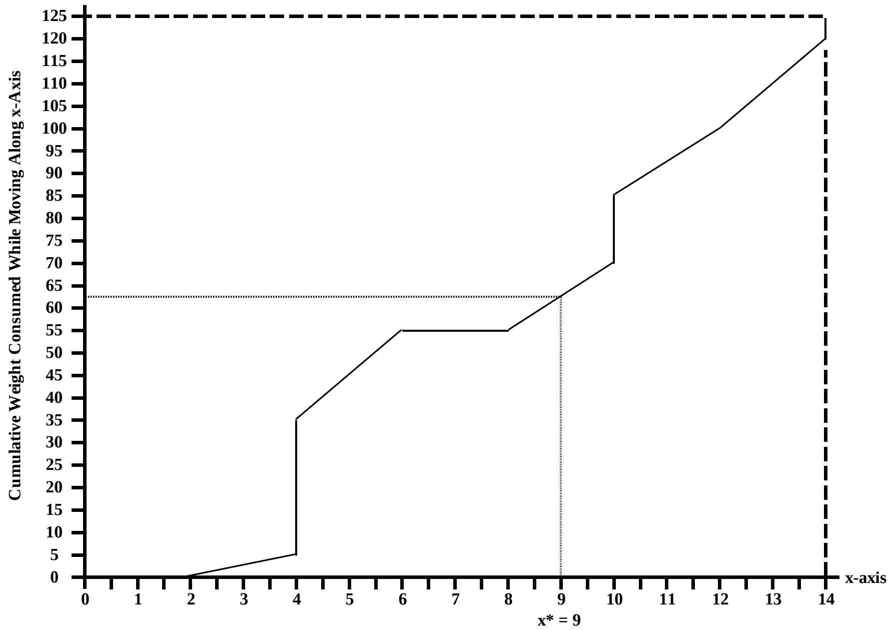
**10.8** It is desired to determine the unweighted minimax location, i.e., it is important that the farthest house be as close as possible, regardless of the monetary value of the house.

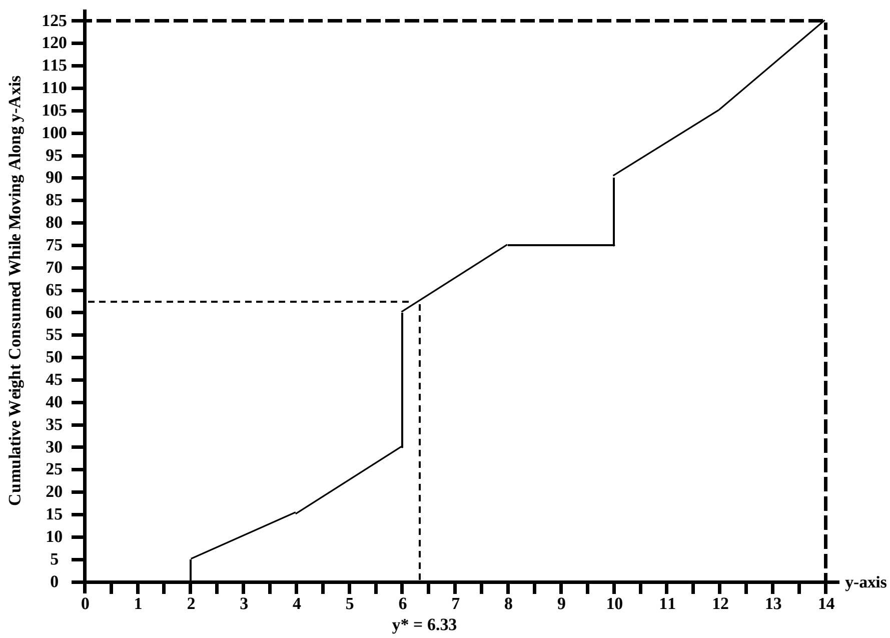
| a | b | a + b | -a + b |
|---|---|---|---|
| 20 | 15 | 35 | -5 |
| 25 | 25 | 50 | 0 |
| 13 | 32 | 45 | 19 |
| 25 | 14 | 39 | -11 |
| 4 | 21 | 25 | 17 |
| 18 | 8 | 26 | -10 |

$c_1 = 25$, $c_2 = 50$, $c_3 = -11$, $c_4 = 19$, $c_5 = \max(c_2-c_1, c_4-c_3) = \max(25, 30) = 30$

The minimax locations are all the points on the line segment joining the points:
$(x_1^*, y_1^*) = ½(c_1-c_3, c_1+c_3+c_5) = ½(36,44) = (18,22)$
$(x_2^*, y_2^*) = ½(c_2-c_4, c_2+c_4-c_5) = ½(31,39) = (15.5,19.5)$

**10.9a** Since the weights are uniformly distributed over the indicated regions, the optimum x-coordinate location will be the point at which no more than half the weight is to the left of the point and no more than half the weight is to the right of the point; likewise, the optimum y-coordinate is the point at which no more than half the weight is below the point and no more than half is above the point. Show below are plots of the cumulative weights along each axis.




The optimum location (x*,y*) = (9, 6.33) is located in A4.

**10.9b** Since the facility cannot be located in A4, we search the border of A4. First, consider y = 4; we know x* = 9 because of the separability of the objective function in x and y. We evaluate f(9,4) =417.5 + 515 = 932.5. Next, consider y = 8 and x = 9, f(9,8) = 417.5 + 375 = 792.5. Finally, consider x = 8 and y = 19/3, f(8,19/3) = 425 + 354.1667 = 779.1667. Therefore, x* = 8 and y* = 6.333.

**10.9c** Using Excel's SOLVER tool,

| EF | Coordinates | | Absolute Difference | | Rectilinear | Weight | |
|---|---|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $\lvert x_1 - a_i \rvert$ | $\lvert y_1 - b_i \rvert$ | Distance | $w_i$ | $w_i d_i$ |
| A1 | 1.5 | 1.5 | 9.10 | 5.10 | 14.20 | 10 | 142.0 |
| A2 | 4 | 12 | 6.60 | 5.40 | 12.00 | 15 | 180.0 |
| A3 | 14 | 14 | 3.40 | 7.40 | 10.80 | 20 | 216.0 |
| A4 | 12 | 4 | 1.40 | 2.60 | 4.00 | 30 | 120.0 |
| P1 | 4 | 6 | 6.60 | 0.60 | 7.20 | 30 | 216.0 |
| P2 | 10 | 10 | 0.60 | 3.40 | 4.00 | 15 | 60.0 |
| P3 | 14 | 2 | 3.40 | 4.60 | 8.00 | 5 | 40.0 |

| Decision Variables | | Obj Fn |
|---|---|---|
| $x$ | $y$ | f(X*) |
| 10.60 | 6.60 | 216.0 |

Note: Excel's SOLVER tool often converges to a location that is not the optimal location. For the example, an optimal location was obtained. However, there are multiple optimum solutions, as can be determined by using various starting solutions with SOLVER.

**10.9d** Since the minimax location is to be determined with all points equally weighted, the rectangular regions can be represented by their corners. Hence,

| a | b | a + b | -a + b |
|---|---|---|---|
| 2 | 2 | 4 | 0 |
| 2 | 4 | 6 | 2 |
| 4 | 6 | 10 | 2 |
| 4 | 10 | 14 | 6 |
| 4 | 12 | 16 | 8 |
| 6 | 2 | 8 | -4 |
| 6 | 4 | 10 | -2 |
| 6 | 10 | 16 | 4 |
| 6 | 12 | 18 | 6 |

| a | b | a + b | -a + b |
|---|---|---|---|
| 8 | 4 | 12 | -4 |
| 8 | 8 | 16 | 0 |
| 10 | 10 | 20 | 0 |
| 12 | 4 | 16 | -8 |
| 12 | 8 | 20 | -4 |
| 12 | 12 | 24 | 0 |
| 12 | 14 | 26 | 2 |
| 14 | 2 | 16 | -12 |
| 14 | 12 | 26 | -2 |
| 14 | 14 | 28 | 0 |

$c_1 = 4$, $c_2 = 28$, $c_3 = -12$, $c_4 = 8$, $c_5 = \max(c_2-c_1, c_4-c_3) = \max(24, 20) = 24$
The minimax locations are all the points on the line segment joining the points:
$(x_1^*, y_1^*) = ½(c_1-c_3, c_1+c_3+c_5) = ½(16,16) = (8,8)$
$(x_2^*, y_2^*) = ½(c_2-c_4, c_2+c_4-c_5) = ½(20,12) = (10,6)$
Hence, the optimum location is inside region A4.

**10.10a** Since there are only 3 sites to consider, enumeration is the quickest means of solving the problem.

f(50,50) = 600(|50-20|+|50-25|) + 400(|50-36|+|50-18|) + 500(|50-62|+|50-37|)
+ 300(|50-50|+|50-56|) + 200(|50-25|+|50-0|) = 80,700
f(30,45) = 600(|30-20|+|45-25|) + 400(|30-36|+|45-18|) + 500(|30-62|+|45-37|)
+ 300(|30-50|+|45-56|) + 200(|30-25|+|40-0|) = **70,500**
f(65,28) = 600(|65-20|+|28-25|) + 400(|65-36|+|28-18|) + 500(|65-62|+|28-37|)
+ 300(|65-50|+|28-56|) + 200(|65-25|+|28-0|) = 76,900
(**x***,**y***) = (30,45)

**10.10b** With weights:
f(50,50) = max[600(|50-20|+|50-25|), 400(|50-36|+|50-18|), 500(|50-62|+|50-37|),
300(|50-50|+|50-56|), 200(|50-25|+|50-0|)] = 33,000
f(30,45) = max[600(|30-20|+|45-25|), 400(|30-36|+|45-18|), 500(|30-62|+|45-37|),
300(|30-50|+|45-56|), 200(|30-25|+|40-0|)] = **20,000**
f(65,28) = max[600(|65-20|+|28-25|), 400(|65-36|+|28-18|), 500(|65-62|+|28-37|),
300(|65-50|+|28-56|), 200(|65-25|+|28-0|)] = 28,800
(**x***,**y***) = (30,45)

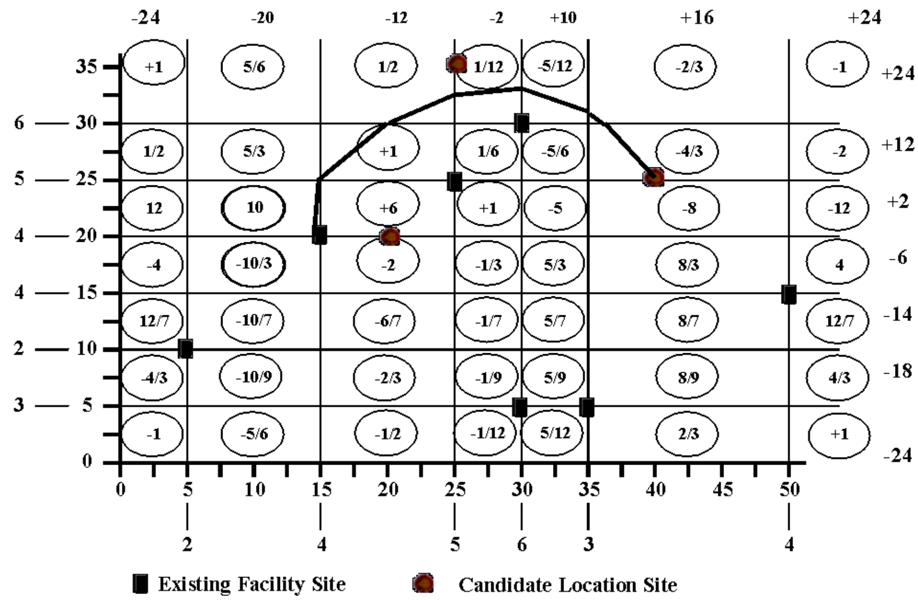
Without weights:
f(50,50) = max[(|50-20|+|50-25|), (|50-36|+|50-18|), (|50-62|+|50-37|), (|50-50|+|50-56|),
(|50-25|+|50-0|)] = 75
f(30,45) = max[(|30-20|+|45-25|), (|30-36|+|45-18|), (|30-62|+|45-37|), (|30-50|+|45-56|),
(|30-25|+|40-0|)] = **50**
f(65,28) = max[(|65-20|+|28-25|), (|65-36|+|28-18|), (|65-62|+|28-37|), (|65-50|+|28-56|),
(|65-25|+|28-0|)] = 68
(**x***,**y***) = (30,45)

**10.11a** Using the half-sum method: half of the sum of the weights equals 1200.

| **x-coord** | **$w_i$** | **$\sum w_i$** | **y-coord** | **$w_i$** | **$\sum w_i$** |
|---|---|---|---|---|---|
| 5 | 200 | 200 | 5 | 300 | 300 |
| 15 | 400 | 600 | 10 | 200 | 500 |
| 25 | 500 | 1100 | 15 | 400 | 900 |
| 30 | 600 | 1700 > 1200 | 20 | 400 | 1300 > 1200 |
| 35 | 300 | 200 | 25 | 500 | 1800 |
| 50 | 400 | 2400 | 30 | 600 | 2400 |

(**x***, **y***) = (30,20)

**10.11b** Shown below is a partial contour line passing through the Building 2 location. Building 1 is inside the contour line and Building 3 is outside the contour line. The least cost location is Building 1: (20,20).




**10.11c** Enumerating the weighted maximum rectilinear distance for each of the three feasible locations gives:

| Building | Decision Variables *x* | *y* | Obj. Fcn *f(X*)* |
|---|---|---|---|
| 1 | 20 | 20 | 14,000 |
| 2 | 40 | 25 | 12,000 |
| 3 | 25 | 35 | 18,000 |

Building 2 is preferred. Note: if the print shop could be placed at any location, using various starting solutions with SOLVER, we obtained the following solutions having a maximum weighted distance of 8,400: (**x***, **y***) = (30.0,16.0); (**x***, **y***) = (31.0,17.0; (**x***, **y***) = (32.0,18.0); (**x***, **y***) = (33.0,19.0); and (**x***, **y***) = (35.00,21.00). All points on the straight line connecting (30,16) and (35,21) are optimal solutions, as confirmed by an exact solution procedure not included in the text.

Enumerating the unweighted maximum rectilinear distance for each of the three feasible locations gives:

| Building | Decision Variables *x* | *y* | Obj. Fcn *f(X*)* |
|---|---|---|---|
| 1 | 20 | 20 | 35 |
| 2 | 40 | 25 | 50 |
| 3 | 25 | 35 | 45 |

Building 1 is preferred. Note: solving for the unconstrained optimum location gives any point on the line segment connecting (25,15) and (30,10) for a maximum unweighted distance of 25.

| Center ($i$) | Coordinates $a_i$ | $b_i$ | $a_i + b_i$ | $-a_i + b_i$ |
|---|---|---|---|---|
| A | 5 | 10 | 15 | 5 |
| B | 50 | 15 | 65 | -35 |
| C | 25 | 25 | 50 | 0 |
| D | 35 | 5 | 40 | -30 |
| E | 15 | 20 | 35 | 5 |
| F | 30 | 30 | 60 | 0 |
| | | Min: | 15 | -35 |
| | | Max: | 65 | 5 |

$c_1 = 15$, $c_2 = 65$, $c_3 = -35$, $c_4 = 5$, $c_5 = 50$, $x_1^* = 25$, $x_2^* = 30$, $y_1^* = 15$, $y_2^* = 10$.

**10.11d** Enumerating the weighted sum of squared Euclidean distances for each of the three feasible locations gives:

| Building | Decision Variables $x$ | $y$ | Obj. Fcn $f(X^*)$ |
|---|---|---|---|
| 1 | 20 | 20 | 725,000 |
| 2 | 40 | 25 | 945,000 |
| 3 | 25 | 35 | 1,125,000 |

Building 1 is preferred. Note: the unconstrained squared Euclidean optimum location is (**x***, **y***) = (28.33,20.00).

**10.11e** Enumerating the weighted sum of Euclidean distances for each of the three feasible locations gives:

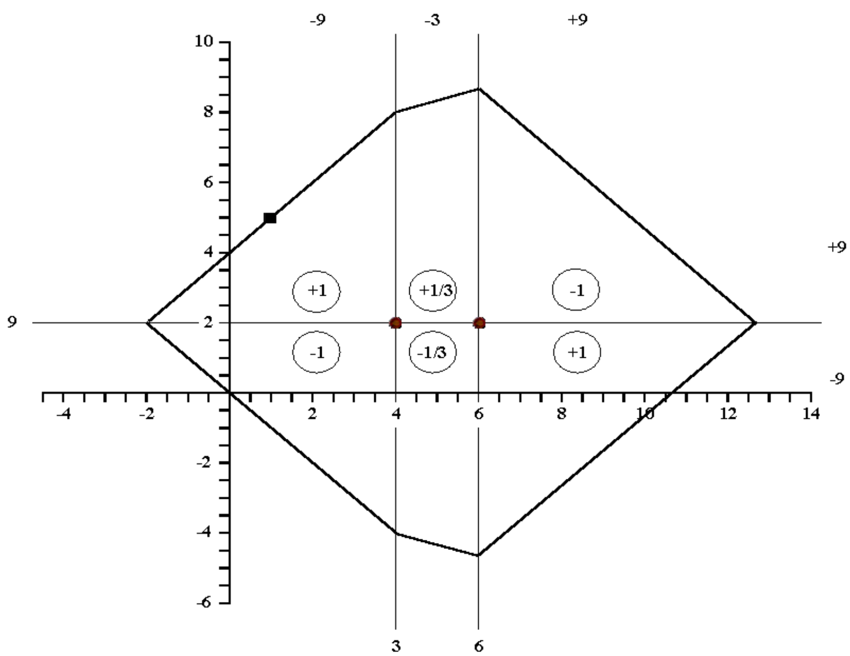
| Building | Decision Variables $x$ | $y$ | Obj. Fcn $f(X^*)$ |
|---|---|---|---|
| 1 | 20 | 20 | 36,156 |
| 2 | 40 | 25 | 43,864 |
| 3 | 25 | 35 | 45,150 |

Building 1 is preferred. Note: the unconstrained minisum Euclidean distance location obtained using SOLVER is (**x***, **y***) = (25.03,24.98).

| **Center** | **Coordinates** | | **Squared Difference** | | **Euclidean** | **Weight** | |
|---|---|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $(x_1 - a_i)^2$ | $(y_1 - b_i)^2$ | **Distance** | $w_i$ | $w_i d_i$ |
| A | 5 | 10 | 401 | 224 | 25.012 | 200 | 5002 |
| B | 50 | 15 | 623 | 100 | 26.888 | 400 | 10755 |
| C | 25 | 25 | 0 | 0 | 0.039 | 500 | 20 |
| D | 35 | 5 | 99 | 399 | 22.326 | 300 | 6698 |
| E | 15 | 20 | 101 | 25 | 11.199 | 400 | 4480 |
| F | 30 | 30 | 25 | 25 | 7.064 | 600 | 4239 |

| **Decision Variables** | | **Obj Fn** |
|---|---|---|
| $x$ | $y$ | **f(X*)** |
| 25.032 | 24.977 | 31,193 |

**10.12** The contour line passing through (1,5) is shown below.




**10.13a** The objective function for the optimum x-coordinate is

$$f(x_1, x_2) = 10|x_1 - 10| + 6|x_1 - 10| + 5|x_1 - 15| + 4|x_1 - 20| + 3|x_1 - 25| + 2|x_2 - 10| + 3|x_2 - 10| + 4|x_2 - 15| + 6|x_2 - 20| + 12|x_2 - 25| + 4|x_1 - x_2|$$

Likewise, the objective function for the optimum y-coordinate is

$$f(y_1, y_2) = 10|y_1 - 25| + 6|y_1 - 15| + 5|y_1 - 30| + 4|y_1 - 10| + 3|y_1 - 25| + 2|y_2 - 25| + 3|y_2 - 15| + 4|y_2 - 30| + 6|y_2 - 10| + 12|y_2 - 25| + 4|y_1 - y_2|$$

Although not requested, as shown below, using Excel's SOLVER tool, $(x_1^*, y_1^*) = (15, 25)$ and $(x_2^*, y_2^*) = (20, 25)$ are optimal solutions to a minisum formulation of the rectilinear distance problem.

| Machine | Coordinates | | Absolute Difference | | | | Weight | |
|---|---|---|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $\lvert x_1 - a_i\rvert$ | $\lvert y_1 - b_i\rvert$ | $\lvert x_2 - a_i\rvert$ | $\lvert y_2 - b_i\rvert$ | $w_{1i}$ | $w_{2i}$ |
| 1 | 10 | 25 | 5 | 0 | 10 | 0 | 10 | 2 |
| 2 | 10 | 15 | 5 | 10 | 10 | 10 | 6 | 3 |
| 3 | 15 | 30 | 0 | 5 | 5 | 5 | 5 | 4 |
| 4 | 20 | 10 | 5 | 15 | 0 | 15 | 4 | 6 |
| 5 | 25 | 25 | 10 | 0 | 5 | 0 | 3 | 12 |

| Absolute Difference | | Weight |
|---|---|---|
| $\lvert x_1 - x_2\rvert$ | $\lvert y_1 - y_2\rvert$ | $v_{12}$ |
| 5 | 0 | 4 |

| Decision Variables | | | | Obj Fn |
|---|---|---|---|---|
| $x_1$ | $y_1$ | $x_2$ | $y_2$ | f(X*) |
| 15 | 25 | 20 | 25 | 565 |

**10.13b** The objective function for the optimum x and y coordinates is

$$f(X,Y) = 10[(x_1 - 10)^2 + (y_1 - 25)^2]^{½} + 6[(x_1 - 10)^2 + (y_1 - 15)^2]^{½} + 5[(x_1 - 15)^2 + 5(y_1 - 30)^2]^{½} + 4[(x_1 - 20)^2 + (y_1 - 10)^2]^{½} + 3[(x_1 - 25)^2 + (y_1 - 25)^2]^{½} + 2[(x_2 - 10)^2 + (y_2 - 25)^2]^{½} + 3[(x_2 - 10)^2 + 3(y_2 - 15)^2]^{½} + 4[(x_2 - 15)^2 + 4(y_2 - 30)^2]^{½} + 6[(x_2 - 20)^2 + 6(y_2 - 10)^2]^{½} + 12[(x_2 - 25)^2 + 12(y_2 - 25)^2]^{½} + 4[(x_1 - x_2)^2 + 4(y_1 - y_2)^2]^{½}$$

Although not requested, as shown below, using Excel's SOLVER tool, $(x_1^*, y_1^*) = (10, 25)$ and $(x_2^*, y_2^*) = (25, 25)$ are optimal solutions to a minisum formulation of the Euclidean distance problem. Both locations coincide with an existing facility location.

| Machine | Coordinates | | Squared Difference | | | | Euclidean Distance | | Weight | |
|---|---|---|---|---|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $(x_1 - a_i)^2$ | $(y_1 - b_i)^2$ | $(x_2 - a_i)^2$ | $(y_2 - b_i)^2$ | $d(X_1,P_i)$ | $d(X_2,P_i)$ | $w_{1i}$ | $w_{2i}$ |
| 1 | 10 | 25 | 0 | 0 | 15 | 0 | 0.003 | 3.873 | 10 | 2 |
| 2 | 10 | 15 | 0 | 10 | 15 | 10 | 3.162 | 5.000 | 6 | 3 |
| 3 | 15 | 30 | 5 | 5 | 10 | 5 | 3.162 | 3.873 | 5 | 4 |
| 4 | 20 | 10 | 10 | 15 | 5 | 15 | 5.000 | 4.472 | 4 | 6 |
| 5 | 25 | 25 | 15 | 0 | 0 | 0 | 3.873 | 0.003 | 3 | 12 |

| Squared Difference | | Distance | Weight |
|---|---|---|---|
| $(x_1 - x_2)^2$ | $(y_1 - y_2)^2$ | $d(X_1,X_2)$ | $v_{12}$ |
| 225 | 0 | 15.000 | 4 |

| Decision Variables | | | | Obj Fn |
|---|---|---|---|---|
| $x_1$ | $y_1$ | $x_2$ | $y_2$ | f(X*) |
| 10.0 | 25.0 | 25.0 | 25.0 | 191.541 |

**10.14a**

x-coordinates — Decision Variables

| | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | AA | AB | AC |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $p_{12}$ | $q_{12}$ | $p_{13}$ | $q_{13}$ | $p_{23}$ | $q_{23}$ | $r_{11}$ | $s_{11}$ | $r_{12}$ | $s_{12}$ | $r_{14}$ | $s_{14}$ | $r_{21}$ | $s_{21}$ | $r_{23}$ | $s_{23}$ | $r_{26}$ | $s_{26}$ | $r_{33}$ | $s_{33}$ | $r_{34}$ | $s_{34}$ | $r_{35}$ | $s_{35}$ |
| Value | 5 | 10 | 10 | 0 | 5 | 0 | 5 | 0 | 0 | 5 | 0 | 0 | 10 | 0 | 0 | 10 | 0 | 0 | 0 | 0 | 15 | 0 | 0 | 5 | 0 | 0 | 10 |

f(X) = 245 =K11*(F3+G3)+L11*(H3+I3)+L12*(J3+K3)+J7*(L3+M3)+L7*(N3+O3)+N7*(P3+Q3)+O7*(R3+S3)
+N8*(T3+U3)+O8*(V3+W3)+L9*(X3+Y3)+M9*(Z3+AA3)+N9*(AB3+AC3)

| | | |
|---|---|---|
| 1st constraint | 0 | =C3-L3+M3-T8 |
| 2nd constraint | 0 | =C3-N3+O3-U8 |
| 3rd constraint | 0 | =C3-P3+Q3-W8 |
| 4th constraint | 0 | =D3-R3+S3-T8 |
| 5th constraint | 0 | =D3-T3+U3-V8 |
| 6th constraint | 0 | =D3-V3+W3-Y8 |
| 7th constraint | 0 | =E3-X3+Y3-V8 |
| 8th constraint | 0 | =E3-Z3+AA3-W8 |
| 9th constraint | 0 | =E3-AB3+AC3-X8 |
| 10th constraint | 0 | =C3-D3-F3+G3 |
| 11th constraint | 0 | =C3-E3-H3+I3 |
| 12th constraint | 0 | =D3-E3-J3+K3 |

$$W = \begin{pmatrix} 4 & 2 & 0 & 4 & 0 & 0 \\ 2 & 0 & 4 & 0 & 0 & 7 \\ 0 & 0 & 4 & 2 & 5 & 0 \end{pmatrix} \qquad a = \begin{pmatrix} 0 & 15 & 10 & 5 & 20 & 25 \end{pmatrix}$$

$$V = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 0 \end{pmatrix}$$

y-coordinates — Decision Variables

| | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | AA | AB | AC |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $y_1$ | $y_2$ | $y_3$ | $p_{12}$ | $q_{12}$ | $p_{13}$ | $q_{13}$ | $p_{23}$ | $q_{23}$ | $r_{11}$ | $s_{11}$ | $r_{12}$ | $s_{12}$ | $r_{15}$ | $s_{15}$ | $r_{16}$ | $s_{16}$ | $r_{25}$ | $s_{25}$ | $r_{26}$ | $s_{26}$ | $r_{33}$ | $s_{33}$ | $r_{34}$ | $s_{34}$ | $r_{35}$ | $s_{35}$ |
| Value | 10 | 10 | 20 | 0 | 0 | 0 | 10 | 0 | 10 | 0 | 0 | 10 | 0 | 0 | 5 | 0 | 0 | 0 | 15 | 5 | 0 | 0 | 5 | 5 | 0 | 0 | 0 |

f(Y) = 215 =L11*(F3+G3)+M11*(H3+I3)+M12*(J3+K3)+K7*(L3+M3)+L7*(N3+O3)+N7*(P3+Q3)
+K8*(R3+S3)+M8*(T3+U3)+P8*(V3+W3)+M9*(X3+Y3)+N9*(Z3+AA3)+O9*(AB3+AC3)

| | | |
|---|---|---|
| 1st constraint | 0 | =C3-L3+M3-T8 |
| 2nd constraint | 0 | =C3-N3+O3-U8 |
| 3rd constraint | 0 | =C3-P3+Q3-W8 |
| 4th constraint | 0 | =D3-R3+S3-T8 |
| 5th constraint | 0 | =D3-T3+U3-V8 |
| 6th constraint | 0 | =D3-V3+W3-Y8 |
| 7th constraint | 0 | =E3-X3+Y3-V8 |
| 8th constraint | 0 | =E3-Z3+AA3-W8 |
| 9th constraint | 0 | =E3-AB3+AC3-X8 |
| 10th constraint | 0 | =C3-D3-F3+G3 |
| 11th constraint | 0 | =C3-E3-H3+I3 |
| 12th constraint | 0 | =D3-E3-J3+K3 |

$$W = \begin{pmatrix} 4 & 2 & 0 & 4 & 0 & 0 \\ 2 & 0 & 4 & 0 & 0 & 7 \\ 0 & 0 & 4 & 2 & 5 & 0 \end{pmatrix} \qquad b = \begin{pmatrix} 10 & 0 & 25 & 15 & 20 & 5 \end{pmatrix}$$

$$V = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 0 \end{pmatrix}$$

**10.14b**

| | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Machine | Coordinates | | Absolute Difference | | | | | | Weight | | |
| 2 | $i$ | $a_i$ | $b_i$ | $\lvert x_1 - a_i \rvert$ | $\lvert y_1 - b_i \rvert$ | $\lvert x_2 - a_i \rvert$ | $\lvert y_2 - b_i \rvert$ | $\lvert x_3 - a_i \rvert$ | $\lvert y_3 - b_i \rvert$ | $w_{1i}$ | $w_{2i}$ | $w_{2i}$ |
| 3 | 1 | 0 | 10 | 5.00 | 5.00 | 10.00 | 0.00 | 10.00 | 10.00 | 4 | 2 | 0 |
| 4 | 2 | 15 | 0 | 10.00 | 15.00 | 5.00 | 10.00 | 5.00 | 20.00 | 2 | 0 | 0 |
| 5 | 3 | 10 | 25 | 5.00 | 10.00 | 0.00 | 15.00 | 0.00 | 5.00 | 0 | 4 | 4 |
| 6 | 4 | 5 | 15 | 0.00 | 0.00 | 5.00 | 5.00 | 5.00 | 5.00 | 4 | 0 | 2 |
| 7 | 5 | 20 | 20 | 15.00 | 5.00 | 10.00 | 10.00 | 10.00 | 0.00 | 0 | 0 | 5 |
| 8 | 6 | 25 | 5 | 20.00 | 10.00 | 15.00 | 5.00 | 15.00 | 15.00 | 0 | 7 | 0 |
| 10 | Absolute Difference | | | | | | Weight | Weight | Weight | | | |
| 11 | $\lvert x_1 - x_2 \rvert$ | $\lvert y_1 - y_2 \rvert$ | $\lvert x_1 - x_3 \rvert$ | $\lvert y_1 - y_3 \rvert$ | $\lvert x_2 - x_3 \rvert$ | $\lvert y_2 - y_3 \rvert$ | $v_{12}$ | $v_{13}$ | $v_{23}$ | | | |
| 12 | 5.00 | 5.00 | 5 | 5 | 0 | 10 | 1 | 3 | 2 | | | |
| 14 | Decision Variables | | | | | | Obj Fn | | | | | |
| 15 | $x_1$ | $y_1$ | $x_2$ | $y_2$ | $x_3$ | $y_3$ | f(X*) | | | | | |
| 16 | 5.00 | 15.00 | 10.00 | 10.00 | 10.00 | 20.00 | 460.00 | | | | | |

=SUMPRODUCT(J3:J8,D3:D8)+SUMPRODUCT(J3:J8,E3:E8)+SUMPRODUCT(K3:K8,F3:F8)+SUMPRODUCT(K3:K8,G3:G8)+SUMPRODUCT(L3:L8,H3:H8)+SUMPRODUCT(L3:L8,I3:I8)+G12*(A12+B12)+H12*(C12+D12)+I12*(E12+F12)

**10.14c**

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Machine | Coordinates | | Squared Difference | | | | | | Euclidean Distance | | | Weight | | |
| 2 | $i$ | $a_i$ | $b_i$ | $(x_1 - a_i)^2$ | $(y_1 - b_i)^2$ | $(x_2 - a_i)^2$ | $(y_2 - b_i)^2$ | $(x_3 - a_i)^2$ | $(y_3 - b_i)^2$ | $d(X_1,P_i)$ | $d(X_2,P_i)$ | $d(X_3,P_i)$ | $w_{1i}$ | $w_{2i}$ | $w_{2i}$ |
| 3 | 1 | 0 | 10 | 26.23 | 17.16 | 204.61 | 106.77 | 26.23 | 17.16 | 6.59 | 14.89 | 17.65 | 4 | 2 | 0 |
| 4 | 2 | 15 | 0 | 97.58 | 200.03 | 0.48 | 413.43 | 97.57 | 200.02 | 17.25 | 14.16 | 20.35 | 2 | 0 | 0 |
| 5 | 3 | 10 | 25 | 23.80 | 117.87 | 18.53 | 21.78 | 23.79 | 117.88 | 11.90 | 11.68 | 6.35 | 0 | 4 | 4 |
| 6 | 4 | 5 | 15 | 0.01 | 0.73 | 86.57 | 28.44 | 0.01 | 0.73 | 0.87 | 9.34 | 10.72 | 4 | 0 | 2 |
| 7 | 5 | 20 | 20 | 221.36 | 34.30 | 32.44 | 0.11 | 221.35 | 34.31 | 15.99 | 8.17 | 5.71 | 0 | 0 | 5 |
| 8 | 6 | 25 | 5 | 395.14 | 83.59 | 114.40 | 235.10 | 395.13 | 83.59 | 21.88 | 14.07 | 18.70 | 0 | 7 | 0 |
| 10 | Squared Difference | | | | | | Weight | | | | | | | | |
| 11 | $(x_1 - x_2)^2$ | $(y_1 - y_2)^2$ | $(x_1 - x_3)^2$ | $(y_1 - y_3)^2$ | $(x_2 - x_3)^2$ | $(y_2 - y_3)^2$ | $v_{12}$ | $v_{13}$ | $v_{23}$ | | | | | | |
| 12 | 84.31 | 38.32 | 0.00 | 0.00 | 84.31 | 38.32 | 1 | 3 | 2 | | | | | | |
| 14 | Decision Variables | | | | | | Obj Fn | | | | | | | | |
| 15 | $x_1$ | $y_1$ | $x_2$ | $y_2$ | $x_3$ | $y_3$ | f(X*) | | | | | | | | |
| 16 | 5.12 | 14.14 | 14.30 | 20.33 | 5.12 | 14.14 | 347.91 | | | | | | | | |

=SUMPRODUCT(M3:M8,J3:J8)+SUMPRODUCT(N3:N8,K3:K8)+SUMPRODUCT(O3:O8,L3:L8)+G12*(A12+B12)^0.5+H12*(C12+D12)^0.5+I12*(E12+F12)^0.5

**10.14d**

| Machine | Coordinates | | Squared Difference | | | | | | Squared Euclidean Distance | | | Weight | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $(x_1 - a_i)^2$ | $(y_1 - b_i)^2$ | $(x_2 - a_i)^2$ | $(y_2 - b_i)^2$ | $(x_3 - a_i)^2$ | $(y_3 - b_i)^2$ | $d(X_1,P_i)$ | $d(X_2,P_i)$ | $d(X_3,P_i)$ | $w_{1i}$ | $w_{2i}$ | $w_{2i}$ |
| 1 | 0 | 10 | 45.05 | 3.28 | 210.10 | 88.23 | 96.54 | 23.47 | 48.34 | 213.39 | 298.33 | 4 | 2 | 0 |
| 2 | 15 | 0 | 68.69 | 139.53 | 0.26 | 376.09 | 26.78 | 220.36 | 208.21 | 139.78 | 376.34 | 2 | 0 | 0 |
| 3 | 10 | 25 | 10.81 | 173.92 | 20.20 | 31.44 | 0.03 | 103.14 | 184.73 | 194.13 | 51.64 | 0 | 4 | 4 |
| 4 | 5 | 15 | 2.93 | 10.16 | 90.15 | 19.30 | 23.28 | 0.02 | 13.09 | 100.32 | 109.45 | 4 | 0 | 2 |
| 5 | 20 | 20 | 176.57 | 67.04 | 30.31 | 0.37 | 103.52 | 26.58 | 243.61 | 97.35 | 30.67 | 0 | 0 | 5 |
| 6 | 25 | 5 | 334.44 | 46.40 | 110.36 | 207.16 | 230.27 | 96.91 | 380.85 | 156.76 | 317.51 | 0 | 7 | 0 |

| Squared Difference | | | | | | Weight | | |
|---|---|---|---|---|---|---|---|---|
| $(x_1 - x_2)^2$ | $(y_1 - y_2)^2$ | $(x_1 - x_3)^2$ | $(y_1 - y_3)^2$ | $(x_2 - x_3)^2$ | $(y_2 - y_3)^2$ | $v_{12}$ | $v_{13}$ | $v_{23}$ |
| 60.57 | 57.47 | 9.69 | 9.20 | 21.81 | 20.69 | 1 | 3 | 2 |

| Decision Variables | | | | | | Obj Fn |
|---|---|---|---|---|---|---|
| $x_1$ | $y_1$ | $x_2$ | $y_2$ | $x_3$ | $y_3$ | f(X*) |
| 6.71 | 11.81 | 14.49 | 19.39 | 9.83 | 14.84 | 3801.29 |

```
=SUMPRODUCT(M3:M8,J3:J8)+SUMPRODUCT(N3:N8,K3:K8)+SUMPRODUCT(O3:O8,L3:L8)+G12*(A12+B12)+H12*(C12+D12)+I12*(E12+F12)
```

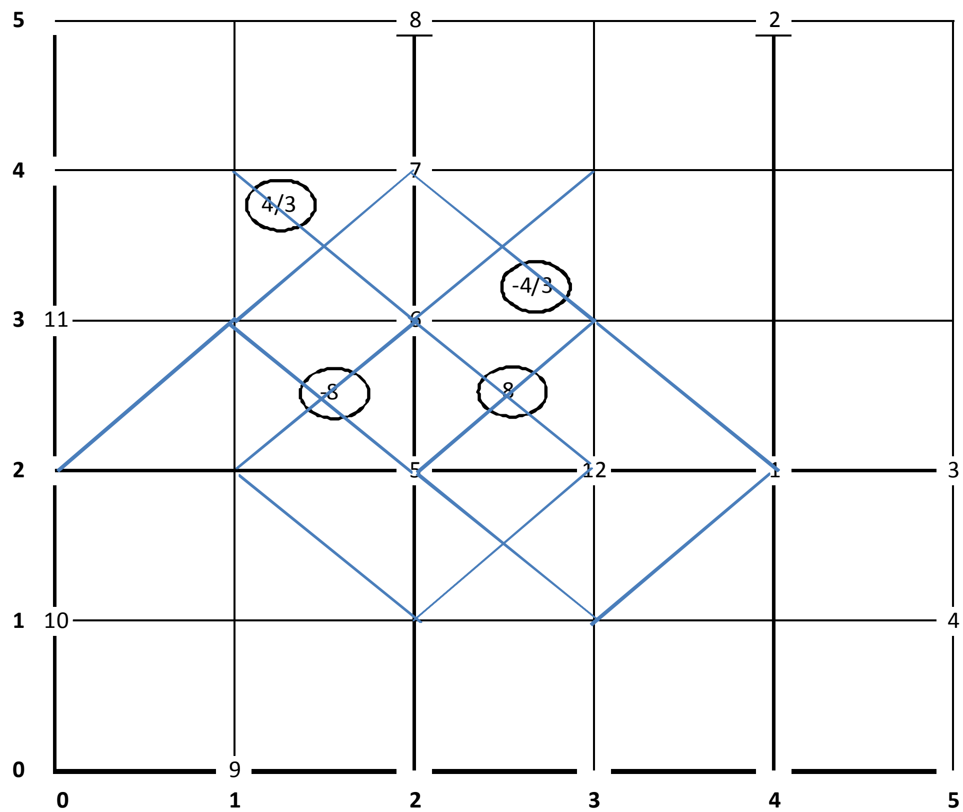
**10.15a** Using the half-sum method with a total weight of 30,

| Store $i$ | x-coord $a_i$ | weight $w_i$ | Cum Sum | |
|---|---|---|---|---|
| 10 | 0 | 1 | 1 | |
| 11 | 0 | 2 | 3 | |
| 9 | 1 | 4 | 7 | |
| 5 | 2 | 3 | 10 | |
| 6 | 2 | 5 | 15 | = 30/2 |
| 7 | 2 | 6 | 21 | |
| 8 | 2 | 2 | 23 | |
| 12 | 3 | 3 | 26 | |
| 1 | 4 | 1 | 27 | |
| 2 | 4 | 1 | 28 | |
| 3 | 5 | 1 | 29 | |
| 4 | 5 | 1 | 30 | |
| | Sum = | 30 | | |
| | | x* = 2 | | |

| Store $i$ | y-coord $b_i$ | weight $w_i$ | Cum Sum | |
|---|---|---|---|---|
| 9 | 0 | 4 | 4 | |
| 4 | 0 | 1 | 5 | |
| 10 | 1 | 1 | 6 | |
| 1 | 2 | 1 | 7 | |
| 3 | 2 | 1 | 8 | |
| 5 | 2 | 3 | 11 | |
| 12 | 2 | 3 | 14 | < 15 |
| 6 | 3 | 5 | 19 | > 15 |
| 11 | 3 | 2 | 21 | |
| 7 | 4 | 6 | 27 | |
| 2 | 5 | 1 | 28 | |
| 8 | 5 | 2 | 30 | |
| | Sum = | 30 | | |
| | | y* = 3 | | |

The optimum location coincides with the location of store #6.

**10.15b** As shown below, we plot diamonds around each existing facility, with the radius of each diamond equal to 1 mile. We also compute the slopes of contour lines in the grid squares adjacent to the optimum location. In so doing, we find that (1.5, 3.5) and (2.5, 3.5) are coordinate locations that are 1 mile from any store and have the smallest value of the objective function. Therefore, either (x*, y*) = (1.5, 3.5) or (x*, y*) = (2.5, 3.5) is the best location for the new facility.



**10.15c** The optimum unweighted minimax location is obtained as follows:

| Store ($i$) | Coordinates $a_i$ | $b_i$ | $a_i + b_i$ | $-a_i + b_i$ |
|---|---|---|---|---|
| 1 | 4 | 2 | 6 | -2 |
| 2 | 4 | 5 | 9 | 1 |
| 3 | 5 | 2 | 7 | -3 |
| 4 | 5 | 1 | 6 | -4 |
| 5 | 2 | 2 | 4 | 0 |
| 6 | 2 | 3 | 5 | 1 |
| 7 | 2 | 4 | 6 | 2 |
| 8 | 2 | 5 | 7 | 3 |
| 9 | 1 | 0 | 1 | -1 |
| 10 | 0 | 1 | 1 | 1 |
| 11 | 0 | 3 | 3 | 3 |
| 12 | 3 | 2 | 5 | -1 |
| | | Min: | 1 | -4 |
| | | Max: | 9 | 3 |

$c_1 = 1$, $c_2 = 9$, $c_3 = -4$, $c_4 = 3$, $c_5 = 8$, $x_1^* = 2.5$, $x_2^* = 3$, $y_1^* = 2.5$, $y_2^* = 2$.

Hence, the optimum minimax location is any point on the line segment connecting the (2.5,2.5) and (3,2).

**10.16a** From the data given, it is a 4-story building, including a basement at $z = -8$. Three departments (2, 3, and 4) are located on the first floor at $z = 2$, two departments are located on the second floor at $z = 12$, and two departments are located on the third floor at $z = 22$.

Using the half-sum method, half of the sum of the weights equals 7. As shown below, the optimum unconstrained location is $(x^*,y^*,z^*) = (25,35,12)$, which is the entrance to department 5.

| $a_i$ | $w_i$ | *Sum* | $b_i$ | $w_i$ | *Sum* | $c_i$ | $w_i$ | *Sum* |
|---|---|---|---|---|---|---|---|---|
| 8 | 2 | 2 | 10 | 1 | 1 | -8 | 2 | 2 |
| 12 | 1 | 3 | 20 | 2 | 3 | 2 | 1 | 3 |
| 15 | 1 | 4 | 20 | 2 | 5 | 2 | 1 | 4 |
| 25 | 1 | 5 | 25 | 1 | 6 | 2 | 2 | 6 |
| 25 | 2 | 7 | 35 | 1 | 7 | 12 | 2 | 8 |
| 25 | 2 | 9 | 35 | 2 | 9 | 12 | 3 | 11 |
| 30 | 3 | 12 | 35 | 2 | 11 | 22 | 1 | 12 |
| 40 | 2 | 14 | 40 | 3 | 14 | 22 | 2 | 14 |
| | x* = | 25 | | y* = | 35 | | z* = | 12 |

**10.16b** Since the unconstrained optimum location coincides with the constrained values for x and y, it will be the constrained location.

**10.16c** Using SOLVER to obtain the weighted minimax location gives:

| **EF** | **Coordinates** | | | ***Absolute Difference*** | | | **Rectilinear** | **Weight** | |
|---|---|---|---|---|---|---|---|---|---|
| ***i*** | $a_i$ | $b_i$ | $c_i$ | $\lvert x - a_i \rvert$ | $\lvert y - b_i \rvert$ | $\lvert z - c_i \rvert$ | **Distance** | $w_i$ | $w_i d_i$ |
| 1 | 8 | 20 | -8 | 16.73 | 0.73 | 19.73 | 37.20 | 2 | 74.4 |
| 2 | 12 | 10 | 2 | 12.73 | 10.73 | 9.73 | 33.20 | 1 | 33.2 |
| 3 | 25 | 35 | 2 | 0.27 | 14.27 | 9.73 | 24.27 | 1 | 24.3 |
| 4 | 40 | 20 | 2 | 15.27 | 0.73 | 9.73 | 25.73 | 2 | 51.5 |
| 5 | 25 | 35 | 12 | 0.27 | 14.27 | 0.27 | 14.80 | 2 | 29.6 |
| 6 | 30 | 40 | 12 | 5.27 | 19.27 | 0.27 | 24.80 | 3 | 74.4 |
| 7 | 15 | 25 | 22 | 9.73 | 4.27 | 10.27 | 24.27 | 1 | 24.3 |
| 8 | 25 | 35 | 22 | 0.27 | 14.27 | 10.27 | 24.80 | 2 | 49.6 |

| **Decision Variables** | | | **Obj Fn** |
|---|---|---|---|
| ***x*** | ***y*** | ***z*** | **f(X*)** |
| 24.73 | 20.73 | 11.73 | 74.4 |

**10.16d** With the constrained minimax location, fixed distances are traveled by departments that are not on the same floor as the mail room, with the fixed distance being the weighted distance from the department location to (25,35,z) where z is the z-coordinate for the floor on which the mail room is located. In general, we would need to consider each floor and, then, determine which floor yielded the minimum maximum weighted distance traveled. However, because the unconstrained optimum z-coordinate is 11.73, let's consider, first, z = 12 as the optimum location. As shown below, x* = 25, y* = 35, and z* = 12.

| Dept | coordinates | | | $w_i$ | $g_i$ | Absolute Difference | | | Rectilinear Distance | $g_i+w_id_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | $a_i$ | $b_i$ | $c_i$ | | | $\|x-a_i\|$ | $\|y-b_i\|$ | $\|z-c_i\|$ | | |
| 1 | 25 | 35 | 12 | 2 | 52 | 5.00 | 6.50 | 0.00 | 11.50 | 75.0 |
| 2 | 25 | 35 | 12 | 1 | 48 | 5.00 | 35.00 | 12.00 | 40.00 | 88.0 |
| 3 | 25 | 35 | 12 | 1 | 10 | 5.00 | 35.00 | 12.00 | 40.00 | 50.0 |
| 4 | 25 | 35 | 12 | 2 | 40 | 5.00 | 35.00 | 12.00 | 40.00 | 120.0 |
| 5 | 25 | 35 | 12 | 2 | 0 | 5.00 | 35.00 | 12.00 | 40.00 | 80.0 |
| 6 | 30 | 40 | 12 | 3 | 0 | 0.00 | 40.00 | 12.00 | 40.00 | 120.0 |
| 7 | 25 | 35 | 12 | 1 | 30 | 5.00 | 35.00 | 12.00 | 40.00 | 70.0 |
| 8 | 25 | 35 | 12 | 2 | 10 | 5.00 | 35.00 | 12.00 | 40.00 | 90.0 |

| Decision Variables | | | Obj Fn |
|---|---|---|---|
| x | y | z | f(X*) |
| 30 | 29 | 12 | 120.0 |

Now, assume the mail room is located on the first floor.

| Dept | coordinates | | | $w_i$ | $g_i$ | Absolute Difference | | | Rectilinear Distance | $g_i+w_id_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | $a_i$ | $b_i$ | $c_i$ | | | $\|x-a_i\|$ | $\|y-b_i\|$ | $\|z-c_i\|$ | | |
| 1 | 25 | 35 | 2 | 2 | 84 | 0.00 | 0.00 | 0.00 | 0.00 | 84.0 |
| 2 | 12 | 10 | 2 | 1 | 0 | 13.00 | 25.00 | 0.00 | 38.00 | 38.0 |
| 3 | 25 | 35 | 2 | 1 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0 |
| 4 | 40 | 20 | 2 | 2 | 0 | 15.00 | 15.00 | 0.00 | 30.00 | 60.0 |
| 5 | 25 | 35 | 2 | 2 | 20 | 0.00 | 0.00 | 0.00 | 0.00 | 20.0 |
| 6 | 30 | 40 | 2 | 3 | 30 | 5.00 | 5.00 | 0.00 | 10.00 | 60.0 |
| 7 | 25 | 35 | 2 | 1 | 40 | 0.00 | 0.00 | 0.00 | 0.00 | 40.0 |
| 8 | 25 | 35 | 2 | 2 | 40 | 0.00 | 0.00 | 0.00 | 0.00 | 40.0 |

| Decision Variables | | | Obj Fn |
|---|---|---|---|
| x | y | z | f(X*) |
| 25 | 35 | 2 | 84.0 |

The maximum weighted distance is reduced to 84. What if the mail room is located in the basement? As shown, using SOLVER, f* = 84 with x* = 26, y* = 36, and z* = -8.

| Dept | coordinates | | | $w_i$ | $g_i$ | Absolute Difference | | | Rectilinear Distance | $g_i+w_id_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | $a_i$ | $b_i$ | $c_i$ | | | $\|x-a_i\|$ | $\|y-b_i\|$ | $\|z-c_i\|$ | | |
| 1 | 25 | 35 | 2 | 2 | 84 | 0.00 | 0.00 | 0.00 | 0.00 | 84.0 |
| 2 | 12 | 10 | 2 | 1 | 0 | 13.00 | 25.00 | 0.00 | 38.00 | 38.0 |
| 3 | 25 | 35 | 2 | 1 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0 |
| 4 | 40 | 20 | 2 | 2 | 0 | 15.00 | 15.00 | 0.00 | 30.00 | 60.0 |
| 5 | 25 | 35 | 2 | 2 | 20 | 0.00 | 0.00 | 0.00 | 0.00 | 20.0 |
| 6 | 30 | 40 | 2 | 3 | 30 | 5.00 | 5.00 | 0.00 | 10.00 | 60.0 |
| 7 | 25 | 35 | 2 | 1 | 40 | 0.00 | 0.00 | 0.00 | 0.00 | 40.0 |
| 8 | 25 | 35 | 2 | 2 | 40 | 0.00 | 0.00 | 0.00 | 0.00 | 40.0 |

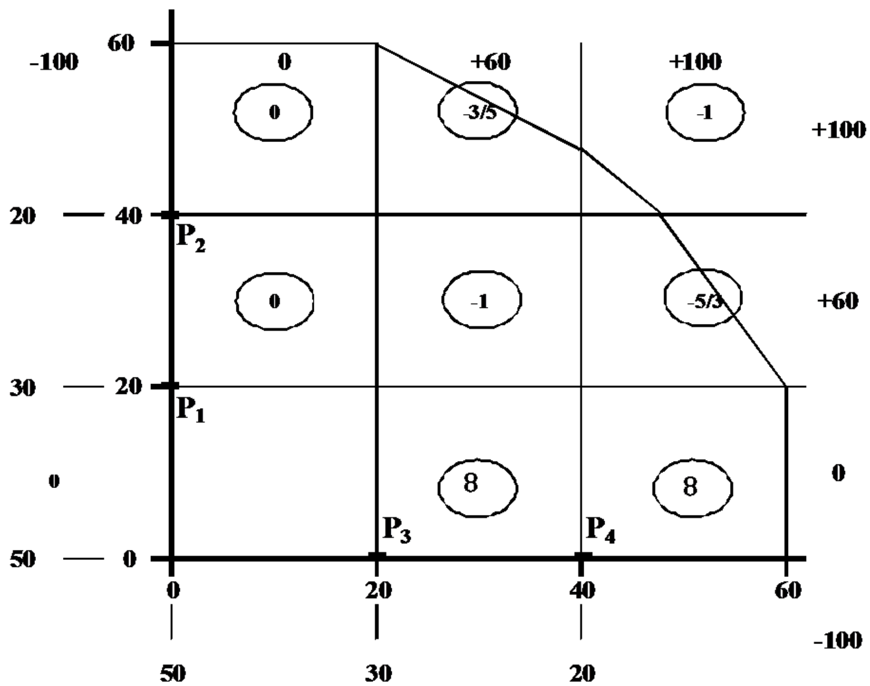
| Decision Variables | | | Obj Fn |
|---|---|---|---|
| x | y | z | f(X*) |
| 25 | 35 | 2 | 84.0 |

Finally, consider locating the mail room on the 3$^{rd}$ floor. As shown, f* = 124. Therefore, locate on either the first floor or in the basement to obtain f* = 84.

| Dept | coordinates | | | $w_i$ | $g_i$ | Absolute Difference | | | Rectilinear Distance | $g_i+w_id_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | $a_i$ | $b_i$ | $c_i$ | | | $\|x - a_i\|$ | $\|y - b_i\|$ | $\|z - c_i\|$ | | |
| 1 | 25 | 35 | 22 | 2 | 124 | 0.00 | 0.00 | 0.00 | 0.00 | 124.0 |
| 2 | 25 | 35 | 22 | 1 | 58 | 0.00 | 0.00 | 0.00 | 0.00 | 58.0 |
| 3 | 25 | 35 | 22 | 1 | 20 | 0.00 | 0.00 | 0.00 | 0.00 | 20.0 |
| 4 | 25 | 35 | 22 | 2 | 100 | 0.00 | 0.00 | 0.00 | 0.00 | 100.0 |
| 5 | 25 | 35 | 22 | 2 | 20 | 0.00 | 0.00 | 0.00 | 0.00 | 20.0 |
| 6 | 30 | 40 | 22 | 3 | 30 | 5.00 | 5.00 | 0.00 | 10.00 | 60.0 |
| 7 | 15 | 25 | 22 | 1 | 0 | 10.00 | 10.00 | 0.00 | 20.00 | 20.0 |
| 8 | 25 | 35 | 22 | 2 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0 |

| Decision Variables | | | Obj Fn |
|---|---|---|---|
| *x* | *y* | *z* | f(X*) |
| 25 | 35 | 22 | 124.0 |

**10.17** There was a mistake in the statement of the problem. It should have read, "Thirty percent of the item movement between the existing machines and a new machine is between the new machine and each of the existing machines at (0, 20) and (20, 0). Twenty percent is between the new machine and each of the existing machines at (40, 0) and (0, 40)." The following solution is based on the intended problem.




Existing machine locations are dock locations, the percentages represent the movement between a dock and the storage location for an item, and the contour line is a continuous representation of the storage boundary for the item. See Figure 7.19 for a discrete representation of the storage boundary.

**10.18a** When one existing facility has at least half the total weight, its location is the optimum location. This is knows as the Majority Theorem. Hence, $X^* = P_2 = (5,0)$

When m = 3 and all weights are equal, if the convex hull formed by the existing facility locations is a triangle with all angles less than 120 degrees, the optimum location is the point from which straight lines drawn to each existing facility form three angles of 120 degrees; if the triangle contains an angle greater than or equal to 120 degrees, then the optimum location coincides with that vertex.

**10.18b** When all weights are equal and m = 4, if the convex hull is a quadrilateral, the optimum location is at the intersection of lines connecting non-adjacent vertices of the quadrilateral; if the convex hull is a triangle, the interior existing facility location is the optimum location. Here, the four existing facilities form a quadrilateral. The optimum location is x* = 10 and y* = 10.

**10.18c** Since the convex hull is a straight line, the problem is a "location on a line" problem. The half-sum method can be used. Or, since one existing facility has a weight that is greater than half the total, by the Majority Theorem, it will be the optimum location. Hence, $X^* = P_2 = (0, 5)$.

**10.19a**

| EF | Coordinates | | Squared Difference | | Euclidean | Weight | |
|---|---|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $(x - a_i)^2$ | $(y - b_i)^2$ | Distance | $w_i$ | $w_i d_i$ |
| 1 | 0 | 0 | 14 | 2 | 3.953 | 1 | 3.953 |
| 2 | 5 | 0 | 2 | 2 | 1.768 | 3 | 5.303 |
| 3 | 0 | 5 | 14 | 14 | 5.303 | 1 | 5.303 |

| Decision Variables | | Obj Fn |
|---|---|---|
| $x$ | $y$ | f(X*) |
| 3.750 | 1.250 | 5.303 |

**10.19b**

| EF | Coordinates | | Squared Difference | | Euclidean | Weight | |
|---|---|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $(x - a_i)^2$ | $(y - b_i)^2$ | Distance | $w_i$ | $w_i d_i$ |
| 1 | 10 | 0 | 0 | 100 | 10.000 | 1 | 10.000 |
| 2 | 20 | 10 | 100 | 0 | 10.000 | 1 | 10.000 |
| 3 | 10 | 20 | 0 | 100 | 10.000 | 1 | 10.000 |
| 4 | 0 | 10 | 100 | 0 | 10.000 | 1 | 10.000 |

| Decision Variables | | Obj Fn |
|---|---|---|
| $x$ | $y$ | f(X*) |
| 10.000 | 10.000 | 10.00 |

**10.19c**

| EF | Coordinates | | Squared Difference | | Euclidean | Weight | |
|---|---|---|---|---|---|---|---|
| $i$ | $a_i$ | $b_i$ | $(x - a_i)^2$ | $(y - b_i)^2$ | Distance | $w_i$ | $w_i d_i$ |
| 1 | 0 | 0 | 0 | 56 | 7.500 | 1 | 7.500 |
| 2 | 0 | 5 | 0 | 6 | 2.500 | 3 | 7.500 |
| 3 | 0 | 15 | 0 | 56 | 7.500 | 1 | 7.500 |

| Decision Variables | | Obj Fn |
|---|---|---|
| $x$ | $y$ | f(X*) |
| 0.000 | 7.500 | 7.50 |

**10.20a**

| Magisterial | Potential Sites | | | | | |
|---|---|---|---|---|---|---|
| District | 1 | 2 | 3 | 4 | 5 | Population |
| 1 | 45 | 30 | 75 | 60 | 90 | 6,000 |
| 2 | 90 | 30 | 60 | 60 | 105 | 9,000 |
| 3 | 90 | 45 | 30 | 45 | 90 | 5,000 |
| 4 | 75 | 0 | 45 | 30 | 75 | 10,000 |
| 5 | 60 | 30 | 15 | 30 | 60 | 4,000 |
| 6 | 90 | 60 | 15 | 30 | 60 | 8,000 |
| 7 | 45 | 75 | 60 | 15 | 30 | 12,000 |
| 8 | 0 | 75 | 90 | 60 | 45 | 14,000 |
| 9 | 60 | 75 | 60 | 30 | 15 | 6,000 |
| 10 | 30 | 50 | 90 | 60 | 30 | 10,000 |

| Magisterial | Potential Sites | | | | | |
|---|---|---|---|---|---|---|
| District | 1 | 2 | 3 | 4 | 5 | Population |
| 1 | 270 | 180 | 450 | 360 | 540 | 6,000 |
| 2 | 810 | 270 | 540 | 540 | 945 | 9,000 |
| 3 | 450 | 225 | 150 | 225 | 450 | 5,000 |
| 4 | 750 | 0 | 450 | 300 | 750 | 10,000 |
| 5 | 240 | 120 | 60 | 120 | 240 | 4,000 |
| 6 | 720 | 480 | 120 | 240 | 480 | 8,000 |
| 7 | 540 | 900 | 720 | 180 | 360 | 12,000 |
| 8 | 0 | 1,050 | 1,260 | 840 | 630 | 14,000 |
| 9 | 360 | 450 | 360 | 180 | 90 | 6,000 |
| 10 | 300 | 500 | 900 | 600 | 300 | 10,000 |
| Sum: | 4,400 | 4,175 | 5,010 | 3,585 | 4,785 | |

With once clinic, locate it at site 4.

**10.20b**

| 1,2 | 1,3 | 1,4 | 1,5 | 2,3 | 2,4 | 2,5 | 3,4 | 3,5 | 4,5 |
|---|---|---|---|---|---|---|---|---|---|
| 180 | 270 | 270 | 270 | 180 | 180 | 180 | 360 | 450 | 360 |
| 270 | 540 | 540 | 810 | 270 | 270 | 270 | 540 | 540 | 540 |
| 225 | 150 | 225 | 450 | 150 | 225 | 225 | 150 | 150 | 225 |
| 0 | 450 | 300 | 750 | 0 | 0 | 0 | 300 | 450 | 300 |
| 120 | 60 | 120 | 240 | 60 | 120 | 120 | 60 | 60 | 120 |
| 480 | 120 | 240 | 480 | 120 | 240 | 480 | 120 | 120 | 240 |
| 540 | 540 | 180 | 360 | 720 | 180 | 360 | 180 | 360 | 180 |
| 0 | 0 | 0 | 0 | 1,050 | 840 | 630 | 840 | 630 | 630 |
| 360 | 360 | 180 | 90 | 360 | 180 | 90 | 180 | 90 | 90 |
| 300 | 300 | 300 | 300 | 500 | 500 | 300 | 600 | 300 | 300 |
| 2,475 | 2,790 | 2,355 | 3,750 | 3,410 | 2,735 | 2,655 | 3,330 | 3,150 | 2,985 |

With two clinics, locate them at sites 1 and 4.

Using the Ignizio algorithm: theta x = 4:

| Magisterial | Potential Sites | | | | | |
|---|---|---|---|---|---|---|
| District | 1 | 2 | 3 | 5 | a* | New a* |
| 1 | 90 | 180 | 0 | 0 | 360 | 270 |
| 2 | 0 | 270 | 0 | 0 | 540 | 540 |
| 3 | 0 | 0 | 75 | 0 | 225 | 225 |
| 4 | 0 | 300 | 0 | 0 | 300 | 300 |
| 5 | 0 | 0 | 60 | 0 | 120 | 120 |
| 6 | 0 | 0 | 120 | 0 | 240 | 240 |
| 7 | 0 | 0 | 0 | 0 | 180 | 180 |
| 8 | 840 | 0 | 0 | 210 | 840 | 0 |
| 9 | 0 | 0 | 0 | 90 | 180 | 180 |
| 10 | 300 | 100 | 0 | 300 | 600 | 300 |
| Sum: | 1,230 | 850 | 255 | 600 | 3,585 | 2355 |

The same answer results: sites 1 and 4.

**10.20c**

| Magisterial District | Potential Sites 2 | 3 | 5 | a* | New a* |
|---|---|---|---|---|---|
| 1 | 90 | 0 | 0 | 270 | 180 |
| 2 | 270 | 0 | 0 | 540 | 270 |
| 3 | 0 | 75 | 0 | 225 | 225 |
| 4 | 300 | 0 | 0 | 300 | 0 |
| 5 | 0 | 60 | 0 | 120 | 120 |
| 6 | 0 | 120 | 0 | 240 | 240 |
| 7 | 0 | 120 | 0 | 180 | 180 |
| 8 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 90 | 180 | 180 |
| 10 | 0 | 0 | 0 | 300 | 300 |
| Sum: | 850 | 255 | 600 | 2,355 | 1,695 |

$R =$

| | 4<br>$j_1$ | 1<br>$j_2$ | 2<br>$j_3$ | $a*$ |
|---|---|---|---|---|
| | 360 | 270 | 180 | 180 |
| | 540 | 810 | 270 | 270 |
| | 225 | 450 | 225 | 225 |
| | 300 | 750 | 0 | 0 |
| | 120 | 240 | 120 | 120 |
| | 240 | 720 | 480 | 240 |
| | 180 | 540 | 900 | 180 |
| | 840 | 0 | 1,050 | 0 |
| | 180 | 360 | 450 | 180 |
| | 600 | 300 | 500 | 300 |

| | $a_{it}$ 4 | 1 | 2 | $a*$ |
|---|---|---|---|---|
| | 0 | 0 | 90 | 180 |
| | 0 | 0 | 270 | 270 |
| | 0 | 0 | 0 | 225 |
| | 0 | 0 | 300 | 0 |
| | 0 | 0 | 0 | 120 |
| | 240 | 0 | 0 | 240 |
| | 360 | 0 | 0 | 180 |
| | 0 | 840 | 0 | 0 |
| | 180 | 0 | 0 | 180 |
| | 0 | 200 | 0 | 300 |
| $DTC_t =$ | 780 | 1,040 | 660 | |

Since the once that is to be removed is the last one chosen, no removal is performed. Also, since h = k, we stop.

| Magisterial District | Potential Sites 1 | 2 | 4 |
|---|---|---|---|
| 1 | 270 | **180** | 360 |
| 2 | 810 | **270** | 540 |
| 3 | 450 | **225** | 225 |
| 4 | 750 | **0** | 300 |
| 5 | 240 | **120** | 120 |
| 6 | 720 | 480 | **240** |
| 7 | 540 | 900 | **180** |
| 8 | **0** | 1,050 | 840 |
| 9 | 360 | 450 | **180** |
| 10 | **300** | 500 | 600 |

**10.21a**

| Magisterial | Potential Sites | | | | |
|---|---|---|---|---|---|
| District | 1 | 2 | 3 | 4 | 5 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 |

All sites cannot be covered, regardless of the number of clinics

**10.21b** Same answer as for (a)

**10.21c**

| Magisterial | Potential Sites | | | | |
|---|---|---|---|---|---|
| District | 1 | 2 | 3 | 4 | 5 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 0 | 1 | 0 | 0 |
| 6 | 0 | 0 | 1 | 0 | 0 |
| 7 | 0 | 0 | 0 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 |

Placing a clinic at every site only covers 6 of the districts.

**10.21d** Same answer as for (c)

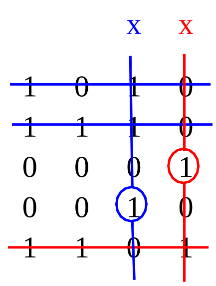
With X = 30 miles, every district can be covered with 4 clinics; there are multiple optimal solutions. With X = 45 miles, every district can be covered with 3 clinics; again, multiple optimal solutions exist. Finally, with X = 60, there are multiple ways to cover every district with one clinic.

**10.22** The problem can be solved by inspection. From the cover matrix shown below, there are multiple optimal solutions. Four patrol operators are required to cover all 30 squares. As an example, locating a patrol operator at squares 7, 9, 22, and 24 will cover all squares.

| | 1 | 2 | 3 | 4 | 5 | 6 | **7** | 8 | **9** | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | **22** | 23 | **24** | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | | | 1 | **1** | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 1 | 1 | 1 | | | 1 | **1** | 1 | | | | | | | | | | | | | | | | | | | | | | |
| 3 | | 1 | 1 | 1 | | | **1** | 1 | **1** | | | | | | | | | | | | | | | | | | | | | |
| 4 | | | 1 | 1 | 1 | | | 1 | **1** | 1 | | | | | | | | | | | | | | | | | | | | |
| 5 | | | | 1 | 1 | | | | **1** | 1 | | | | | | | | | | | | | | | | | | | | |
| 6 | 1 | 1 | | | | 1 | **1** | | | | 1 | 1 | | | | | | | | | | | | | | | | | | |
| 7 | 1 | 1 | 1 | | | 1 | **1** | 1 | | | 1 | 1 | 1 | | | | | | | | | | | | | | | | | |
| 8 | | 1 | 1 | 1 | | | **1** | 1 | **1** | | | 1 | 1 | 1 | | | | | | | | | | | | | | | | |
| 9 | | | 1 | 1 | 1 | | | 1 | **1** | 1 | | | 1 | 1 | 1 | | | | | | | | | | | | | | | |
| 10 | | | | 1 | 1 | | | | **1** | 1 | | | | 1 | 1 | | | | | | | | | | | | | | | |
| 11 | | | | | | 1 | **1** | | | | 1 | 1 | | | | 1 | 1 | | | | | | | | | | | | | |
| 12 | | | | | | 1 | **1** | 1 | | | 1 | 1 | 1 | | | 1 | 1 | 1 | | | | | | | | | | | | |
| 13 | | | | | | | **1** | 1 | **1** | | | 1 | 1 | 1 | | | 1 | 1 | 1 | | | | | | | | | | | |
| 14 | | | | | | | | 1 | **1** | 1 | | | 1 | 1 | 1 | | | 1 | 1 | 1 | | | | | | | | | | |
| 15 | | | | | | | | | **1** | 1 | | | | 1 | 1 | | | | 1 | 1 | | | | | | | | | | |
| 16 | | | | | | | | | | | 1 | 1 | | | | 1 | 1 | | | | 1 | **1** | | | | | | | | |
| 17 | | | | | | | | | | | 1 | 1 | 1 | | | 1 | 1 | 1 | | | 1 | **1** | 1 | | | | | | | |
| 18 | | | | | | | | | | | | 1 | 1 | 1 | | | 1 | 1 | 1 | | | **1** | 1 | **1** | | | | | | |
| 19 | | | | | | | | | | | | | 1 | 1 | 1 | | | 1 | 1 | 1 | | | 1 | **1** | 1 | | | | | |
| 20 | | | | | | | | | | | | | | 1 | 1 | | | | 1 | 1 | | | | **1** | 1 | | | | | |
| 21 | | | | | | | | | | | | | | | | 1 | 1 | | | | 1 | **1** | | | | 1 | 1 | | | |
| 22 | | | | | | | | | | | | | | | | 1 | 1 | 1 | | | 1 | **1** | 1 | | | 1 | 1 | 1 | | |
| 23 | | | | | | | | | | | | | | | | | 1 | 1 | 1 | | | **1** | 1 | **1** | | | 1 | 1 | 1 | |
| 24 | | | | | | | | | | | | | | | | | | 1 | 1 | 1 | | | 1 | **1** | 1 | | | 1 | 1 | 1 |
| 25 | | | | | | | | | | | | | | | | | | | 1 | 1 | | | | **1** | 1 | | | | 1 | 1 |
| 26 | | | | | | | | | | | | | | | | | | | | | 1 | **1** | | | | 1 | 1 | | | |
| 27 | | | | | | | | | | | | | | | | | | | | | 1 | **1** | 1 | | | 1 | 1 | 1 | | |
| 28 | | | | | | | | | | | | | | | | | | | | | | **1** | 1 | **1** | | | 1 | 1 | 1 | |
| 29 | | | | | | | | | | | | | | | | | | | | | | | 1 | **1** | 1 | | | 1 | 1 | 1 |
| 30 | | | | | | | | | | | | | | | | | | | | | | | | **1** | 1 | | | | 1 | 1 |

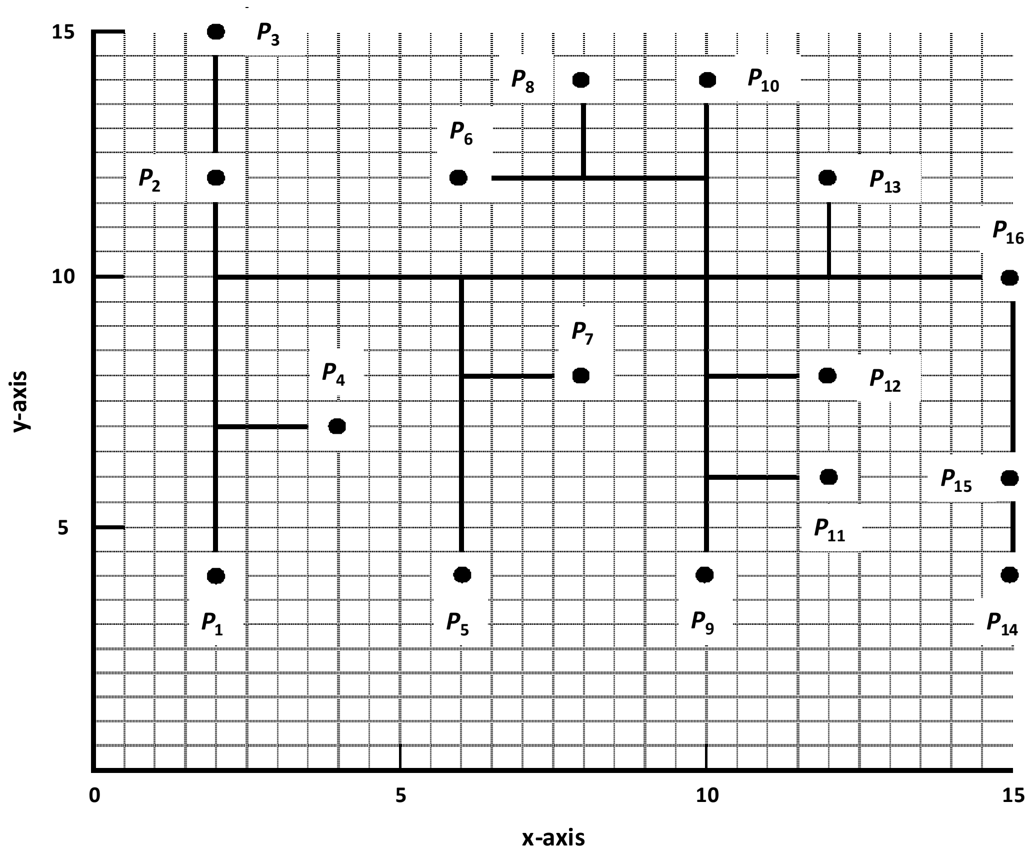
**10.23**

| | | | |
|---|---|---|---|
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |




Two branch banks required.

**10.24** Unfortunately, the grid lines do not show in the printed version of the figure for this problem. We have added grid lines, below. Student would benefit by receiving a copy of the figure.




**10.24a** Using the majority algorithm, locate at x = 10 and y = 10.

**10.24b** Choose any node (say $P_8$) and find the farthest node (which is node $P_1$) and call it *anchor node 1*. Find the farthest node from *anchor node 1* (node $P_{14}$) and call it *anchor node 2*. The *mid-point* between the anchor nodes is the minimax location. For the problem, x* = 8.5, y* = 10; the maximum distance is 25/2 = 12.5.

**10.25a**

| | Potential Sites | | | | | # Trips |
|---|---|---|---|---|---|---|
| Customer | 1 | 2 | 3 | 4 | 5 | Weekly |
| 1 | 5 | 10 | 15 | 20 | 25 | 80 |
| 2 | 10 | 5 | 10 | 15 | 20 | 200 |
| 3 | 15 | 10 | 5 | 10 | 15 | 150 |
| 4 | 20 | 15 | 10 | 5 | 10 | 400 |
| 5 | 25 | 20 | 15 | 10 | 5 | 600 |
| 6 | 30 | 25 | 20 | 15 | 10 | 250 |
| 7 | 10 | 15 | 20 | 25 | 30 | 50 |
| 8 | 30 | 10 | 25 | 5 | 20 | 500 |

| | Potential Sites | | | | |
|---|---|---|---|---|---|
| Customer | 1 | 2 | 3 | 4 | 5 |
| 1 | 400 | 800 | 1,200 | 1,600 | 2,000 |
| 2 | 2,000 | 1,000 | 2,000 | 3,000 | 4,000 |
| 3 | 2,250 | 1,500 | 750 | 1,500 | 2,250 |
| 4 | 8,000 | 6,000 | 4,000 | 2,000 | 4,000 |
| 5 | 15,000 | 12,000 | 9,000 | 6,000 | 3,000 |
| 6 | 7,500 | 6,250 | 5,000 | 3,750 | 2,500 |
| 7 | 500 | 750 | 1,000 | 1,250 | 1,500 |
| 8 | 15,000 | 5,000 | 12,500 | 2,500 | 10,000 |
| Sum: | 50,650 | 33,300 | 35,450 | 21,600 | 29,250 |

Locate at site 4.

| | Potential Sites | | | | | |
|---|---|---|---|---|---|---|
| Customer | 1 | 2 | 3 | 5 | a* | New a* |
| 1 | 1,200 | 800 | 400 | 0 | 1,600 | 1,600 |
| 2 | 1,000 | 2,000 | 1,000 | 0 | 3,000 | 3,000 |
| 3 | 0 | 0 | 750 | 0 | 1,500 | 1,500 |
| 4 | 0 | 0 | 0 | 0 | 2,000 | 2,000 |
| 5 | 0 | 0 | 0 | 3,000 | 6,000 | 3,000 |
| 6 | 0 | 0 | 0 | 1,250 | 3,750 | 2,500 |
| 7 | 750 | 500 | 250 | 0 | 1,250 | 1,250 |
| 8 | 0 | 0 | 0 | 0 | 2,500 | 2,500 |
| Sum: | 2,950 | 3,300 | 2,400 | 4,250 | | 17,350 |

Locate at sites 4 and 5.

**10.25b**

| | Potential Sites | | | | |
|---|---|---|---|---|---|
| Customer | 1 | 2 | 3 | a* | New a* |
| 1 | 1,200 | 800 | 400 | 1,600 | 800 |
| 2 | 1,000 | 2,000 | 1,000 | 3,000 | 1,000 |
| 3 | 0 | 0 | 750 | 1,500 | 1,500 |
| 4 | 0 | 0 | 0 | 2,000 | 2,000 |
| 5 | 0 | 0 | 0 | 3,000 | 3,000 |
| 6 | 0 | 0 | 0 | 2,500 | 2,500 |
| 7 | 750 | 500 | 250 | 1,250 | 750 |
| 8 | 0 | 0 | 0 | 2,500 | 2,500 |
| Sum: | 2,950 | 3,300 | 600 | 17,350 | 14,050 |

Locate at sites 2, 4, and 5.

| | 4 | 5 | 2 | | |
|---|---|---|---|---|---|
| | $j_1$ | $j_2$ | $j_3$ | | $a^*$ |
| $R =$ | 1,600 | 2,000 | 800 | | 800 |
| | 3,000 | 4,000 | 1,000 | | 1,000 |
| | 1,500 | 2,250 | 1,500 | | 1,500 |
| | 2,000 | 4,000 | 6,000 | | 2,000 |
| | 6,000 | 3,000 | 12,000 | | 3,000 |
| | 3,750 | 2,500 | 6,250 | | 2,500 |
| | 1,250 | 1,500 | 750 | | 750 |
| | 2,500 | 10,000 | 5,000 | | 2,500 |
| | | | | | 14,050 |

| | $a_{it}$ | | | | |
|---|---|---|---|---|---|
| | 4 | 5 | 2 | | $a^*$ |
| | 0 | 0 | 800 | | 800 |
| | 0 | 0 | 2,000 | | 1,000 |
| | 0 | 0 | 0 | | 1,500 |
| | 2,000 | 0 | 0 | | 2,000 |
| | 0 | 3,000 | 0 | | 3,000 |
| | 0 | 1,250 | 0 | | 2,500 |
| | 0 | 0 | 500 | | 750 |
| | 2,500 | 0 | 0 | | 2,500 |
| $DTC_t =$ | 4,500 | 4,250 | 3,300 | | 14,050 |

Since site 2 was just added to the set, we stop. Locate at 2, 4, and 5. Obj. fcn. = 14,050.

| | Potential Sites | | | | |
|---|---|---|---|---|---|
| Customer | 1 | 2 | 3 | 4 | 5 |
| 1 | 400 | **800** | 1,200 | 1,600 | 2,000 |
| 2 | 2,000 | **1,000** | 2,000 | 3,000 | 4,000 |
| 3 | 2,250 | 1,500 | 750 | **1,500** | 2,250 |
| 4 | 8,000 | 6,000 | 4,000 | **2,000** | 4,000 |
| 5 | 15,000 | 12,000 | 9,000 | 6,000 | **3,000** |
| 6 | 7,500 | 6,250 | 5,000 | 3,750 | **2,500** |
| 7 | 500 | **750** | 1,000 | 1,250 | 1,500 |
| 8 | 15,000 | 5,000 | 12,500 | **2,500** | 10,000 |

**10.26**

| Logging Area | Potential Sites 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | 300 | 750 | 450 | 1,200 |
| 2 | 120 | 150 | 1,500 | 900 | 90 |
| 3 | 1,200 | 600 | 150 | 1,050 | 480 |
| 4 | 300 | 450 | 180 | 30 | 750 |
| Sum: | 1,620 | 1,500 | 2,580 | 2,430 | 2,520 |

Locate at site 2.

| Logging Area | Potential Sites 1 | 3 | 4 | 5 | a* | New a* |
|---|---|---|---|---|---|---|
| 1 | 300 | 0 | 0 | 0 | 300 | 300 |
| 2 | 30 | 0 | 0 | 60 | 150 | 150 |
| 3 | 0 | 450 | 0 | 120 | 600 | 150 |
| 4 | 150 | 270 | 420 | 0 | 450 | 180 |
| Sum: | 480 | 720 | 420 | 180 | 1,500 | 780 |

Locate at sites 2 and 3.

| Logging Area | Potential Sites 1 | 4 | 5 | a* | New a* |
|---|---|---|---|---|---|
| 1 | 300 | 0 | 0 | 300 | 0 |
| 2 | 30 | 0 | 60 | 150 | 120 |
| 3 | 0 | 0 | 0 | 150 | 150 |
| 4 | 0 | 150 | 0 | 180 | 180 |
| Sum: | 330 | 150 | 60 | 780 | 450 |

Locate at sites 1, 2, and 3.

$$R = \begin{pmatrix} \overset{2}{j_1} & \overset{3}{j_2} & \overset{1}{j_3} \\ 300 & 750 & 0 \\ 150 & 1{,}500 & 120 \\ 600 & 150 & 1{,}200 \\ 450 & 180 & 300 \end{pmatrix}$$

| $a^*$ |
|---|
| 0 |
| 120 |
| 150 |
| 180 |
| 450 |

| | $a_{it}$ | | | | $a^*$ |
|---|---|---|---|---|---|
| | 2 | 3 | 1 | | |
| | 0 | 0 | 300 | | 0 |
| | 0 | 0 | 30 | | 120 |
| | 0 | 450 | 0 | | 150 |
| | 0 | 120 | 0 | | 180 |
| $DTC_t$ = | 0 | 570 | 330 | | 450 |

Remove site 2 from solution set.

| Logging Area | Potential Sites 2 | 4 | 5 | New a* |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 30 | 120 |
| 3 | 0 | 0 | 0 | 150 |
| 4 | 0 | 150 | 0 | 30 |
| Sum: | 0 | 150 | 30 | 300 |

Locate at sites 1, 3, and 4.

$$R = \begin{pmatrix} \overset{3}{j_1} & \overset{1}{j_2} & \overset{4}{j_3} \\ 750 & 0 & 450 \\ 1{,}500 & 120 & 900 \\ 150 & 1{,}200 & 1{,}050 \\ 180 & 300 & 30 \end{pmatrix}$$

| $a^*$ |
|---|
| 0 |
| 120 |
| 150 |
| 30 |
| 300 |

| $a_{it}$ | | |
|---|---|---|
| 3 | 1 | 4 |
| 0 | 450 | 0 |
| 0 | 780 | 0 |
| 900 | 0 | 0 |
| 0 | 0 | 150 |
| 900 | 1,230 | 150 |

Since site 4 was just added to the set, we stop. Locate at sites 1, 3, and 4.
Obj. fcn. = 300.

| Logging Area | Potential Sites 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | **0** | 300 | 750 | 450 | 1,200 |
| 2 | **120** | 150 | 1,500 | 900 | 90 |
| 3 | 1,200 | 600 | **150** | 1,050 | 480 |
| 4 | 300 | 450 | 180 | **30** | 750 |

**10.27**

| Logging | Potential Sites | | | | |
|---|---|---|---|---|---|
| Area | 1 | 2 | 3 | 4 | 5 |
| 1 | 0 | 450 | 1,125 | 675 | 1,800 |
| 2 | 180 | 225 | 2,250 | 1,350 | 135 |
| 3 | 1,800 | 900 | 225 | 1,575 | 720 |
| 4 | 450 | 675 | 270 | 45 | 1,125 |
| Fixed Cost: | $1,000 | $500 | $250 | $750 | $1,000 |
| Total Cost: | $3,430 | $2,750 | $4,120 | $4,395 | $4,780 |

Locate at site 2.

| Logging | Potential Sites | | | | | |
|---|---|---|---|---|---|---|
| Area | 1 | 3 | 4 | 5 | a* | New a* |
| 1 | 450 | 0 | 0 | 0 | 450 | 450 |
| 2 | 45 | 0 | 0 | 90 | 225 | 225 |
| 3 | 0 | 675 | 0 | 180 | 900 | 225 |
| 4 | 225 | 405 | 630 | 0 | 675 | 270 |
| Net Cost Δ: | -280 | 580 | 380 | -480 | | 1,420 |

Locate at sites 2 and 3.

| Logging | Potential Sites | | | | |
|---|---|---|---|---|---|
| Area | 1 | 4 | 5 | a* | New a* |
| 1 | 450 | 0 | 0 | 450 | 0 |
| 2 | 45 | 0 | 90 | 225 | 180 |
| 3 | 0 | 0 | 0 | 225 | 225 |
| 4 | 0 | 225 | 0 | 270 | 270 |
| Net Cost Δ: | -505 | -525 | -910 | | 1,100 |

Adding a new site will not reduce cost. Since the addition of any other site will not decrease cost, we stop. Locate only at 2 and 3. Total cost = 500 + 250 + 450 + 225 + 225 + 270 = 1,920.

| Logging | Potential Sites | | | | |
|---|---|---|---|---|---|
| Area | 1 | 2 | 3 | 4 | 5 |
| 1 | 0 | **450** | 1,125 | 675 | 1,800 |
| 2 | 180 | **225** | 2,250 | 1,350 | 135 |
| 3 | 1,800 | 900 | **225** | 1,575 | 720 |
| 4 | 450 | 675 | **270** | 45 | 1,125 |

**SECTION 10.3**

**10.28a** Order the values in V and D in a decreasing and increasing order, respectively.

v = (9,8,7,6,6,5,4,3,2,1), d = (28,35,45,47,57,62,62,70,71,91)

Lower Bound = vd' = 2,472.

**10.28b** Suppose that department *i* is located at site *i* in the current layout design:

Material Handling Cost = 4(70) + 8(45) + 6(62) + 15(35) + 3(57)
+ 7(47) + 9(91) + 2(28) + 1(71) + 6(62)
= 3,005

**10.29a** Assuming rectilinear distance

Decreasing order of flow rate: v = (250,200,150,150,150,100,50,30,20,10,10,0,0,0,0)

Distance Matrix:

| $d_{ij}$ | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | | 1 | 1 | 2 | 2 | 3 |
| B | | | 2 | 1 | 3 | 2 |
| C | | | | 1 | 1 | 2 |
| D | | | | | 2 | 1 |
| E | | | | | | 1 |

Increasing order of distance: d = (1,1,1,1,1,1,1,2,2,2,2,2,2,3,3)

Lower Bound = vd' = 1,190.

**10.29b** Material Handling Cost = 1,740

**10.29c** There are 15 possible pairwise exchanges. For illustration purposes, only the first iteration is shown.

Iteration 1:

| Pairwise Exchange | Savings | Pairwise Exchange | Savings | Pairwise Exchange | Savings |
|---|---|---|---|---|---|
| 1-2 | 110 | 2-3 | -100 | 3-5 | 40 |
| 1-3 | -360 | 2-4 | 140 | 3-6 | -400 |
| 1-4 | -100 | 2-5 | -350 | 4-5 | 100 |
| 1-5 | 80 | 2-6 | 80 | 4-6 | -550 |
| 1-6 | -60 | 3-4 | -190 | 5-6 | 0 |

The largest cost reduction occurs when we exchange cells 4 and 5. The new material handling cost is 1,460.

This problem illustrates the issues with choosing a starting point for the algorithm. The algorithm will terminate in the second iteration without improving the solution obtained from iteration 1. For instance, starting with the following initial arrangement will result in a solution at the lower bound during the second iteration.

| | | |
|---|---|---|
| 1 | 3 | 6 |
| 4 | 5 | 2 |

**10.30a** Assuming rectilinear distance

Decreasing order of flow rate: v = (250,200,150,150,150,100,50,30,20,10,10,0,0,0,0)

Distance Matrix:

| $d_{ij}$ | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | | 1 | 2 | 1 | 2 | 3 |
| B | | | 1 | 2 | 1 | 2 |
| C | | | | 3 | 2 | 1 |
| D | | | | | 1 | 2 |
| E | | | | | | 1 |

Increasing order of distance: d = (1,1,1,1,1,1,1,2,2,2,2,2,2,3,3)

Lower Bound = vd’ = 1,190.

**10.30b** Material Handling Cost = 1,690

**10.30c** There are 15 possible pairwise exchanges. For illustration purposes, only the first iteration is shown.

Iteration 1:

| Pairwise Exchange | Savings | Pairwise Exchange | Savings | Pairwise Exchange | Savings |
|---|---|---|---|---|---|
| 1-2 | -110 | 2-3 | 70 | 3-5 | 60 |
| 1-3 | -540 | 2-4 | 20 | 3-6 | 0 |
| 1-4 | -310 | 2-5 | 130 | 4-5 | -470 |
| 1-5 | -20 | 2-6 | -200 | 4-6 | -540 |
| 1-6 | -160 | 3-4 | 190 | 5-6 | 0 |

The largest cost reduction occurs when we exchange cells 3 and 4. The new material handling cost is 1,500.

**10.31a** Assuming rectilinear distance

Decreasing order of flow rate: v = (250,200,150,150,150,100,50,30,20,10,10,0,0,0,0)

Distance Matrix:

| $d_{ij}$ | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | | 1 | 2 | 3 | 4 | 5 |
| B | | | 1 | 2 | 3 | 4 |
| C | | | | 1 | 2 | 3 |
| D | | | | | 1 | 2 |
| E | | | | | | 1 |

Increasing order of distance: d = (1,1,1,1,1,2,2,2,2,3,3,3,4,4,5)

Lower Bound = vd’ = 1,360.

**10.31b** Material Handling Cost = 2,530

**10.31c** There are 15 possible pairwise exchanges. For illustration purposes, only the first iteration is shown.

Iteration 1:

| Pairwise Exchange | Savings | Pairwise Exchange | Savings | Pairwise Exchange | Savings |
|---|---|---|---|---|---|
| 1-2 | 170 | 2-3 | -170 | 3-5 | 140 |
| 1-3 | -560 | 2-4 | 300 | 3-6 | -400 |
| 1-4 | -70 | 2-5 | -90 | 4-5 | 250 |
| 1-5 | 410 | 2-6 | 140 | 4-6 | -80 |
| 1-6 | -280 | 3-4 | 290 | 5-6 | -200 |

The largest cost reduction occurs when we exchange cells 1 and 5. The new material handling cost is 2,120.

**10.32a** Assuming rectilinear distance

Decreasing order of flow rate: v = (250,200,150,150,150,100,50,30,20,10,10,0,0,0,0)

Distance Matrix:

| $d_{ij}$ | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | | 2 | 1 | 3 | 1 | 2 |
| B | | | 1 | 5 | 3 | 4 |
| C | | | | 4 | 2 | 3 |
| D | | | | | 2 | 1 |
| E | | | | | | 1 |

Increasing order of distance: d = (1,1,1,1,1,2,2,2,2,3,3,3,4,4,5)

Lower Bound = vd' = 1,360.

**10.32b** Material Handling Cost = 2,500

**10.32c** There are 15 possible pairwise exchanges. For illustration purposes, only the first iteration is shown.

Iteration 1:

| Pairwise Exchange | Savings |
|---|---|
| 1-2 | 20 |
| 1-3 | 360 |
| 1-4 | -830 |
| 1-5 | 390 |
| 1-6 | -100 |

| Pairwise Exchange | Savings |
|---|---|
| 2-3 | -370 |
| 2-4 | 80 |
| 2-5 | 330 |
| 2-6 | 380 |
| 3-4 | 60 |

| Pairwise Exchange | Savings |
|---|---|
| 3-5 | 600 |
| 3-6 | -70 |
| 4-5 | -420 |
| 4-6 | 30 |
| 5-6 | -300 |

The largest cost reduction occurs when we exchange cells 3 and 5. The new material handling cost is 1,900.

**10.33a** Average distance item i travels between dock k and its storage region, $R_i = \sum_{j \in R_i} d_{jk}/A_i$

**10.33b** Average cost of transporting item *i* between dock *k* and its storage region, $R_i = w_{ik} \sum_{j \in R_i} d_{jk}/A_i$

**10.33c** Integer programming formulation:

Minimize $\sum_{i=1}^{m} \sum_{k=1}^{p} w_{ik} \left( \sum_{j \in R_i} \frac{d_{jk}}{A_i} \right)$

Subject to: $\sum_{i=1}^{m} A_i = n$

$\sum_{j=1}^{n} x_{ij} = A_i, \forall i$

$\sum_{i=1}^{m} x_{ij} = 1, \forall j$

$x_{ij} = 0 \text{ or } 1$

**10.34a** Minimize $\sum_{j=1}^{4}\sum_{k=1}^{32}\frac{T_j}{S_j}\sum_{i=1}^{3}p_i d_{ik}x_{jk}$

Subject to $\sum_{j=1}^{4}x_{jk} = 1, k = 1, \ldots, 32$

$\sum_{k=1}^{32}x_{jk} = S_j, j = 1, \ldots, 4$

$x_{ij} = 0 \text{ or } 1, \forall j, \forall k$

**10.34b** Product Ranking: C > A > B > D.

| Product (j) | Area ($ft^2$) | # of Bays ($S_j$) | Load Rate ($T_j$) | $T_j/S_j$ |
|---|---|---|---|---|
| A | 4,375 | 7 | 500 | 71.43 |
| B | 7,500 | 12 | 600 | 50 |
| C | 1,500 | 3 | 700 | 23.33 |
| D | 6,250 | 10 | 200 | 20 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 137.5 | 137.5 | 137.5 | 137.5 | 137.5 | 137.5 | 137.5 | 137.5 |
| 118.75 | 118.75 | 118.75 | 118.75 | 118.75 | 118.75 | 118.75 | 118.75 |
| 118.75 | 118.75 | 118.75 | 118.75 | 118.75 | 118.75 | 118.75 | 118.75 |
| 137.5 | 137.5 | 137.5 | 137.5 | 137.5 | 137.5 | 137.5 | 137.5 |

Doors: 0.5 (left side, between rows 2 and 3); 0.25 (right side, between rows 1 and 2); 0.25 (right side, between rows 3 and 4)

Note: Each block is 25' x 25'.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| B | B | B | D | D | D | D | D |
| C | C | B | B | B | A | A | A |
| C | B | B | B | A | A | A | A |
| B | B | B | D | D | D | D | D |

Doors: 0.5 (left side, between rows 2 and 3); 0.25 (right side, between rows 1 and 2); 0.25 (right side, between rows 3 and 4)

Note: Multiple optimal layouts exist

**10.35a** Minimize $\sum_{j=1}^{6}\sum_{k=1}^{40}\frac{T_j}{S_j}\sum_{i=1}^{2} p_i d_{ik} x_{jk}$

Subject to $\sum_{j=1}^{6} x_{jk} = 1, k = 1, \ldots, 32$

$\sum_{k=1}^{40} x_{jk} = S_j, j = 1, \ldots, 6$

$x_{ij} = 0 \text{ or } 1, \forall j, \forall k$

**10.35b** Product Ranking: 6 > 3 > 1 > 4 > 2 > 5

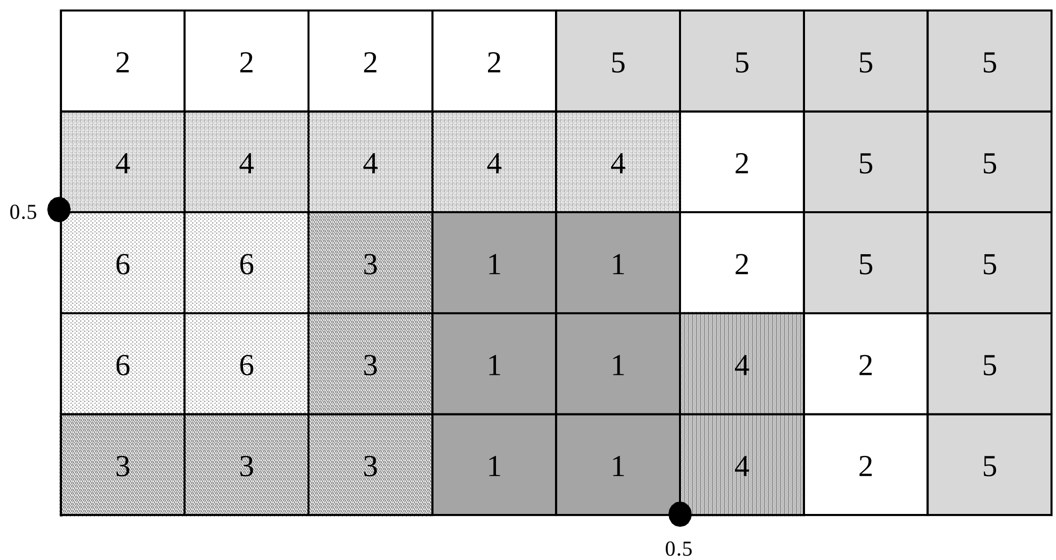
| Product (j) | Area ($ft^2$) | # of Bays ($S_j$) | Load Rate ($T_j$) | $T_j/S_j$ |
|---|---|---|---|---|
| 1 | 2,400 | 6 | 600 | 100 |
| 2 | 3,200 | 8 | 400 | 50 |
| 3 | 2,000 | 5 | 800 | 160 |
| 4 | 2,800 | 7 | 400 | 57.14 |
| 5 | 4,000 | 10 | 400 | 40 |
| 6 | 1,600 | 4 | 800 | 200 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 110 | 110 | 110 | 110 | 110 | 120 | 140 | 160 |
| 90 | 90 | 90 | 90 | 90 | 100 | 120 | 140 |
| 80 | 80 | 80 | 80 | 80 | 90 | 110 | 130 |
| 80 | 80 | 80 | 80 | 80 | 90 | 110 | 130 |
| 80 | 80 | 80 | 80 | 80 | 90 | 110 | 130 |

0.5 (dock point on the left edge, between the second and third rows)

0.5 (dock point on the bottom edge, between the fifth and sixth columns)

Note: Each block is 20' x 20'.




Note: Multiple optimal layouts exist

**10.36a** Minimize $\sum_{j=1}^{3}\sum_{k=1}^{36}\frac{T_j}{S_j}\sum_{i=1}^{3} p_i d_{ik} x_{jk}$

Subject to $\sum_{j=1}^{3} x_{jk} = 1, k = 1, \ldots, 36$

$\sum_{k=1}^{36} x_{jk} = S_j, j = 1, \ldots, 3$

$x_{ij} = 0 \text{ or } 1, \forall j, \forall k$

**10.36b** Product Ranking: B > A > C

| Product (j) | # of Bays ($S_j$) | Load Rate ($T_j$) | $T_j/S_j$ |
|---|---|---|---|
| A | 15 | 1 | 0.0667 |
| B | 5 | 1 | 0.2 |
| C | 16 | 1 | 0.0625 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.5 | 54.375 | 61.875 | 69.375 | 76.875 | 84.375 | 91.875 | |
| | 54.375 | 61.875 | 69.375 | 76.875 | 84.375 | 91.875 | |
| | 54.375 | 61.875 | 69.375 | 76.875 | 84.375 | 91.875 | 0.25 |
| | 58.125 | 65.625 | 73.125 | 80.625 | 88.125 | 95.625 | |
| | 65.625 | 73.125 | 80.625 | 88.125 | 95.625 | 103.125 | |
| 0.25 | 73.125 | 80.625 | 88.125 | 95.625 | 103.125 | 110.625 | |

Note: Each block is 15’ x 15’

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.5 | B | B | A | A | C | C | |
| | B | A | A | A | C | C | |
| | B | A | A | A | C | C | 0.25 |
| | B | A | A | C | C | C | |
| | A | A | A | C | C | C | |
| 0.25 | A | A | C | C | C | C | |

Note: Multiple optimal layouts exist

**10.37a** Minimize $\sum_{j=1}^{3}\sum_{k=1}^{44}\frac{T_j}{S_j}\sum_{i=1}^{2} p_i d_{ik} x_{jk}$

Subject to $\sum_{j=1}^{3} x_{jk} = 1, k = 1, \dots, 44$

$\sum_{k=1}^{44} x_{jk} = S_j, j = 1, \dots, 3$

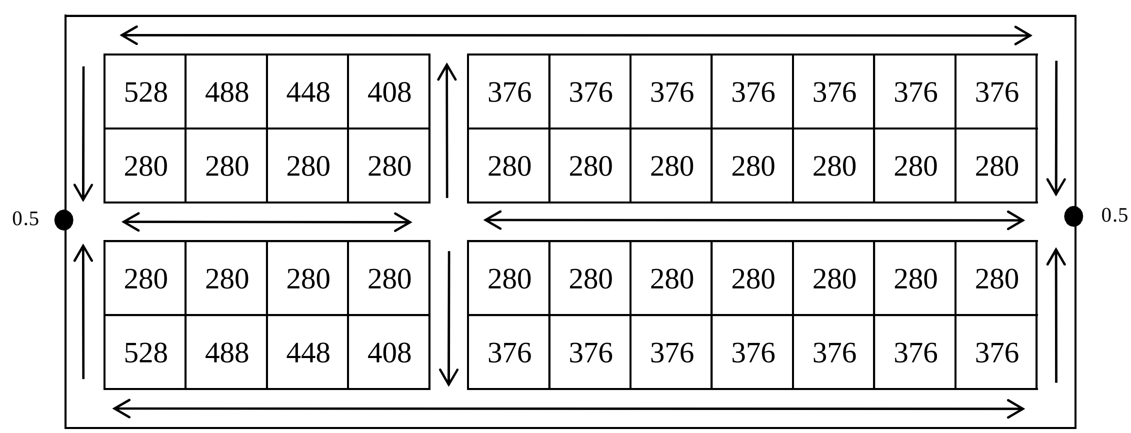
$x_{ij} = 0 \text{ or } 1, \forall j, \forall k$

**10.37b** Product Ranking: Z > X > Y

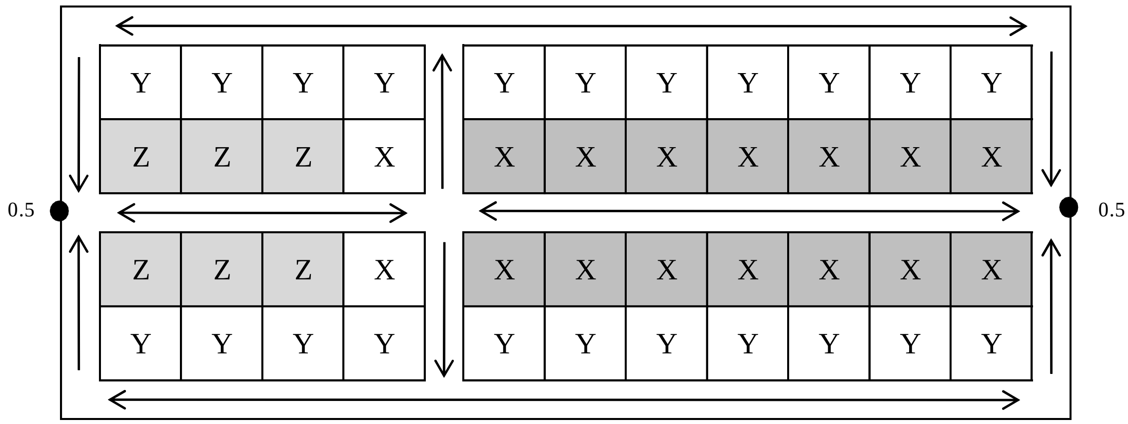
| Product (j) | Area ($ft^2$) | # of Bays ($S_j$) | Load Rate ($T_j$) | $T_j/S_j$ |
|---|---|---|---|---|
| X | 6,400 | 16 | 400 | 25 |
| Y | 8,800 | 22 | 400 | 18.18 |
| Z | 2,400 | 6 | 600 | 100 |

Distances to/from storage bays are determined by summing the distance traveled from the receiving door (A) to the center of the storage bay and from the center of the storage bay to the shipping door (B). Consider the shaded storage bay, shown below. Travel from the receiving door to the midpoint of the cross-aisle is 92'; travel up the cross-aisle to the center of the two-way aisle is 52'; travel along the two-way aisle to the center point of the storage bay is 54'; travel from the mid-point of the aisle to the mid-point of the storage bay is 14'. Hence, a total of 212' is required to storage a load in the storage bay. Travel from the storage bay to the shipping door requires 14' from bay to aisle, 202' along the aisle to the mid-point of the farthest one-way aisle, 52' down the aisle to the mid-point of the center aisle, and 4' to the shipping door for a total distance of 272'. The total distance traveled to/from the storage bay is 488', as shown. We do not compute the distance traveled by the lift truck after storage and before retrieval since we do not know its destination and origin points, respectively. Once the distance is known for one storage bay, the other distances can be easily calculated.

The distances to/from storage bays located along the center aisle are determined as follows: the distance from from door A to door B is 244'; the distance from the mid-point of the center aisle to the mid-point of a storage bay is 18'. Hence, the total distance equals 244' + 18' + 18', or 280'.




Note: Each block is approximately 20’ x 20’




## SECTION 10.4

**10.38** Initial Layout:

| | A | B | C | D |
|---|---|---|---|---|
| P/D Location | 30 | 110 | 220 | 305 |

| $f_{ij}$ | A | B | C | D |
|---|---|---|---|---|
| A | | 2,100 | 1,100 | 1,000 |
| B | | | 400 | 1,300 |
| C | | | | 2,700 |

| $d_{ij}$ | A | B | C | D |
|---|---|---|---|---|
| A | | 80 | 190 | 275 |
| B | | | 110 | 195 |
| C | | | | 85 |

Total Cost (TC) = 1,179,000

Improvement Procedure: (Note: not all pairwise interchanges are illustrated for each iteration)

ITERATION 1:
Exchange A and B: TC = 1,071,000, Savings = 108,000

| | B | A | C | D |
|---|---|---|---|---|
| P/D Location | 50 | 130 | 220 | 305 |

Exchange: B and C: TC = 1,475,000, Savings = -296,000

| | A | C | B | D |
|---|---|---|---|---|
| P/D Location | 30 | 100 | 230 | 305 |

Exchange: C and D: TC = 978,000, Savings = 201,000

| | A | B | D | C |
|---|---|---|---|---|
| P/D Location | 30 | 110 | 185 | 270 |

Best interchange is C and D.

ITERATION 2:
Exchange A and B: TC = 870,000, Savings = 108,000

| | B | A | D | C |
|---|---|---|---|---|
| P/D Location | 50 | 130 | 185 | 270 |

Exchange B and D: TC = 1,233,000, Savings = -255,000

| | A | D | B | C |
|---|---|---|---|---|
| P/D Location | 30 | 85 | 160 | 270 |

Best interchange is A and B.

ITERATION 3:
Exchange A and D: TC = 1,004,000, Savings: -134,000

| | B | D | A | C |
|---|---|---|---|---|
| P/D Location | 50 | 125 | 180 | 270 |

No additional savings. Solution is the layout: B – A – D – C.

**10.39** Initial Layout:

| | A | B | C | D |
|---|---|---|---|---|
| P/D Location | 30 | 110 | 220 | 305 |

| $f_{ij}$ | A | B | C | D |
|---|---|---|---|---|
| A | | 1,800 | 1,200 | 1,200 |
| B | | | 600 | 1,200 |
| C | | | | 1,800 |

| $d_{ij}$ | A | B | C | D |
|---|---|---|---|---|
| A | | 80 | 190 | 275 |
| B | | | 110 | 195 |
| C | | | | 85 |

Total Cost = 1,155,000

Improvement Procedure:

ITERATION 1:
Exchange A and B: TC = 1,023,000, Savings = 132,000

| | B | A | C | D |
|---|---|---|---|---|
| P/D Location | 50 | 130 | 220 | 305 |

Exchange A and C: TC = 1,107,000, Savings = 48,000

| | C | B | A | D |
|---|---|---|---|---|
| P/D Location | 60 | 170 | 250 | 305 |

Exchange A and D: TC = 1,287,000, Savings = -132,000

| | D | B | C | A |
|---|---|---|---|---|
| P/D Location | 25 | 100 | 210 | 300 |

Exchange B and C: Same TC as interchanging A and D

Exchange B and D: TC = 1,137,000, Savings = 18,000

| | A | D | C | B |
|---|---|---|---|---|
| P/D Location | 30 | 85 | 170 | 280 |

Exchange C and D: TC = 957,000, Savings = 198,000

| | A | B | D | C |
|---|---|---|---|---|
| P/D Location | 30 | 110 | 185 | 270 |

Best interchange is C and D.

ITERATION 2:
Exchange A and B: TC = 825,000, Savings = 132,000

| | B | A | D | C |
|---|---|---|---|---|
| P/D Location | 50 | 130 | 185 | 270 |

Exchange A and C: TC = 1,077,000, Savings = -120,000

| | C | B | D | A |
|---|---|---|---|---|
| P/D Location | 60 | 170 | 245 | 300 |

Exchange A and D: TC = 1,071,000, Savings = -114,000

| | D | B | A | C |
|---|---|---|---|---|
| P/D Location | 25 | 100 | 180 | 270 |

Exchange B and C: TC = 1,107,000, Savings = -150,000

| | A | C | D | B |
|---|---|---|---|---|
| P/D Location | 30 | 120 | 205 | 280 |

Exchange B and D: TC = 1,077,000, Savings = -120,000

| | A | D | B | C |
|---|---|---|---|---|
| P/D Location | 30 | 85 | 160 | 270 |

Exchange C and D: No need to calculate.  Interchange would return starting point.

Best interchange is A and B.

ITERATION 3: The only exchanging B and C will result in a layout that hasn't been obtained during the first two iterations.

Exchange B and C: TC = 891,000, Savings = -66,000

| | C | A | D | B |
|---|---|---|---|---|
| P/D Location | 60 | 150 | 205 | 280 |

No additional savings. Solution is the layout: B – A – D – C.

**10.40** Initial Layout:

| | A | B | C | D |
|---|---|---|---|---|
| P/D Location | 30 | 110 | 220 | 305 |

| $f_{ij}$ | A | B | C | D |
|---|---|---|---|---|
| A | | 1,700 | 1,300 | 1,000 |
| B | | | 1000 | 1,100 |
| C | | | | 1,100 |

| $d_{ij}$ | A | B | C | D |
|---|---|---|---|---|
| A | | 80 | 190 | 275 |
| B | | | 110 | 195 |
| C | | | | 85 |

Total Cost = 1,076,000

Improvement Procedure:

ITERATION 1:
Exchange A and B: TC = 972,000, Savings = 104,000

| | B | A | C | D |
|---|---|---|---|---|
| P/D Location | 50 | 130 | 220 | 305 |

Exchange A and C: TC = 966,000, Savings = 110,000

| | C | B | A | D |
|---|---|---|---|---|
| P/D Location | 60 | 170 | 250 | 305 |

Exchange A and D: TC = 1,128,000, Savings = -52,000

| | D | B | C | A |
|---|---|---|---|---|
| P/D Location | 25 | 100 | 210 | 300 |

Exchange B and C: Same TC as interchanging A and D

Exchange B and D: TC = 1,080,000, Savings = -4,000

| | A | D | C | B |
|---|---|---|---|---|
| P/D Location | 30 | 85 | 170 | 280 |

Exchange C and D: TC = 939,000, Savings = 137,000

| | A | B | D | C |
|---|---|---|---|---|
| P/D Location | 30 | 110 | 185 | 270 |

Best interchange is C and D.

ITERATION 2:

Exchange A and B: TC = 835,000, Savings = 104,000

| | B | A | D | C |
|---|---|---|---|---|
| P/D Location | 50 | 130 | 185 | 270 |

Exchange A and C: TC = 984,000, Savings = -45,000

| | C | B | D | A |
|---|---|---|---|---|
| P/D Location | 60 | 170 | 245 | 300 |

Exchange A and D: TC = 930,000, Savings = 9,000

| | D | B | A | C |
|---|---|---|---|---|
| P/D Location | 25 | 100 | 180 | 270 |

Exchange B and C: TC = 1,053,000, Savings = -114,000

| | A | C | D | B |
|---|---|---|---|---|
| P/D Location | 30 | 120 | 205 | 280 |

Exchange B and D: TC = 984,000, Savings = -45,000

| | A | D | B | C |
|---|---|---|---|---|
| P/D Location | 30 | 85 | 160 | 270 |

Exchange C and D: No need to calculate. Interchange would return starting point.

Best interchange is A and B.

ITERATION 3: The only exchanging B and C will result in a layout that hasn't been obtained during the first two iterations.

Exchange B and C: TC = 855,000, Savings = -20,000

| | C | A | D | B |
|---|---|---|---|---|
| P/D Location | 60 | 150 | 205 | 280 |

No additional savings. Solution is the layout: B – A – D – C.

**10.41**

| $f_{ij}$ | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | | 75 | 150 | 0 | 75 | 150 |
| B | | | 75 | 75 | 0 | 75 |
| C | | | | 150 | 75 | 0 |
| D | | | | | 75 | 150 |
| E | | | | | | 75 |

Total Cost (TC) = 165,000

Improvement Procedure: (Due to the structure of the problem, it is not necessary to consider all possible pairs. For example, interchanges directly "across the aisle" need not be considered, since exchanging A&D in the original layout will not change any distances between any pairs.)

ITERATION 1:

| | | |
|---|---|---|
| A and B interchange | BAC-DEF | TC = 143,250 < 165,000 |
| A and C interchange | CBA-DEF | TC = 153,000 > 143,250 |
| A and F interchange | FBC-DEA | TC = 104,250 < 143,250 ←Minimum |
| B and C interchange | ACB-DEF | TC = 166,875 > 104,250 |
| C and D interchange | FBC-DEA | same as A&F |
| D and E interchange | ABC-EDF | TC = 157,000 > 104,250 |
| E and F interchange | ABC-DFE | TC = 148,125 > 104,250 |

The best pairwise interchange is either A and F or C and D. Since we considered A and F first, it will be taken to give the first improved layout: FBC-DEA

ITERATION 2:

| | | |
|---|---|---|
| B and F interchange | BFC-DEA | TC = 114,250 > 104,250 |
| B and C interchange | FCB-DEA | TC = 145,500 > 104,250 |
| D and E interchange | FBC-EDA | TC = 117,250 > 104,250 |
| A and E interchange | FBC-DAE | TC = 138,500 > 104,250 |

No savings derived from further interchanges. The layout with the lowest total cost ($104,250) is FBC-DEA.

**10.42a** We assume that $c_{ij} = 1\ \forall i,j$. Since the from-to matrix is symmetric there is no need to convert to a flow-between matrix to start the construction algorithm.

ITERATION 1:
Step 1: Since $f_{BC}$ is the maximum flow value, these are the first two machines to enter the layout.

Step 2: Although we don't have the case where the P/D locations are at the centroid, there is symmetry between B and C so the layout order B – C is arbitrarily chosen. Total cost = 12,000.

Step3:

A – B – C: Total Cost (TC) = 100(10) + 150(40) + 100(30) + 150(40) = 16,000
B – C – A: TC = 100(50) + 150(20) + 100(30) + 150(20) = 14,000
D – B – C: TC = 50(20) + 50(40) = 3,000
B – C – D: TC = 50(40) + 50(20) = 3,000

Choose the layout order B – C – D arbitrarily.

ITERATION 2:
Step 3:

A – B – C – D: TC = 100(10) + 150(40) + 100(50) + 100(30) + 150(40) + 100(50)
= 26,000
B – C – D – A: TC = 100(70) + 150(40) + 100(30) + 100(70) + 150(40) + 100(30)
= 30,000

Stop. Final layout order is A – B – C – D at a total cost of 12,000 + 3,000 + 26,000 = 41,000.

**10.42b** We assume that $c_{ij} = 1\ \forall i,j$. Since the from-to matrix is symmetric there is no need to convert to a flow-between matrix to start the construction algorithm.

ITERATION 1:
Step 1: Since $f_{BC}$ is the maximum flow value, these are the first two machines to enter the layout.

Step 2: Although we don't have the case where the P/D locations are at the centroid, there is symmetry between B and C so the order B – C is arbitrarily chosen. Total cost = 16,800.

Step3:

A – B – C: Total Cost (TC) = 100(18) + 150(56) + 100(38) + 150(56) = 22,400
B – C – A: TC = 100(66) + 150(28) + 100(46) + 150(28) = 19,600
D – B – C: TC = 50(28) + 50(48) = 3,800
B – C – D: TC = 50(56) + 50(36) = 4,600

Choose the layout order D – B – C.

ITERATION 2:
Step 3:

A – D – B – C: TC = 100(46) + 150(84) + 100(18) + 100(66) + 150(84) + 100(18)
= 40,000
D – B – C – A: TC = 100(66) + 150(28) + 100(94) + 100(46) + 150(28) + 100(94)
= 38,400

Stop. Final layout is D – B – C – A at a total cost of 16,800 + 3,800 + 38,400 = 59,000.

**10.43** The design of the flow chart will depend on the student. However, the flow chart should include at least the four steps listed in the algorithm described on pages 578 and 579. The student will have to rewrite step 2 since it assumes that the P/D points are located at the midpoint along the aisle, which is not assumed in the problem statement.

**10.44** The design of the flow chart will depend on the student. However, the flow chart should include at least the four steps listed in the algorithm described on pages 578 and 579. The student will have to rewrite step 2 since it assumes that the P/D points are located at the midpoint along the aisle, which is not assumed in the problem statement. Also, step 3 must also be suitably modified to account for locations on both sides of the aisle.

**10.45** Research question. The report will depend on the research articles used for the report. Machine layout articles are often published in the journals *European Journal of Operational Research* and the *International Journal of Production Research*.

**SECTION 10.5**

**10.46a** Shown below is an Excel spreadsheet solution of the problem. To compute the number of storage rows required to accommodate the designated inventory level, the ROUNDUP Function is used, e.g., if the inventory level is shown in cell D6 and the row depth is given in cell F5, then if the product is stacked 4 levels high, the following entry would be entered: =ROUNDUP(D6/(4*F5),0).

Q = 60 L = 48 W = 52 c = 8 A = 144

**Number of Storage Rows Required to Accommodate the Inventory Level**

| | | Row Depth (x) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | Inv | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 60 | 15 | 8 | 5 | 4 | 3 | 3 | 3 | 2 | 2 | 2 |
| 2 | 59 | 15 | 8 | 5 | 4 | 3 | 3 | 3 | 2 | 2 | 2 |
| 3 | 58 | 15 | 8 | 5 | 4 | 3 | 3 | 3 | 2 | 2 | 2 |
| 4 | 57 | 15 | 8 | 5 | 4 | 3 | 3 | 3 | 2 | 2 | 2 |
| 5 | 56 | 14 | 7 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| 6 | 55 | 14 | 7 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| 7 | 54 | 14 | 7 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| 8 | 53 | 14 | 7 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| 9 | 52 | 13 | 7 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| 10 | 51 | 13 | 7 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| 11 | 50 | 13 | 7 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| 12 | 49 | 13 | 7 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| 13 | 48 | 12 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 14 | 47 | 12 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 15 | 46 | 12 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 16 | 45 | 12 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 17 | 44 | 11 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 18 | 43 | 11 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 19 | 42 | 11 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 20 | 41 | 11 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 21 | 40 | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 |
| 22 | 39 | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 |
| 23 | 38 | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 |
| 24 | 37 | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 |
| 25 | 36 | 9 | 5 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1 |
| 26 | 35 | 9 | 5 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1 |
| 27 | 34 | 9 | 5 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1 |
| 28 | 33 | 9 | 5 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1 |
| 29 | 32 | 8 | 4 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 30 | 31 | 8 | 4 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 31 | 30 | 8 | 4 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 32 | 29 | 8 | 4 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 33 | 28 | 7 | 4 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| 34 | 27 | 7 | 4 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| 35 | 26 | 7 | 4 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |

**Number of Storage Rows Required to Accommodate the Inventory Level**

| | | Row Depth (x) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | Inv | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 36 | 25 | 7 | 4 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| 37 | 24 | 6 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 38 | 23 | 6 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 39 | 22 | 6 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 40 | 21 | 6 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 41 | 20 | 5 | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| 42 | 19 | 5 | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| 43 | 18 | 5 | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| 44 | 17 | 5 | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| 45 | 16 | 4 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 46 | 15 | 4 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 47 | 14 | 4 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 48 | 13 | 4 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 49 | 12 | 3 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 50 | 11 | 3 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 51 | 10 | 3 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 52 | 9 | 3 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 53 | 8 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 54 | 7 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 55 | 6 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 56 | 5 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 57 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 58 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 59 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 60 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | $\eta$ = | 8.00 | 4.27 | 3.00 | 2.40 | 2.00 | 1.80 | 1.60 | 1.47 | 1.40 | 1.33 |
| | S = | 57,600 | 43,008 | 38,880 | 38,016 | 37,440 | 38,880 | 39,168 | 40,128 | 42,336 | 44,160 |
| | | | | | | x* = 5 | | | | | |

To determine the average number of storage rows required over the life of a storage lot ($\eta$), the value obtained using the SUM function over the range of storage rows required is divided by the number of entries, e.g., =SUM(F6:F65)/60.

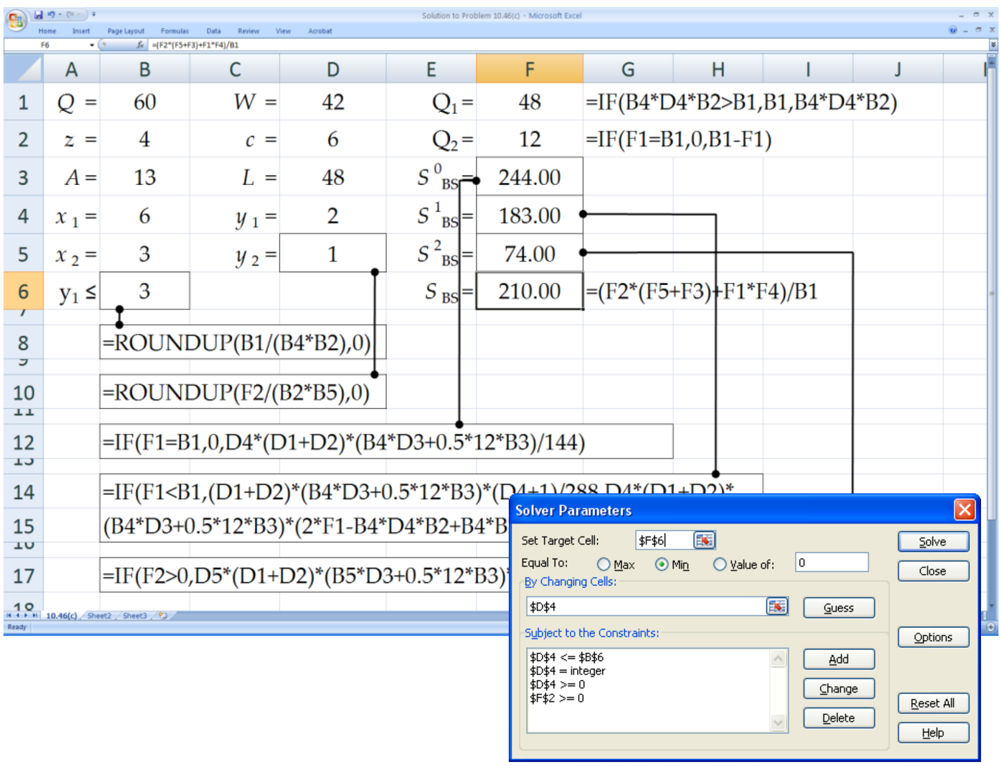
Likewise, to determine the average amount of floor space required during the life of the storage lot (S), the following was used: =F69*(K1+M1)*(F5*I1+0.5*O1), where F69 is the value of $\eta$, K1 is the value of W, M1 is the value of c, F5 is the value of x, I1 is the value of L, and O1 is the value of A.

Alternately, the following equation can be used to compute the value of $S_{BS}$: $S_{BS} = y(W + c)(xL + 0.5A)[2Q - xyz + xz]/2Q$, where y is obtained using the ROUNDUP function. Shown below is an Excel solution to the problem. In enumerating over x, $S_{BS}$ is minimized with x equal to 5.

| | Q = 60 | | L = 48 | | W = 52 | | c = 8 | | A = 144 | |
|---|---|---|---|---|---|---|---|---|---|---|
| z = | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| x = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| xz = | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| y = | 15 | 8 | 5 | 4 | 3 | 3 | 3 | 2 | 2 | 2 |
| $S_{BS}$ = | 57,600 | 43,008 | 38,880 | 38,016 | 37,440 | 38,880 | 39,168 | 40,128 | 42,336 | 44,160 |
| | | | | | x* = 5 | | | | | |

**10.46b** Using a continuous approximation, $x^c_{BS} = [AQ/2Lz]^{½} = [156(60)/2(48)(4)]^{½} = 4.937$
Therefore, rounding off the value to the nearest integer gives $x^c_{BS} = 5$.

**10.46c**




$x_1 = 6$, $y_1 = 2$, $x_2 = 3$, $y_2 = 1$, $S_{BS}$ = 2,10.00 sq. ft.

**10.47a** L = 48", W = 40", c = 8", A = 156", z = 5
Q = 300
$x^c_{BS} = [AQ/2Lz]^{½} = [156(300)/2(48)(5)]^{½} = 9.87$
Therefore, rounding off the value to the nearest integer gives $x^c_{BS} = 10$.
Using Excel to solve by enumeration gives $x^*_{BS} = 10$, as shown below.

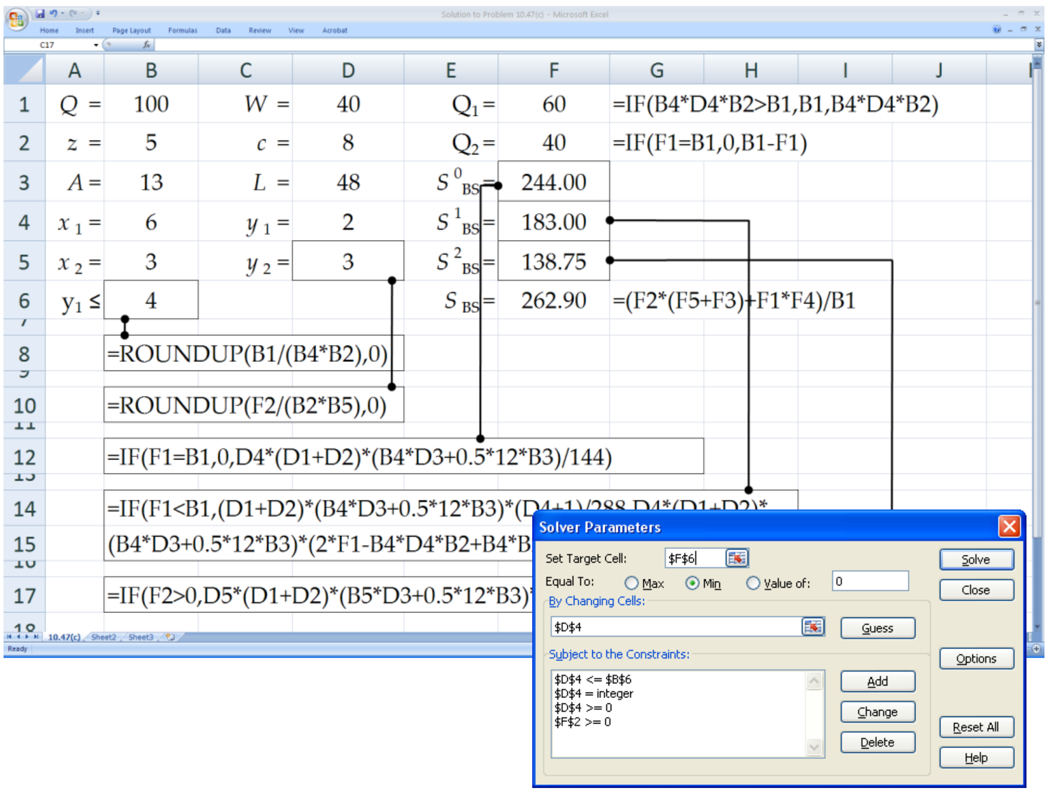
| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x = | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| xz = | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 |
| y = | 9 | 8 | 7 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 4 | 4 |
| $S_{BS}$ = | 96,230 | 95,095 | 94,433 | 93,688 | 94,276 | 93,744 | 94,883 | 95,275 | 94,920 | 96,499 | 97,630 | 98,314 |
| | | | | $x^*_{BS}$ = 10 | | | | | | | | |

Excel's SUMPRODUCT function was used to calculate the value of η; the values in the percentage column were multiplied by the number of storage rows required to accommodate the inventory level; the sum of the products is displayed in the η row. Knowing the value of η, S is easily determined by multiplying by the size of the foot-print of a row, (W + c)(xL + 0.5A). Developing a spreadsheet allows analyses of changes in inventory profiles over lifetimes of production lots to be performed easily.

**10.47b** Q = 50. Using enumeration yields the results shown below.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| xz = | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| y = | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| $S_{BS}$ = | 36,344 | 26,544 | 24,394 | 23,990 | 23,352 | 24,931 | 26,062 | 26,746 | 26,981 | 26,768 | 29,008 | 31,248 |
| | | | | | $x^*_{BS} = 5$ | | | | | | | |

**10.47c** Q = 100, $x_1 = 6$, $x_2 = 3$. As shown, $y_1 = 2$, $y_2 = 3$, and $S_{BS} = 262.90$ sq. ft.



**10.47d** Q = 50 and x = 3.

Q = 50 L = 40 W = 48 c = 8 A = 156 z = 5

Number of Storage Rows Required to Accommodate the Inventory Level

| | | Row Depth (x) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Inv | Percent | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 50 | 20% | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 |
| 45 | 20% | 9 | 5 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1 |
| 40 | 15% | 8 | 4 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 35 | 10% | 7 | 4 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| 30 | 10% | 6 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 25 | 5% | 5 | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| 20 | 5% | 4 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 15 | 5% | 3 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | 5% | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 5% | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | $\eta$ = | 7.05 | 3.75 | 2.70 | 2.20 | 1.75 | 1.65 | 1.55 | 1.40 | 1.20 | 1.00 |
| | S = | 46,586 | 33,180 | 29,938 | 29,322 | 27,244 | 29,383 | 31,074 | 31,203 | 29,434 | 26,768 |
| | | | | | | $x^*_{BS}$ = 5 | | | | | |

**10.47e** S = 28,512 sq. ft.

**10.48a** Q = 30, L = 42", W = 48", c = 8" A = 156", z = 3.

Q = 30 L = 42 W = 48 c = 8 A = 156 z = 3

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| xz = | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| y = | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| $S_{BS}$ = | 36,960 | 27,216 | 25,133 | 24,797 | 24,192 | 25,872 | 27,082 | 27,821 | 28,090 | 27,888 | 30,240 | 32,592 |
| | | | | | $x^*_{BS}$ = 5 | | | | | | | |

**10.48b** Using a continuous approximation, with Q = 300,
$x^c_{BS} = [AQ/2Lz]^{½} = [156(300)/2(42)(3)]^{½} = 13.63$
Therefore, rounding off the value to the nearest integer gives $x^c_{BS}$ = 14. Enumerating over values of x, as shown below the optimum value is actually 14.

Q = 300 L = 42 W = 48 c = 8 A = 156 z = 3

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x = | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| xz = | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 | 54 | 57 | 60 | 63 |
| y = | 10 | 10 | 9 | 8 | 8 | 7 | 7 | 6 | 6 | 6 | 5 | 5 |
| $S_{BS}$ = | 153,384 | 152,712 | 152,531 | 152,356 | 152,168 | 152,645 | 152,880 | 153,014 | 154,123 | 154,526 | 154,224 | 155,904 |
| | | | | | $x^*_{BS}$ = 14 | | | | | | | |

**10.48c** Using a continuous approximation, with Q = 300 and s = 30

$x^c_{BSSS} = [A(Q + 2s)/2Lz]^{½} = [156(300 + 60)/2(42)(3)]^{½} = 14.93$

Therefore, rounding off the value to the nearest integer gives $x^c_{BSSS} = 15$.

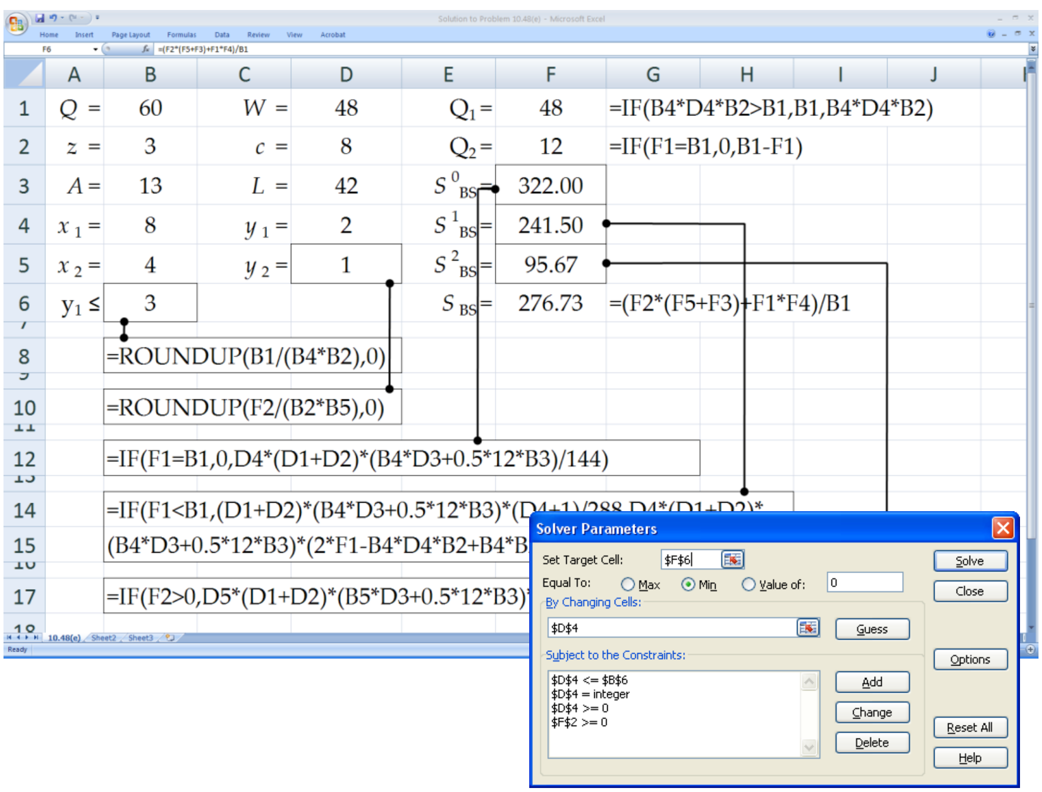
Enumerating over values of x, as shown below the optimum value is 15.

| Q = 300 | | L = 42 | | W = 48 | | c = 8 | | A = 156 | | z = 3 | | s = 30 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x = | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| xz = | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 | 54 | 57 | 60 | 63 | 66 |
| y = | 10 | 10 | 9 | 8 | 8 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 |
| $S_{BS}$ = | 164,793 | 163,696 | 162,960 | 162,502 | 162,262 | 162,196 | 162,273 | 162,466 | 162,756 | 163,129 | 163,571 | 164,073 | 164,626 |
| | | | | | | $x^*_{BSSS} = 15$ | | | | | | | |

**10.48d** Using enumeration for Q =30 and s = 5 gives $x^*_{BSSS} = 5$, as shown below.

| Q = 30 | | L = 42 | | W = 48 | | c = 8 | | A = 156 | | z = 3 | | s = 5 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| xz = | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 |
| y = | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| $S_{BS}$ = | 41,280 | 29,808 | 26,656 | 25,584 | 25,344 | 25,520 | 25,934 | 26,496 | 27,157 | 27,888 | 28,669 | 29,488 | 30,336 |
| | | | | | $x^*_{BSSS} = 5$ | | | | | | | | |

**10.48e** Q = 60, s = 0, $x_1 = 8$, $x_2 = 4$. As shown, $y_1 = 2$, $y_2 = 1$, and $S_{BS} = 276.73$ sq. ft.




**10.49a** Q = 200, L = 5', W = 4', A = 10', z = 5, c = not stated. (Note: it is not necessary to know the values of W and c in order to determine the optimum row depth.)

Using a continuous approximation, with Q = 200,
$x^c_{BS} = [AQ/2Lz]^{½} = [10(200)/2(5)(5)]^{½} = 6.32$
Therefore, rounding off the value to the nearest integer gives $x^c_{BS} = 6$.
Enumerating over values of x, demonstrates this is not a convex function, as shown below. There are two local optima at x = 6 and x = 8. If only values of x = 5, 6, and 7 were considered, the alternate optimum at x = 8 would be missed.

**10.49b**

**Number of Storage Rows Required to Accommodate the Inventory Level**

| Inventory Level | Prob. | Row Depth (x) 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 200 | 0.10 | 40 | 20 | 14 | 10 | 8 | 7 | 6 | 5 | 5 | 4 |
| 190 | 0.02 | 38 | 19 | 13 | 10 | 8 | 7 | 6 | 5 | 5 | 4 |
| 180 | 0.02 | 36 | 18 | 12 | 9 | 8 | 6 | 6 | 5 | 4 | 4 |
| 170 | 0.02 | 34 | 17 | 12 | 9 | 7 | 6 | 5 | 5 | 4 | 4 |
| 160 | 0.02 | 32 | 16 | 11 | 8 | 7 | 6 | 5 | 4 | 4 | 4 |
| 150 | 0.02 | 30 | 15 | 10 | 8 | 6 | 5 | 5 | 4 | 4 | 3 |
| 140 | 0.02 | 28 | 14 | 10 | 7 | 6 | 5 | 4 | 4 | 4 | 3 |
| 130 | 0.04 | 26 | 13 | 9 | 7 | 6 | 5 | 4 | 4 | 3 | 3 |
| 120 | 0.04 | 24 | 12 | 8 | 6 | 5 | 4 | 4 | 3 | 3 | 3 |
| 110 | 0.04 | 22 | 11 | 8 | 6 | 5 | 4 | 4 | 3 | 3 | 3 |
| 100 | 0.04 | 20 | 10 | 7 | 5 | 4 | 4 | 3 | 3 | 3 | 2 |
| 90 | 0.04 | 18 | 9 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 2 |
| 80 | 0.06 | 16 | 8 | 6 | 4 | 4 | 3 | 3 | 2 | 2 | 2 |
| 70 | 0.06 | 14 | 7 | 5 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| 60 | 0.06 | 12 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 50 | 0.08 | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 |
| 40 | 0.08 | 8 | 4 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 30 | 0.08 | 6 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| 20 | 0.08 | 4 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | 0.08 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | $\eta$ = | 17.28 | 8.64 | 6.14 | 4.56 | 3.84 | 3.24 | 2.92 | 2.52 | 2.40 | 2.12 |
| | S = | 691.20 | 518.40 | 491.20 | 456.00 | 460.80 | 453.60 | 467.20 | 453.60 | 480.00 | 466.40 |
| | | | | | | | x* = 6 | | x* = 8 | | |

**10.50a** Q = 30, L = 36", W = 60", c = 12", A = 156", z = 3

| x = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| xz = | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| y = | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| $S_{BS}$ = | 313.5 | 225.0 | 204.6 | 199.8 | 193.5 | 205.8 | 214.5 | 219.6 | 221.1 | 219.0 | 237.0 | 255.0 |
| | | | | | $x^*_{BS} = 5$ | | | | | | | |

**10.50b** Q = 300, L = 36", W = 60", c = 12", A = 156", z = 3

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x = | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| xz = | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 51 | 54 | 57 | 60 | 63 |
| y = | 10 | 10 | 9 | 8 | 8 | 7 | 7 | 6 | 6 | 6 | 5 | 5 |
| $S_{BS}$ = | 1,204.5 | 1,196.9 | 1,193.4 | 1,190.3 | 1,187.3 | 1,189.7 | 1,190.3 | 1,190.3 | 1,197.9 | 1,200.2 | 1,197.0 | 1,209.3 |
| | | | | | $x^*_{BS}$ = 14 | | | | | | | |

**10.50c** Q = 300, L = 36", W = 60", c = 12", A = 156", z = 3, s = 30.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x = | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| xz = | 36 | 39 | 42 | 45 | 48 | 51 | 54 | 57 | 60 | 63 | 66 | 69 | 72 |
| y = | 9 | 8 | 8 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 |
| $S_{BS}$ = | 1,275.0 | 1,269.5 | 1,266.0 | 1,264.1 | 1,263.4 | 1,263.8 | 1,265.0 | 1,267.0 | 1,269.5 | 1,272.7 | 1,276.2 | 1,280.2 | 1,284.5 |
| | | | | | $x^*_{BSSS}$ = 16 | | | | | | | | |

**10.50d** Q = 30, L = 60", W = 36", c = 24", A = 156", z = 3, s = 0.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| xz = | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 |
| y = | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| $S_{BS}$ = | 316.3 | 247.5 | 232.9 | 231.9 | 236.3 | 243.3 | 252.0 | 261.6 | 271.8 | 282.5 | 293.5 | 304.8 | 316.3 |
| | | | | $x^*_{BSSS}$ = 4 | | | | | | | | | |

The best configuration is 36" x 60" x 48".

**10.51a** Q = 30, L = 3', W = 4', c = 1', A = 8', z = 3
For x = 2, the average amount of floor space required in the freezer is 194.4 sq. ft.

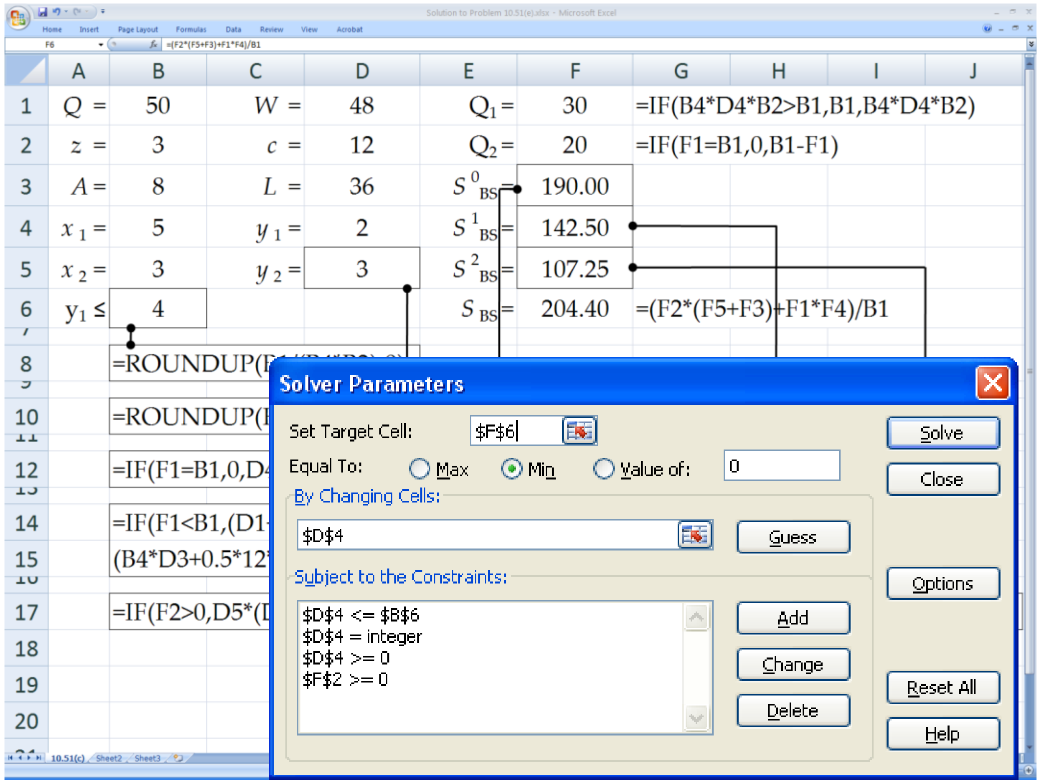
**10.51b** For x = 3, the average amount of floor space required in the freezer is 195.0 sq. ft.

**10.51c** The optimum storage depth is 5, with an average floor space requirement of 158.3 sq. ft.

**Number of Storage Rows Required to Accommodate the Inventory Level**

| | | Row Depth (x) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Inv | % | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 30 | 44% | 10 | 5 | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 1 |
| 25 | 11% | 9 | 5 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1 |
| 20 | 11% | 7 | 4 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| 15 | 11% | 5 | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | 11% | 4 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 11% | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | $\eta$ = | 7.44 | 3.89 | 3.00 | 2.33 | 1.67 | 1.67 | 1.56 | 1.56 | 1.44 | 1.00 |
| | S = | 260.6 | 194.4 | 195.0 | 186.7 | 158.3 | 183.3 | 194.4 | 217.8 | 223.9 | 170.0 |
| | | | | | | x* = 5 | | | | | |

**10.51d** With Q = 50, we assume the inventory pattern is 51 - k, for days 1, ... 50. In other words, no safety stock exists for this production run and single pallet loads are shipped daily. As shown, y1 = 2, y2 = 3, and $S_{BS}$ = 204.40 sq. ft.




**10.52a** Q = 300

$x^c_{DLSS} = [(A + f)(Q)/2L]^{½} = [(96 + 12)(300)/2(42)]^{½} = 19.64$

$x^c_{DLSS} = 20$

Using Excel to solve by enumerating over x gives the same result, as shown below.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Q = 300 | | L = 42 | | W = 48 | | c = 4 | | | |
| | A = 96 | | f = 12 | | r = 6 | | z = 6 | | | |
| x = | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| ν = | 20 | 19 | 18 | 17 | 16 | 15 | 15 | 14 | 14 | 13 |
| $S_{BS}$ = | 74,214 | 74,120 | 74,043 | 73,991 | 73,954 | 73,904 | 73,991 | 74,043 | 74,026 | 74,184 |

$x_{DLSS} = 20$

**10.52b** Q = 35, s = 0. Using Excel to solve by enumerating over x gives the following result.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Q = 35 | | L = 42 | | W = 48 | | c = 4 | | | |
| | A = 96 | | f = 12 | | r = 6 | | z = 6 | | | |
| x = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ν = | 35 | 18 | 12 | 9 | 7 | 6 | 5 | 5 | 4 | 4 |
| $S_{BS}$ = | 17,856 | 13,201 | 11,798 | 11,208 | 10,912 | 10,841 | 10,788 | 10,939 | 10,969 | 11,195 |
| | | | | | | | $x_{DLSS}$ = 7 | | | |

**10.53a** Q = 30, L = 48", W = 40", c = 4", r = 4", f = 9", A = 96", z = 7.
$S_{SD}$ = (W + 1.5c + 0.5r)[L + 0.5(A + f)](Q + 1)/2z
$S_{SD}$ = (40 + 6 + 2)[48 + 0.5(96 + 9)](31)/2(7)(144)
$S_{SD}$ = 74.18 sq. ft.

**10.53b** Q = 30, L = 48", W = 40", c = 4", r = 4", f = 9", A = 96", z = 7.
$S_{DD}$ = (W + 1.5c + 0.5r)[2L + 0.5(A + f)](Q + 2)/4z
$S_{DD}$ = (40 + 6 + 2)[96 + 0.5(96 + 9)](32)/4(7)(144)
$S_{DD}$ = 56.57 sq. ft.

**10.53c** Q = 30, L = 48", W = 40", c = 4", r = 4", f = 9", A = 96", z = 7.
$S_{TD}$ = ν(W + 1.5c + 0.5r)[3L + 0.5(A + f)](2Q - 3ν + 3)/2Qz
where ν = ⌈Q/3⌉ = ⌈30/3⌉ = 10, (note: ⌈xx⌉ denotes the largest integer ≤ xx).
$S_{TD}$ = 10(40 + 6 + 2)[144 + 0.5(96 + 9)](60 - 30 + 3)/60(7)(144)
$S_{TD}$ = 51.46 sq. ft.

**10.54a**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Q = 30 | | L = 60 | | W = 38 | | c = 10 | | A = 156 | | z = 4 | |
| x = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| xz = | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| y = | 8 | 4 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| $S_{BS}$ = | 196.3 | 158.4 | 154.8 | 155.5 | 168.0 | 175.2 | 177.1 | 186.0 | 206.0 | 226.0 | 246.0 | 266.0 |
| | | | $x^*_{BS}$ = 3 | | | | | | | | | |

**10.54b** Q = 30, L = 60", W = 38", c = 3", f = 12", r = 4", A = 72", z = 8
$x^c_{DL}$ = [(A + f)(Q)/2L]$^{½}$ = [(72 + 12)(30)/2(60)]$^{½}$ = 4.58
$x^c_{DL}$ = 5 (Enumeration yields the same result, as shown below.)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ν = | 30 | 15 | 10 | 8 | 6 | 5 | 5 | 4 | 4 | 3 |
| $S_{DL}$ = | 65.9 | 54.0 | 50.9 | 50.1 | 49.9 | 50.3 | 51.3 | 52.2 | 53.4 | 53.5 |
| | | | | | $x_{DL}$ = 5 | | | | | |

**10.54c** Q = 30, L = 60", W = 38", c = 5", f = 12", r = 5", A = 96", z = 6
$S_{SD}$ = (W + 1.5c + 0.5r)[L + 0.5(A + f)](Q + 1)/2z

$S_{SD}$ = (38 + 7.5 + 2.5)[60 + 0.5(96 + 12)](31)/2(6)(144)
$S_{SD}$ = 98.17 sq. ft.

**10.54d** $S_{DD}$ = (W + 1.5c + 0.5r)[2L + 0.5(A + f)](Q + 2)/4z
$S_{DD}$ = (38 + 7.5 + 2.5)[120 + 0.5(96 + 12)](32)/4(6)(144)
$S_{DD}$ = 77.33 sq. ft.

**10.54e** $S_{TD}$ = ν(W + 1.5c + 0.5r)[3L + 0.5(A + f)](2Q - 3ν + 3)/2Qz
where $\nu = \lceil Q/3 \rceil = \lceil 30/3 \rceil = 10$
$S_{TD}$ = 10(38 + 7.5 + 2.5)[180 + 0.5(96 + 12)](60 - 30 + 3)/60(6)(144)
$S_{TD}$ = 71.50 sq. ft.

**10.55a** Q = 250, L = 36", W = 48", c = 4", r = 4", f = 12", A = 60", z = 15. Computing the value of $S_{DL}$ yields the following results. The lane depth of 5 is the worst of the three.

| | | |
|---|---|---|
| Q = 250 | L = 36 | W = 48 |
| c = 4 | A = 60 | f = 12 |
| s = 0 | r = 4 | z = 15 |

| | | | |
|---|---|---|---|
| x = | 5 | 10 | 15 |
| ν = | 50 | 25 | 17 |
| $S_{DL}$ = | 153.00 | 143.00 | 141.44 |
| | Worst! | | |

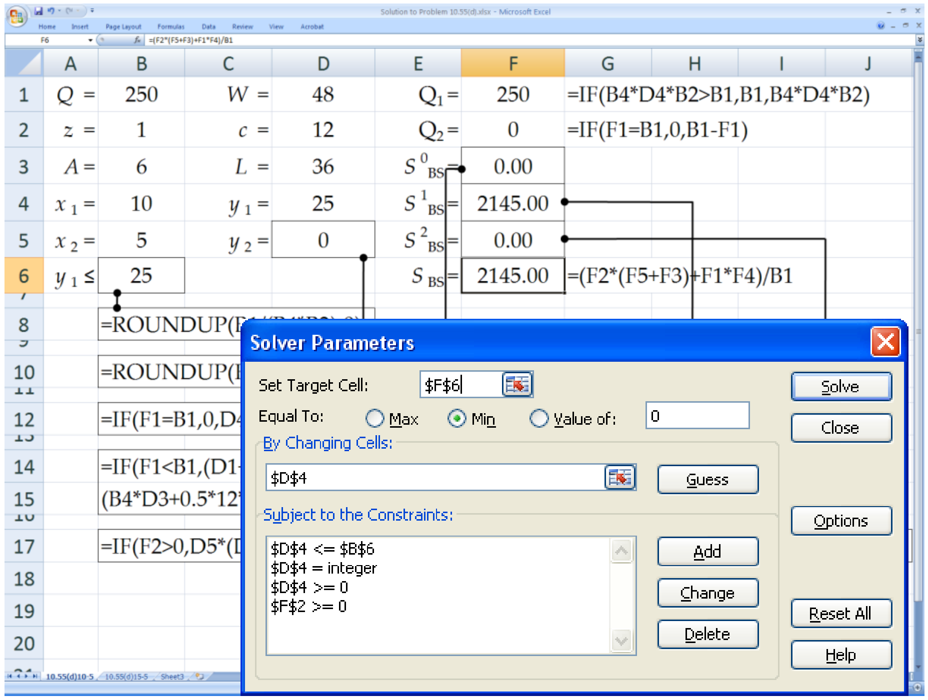
**10.55b** As shown below, 15 is the best lane depth of the three choices.

| | | |
|---|---|---|
| Q = 250 | L = 36 | W = 48 |
| c = 4 | A = 96 | z = 6 |
| s = 50 | f = 12 | r = 4 |

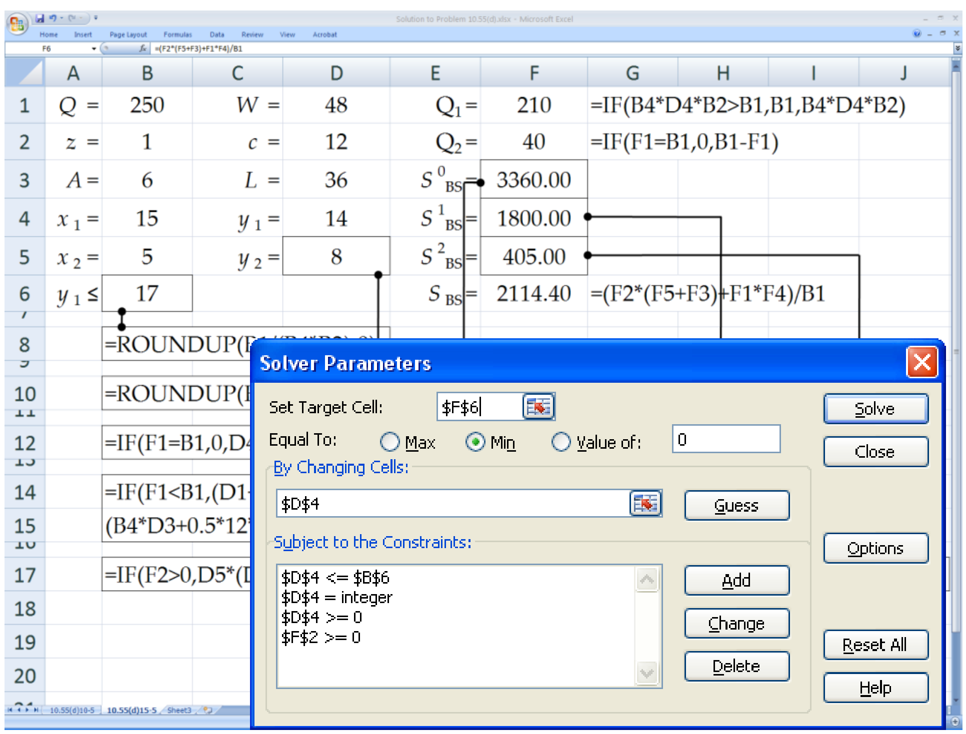
| | | | |
|---|---|---|---|
| x = | 5 | 10 | 15 |
| ν = | 50 | 25 | 17 |
| $S_{DLSS}$ = | 480.7 | 431.3 | 420.8 |
| | | Best! | |

**10.55c** Q = 250, s = 50, L = 36", W = 48", c = 4", f = 12", r = 4", A = 96", z = 6
$S_{DD}$ = (Q/2)(W+1.5c+0.5r)[2L+0.5(A+f)][2(Q+s)-Q+2]/2(Q+s)z
$S_{DD}$ = 125(48 + 6 + 2)[72 + 0.5(96 + 12)](352)/2(300)(6)(144)
$S_{DD}$ = 598.89 sq. ft.

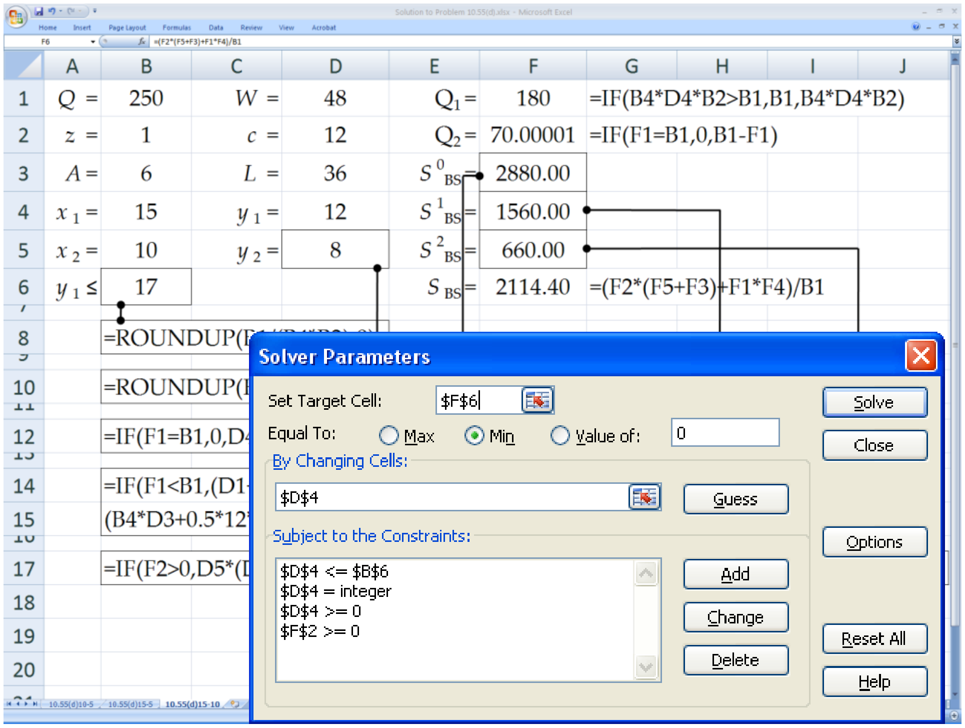
**10.55d** Recall, the deep lane storage equation for floor space required can be obtained directly from the block stacking equation for floor space required . Specifically, replace c with 2c + r ,replace A with A + f, let z = 1, and divide the resulting floor space by the number of levels of deep lane storage. Hence, using SOLVER , when $x_1$ = 10 and $x_2$ = 5, we obtain $y_1$ = 25, $y_2$ = 0, and $S_{DL}$ = 2,145/15 = 143 sq. ft., as obtained in part b).




When $x_1$ = 15 and $x_2$ = 5, we obtain $y_1$ = 14, $y_2$ = 8, and $S_{DL}$ = 2,114.40/15 = 140.96 sq. ft.

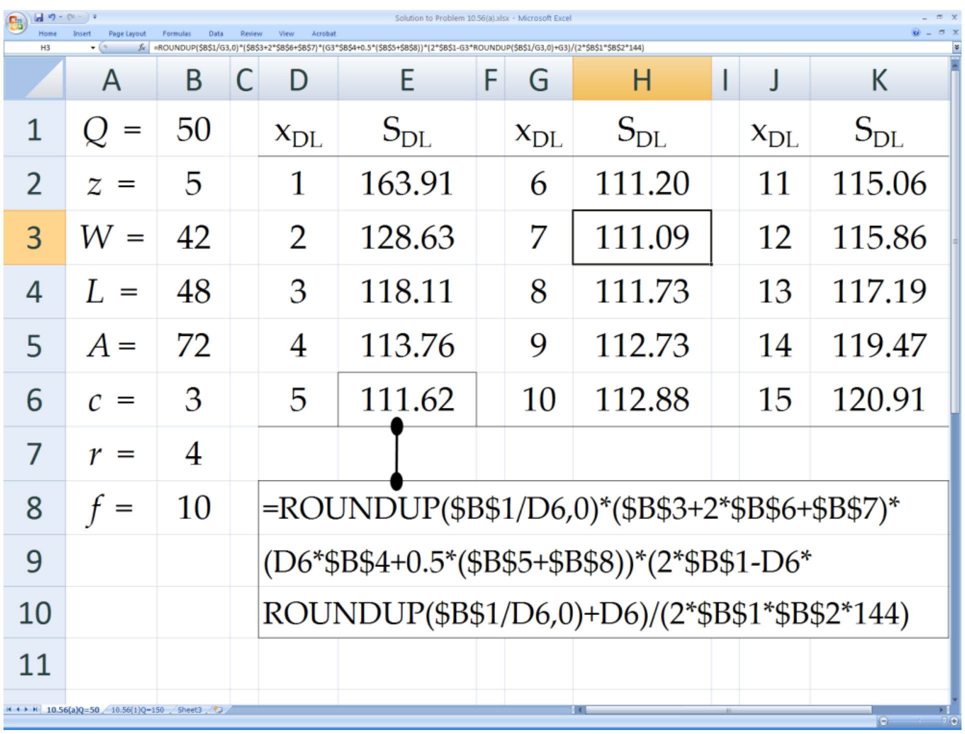



When $x_1$ = 15 and $x_2$ = 10, we obtain $y_1$ = 12, $y_2$ = 8, and $S_{DL}$ = 2,114.40/15 = 140.96 sq. ft.




Hence, the product should be stored in either 15-deep and 5-deep or 15-deep and 10-deep storage lanes.

**10.56a**



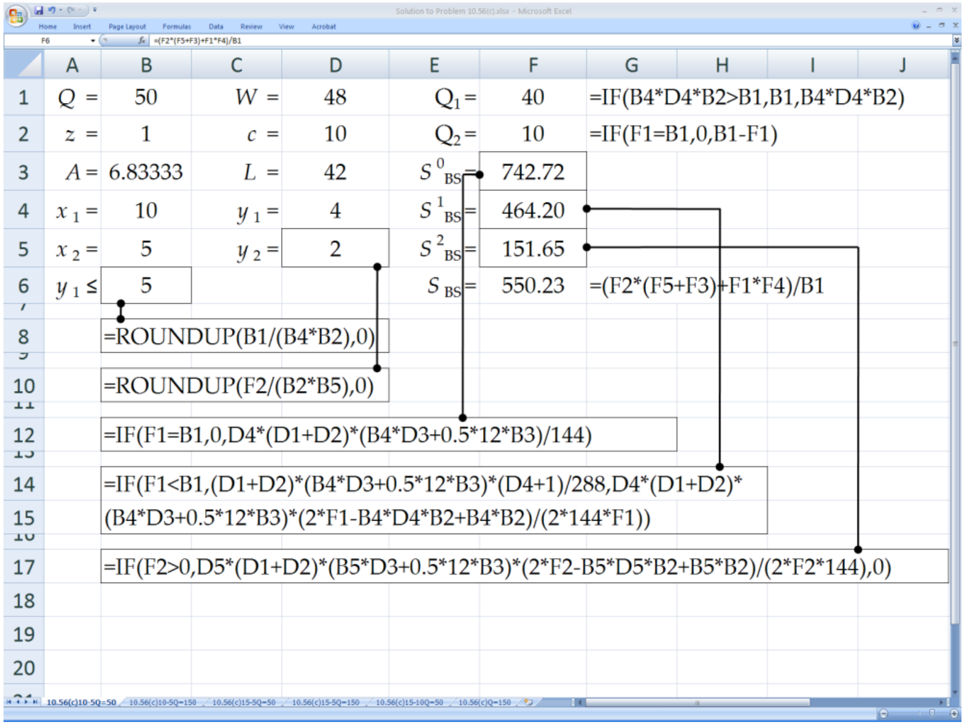
| | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $Q$ = | 150 | | $x_{DL}$ | $S_{DL}$ | | $x_{DL}$ | $S_{DL}$ | | $x_{DL}$ | $S_{DL}$ |
| 2 | $z$ = | 5 | | 1 | 485.30 | | 6 | 308.89 | | 11 | 301.09 |
| 3 | $W$ = | 42 | | 2 | 375.99 | | 7 | 305.50 | | 12 | 301.23 |
| 4 | $L$ = | 48 | | 3 | 340.71 | | 8 | 303.26 | | 13 | 301.61 |
| 5 | $A$ = | 72 | | 4 | 323.99 | | 9 | 301.98 | | 14 | 302.10 |
| 6 | $c$ = | 3 | | 5 | 314.56 | | 10 | 301.02 | | 15 | 302.29 |
| 7 | $r$ = | 4 | | | | | | | | | |
| 8 | $f$ = | 10 | | =ROUNDUP($B$1/D6,0)*($B$3+2*$B$6+$B$7)* | | | | | | | |
| 9 | | | | (D6*$B$4+0.5*($B$5+$B$8))*(2*$B$1-D6* | | | | | | | |
| 10 | | | | ROUNDUP($B$1/D6,0)+D6)/(2*$B$1*$B$2*144) | | | | | | | |
| 11 | | | | | | | | | | | |

Low Calorie Hot Fudge Sundae should be stored in 7-deep lanes; Cardiac Arrest Banana Split should be stored in 10-deep lanes. The total average square footage requirement is 111.09 + 314.56 = 425.65 sq. ft.

**10.56b** $Q_1$ = 50, L = 42", W = 48", c = 3", f = 10", r = 4", A = 72", z = 5
$S_{DD} = (Q/2)(W+1.5c+0.5r)[2L+0.5(A+f)][2(Q+s)-Q+2]/2(Q+s)z$
$S_{DD} = 25(48 + 4.5 + 2)[84 + 0.5(72 + 10)](52)/2(50)(5)(144)$
$S_{DD}$ = 123.00 sq. ft.

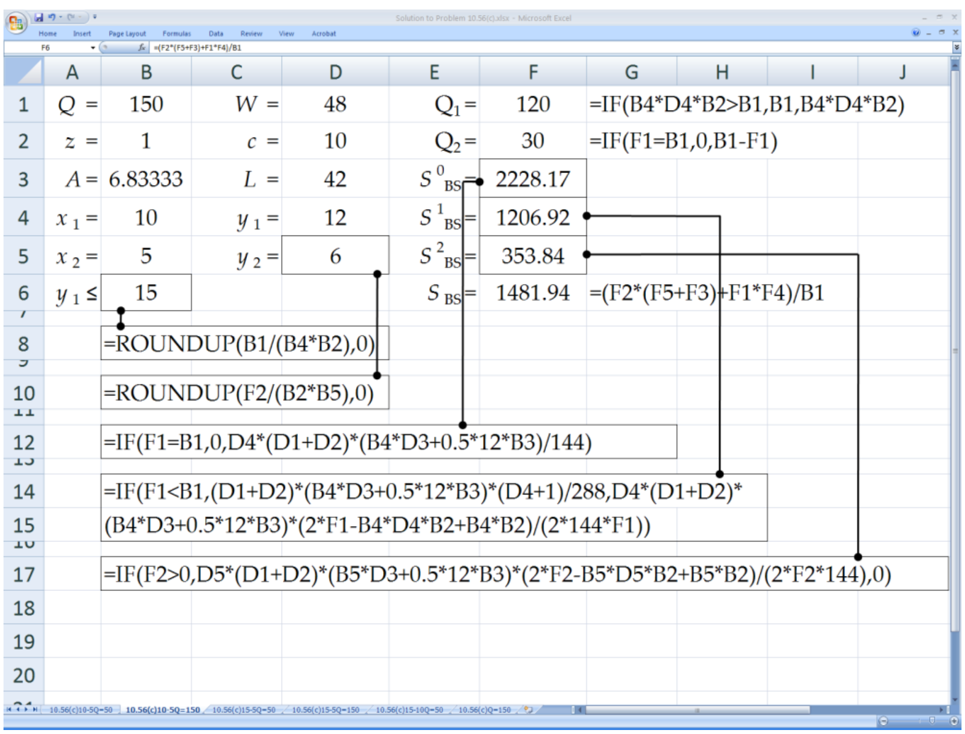
$Q_2$ = 150, L = 42", W = 48", c = 3", f = 10", r = 4", A = 72", z = 5
$S_{DD} = (Q/2)(W+1.5c+0.5r)[2L+0.5(A+f)][2(Q+s)-Q+2]/2(Q+s)z$
$S_{DD} = 75(48 + 4.5 + 2)[84 + 0.5(72 + 10)](152)/2(150)(5)(144)$
$S_{DD}$ = 359.55 sq. ft.

**10.56c** Refer to the solution to 10.55(d). For Q = 50, $x_1$ = 10, and $x_2$ = 5, using SOLVER we obtain $y_1$ = 4, $y_2$ = 2, and $S_{DL}$ = 550.23/5 = 110.05 sq. ft.



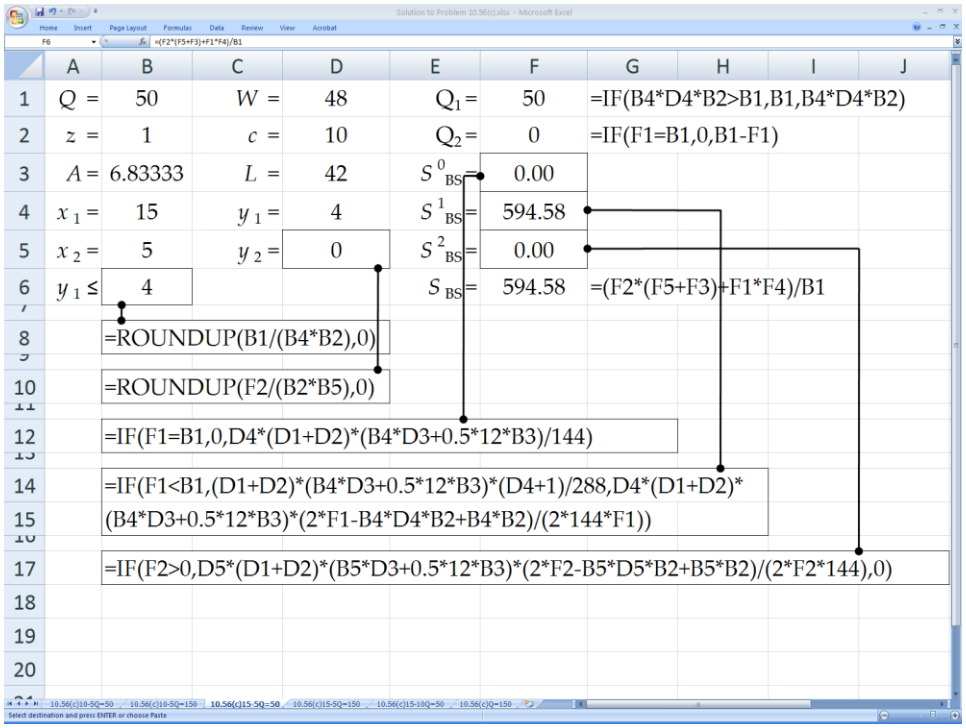
| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | Q = | 50 | W = | 48 | $Q_1$ = | 40 | =IF(B4*D4*B2>B1,B1,B4*D4*B2) |
| 2 | z = | 1 | c = | 10 | $Q_2$ = | 10 | =IF(F1=B1,0,B1-F1) |
| 3 | A = | 6.83333 | L = | 42 | $S^0_{BS}$ = | 742.72 | |
| 4 | $x_1$ = | 10 | $y_1$ = | 4 | $S^1_{BS}$ = | 464.20 | |
| 5 | $x_2$ = | 5 | $y_2$ = | 2 | $S^2_{BS}$ = | 151.65 | |
| 6 | $y_1$ ≤ | 5 | | | $S_{BS}$ = | 550.23 | =(F2*(F5+F3)+F1*F4)/B1 |
| 8 | | =ROUNDUP(B1/(B4*B2),0) | | | | | |
| 10 | | =ROUNDUP(F2/(B2*B5),0) | | | | | |
| 12 | | =IF(F1=B1,0,D4*(D1+D2)*(B4*D3+0.5*12*B3)/144) | | | | | |
| 14 | | =IF(F1<B1,(D1+D2)*(B4*D3+0.5*12*B3)*(D4+1)/288,D4*(D1+D2)* | | | | | |
| 15 | | (B4*D3+0.5*12*B3)*(2*F1-B4*D4*B2+B4*B2)/(2*144*F1)) | | | | | |
| 17 | | =IF(F2>0,D5*(D1+D2)*(B5*D3+0.5*12*B3)*(2*F2-B5*D5*B2+B5*B2)/(2*F2*144),0) | | | | | |

For Q = 150, $x_1$ = 10, and $x_2$ = 5, using SOLVER we obtain $y_1$ = 12, $y_2$ = 6, and $S_{DL}$ = 1481.94/5 = 296.39 sq. ft.

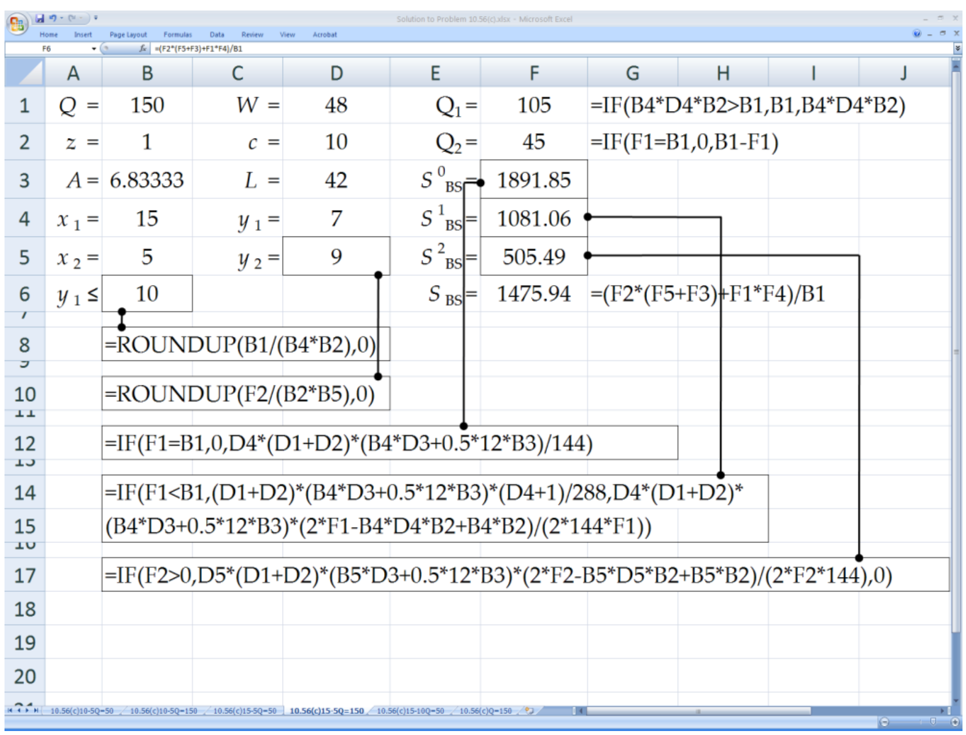


| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | Q = | 150 | W = | 48 | $Q_1$ = | 120 | =IF(B4*D4*B2>B1,B1,B4*D4*B2) |
| 2 | z = | 1 | c = | 10 | $Q_2$ = | 30 | =IF(F1=B1,0,B1-F1) |
| 3 | A = | 6.83333 | L = | 42 | $S^0_{BS}$ = | 2228.17 | |
| 4 | $x_1$ = | 10 | $y_1$ = | 12 | $S^1_{BS}$ = | 1206.92 | |
| 5 | $x_2$ = | 5 | $y_2$ = | 6 | $S^2_{BS}$ = | 353.84 | |
| 6 | $y_1$ ≤ | 15 | | | $S_{BS}$ = | 1481.94 | =(F2*(F5+F3)+F1*F4)/B1 |
| 8 | | =ROUNDUP(B1/(B4*B2),0) | | | | | |
| 10 | | =ROUNDUP(F2/(B2*B5),0) | | | | | |
| 12 | | =IF(F1=B1,0,D4*(D1+D2)*(B4*D3+0.5*12*B3)/144) | | | | | |
| 14 | | =IF(F1<B1,(D1+D2)*(B4*D3+0.5*12*B3)*(D4+1)/288,D4*(D1+D2)* | | | | | |
| 15 | | (B4*D3+0.5*12*B3)*(2*F1-B4*D4*B2+B4*B2)/(2*144*F1)) | | | | | |
| 17 | | =IF(F2>0,D5*(D1+D2)*(B5*D3+0.5*12*B3)*(2*F2-B5*D5*B2+B5*B2)/(2*F2*144),0) | | | | | |

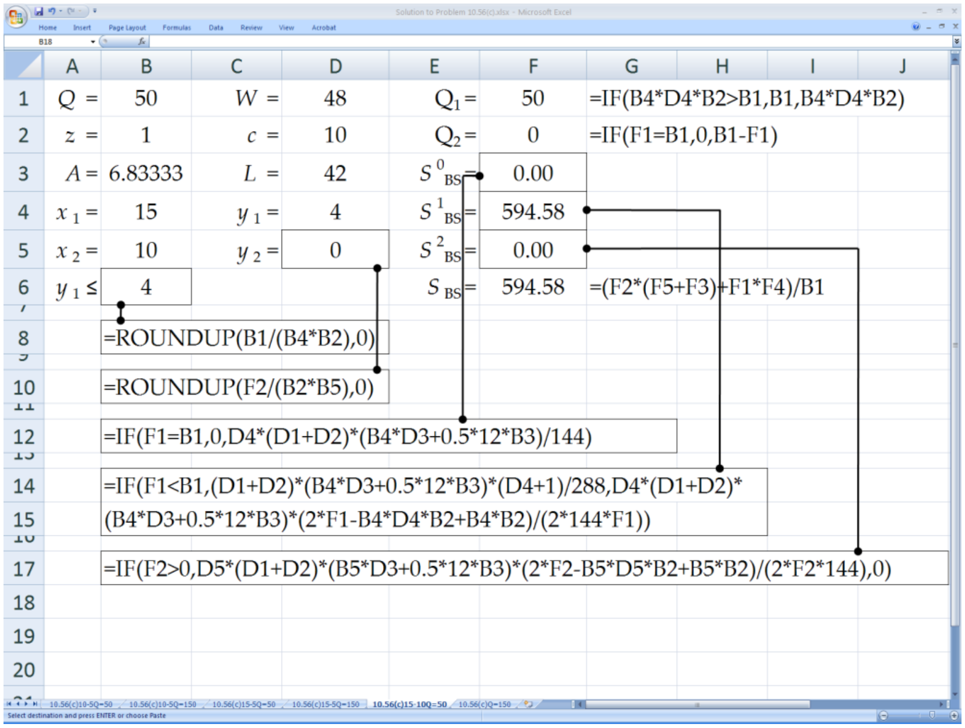
For Q = 50, $x_1$ = 15, and $x_2$ = 5, using SOLVER we obtain $y_1$ = 4, $y_2$ = 0, and $S_{DL}$ = 594.58/5 = 118.92 sq. ft.



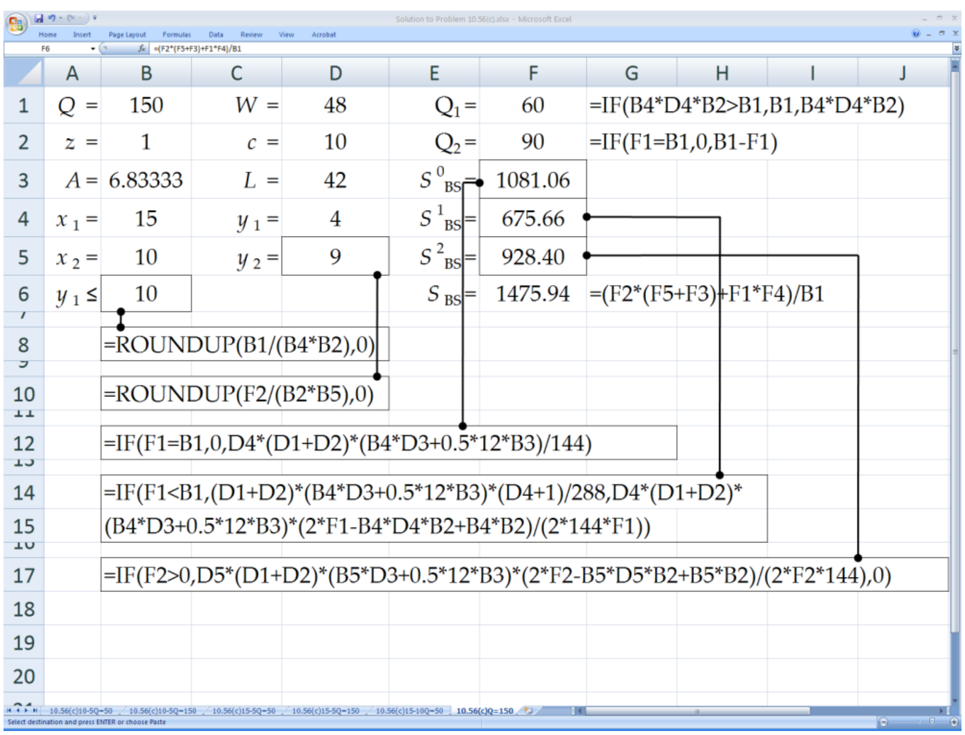

For Q = 150, $x_1$ = 15, and $x_2$ = 5, using SOLVER we obtain $y_1$ = 7, $y_2$ = 9, and $S_{DL}$ = 1475.94/5 = 295.19 sq. ft.




For Q = 50, $x_1$ = 15, and $x_2$ = 10, using SOLVER we obtain $y_1$ = 4, $y_2$ = 0, and $S_{DL}$ = 594.58/5 = 118.92 sq. ft.




For Q = 150, $x_1$ = 15, and $x_2$ = 10, using SOLVER we obtain $y_1$ = 4, $y_2$ = 9, and $S_{DL}$ = 1475.94/5 = 295.19 sq. ft.




Based on the SOLVER solutions, the best storage configuration for Low Calorie Hot Fudge Sundae is a combination of 5-deep and 10-deep storage lanes. Specifically, with $x_1$

= 10, $y_1$ = 4, $x_2$ = 5, and $y_2$ = 2, 110.05 sq. ft. will be required, on average, to store 50 unit loads of the product.

For Cariac Arrest Banana Split, the best configuration is a combination of 15-deep and either 5-deep or 10-deep storage lanes. Specifically, with either $x_1$ = 15, $y_1$ = 7, $x_2$ = 5, and $y_2$ = 9, or with $x_1$ = 15, $y_1$ = 4, $x_2$ = 10, and $y_2$ = 9, 295.19 sq. ft. will be required, on average, to store 150 unit loads of the product.

A cautionary note: SOLVER does not guarantee an optimal solution. Depending on the starting solution, different answers can be obtained when using SOLVER. It might be possible for a different starting solution to yield better solutions than are shown in this and previous problems, where SOLVER was used.

**10.57** Research Question. Very different results can be obtained from the Web search. A good starting point for such a search is the Material Handling Industry of America Web site. Association, particularly the College-Industry Committee on Material Handling Education site, e.g, https://www.cirrelt.ca/mhmultimediabank/

**SECTION 10.6**

**10.58** We have L = 250 ft, H = 55 ft, $t_h$ = 0.781 min, $t_v$ = 0.733, T = 0.781 min, and Q = 0.939. Therefore, $T_{SC}$ = 1.497 min and $T_{DC}$ = 2.363 min.

Each S/R machine will have to perform (on the average) 10.5 dual command cycles and 9 single-command cycles per hour, which requires 10.5(2.3643) + 9(1.5107) = 38.42145 min per hour. Hence, the utilization of each S/R is (38.42145/60) $\times$100% = 64.04%.

To compute the cost of the building, we first compute the building dimensions and the conversion factor as follows: BW = 164 ft, BL = 281.4 ft, BH = 59 ft, and $\delta$ = 20. We use linear interpolation to obtain CF = 1.6067. Then, by (10.89) BC = \$1,482,971.25

The S/R machine cost is CSR = $3\beta + 2\gamma + 4\phi$. Assuming that $\beta = \gamma = \phi$ = \$25,000, we have CSR = \$225,000.

**10.59** We have $t_h$ = 0.75 min, $t_v$ = 0.625 min, T = 0.75 min, Q = 0.833. Therefore, $T_{SC}$ = 1.523 min per single command cycle and $T_{DC}$ = 2.446 min per dual command cycle.

Since a total of 400 single-command cycles and 400 dual-command cycles are performed by five S/R machines over an 8-hr period, on average, each S/R machine performs 10 single-command cycles and 10 dual-command cycles per hour.

Hence, the system utilization is $\frac{(1.523)10+(2.446)10}{60} \times 100\%$, or 66.15%.

**10.60** We have L = 233.33 ft, H = 47.67 ft, $t_h$ = 0.78 min, $t_v$ = 0.681, T = 0.78 min, and Q = 0.873. Therefore, $T_{SC}$ = 1.678 min and $T_{DC}$ = 2.72 min.

Let x be the maximum percentage of single-command operations performed such that the utilization of the S/R machine does not exceed 85%. To meet the required performance of 18 storages and 18 retrievals per hour, each S/R will have to perform 18x single command cycles per hour for storages, 18(1 – x) single command cycles per hour for retrievals, and 18 dual command cycles per hour for remaining storage and retrieval operations.

Thus, we have $2(18x)(1.497) + 18(1 - x)(2.363) \leq 60(0.85)$, or $x \leq 0.1782$. Therefore, in order to maintain an S/R machine utilization of 85% or less, the percentage of single command operations may not exceed 17.82%.

To compute the cost of the building, we first compute the building dimensions and the conversion factor as follows: BW = 98 ft, BL = 267.43 ft, and $\delta = 20$. We use linear interpolation to obtain CF = 1.4445. Then, by (10.89) BC = $757,153.16.

To compute the rack cost we have: v = 49 $ft^3$, w = 2,000 lb, and n = 11. Assuming $\alpha$ = $30, by (10.87) we have CRO = $149.29, and the total rack cost = CRO(8)(1,100) = $1,313,752.

The S/R machine cost is $CSR = 2\beta + 2\gamma + 3\phi$. Assuming that $\beta = \gamma = \phi$ = $25,000, we have CSR = $175,000; hence, total S/R machine cost is given by CSR(8) = $1,400.000.

**10.61a** We have $t_h = 0.73$ min, $t_v = 0.875$ min, T = 0.875 min, Q = 0.834. Therefore, $T_{SC} = 1.88$ min per single-command cycle and $T_{DC} = 3.05$ min per dual-command cycle. Since each S/R machine is required to perform a total of 10 dual-command cycles and 20 single-command cycles per hour, it will take 10(3.05) + 20(1.88) = 68.1 min per hour for the S/R to complete the requirement. Therefore, we conclude that the system cannot handle the required workload.

**10.61b** Assuming that 100% of the storages are performed with dual-command cycles, each S/R will be required to perform a total of 6 dual-command cycles and 22 single-command cycles per hour. It will take 6(3.05) + 22(1.88) = 59.66 *min* per hour for each S/R to meet the required workload. Thus, the utilization of each S/R is 59.66 60 = 0.994 , or 99.4%.

**10.62a** By (10.59) and (10.60) L = 233.33 ft and H = 58 ft. Therefore, $t_h = 0.78$ min, $t_v = 0.97$ min, T = 0.97 min, Q = 0.804, $T_{SC} = 1.78$ min per single-command cycle, and $T_{DC} = 2.79$ min per dual-command cycle.
Thirty percent of the retrievals and 30% of the storages are single-command operations. Let *x* denote the total number of operations (storages + retrievals) performed per hour. Assuming an equal number of storage and retrieval operations per hour, we obtain the following inequality: $0.35x(2.79) + 0.30x(1.78) \leq 60(0.9)$ , from which we obtain $x \leq$ 35.750 operations/hr. That is, each S/R machine can perform 17.875 storages/hr and 17.875 retrievals/hr without being utilized more than 90%.

**10.62b** Compute the cost of racks for v = 53.33 $ft^3$, w= 3,000 lb, n = 12. Assuming $\alpha$ = \$30, by (10.65) we obtain CRO = \$167.70. Thus, the total rack cost is equal to 167.70×50×12×2×8 = \$1,609,954.30.

Compute the cost of S/R machines for $\beta = \gamma = \phi$ = \$25,000. By (10.66) CSR = $3\beta + 2\gamma + 4\phi$ = 9(\$25,000) = \$225,000. Thus, the cost of S/R machines = 8(\$225,000) = \$1,800,000.

To compute the cost of the building:
By (10.62) BW = aW + 24 in.= 1,128 in.= 94 ft.
By (10.63) and (10.64) BL = L + 12.5 + 0.45y = 267.43 ft.
By (10.61) BH = H + 48 in.= 62 ft.
CF = 1.5 + 0.4(7÷15) = 1.687 and $\delta$ = \$25 / $ft^2$. Therefore, by (10.67) BC = \$1,060,212.86.

**10.63** By (10.59) and (10.60) we can determine the length and height of the racks (in feet) for the three choices:

| Choice | L | H |
|---|---|---|
| (a) | 300 | 36 |
| (b) | 336 | 36 |
| (c) | 376 | 36 |

Next, the single-command and dual-command cycle times (in minutes) for the three choices can be computed as follows:

| Choice | $t_h$ | $t_v$ | T | Q | $T_{SC}$ | $T_{DC}$ |
|---|---|---|---|---|---|---|
| (a) | 0.667 | 0.45 | 0.667 | 0.675 | 1.368 | 2.234 |
| (b) | 0.747 | 0.45 | 0.747 | 0.602 | 1.437 | 2.326 |
| (c) | 0.836 | 0.45 | 0.836 | 0.538 | 1.517 | 2.43 |

Forty percent of the operations will be single-command based. Thus, the system is required to perform a total of 144 single-command cycles and 108 dual-command cycles per hour, and the total time (in minutes) required by the three choices is equal to 438.264, 458.136, and 480.888, respectively.

For choice (a), since 438.264 / 10 < 60(.95) , we conclude that it satisfies the throughput requirement. Similarly, we conclude that choice (b) satisfies the throughput requirement, while choice (c) does not.

Next, for v = 48.89 $ft^3$, w = 2,500 lb, n = 8 (and assuming $\alpha$ = \$30 and $\beta = \gamma = \phi$ = \$25,000), by (10.65) and (10.66) we compute the rack cost and the S/R machine cost for choices (a) and (b) to obtain the following table:

| Choice | Rack Cost | S/R Machine Cost | Total Cost |
|---|---|---|---|
| (a) | $1,706,016 | $2,000,000 | $3,706,016 |
| (b) | $1,719,664 | $1,800,000 | $3,519,664 |

We conclude that choice (b) is the least-cost alternative that satisfies the throughput constraint.

**10.64** By (10.59) and (10.60) we have L = 310 ft and H = 70 ft, for which $t_h$ = 0.729 min, $t_v$ = 0.875 min, T = 0.875 min, Q = 0.833, $T_{SC}$ = 1.877 min, and $T_{DC}$ = 3.053.

During peak activity each S/R is expected to perform a total of 6 dual-command cycles and 18 single-command cycles per hour, and the total time required by each S/R is 6(3.053) + 18(1.877) = 52.104 min per hour. Since 52 , we conclude that the system will satisfy the utilization constraint.

**10.65** By (10.60) and (10.61) BH = H + 4 ft = 64 ft. By (10.58) and (10.62) BW = aW + 2 ft = 74 ft. By (10.59), (10.63), and (10.64) BL = L + 12.5 + 0.45y = 213.2 ft.

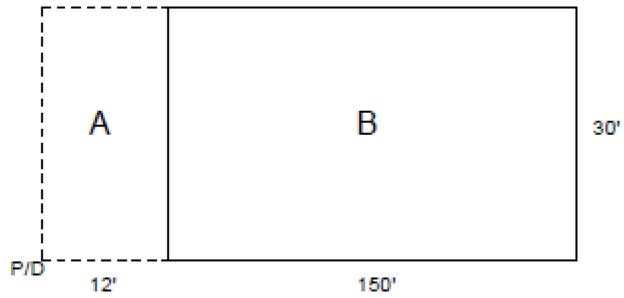
**10.66** Sixty percent of the operations are performed on a dual-command basis. Therefore, the system is required to perform a total of 60 dual-command cycles and 80 single-command cycles per hour. We next determine that $t_h$ = 0.75 min, $t_v$ = 0.75 min, T = 0.75 min, and Q = 1, for which $T_{SC}$ = 1.5 min and $T_{DC}$ = 2.35 min.

Each S/R machine can be utilized at most 75% of the time; i.e., 45 minutes per hour. It takes a total of 1.5(80) + 2.35(60) = 261*min* per hour to perform all the operations. Therefore, a minimum of $\lceil 261/45 \rceil$ = 6 S/R machines are required to maintain a 75% or less equipment utilization.

**10.67** We need to minimize the expected single-command travel time, which is given by the following function: $(1 + Q^2/3)T$, subject to the constraint $QT^2 = A$. (Note that the normalized rack is QT units long in one direction and *T* units long in the other direction; the area is expressed in time units squared.) Hence, we need to minimize the function $(1 + Q^2/3)\sqrt{A/Q}$.

Taking the derivative with respect to Q, and setting it equal to zero, we obtain Q* = 1, which minimizes the expected single-command travel time. In other words, if the sole objective is to minimize the expected single-command travel time, the optimum rack is square-in-time. The same holds true for the expected dual-command travel time although the proof is slightly more involved.

**10.68**




Consider the rack formed by region AB: T = 0.405, Q = 0.926, and $E(SC_{AB})$ = 0.5208 min. Consider next the rack formed by region A: T = 0.375, Q = 0.08, and $E(SC_A)$ = 0.3758 min. Thus, $E(SC_B)$, the expected S/R machine single-command travel time can be obtained by solving the equation $E(SC_{AB}) = E(SC_A)(12/162) + E(SC_B)(150/162)$. We obtain $E(SC_B)$ = 0.5324 min.
The percent difference = (100%)(0.5324-0.5208)/0.5324 = 2.18%

To compute the expected S/R machine travel time for dual-command cycles, note that E(TB) is *not* affected by the location of the P/D station. Hence, for the rack formed by region B: $t_h$ = 0.375, $t_v$ = 0.375, T = 0.375, Q = 1, and $E(TB_B)$ = 0.175. Therefore, $E(DC_B) = E(SC_B) + E(TB_B)$ = 0.7074 min. Next, we compute $E(DC_{AB})$: $t_h$ = 0.405, $t_v$ = 0.375, T = 0.405, Q = 0.926, and by (10.72) $E(DC_{AB})$ = 0.7029. Therefore, the percent difference = (100%)(0.7074 - 0.7029)/0.7074 = 0.636%.

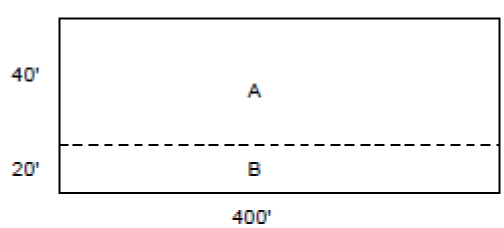
**10.69a** Let the random point be given by (x,y) where $0 \le x \le 1$ and $0 \le y \le Q$. The travel time to point (x,y), say z, is equal to (x + y).

Hence E(z) = E(x + y) = E(x) + E(y) = 1/2 + Q/2 = (Q + 1)/2.
Therefore, E(SC) = 2E(z) = Q + 1.

**10.69b** Let the two random points be given by $(x_1,y_1)$ and $(x_2,y_2)$ where $0 \le x_i \le 1$, and $0 \le y_i \le Q$. Then
$E(TB) = E(|x_1 - x_2| + |y_1 - y_2|) = E(|x_1 - x_2|) + E(|y_1 - y_2|) = 1/3 + Q/3$

Therefore, E(DC) = E(SC) + E(TB) = (Q + 1) + (1/3 + Q/3) = (4/3)(Q + 1).

**10.70a**

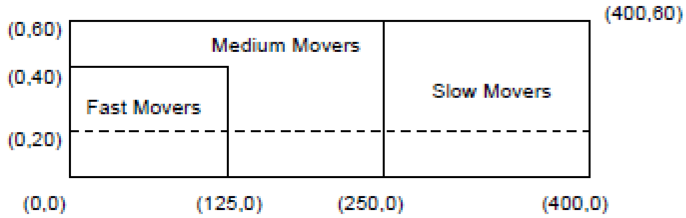



Consider the rack formed by region A: $t_h$ = 0.8, $t_v$ = 0.5, T = 0.8, Q = 0.625. By (10.70) we obtain E(SCA) = 0.9042. Consider next the rack formed by region B; following the

same procedure, we obtain $E(SC_B) = 0.8260$. Therefore, $E(SC_{AB}) = 0.8260$. Therefore, $E(SC_{AB}) = (2/3)(0.9042) + (1/3)(0.8260) = 0.8781$ min, and
$T_{SC} = 0.8781 + 2(0.3) = 1.4781$ MIN.

**10.70b** Considering the entire rack and following the same procedure as in part (a) to compute E(TB) , we obtain E(TB) = 0.3619 . Therefore, $T_{DC} = E(SC_{AB}) + E(TB) + 4T_{P/D} = 2.44$ min.

**10.70c**




For the *fast movers*, consider the "upper" part of the rack (i.e., the 125'×20' region above the dashed line): $t_h = 0.25$, $t_v = 0.25$, T = 0.25, Q = 1. Thus, $E(SC_{Upper}) = 0.3333$. Since the "lower" part of the rack (the 125'x20' region below the dashed line) has the same dimensions as the upper part, we have $E(SC_{Lower}) = 0.3333$. Therefore, $E(SC_F)$, the expected single-command travel time for the rack portion where the fast movers are stored, is given by $E(SC_F) = 0.5E(SC_{Upper}) + 0.5E(SC_{Lower}) = 0.3333$ min.

For the rack portion dedicated to fast and medium movers (a 250'×60' region), following the same procedure, we obtain 0.6667 min and 0.5417 min for the expected single-command travel time for the upper and lower part of the rack, respectively. Thus, $E(SC_{F\&M})$, the expected single-command travel time for the above 250'×60' rack is obtained as follows

$E(SC_{F\&M}) = 0.6667(2/3) + 0.5417(1/3) = 0.6250$ min. Therefore, $E(SC_M)$, the expected single-command travel time for the rack portion where the medium movers are stored, is obtained by solving the following equation:

$$E(SC_{F\&M}) = \frac{40\times125}{60\times250}E(SC_F) + \frac{60\times250-40\times125}{60\times250}E(SC_M),$$

from which we have $E(SC_M) = 0.7708$ min.

Consider the entire rack, we have $E(SC_{Entire}) = 1.0344$ min.

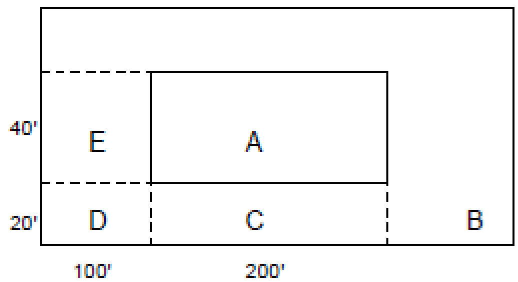
Therefore, $E(SC_S)$, the expected single-command travel time for the rack portion where the slow movers are stored, is obtained by solving the following equation:

$$E(SC_{Entire}) = \frac{60\times250}{60\times400}E(SC_{F\&M}) + \frac{60\times150}{60\times400}E(SC_S),$$

from which we have $E(SC_S) = 1.717$ min.

Finally, $E(SC_{Average})$, the average single-command travel time, is obtained as $E(SC_{Average}) = 0.60E(SC_F) + 0.30E(SC_M) + 0.10E(SC_S) = 0.60292$ min, and the average single-command cycle time (including P/D times) is equal to $0.60292 + 2T_{P/D} = 1.20292$ min.

**10.71a**




Consider first the rack formed by region DC (a 20 ft by 300 ft region): T = 0.666, Q = 0.333, $E(SC_{DC}) = 0.691358$ min.

Consider next the rack formed by region D (a 20 ft by 100 ft region): T = 0.222, Q = 1, $E(SC_D) = 0.2963$ min.

We obtain $E(SC_C) = 0.888$ min from the following equation:
$E(SC_{DC}) = 0.691358 = (1/3)E(SC_D) + (2/3)E(SC_C) = (1/3)(2.2963) + (2/3)E(SC_C)$

Following the same procedure, we obtain $E(SC_{AC}) = 0.9877$ min from the following equation: $E(SC_{EDAC})= (1/3)E(SC_{ED}) + (2/3)E(SC_{AC})$ where $E(SC_{ED}) = 0.691358$ min and $E(SC_{EDAC}) = 0.888$ min.

Finally, the value of $E(SC_A)$ is obtained by solving the equation:
$E(SC_{AC}) = (1/3)E(SC_C) + (2/3)E(SC_A)$. Thus, $E(SC_A) = 1.0370$ min.

**10.71b** For the entire rack we have: T = 0.888, Q = 1, E(SC) = 1.1851852 min. Solving the equation $E(SC) = (1/4)E(SC_A) + (3/4)E(SC_B)$, we obtain $E(SC_B) = 1.2345679$ min.

**10.72** Let the shaded area be region C. Consider first the entire rack: T = 1.25, Q = 0.30, 0.5E(SC) = 0.64375 min; next the rack is formed by region A: T = 0.625, Q = 0.60, $0.5E(SC_A) = 0.35$ min; now the rack formed by region AC: T = 1.04166, Q = 0.36, $0.5E(SC_{AC}) = 0.5433$ min. Therefore, for the rack formed by region B: $0.5E(SC_B) = 1.1458$ min.

Let $(x_1,y_1)$ be any storage point in region A and let $(x_2,y_2)$ be any retrieval point in region B for a dual command trip performed by the S/R machine. Then the expected travel time, E(TB), the S/R machine travels from region A to region B is given by
$E(TB) = E(\max((x_2 - x_1)/400, |y_1 - y_2|/120))$ (we assume that the origin (0,0) is located at the P/D point).

Considering the rack formed by the shaded area, we have 166.666/400 = 0.4167 min for the minimum horizontal travel time. Considering the entire rack, we have 45/120 = 0.375 min for the maximum vertical travel time. Thus, the expected time E(TB) is "dominated" by the horizontal S/R machine travel; that is $(x_2 - x_1)/400 > |y_1 - y_2|/120$.

Hence, have $E(TB) = E((x_2 - x_1)/400) = 0.833$ min and the expected dual command travel time = 0.35 + 0.833 + 1.1458 = 2.33 min.

**10.73** To show that the system will meet the required throughput we need to first compare $\lambda_r$ and $\lambda_s$. Since $\lambda_r > \lambda_s$ we simply need to satisfy (10.98). Using (10.98) we have (32/60)(1.3333) + (8/60)(0.4666) + ((8 + 32)/60)(0.1 + 0.1) = 0.90664 < 1, so the condition for stability is satisfied.

For this part $\lambda_r = 32/60 = 0.5333$ and $\lambda_s = 8/60 = 0.1333$.

$k_4 = 1.3333 – 0.4666 = 0.8667$
$\varphi_1 = [1/(0.5333(0.8667)] – (0.1333/0.6666) = 1.9635$

$k_1 = 0.5(1.3333) = 0.6665$
$k_2 = 0.1 + 0.1 = 0.2$
$k_3 = 0.4666$
$\varphi_2 = [(2(0.1333)(0.6665) + 0.5333(0.4666) + 0.6666(0.2))/(0.5333(0.8667))] – (0.1333/0.6666)$
$= 1.0112$

$A = (4)(1.9635)^2 = 15.42$
$B = 1.9635 + 1 – [8(1.9635)(1.0112)] – 4(1.9635) = -20.7744$
$C = (1.0112 + 1)[4(1.0112) – 1] = 6.1237$

The roots of the equation $A\rho^2 + B\rho + C$ are 0.912 and 0.436. We now test to see which root satisfies the condition that $q_s$ and $q_r$ are non-negative.

$\rho = 0.912$:

$\varphi_3 = [1/(0.1333(0.8667)] – (0.5333/0.6666) = 7.8557$
$\varphi_4 = [(2(0.5333)(0.6665) + 0.1333(0.4666) + 0.6666(0.2))/(0.1333(0.8667))] – (0.5333/0.6666)$
$= 7.0476$

$q_s = 1.9635(0.912) – 1.0112 = 0.8973$
$q_r = 7.8557(0.912) – 7.0476 = 0.1168$

There is no need to check the other root since we know that only one root can satisfy the non-negativity requirement of $q_s$ and $q_r$. The utilization is 0.912, or 91.2%.

**SECTION 10.7**

**10.74a** We have th = 0.8889, tv = 0.8889, T = 0.8889, Q = 1. By (10.75) $E(P, X_1, X_2, ..., X_8, D) \approx 2.4424$ min.

**10.74b** For the two-band heuristic: $C = Q/k^2 = 0.25$, A = 0.1669, B = 0.14126, and E(2,8,0.8889,1) = 2.796 min.

**10.74c** For the four-band heuristic: C = 0.0625, A = 0.1158, B = 0.1135, and E(4,8,0.8889,1) = 4.3151 min.

**10.75a** Average time per trip - 2.4424 + ((20)8)/60 + 0.8 = 5.9091 min. Throughput capacity = (60/5.9091)(8) = 81.23 picks/hour.

**10.75b** Similar to part (a), average time per trip is 6.2627 min, and the throughput capacity is 76.64 picks/hour.

**10.75c** Similar to part (a), average time per trip is 7.7818 min, and the throughput capacity is 61.68 picks/hour.

**10.76** Alternative (a): $t_h = 0.8081$, $t_v = 0.8081$, T = 0.8081, Q = 1. By (10.75) we obtain $E(P, X_1, ..., X_8, D) = 2.2204$ min. Therefore, average time per trip = 2.2204 + ((20)8)/60 + 0.8 = 5.687 min, and the throughput capacity = (60/5.687)(8) = 84.403 picks/hour.

Alternative (b): Average time per trip is 5.5091 min. and the throughput capacity is 87.13 picks/hour. Hence, alternative (b) would increase the throughput capacity of the system more than alternative (a).

**10.77a** S = 7000(18) = 126,000 sq. ft.; R = 215 picks/hr; n = 9 stops/trip; p = 1.5 min/stop; K = 1.20 min; $h_v$ = 400 fpm; $v_v$ = 100 fpm; t = 1.63; q = 1.255; and u = 0.31614.

Using ALGORITHM 1 (pg. 634):
(Iteration 1)
STEP 0. M = 6.
STEP 1. Since $M(60M - Rt)^2 = 547.215 < 727.512 = (Rqu)^2$, go to 2.
STEP 2. M = 7, go to step 1.

(Iteration 2)
STEP 1. Since $M(60M - Rt)^2 = 33860.418 > 7276.512 = (Rqu)^2$, go to 3.
STEP 3. STOP, 7 is the minimum number of pickers and storage aisles.

**10.77b** The surface area for each side of an aisle = 126,000/(7(2)) = 9,000 sq. ft. Assume the rack is "square-in-time" (which is the preferred rack shape when picks are sequenced in an optimum fashion). Let $t = [9{,}000/(400(100))]^{0.5} = 0.4743$. Hence, the rack length, L = 400(0.4743) = 189.72 ft., and the rack height, H = 100(0.4743) = 47.43 ft.

**10.77c** T = 0.4743, Q = 1. By (10.75) we have $E(P,X_1, ...,X_8,D)$ = 1.3031 min. Hence, the average time per trip = 1.3031 + 1.5(8) + 1.2 = 14.5031 min, and the throughput capacity = (60/14.5031)(8)(7) = 231.67 picks/hr. Therefore, the system utilization = (215/231.67)(100%) = 92.8%.

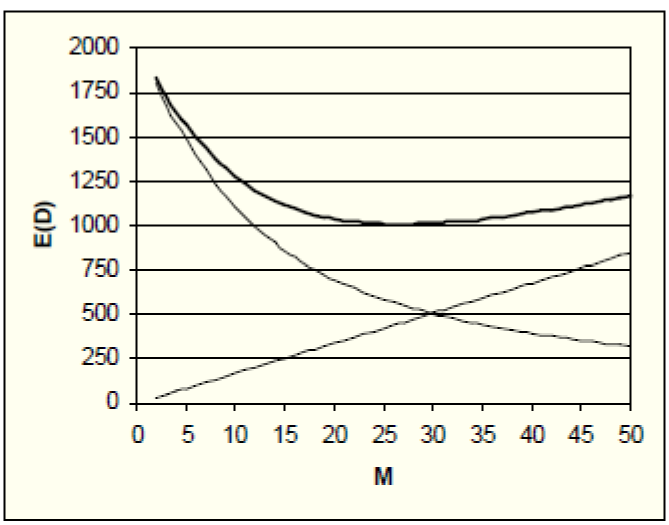
**10.78a-c** Assuming a square-in-time rack (Q = 1), the results are summarized as follows:

| Part | Surface area on one side of an aisle (sq. ft.) | T (Min) | Rack Length (ft) | Rack Height (ft) | E(…) by (10.75) (min) | Travel Time per trip (min) | Throughput capacity (picks/hr) |
|---|---|---|---|---|---|---|---|
| a | 50,000 | 1.118 | 447.2 | 118.8 | 3.28 | 13.68 | 131.58 |
| b | 37,500 | 0.9682 | 387.28 | 96.82 | 2.84 | 13.24 | 181.26 |
| c | 30,000 | 0.866 | 346.4 | 86.6 | 2.54 | 12.94 | 231.82 |

Rack surface area for one side of an aisle is equal to 300,000 divided by (2 times the number of aisles in the system), and *T* is equal to the square root of the surface area for one side of an aisle divided by ($400 \times 100$). Also, the rack length is equal to 400T while the rack height is equal to 100T. The travel time per trip is equal to E(...) from (10.75) plus 0.9(10) min (the time needed for performing 10 picks) plus 1.4 minutes (the time the picker spends at the P/D station between successive trips). Lastly, the throughput capacity is equal to (60 min)(10 picks/trip)(the number of aisles in the system)/(the travel time per trip).

**10.78d** Although seeking a “trend” with three data points is highly undesirable, we might say that the throughput capacity of the system increases with the number of storage aisles (as expected) but it increases at a slower rate as more aisles are added.

**10.79** Using a computer to generate results for M = 1 to 50, E(D) appears to be convex with respect to M.



From the graph, it appears the minimum value of E(D) is in the neighborhood of M = 25 to 30. Summarizing the relevant results, the minimum value of E(D) is 1,008.05 at M = 27.

| M | Warehouse Width | Aisle Length | $E(D_1)$ | $E(D_2)$ | E(D) |
|---|---|---|---|---|---|
| 24 | 240 | 60.00 | 406.15 | 605.90 | 1,012.06 |
| 25 | 250 | 57.60 | 423.08 | 586.50 | 1,009.57 |
| 26 | 260 | 55.38 | 440.00 | 568.27 | 1,008.27 |
| 27 | 270 | 53.33 | 456.92 | 551.13 | 1,008.05 |
| 28 | 280 | 51.43 | 473.85 | 534.97 | 1,008.81 |
| 29 | 290 | 49.66 | 490.77 | 519.72 | 1,010.49 |
| 30 | 300 | 48.00 | 507.69 | 505.30 | 1,012.99 |

**10.80** Let E(D') be the expected distance traveled per trip under the traversal policy with the P/D point located in the corner. As before, we can write $E(D') = E(D_1') + E(D_2')$. Since the P/D point location does not affect in-the-aisle travel, we have $E(D_2') = E(D_2)$. Treating the aisle locations as a continuous random variable (See Hall [31]), and using order statistics, the farthest pick from the P/D point is approximately x/[n/(n+1)] distance units away. Since $E(D_1')$ equals twice the distance to the farthest pick, we obtain:

$$E(D') = yM\left[1 - \left(\frac{M-1}{M}\right)^n\right] + 0.5y + 2x\left(\frac{n}{n+1}\right)$$

**10.81a** S = 7,000(18) = 126,000 sq. ft., R = 215 picks/hr, p = 1.5 min/pick, C = 0.4 min/cycle, Q = 0.9

We use ALGORITHM 2 (pg. 640). In STEP 4 of the first iteration we have E(PU)(60/p)M = 0.9812(60/1.5)6 = 235.488 $\geq$ 215 = R. Thus, we conclude that 6 is the minimum number of pickers required.

**10.81b** The rack length L = 216 ft., and the rack height H = 48.6 ft.

**10.81c** Expected system utilization (= 215/235.488) = 91.3%. Thus, the effective picker utilization (= 0.913(E(PU)) = 0.913(0.9812)) = 89.6% and the effective S/R utilization (= 0.913(E(SRU)) = 0.913(0.8672)) = 79.2%.

**10.82a** $t_h = 0.8$, $t_v = 0.7$, T = 0.8, Q = 0.875, E(DC) = 1.355 min, and $T_{DC}$ = E(DC) + C = 1.955 min.

**10.82b** TDC = 2.3 min. Using the procedure described in STEP 0 and 1 of ALGORITHM 2 (pg. 640), we obtain $\hat{k}_1 = 0.9802846$, $\hat{k}_2 = 2.4073456$, r = 0.8, $k_1 = 0.784228$, $k_2 = 1.925877$, $t_1 = k_1 + C = 1.38423$, $t_2 = k_2 + C = 2.525877$, and p = 2 min/pick. Thus, E(CT) = 2.7177, E(PU) = p/E(CT) = 0.7359. The number of books picked per hour = E(PU)(60/p)(8) = 177 books/hr.

**10.83** In order to obtain a 100% picker utilization, i.e. E(PU) = 1.0, we need $p \geq t_2 = k_2 + C = r\hat{k}_2 + C = (q/\sqrt{M})\,\hat{k}_2 + C$.

Thus, $M \geq \left\lceil \left(\frac{q\hat{k}_2}{p-C}\right)^2 \right\rceil = \left\lceil \left(\frac{1.1785(2.4263)}{1.5-0.42}\right)^2 \right\rceil = \lceil 7.0097 \rceil = 8$ aisles.

**10.84** First, we establish that we need to improve *E(PU)*. Next, we need to identify the current bottleneck; there are two possible bottlenecks: 1. The picker, or 2. The S/R machine. In the first case, we can add a second picker *(served by the same S/R machine)* by either adding a conveyor loop in front of the aisle (to serve both pickers) or by adding a second P/D point at the opposite end (or one side) of the aisle. In the second case, in order to improve the S/R machine cycle time, we can either raise the P/D point or try alternative container sequencing procedures. We may also change the assignment of SKUs to containers to reduce the number of containers retrieved/hr by the S/R machine.

## SECTION 10.8

**10.85a** The least common multiple of the individual periods (5, 2, 4, 3) is 60.

**10.85b** k = 543; p = 60; r = 543mod60 = 3; r/p = 3/60 = 1/20. Since r/p is not a proper fraction, the answer is **no**.

**10.85c** k/p = 543/60 = 9.05. Since k/p is not an integer, the answer is **yes**.

**10.85d** p is not a prime number. Therefore, the answer is **no**.

Note: even though r/p is not a proper fraction, the solution procedure presented in the chapter can be used for this problem. Applying the solution procedure and indexing r carriers yields the following sequence of values for H*1(n): H*1(1), H*1(4), H*1(7), H*1(10), H*1(13), H*1(16), H*1(19), H*1(2), H*1(5), H*1(8), H*1(11), H*1(14), H*1(17), H*1(20), H*1(3), H*1(6), H*1(9), H*1(12), H*1(15), and H*1(18). This is not always the case, as shown in Problem 10.87e.

**10.86a** k = 32; p = 3, r = 32mod3 = 2
$\{f_1(n)\} = (-5, 0, 0)$, $\{f_2(n)\} = (0, -3, 0)$, $\{f_3(n)\} = (0, 0, 4)$, and
$\{f_4(n)\} = (4, 0, 0)$. Therefore, $\{F_1(n)\} = (-1, -3, 4)$.
$H^*_1(1) = 0 \therefore H^*_1(3) = H^*_1(1) + F_1(3) = 0 + 4 = 4$ and $H^*_1(2) = H^*_1(3) + F_1(2) = 1$.
Hence, $\{H^*_1(n)\} = (0, 1, 4)$.
Thus, $\{H^*_2(n)\} = \{H^*_1(n)\} - \{f_1(n)\} = (0, 1, 4) - (-5, 0, 0) = (5, 1, 4)$.
$\{H^*_3(n)\} = \{H^*_2(n)\} - \{f_2(n)\} = (5, 1, 4) - (0, -3, 0) = (5, 4, 4)$.
$\{H^*_4(n)\} = \{H^*_3(n)\} - \{f_3(n)\} = (5, 4, 4) - (0, 0, 4) = (5, 4, 0)$.
Since c = 0, $\{H_i(n)\} = \{H^*_i(n)\}\ \forall$ i, n
**B = 5**

**10.86b** k =33 is not feasible since p = 3; k = 31; p = 3, r = 31mod3 = 1

As before, $\{F_1(n)\} = (-1, -3, 4)$.
$H^*_1(1) = 0 \therefore H^*_1(2) = H^*_1(1) + F_1(2) = 0 - 3 = -3$ and
$H^*_1(3) = H^*_1(2) + F_1(3) = -3 + 4 = 1$. Hence, $\{H^*_1(n)\} = (0, -3, 1)$.
Thus, $\{H^*_2(n)\} = \{H^*_1(n)\} - \{f_1(n)\} = (0, -3, 1) - (-5, 0, 0) = (5, -3, 1)$.
$\{H^*_3(n)\} = \{H^*_2(n)\} - \{f_2(n)\} = (5, -3, 1) - (0, -3, 0) = (5, 0, 1)$.
$\{H^*_4(n)\} = \{H^*_3(n)\} - \{f_3(n)\} = (5, 0, 1) - (0, 0, 4) = (5, 0, -3)$.
Since $c = -3$, $H_i(n) = H^*_i(n) + 3 \ \forall\ i, n$
$\{H_1(n)\} = (3, 0, 4)$, $\{H_2(n)\} = (8, 0, 4)$, $\{H_3(n)\} = (8, 3, 4)$, and
$\{H_4(n)\} = (8, 3, 0)$
B = 8
Inventory = $\sum_{i,n} \{H_i(n)\} = 3 + 0 + 4 + 8 + 0 + 4 + 8 + 3 + 4 + 8 + 3 + 0$
Inventory for k = 31 is 45. From part a), for k = 32, inventory is given by
Inventory = 0 + 1 + 4 + 5 + 1 + 4 + 5 + 4 + 4 + 5 + 4 + 0 = 37
**k = 32 minimizes inventory and minimizes B.**

**10.86c** 1) $\{f_1(n)\} = (0, 0, -4)$, $\{f_2(n)\} = (-4, 0, 0)$, $\{f_3(n)\} = (0, 0, 4)$, and
$\{f_4(n)\} = (4, 0, 0)$. $\{F_1(n)\} = (0, 0, 0) \therefore \{H^*_1(n)\} = (0, 0, 0)$.
$\{H^*_2(n)\} = (0, 0, 0) - (0, 0, -4) = (0, 0, 4)$
$\{H^*_3(n)\} = (0, 0, 4) - (-4, 0, 0) = (4, 0, 4)$
$\{H^*_4(n)\} = (4, 0, 4) - (0, 0, 4) = (4, 0, 0)$.
$c = 0 \therefore \{H_i(n)\} = \{H^*_i(n)\} \ \forall\ i, n$
Inventory = 16; B = 4

2) $\{f_1(n)\} = (-4, 0, 0)$, $\{f_2(n)\} = (0, -4, 0)$, $\{f_3(n)\} = (0, 0, 4)$, and
$\{f_4(n)\} = (4, 0, 0)$. $\{F_1(n)\} = (0, -4, 4) \therefore$
$H^*_1(1) = 0$, $H^*_1(3) = H^*_1(1)+ F_1(3) = 0 + 4 = 4$, $H^*_1(2) = H^*_1(3) + F_1(2) = 4 - 4 = 0$.
$\{H^*_1(n)\} = (0, 0, 4)$. $\{H^*_2(n)\} = (0, 0, 4) - (-4, 0, 0) = (4, 0, 4)$.
$\{H^*_3(n)\} = (4, 0, 4) - (0, -4, 0) = (4, 4, 4)$
$\{H^*_4(n)\} = (4, 4, 4) - (0, 0, 4) = (4, 4, 0)$
$c = 0 \therefore \{H_i(n)\} = \{H^*_i(n)\} \ \forall\ i, n$
Inventory = 32; B = 4
Of the two considered **$\{f_1(n)\} = (0, 0, -4)$** and **$\{f_2(n)\} = (-4, 0, 0)$** are the preferred loading sequences based on inventory on the conveyor.

**10.87a** k = 33, p = 4, r = 33mod4 = 1. Since $r \neq 0$, the answer is **yes**.

**10.87b** k = 33, p = 4, r = 33mod4 = 1. Since r/p = 1/4 is a proper fraction, the answer is **yes**.

**10.87c** $\{f_1(n)\} = (-2, 0, -2, 0)$, $\{f_2(n)\} = (0, -2, 0, -2)$, $\{f_3(n)\} = (0, 0, 4, 0)$, and
$\{f_4(n)\} = (4, 0, 0, 0)$. $\{F_1(n)\} = (2, -2, 2, -2) \therefore H^*_1(1) = 0$.
$H^*_1(2) = H^*_1(1) + F_1(2) = -2$. $H^*_1(3) = H^*_1(2) + F_1(3) = 0$.
$H^*_1(4) = H^*_1(3) + F_1(4) = -2 \therefore \{H^*_1(n)\} = (0, -2, 0, -2)$.
$\{H^*_2(n)\} = (0, -2, 0, -2) - (-2, 0, -2, 0) = (2, -2, 2, -2)$
$\{H^*_3(n)\} = (2, -2, 2, -2) - (0, -2, 0, -2) = (2, 0, 2, 0)$
$\{H^*_4(n)\} = (2, 0, 2, 0) - (0, 0, 4, 0) = (2, 0, -2, 0)$
$c = -2 \therefore H_i(n) = H^*_i(n) + 2 \ \forall\ i, n$

$\{H_1(n)\} = (2, 0, 2, 0)$, $\{H_2(n)\} = (4, 0, 4, 0)$, $\{H_3(n)\} = (4, 2, 4, 2)$, and
$\{H_4(n)\} = (4, 2, 0, 2)$
**$H_3(2) = 2$**

**10.87d** B = 4 (Although not asked for, the inventory totals 4 + 8 + 12 + 8, or 32.)

**10.87e** k = 34, r = 34mod4 = 2. Since r/p is not a proper fraction, it is not clear if the general sequences posed are feasible. To determine the values of $H^*_1(n)$, notice they will be generated in the following sequence: $H^*_1(1)$, $H^*_1(3)$, $H^*_1(5) = H^*_1(1)$. Therefore, values for $H^*_1(2)$ and $H^*_1(4)$ cannot be generated using the solution procedure. Hence, we are unable to solve the problem for k = 34. Likewise, we cannot solve it for k = 32, since r = 0. However, a solution is possible for k = 35 since r = 3 and r/p is a proper fraction.

With r = 3 and $\{F_1(n)\} = (2, -2, 2, -2)$, the following values of $H^*_1(n)$ are obtained: $H^*_1(1) = 0$, $H^*_1(4) = -2$, $H^*_1(3) = 0$, $H^*_1(2) = -2$, or $\{H^*_1(n)\} = (0, -2, 0, -2)$, which is the same sequence obtained for k = 33. Therefore, the same results exist for both k = 33 and k = 35. Hence, **the inventory on the conveyor and the carrier capacity will be the same for any feasible value of k**.

(This problem illustrates the value of Muth's third result, i.e., having p be a prime number. Conveyor compatibility results for all values of k when p is a prime number and $r \neq 0$.)

**10.87f** k = 33, p = 4, r = 1

1) $\{f_1(n)\} = (-2, 0, -2, 0)$, $\{f_2(n)\} = (0, -2, 0, -2)$, $\{f_3(n)\} = (0, 0, 2, 2)$,
$\{f_4(n)\} = (2, 2, 0, 0)$. $\{F_1(n)\} = (0, 0, 0, 0) \therefore \{H^*_1(n)\} = (0, 0, 0. 0)$.
$\{H^*_2(n)\} = (0, 0, 0, 0) - (-2, 0, -2, 0) = (2, 0, 2, 0)$
$\{H^*_3(n)\} = (2, 0, 2, 0) - (0, -2, 0, -2) = (2, 2, 2, 2)$
$\{H^*_4(n)\} = (2, 2, 2, 2) - (0, 0, 2, 2) = (2, 2, 0, 0)$
$c = 0 \therefore H_i(n) = H^*i(n) \ \forall\ i, n$
$\{H_1(n)\} = (0, 0, 0, 0)$, $\{H_2(n)\} = (2, 0, 2, 0)$, $\{H_3(n)\} = (2, 2, 2, 2)$, and
$\{H_4(n)\} = (2, 2, 0, 0)$. Inventory totals 16 and B = 2.

2) $\{f_1(n)\} = (-2, 0, -2, 0)$, $\{f_2(n)\} = (0, -2, 0, -2)$, $\{f_3(n)\} = (2, 0, 2, 0)$,
$\{f_4(n)\} = (0, 2, 0, 2)$. $\{F_1(n)\} = (0, 0, 0, 0) \therefore \{H^*_1(n)\} = (0, 0, 0. 0)$.
$\{H^*_2(n)\} = (0, 0, 0, 0) - (-2, 0, -2, 0) = (2, 0, 2, 0)$
$\{H^*_3(n)\} = (2, 0, 2, 0) - (0, -2, 0, -2) = (2, 2, 2, 2)$
$\{H^*_4(n)\} = (2, 2, 2, 2) - (2, 0, 2, 0) = (0, 2, 0, 2)$
$c = 0 \therefore H_i(n) = H^*_i(n) \ \forall\ i, n$
Inventory totals 16 and B = 2.

The two alternate sequences have identical results, with an inventory of 16 on the conveyor. With the original loading sequences, the inventory totaled 32. Hence, **either of the alternate loading sequences would be preferred to the original sequences**.

**10.88a** k = 62, p = 3, r = 62mod3 = 2, $\{f_1(n)\} = (0, -4, 0)$, $\{f_2(n)\} = (3, 3, 3)$, and
$\{f_3(n)\} = (0, -5, 0) \therefore \{F_1(n)\} = (3, -6, 3)$. $H^*_1(1) = 0$, $H^*_1(3) = 3$, $H^*_1(2) = -3$.
$\{H^*_1(n)\} = (0, -3, 3)$. $\{H^*_2(n)\} = \{H^*_1(n)\} - \{f_1(n)\} = (0, -3, 3) - (0, -4, 0) = (0, 1, 3)$.
$\{H^*_3(n)\} = \{H^*_2(n)\} - \{f_2(n)\} = (0, 1, 3) - (3, 3, 3) = (-3, -2, 0)$.
Since c = -3, $H_i(n) = H^*_i(n) + 3\ \forall$ i, n
$\{H_1(n)\} = (3, 0, 6)$, $\{H_2(n)\} = (3, 4, 6)$, $\{H_3(n)\} = (0, 1, 3)$
**B = 6** (Inventory totals 26.)

**10.88b** 1) $\{f_1(n)\} = (0, -4, 0)$, $\{f_2(n)\} = (3, 3, 3)$, $\{f_3(n)\} = (-5, 0, 0) \therefore \{F_1(n)\} = (-2, -1, 3)$.
$H^*_1(1) = 0$, $H^*_1(3) = 3$, $H^*_1(2) = 2$.
$\{H^*_1(n)\} = (0, 2, 3)$, $\{H^*_2(n)\} = \{H^*_1(n)\} - \{f_1(n)\} = (0, 6, 3)$.
$\{H^*_3(n)\} = \{H^*_2(n)\} - \{f_2(n)\} = (-3, 3, 0)$. c = -3 $\therefore$
$\{H_1(n)\} = (3, 5, 6)$, $\{H_2(n)\} = (3, 9, 6)$, $\{H_3(n)\} = (0, 6, 3)$
**B = 9** (Inventory totals 41.)

2) $\{f_1(n)\} = (0, -4, 0)$, $\{f_2(n)\} = (3, 3, 3)$, $\{f_3(n)\} = (0, 0, -5) \therefore \{F_1(n)\} = (3, -1, -2)$.
$H^*_1(1) = 0$, $H^*_1(3) = -2$, $H^*_1(2) = -3$.
$\{H^*_1(n)\} = (0, -3, -2)$, $\{H^*_2(n)\} = \{H^*_1(n)\} - \{f_1(n)\} = (0, 1, -2)$.
$\{H^*_3(n)\} = \{H^*_2(n)\} - \{f_2(n)\} = (-3, -2, -5)$. c = -5 $\therefore$
$\{H_1(n)\} = (5, 2, 3)$, $\{H_2(n)\} = (5, 6, 3)$, $\{H_3(n)\} = (2, 3, 0)$
**B = 6** (Inventory totals 29.)
To minimize B, use either **$\{f_3(n)\} = (0, -5, 0)$** or **$\{f_3(n)\} = (0, 0, -5)$** for a value of 6.

**10.88c** To minimize inventory on the conveyor, use **$\{f_3(n)\} = (0, -5, 0)$** for a value of 26.

**10.89a** k = 83, p = 3, r = 83mod3 = 2, $\{f_1(n)\} = (1, 0, 0)$, $\{f_2(n)\} = (0, 2, 0)$, $\{f_3(n)\} = (0, 0, 1)$,
$\{f_4(n)\} = (0, -4, 0)$. $\therefore \{F_1(n)\} = (1, -2, 1)$.
$H^*_1(1) = 0$, $H^*_1(3) = H^*_1(1) + F_1(3) = 1$, $H^*_1(2) = H^*_1(3) + F_1(2) = -1$.
$\{H^*_1(n)\} = (0, -1, 1)$. $\{H^*_2(n)\} = (-1, -1, 1)$, $\{H^*_3(n)\} = (-1, -3, 1)$,
$\{H^*_4(n)\} = (-1, 1, 0)$. c = -3 $\therefore H_i(n) = H^*_i(n) + 3\ \forall$ i, n
$\{H_1(n)\} = (3, 2, 4)$, $\{H_2(n)\} = (2, 2, 4)$, $\{H_3(n)\} = (2, 0, 4)$, and
$\{H_4(n)\} = (2, 4, 3) \therefore$ **$H_3(2) = 0$**.

**10.89b** **B = 4**, inventory totals 9 + 8 + 6 + 9, or **32**.

**10.89c** $\{f_1(n)\} = (1, 0, 0)$, $\{f_2(n)\} = (0, 2, 0)$, $\{f_3(n)\} = (0, 0, 1)$, and $\{f_4(n)\} = (0, -a, -b)$.
Therefore, $\{F_1(n)\} = (1, 2-a, 1-b)$.
$H^*_1(1) = 0 \therefore H^*_1(3) = H^*_1(1) + F_1(3) = 1-b$
$H^*_1(2) = H^*_1(3) + F_1(2) = 1-b + 2-a$, or 3 - (a+b). Since a+b = 4,then $H^*_1(2) = -1$. Hence,
$\{H^*_1(n)\} = (0, -1, 1-b)$ and
$\{H^*_2(n)\} = \{H^*_1(n)\} - \{f_1(n)\} = (0, -1, 1-b) - (1, 0, 0) = (-1, -1, 1-b)$.
$\{H^*_3(n)\} = \{H^*_2(n)\} - \{f_2(n)\} = (-1, -1, 1-b) - (0, 2, 0) = (-1, -3, 1-b)$.
$\{H^*_4(n)\} = \{H^*_3(n)\} - \{f_3(n)\} = (-1, -3, 1-b) - (0, 0, 1) = (-1, -3, -b)$.
If b = 4, then c = -4 and $H_i(n) = H^*_i(n) + 4\ \forall$ i, n
$\{H_1(n)\} = (4, 3, 1)$, $\{H_2(n)\} = (3, 3, 1)$, $\{H_3(n)\} = (3, 1, 1)$, and
$\{H_4(n)\} = (3, 1, 0) \therefore$ B = 4 if b = 4.
If b $\leq$ 3, then c = -3 and $H_i(n) = H^*_i(n) + 3\ \forall$ i, n
$\{H_1(n)\} = (3, 2, 4-b)$, $\{H_2(n)\} = (2, 2, 4-b)$, $\{H_3(n)\} = (2, 0, 4-b)$, and

$\{H_4(n)\} = (2, 0, 3\text{-}b) \therefore B = 3$ if $1 \le b \le 3$.
To minimize B, **$1 \le b \le 3$**. Notice that the inventory totals 8 + 7 + 5 + 4, or 24 when b = 4. It totals 9 + 8 + 6 + 5 - 4b, or 28 - 4b when $1 \le b \le 3$. To minimize inventory, let b = 3 for a total of 16. (Note: for any k that results in r = 1, B is minimized when 2 = b = 2.)

**10.89d** $\{f_1(n)\} = (1, 0, 1)$, $\{f_2(n)\} = (0, 2, 0)$, $\{f_3(n)\} = (0, -4, 0)$.
$\{F_1(n)\} = (1, -2, 1)$. $H^*_1(1) = 0$, $H^*_1(3) = H^*_1(1) + F_1(3) = 1$,
$H^*_1(2) = H^*_1(3) + F_1(2) = 1 - 2 = -1$. Hence, $\{H^*_1(n)\} = (0, -1, 1)$ and
$\{H^*_2(n)\} = \{H^*_1(n)\} - \{f_1(n)\} = (-1, -1, 0)$.
$\{H^*_3(n)\} = \{H^*_2(n)\} - \{f_2(n)\} = (-1, -3, 0)$.
Since c = -3, then $H_i(n) = H^*_i(n) + 3 \ \forall$ i, n
$\{H_1(n)\} = (3, 2, 4)$, $\{H_2(n)\} = (2, 2, 3)$, $\{H_3(n)\} = (2, 0, 3)$, and
$\{H_4(n)\} = (2, 0, 3)$ for B = 4. Inventory totals 9 + 7 + 5 + 5, or 26.
**Although inventory is reduced, there is no effect on B.**

**10.90** Using the information and procedure given in Example 10.52, we have the following: $S_1 = (27, 26, ..., 11)$, $S_2 = (34, 33, ..., 28)$, $S_3 = (51, 50, ..., 35)$, $S_4 = (10, 9, ..., 52)$, $\Delta_2 = 5$, and $\Delta_4 = 12$. To ensure stability we need to have $V > \lambda_i + \Delta_i$ for i = 2 and 4. Using the $\lambda_i$ values from the example, and the $\Delta_i$ values from above, we obtain V > 30 + 5 = 35 for i =2 and V > 20 + 12 = 32 for i = 4. Therefore, we must have a conveyor speed greater than 35 windows per minute.

**10.91** a. flat belt driven roller conveyor, TL = 300', WBR = 33", RC = 9", and S = 200 fpm
Based on the WBR and RC combination, **LF = 0.9**.
**BV = 4.60 + 0.445(WBR) = 4.60 + 0.445(33) = 19.29**
LS = load segment length = 24" + 4" = 28"
TW = tote weight, loaded = 25 lbs.
# of full load segments on conveyor = ⌈300(12)/28⌉ = 128
(note: ⌈xx⌉ denotes the largest integer ≤ xx).
length of partial load segment = 300(12) - 128(28) = 16"
weight of partial load segment = 25(16/24) = 16.67 lbs.
**L = 128(25) + 16.67 = 3,216.67 lbs.**

b. roller supported belt conveyor, TL = 100', WBR = 33", RC = 9", and S = 300 fpm
Based on the WBR and RC combination, LF = 0.62.
BV = ⅔(WBR) = 22
For the following calculation, fpm denotes feet per minute, ipf denotes inches per foot, ipt denotes inches per tote, and tpm denotes totes per minute. The rate at which totes exit the roller conveyor is ((250 fpm)(12 ipf))/(28 ipt) = 185.71429 tpm
Therefore, for the belt conveyor, LS (length of a load segment) will be
((300 fpm)(12 ipf))(85.71429 tpm) = 42 ipt
Since a tote is 24" In length, the clearance between totes on the belt conveyor will be 18"
# of full load segments on conveyor = ⌈100(12)/42⌉ = 28

length of partial load segment = 100(12) - 28(42) = 24", which is the length of a tote. At its maximum loaded condition there will be 29 totes on the conveyor at a weight of 29(25) = 725.
∴ **L = 725 lbs**.

**10.92a** chain driven roller conveyor, TL = 150', WBR = 39", RC = 9", S = 90 fpm, length of pallet = 48", clearance between pallets = 24", and weight of loaded pallet = 1,200 lbs.
**FF = 0.05** and **LF = 1.0**. **BV = 4.60 + 0.445(WBR) = 4.60 + 0.445(39) = 21.955**
LS = 48" + 24" = 72"
# of full load segments on conveyor = ⌈150(12)/72⌉ = 25
length of partial load segment = 150(12) - 25(72) = 0
**L = 25(1200) = 30,000 lbs**.

**10.92b** For this calculation, ipf denotes inches per foot and ipls denotes inches per load segment. For the chain belt driven roller conveyor, the rate at which pallets are discharged is ((90 fpm)(12 ipf))/(72 ipls) = 15 pallets per minute. For the roller supported belt conveyor to process 15 pallet loads per minute, its speed must be **S = 15(48" + 36")/12 = 105 fpm**.

**10.93a** V-belt driven roller conveyor, TL = 250', WBR = 30", and RC = 7.5" Using linear interpolation with the data in the table,
for WBR = 27" and RC = 6", LF = 1.0
for WBR = 27" and RC = 9", LF = 0.8 ∴ for WBR = 27" and RC = 7.5", LF ≈ 0.9
for WBR = 33" and RC = 6", LF = 1.2
for WBR = 33" and RC = 9", LF = 0.9 ∴ for WBR = 33" and RC = 7.5", LF ≈ 1.05
**∴ for WBR = 30" and RC = 7.5", LF ≈ 0.975**.

**10.93b** From the weight and length combinations for the cartons shown on page 10-16, notice that the average carton length is 24" and the average carton weight is 15 lbs. Also, notice that the average weight per inch of carton, including the 6" clearance between cartons, is 0.5464 lbs/in. Further, notice that the weight per inch is <u>not</u> the same as the average weight of a carton divided by the average length of a load segment, since the average of a ratio is not equal to the average numerator divided by the average denominator.

An approximation of the average load on the conveyor is the product of the length of the conveyor and the weight per inch of load segment, i.e., 250 x 12 x 0.5464, or 1,639.20 lbs. This is only an approximation because the weight of partial load segments is not uniformly distributed across the length of the partial load segment. (Alternately, by simulating the loading of the conveyor a more accurate estimate can be determined.)

**Carton Combinations**

| Comb. | Length (in.) | Weight (lbs.) | lbs/in* |
|---|---|---|---|
| 1 | 12 | 5 | 0.27778 |
| 2 | 12 | 10 | 0.55556 |
| 3 | 12 | 15 | 0.83333 |
| 4 | 12 | 20 | 1.11111 |
| 5 | 12 | 25 | 1.38889 |
| 6 | 18 | 5 | 0.20833 |
| 7 | 18 | 10 | 0.41667 |
| 8 | 18 | 15 | 0.62500 |
| 9 | 18 | 20 | 0.83333 |
| 10 | 18 | 25 | 1.04167 |
| 11 | 24 | 5 | 0.16667 |
| 12 | 24 | 10 | 0.33333 |
| 13 | 24 | 15 | 0.50000 |
| 14 | 24 | 20 | 0.66667 |
| 15 | 24 | 25 | 0.83333 |
| 16 | 30 | 5 | 0.13889 |
| 17 | 30 | 10 | 0.27778 |
| 18 | 30 | 15 | 0.41667 |
| 19 | 30 | 20 | 0.55556 |
| 20 | 30 | 25 | 0.69444 |
| 21 | 36 | 5 | 0.11905 |
| 22 | 36 | 10 | 0.23810 |
| 23 | 36 | 15 | 0.35714 |
| 24 | 36 | 20 | 0.47619 |
| 25 | 36 | 25 | 0.59524 |
| Avg. | 24 | 15 | 0.54643 |

* Includes 6 in. clearance between cartons

**10.93c** The "worst case" condition occurs when the conveyor is loaded with the shortest and heaviest cartons, e.g., having the conveyor full of 12" cartons weighing 25 lbs., each. With a 6" clearance, the maximum number of 12" cartons that can be placed on the 250' long conveyor can be determined by, first, determining the number of full load segments on the conveyor: $\lfloor(250 \times 12)/(12''+6'')\rfloor = 166$.

The length of the partial load segment is (250 x 12) - (166 x 18) = 12", which is the length of the shortest carton. Hence, there will be 167 cartons on the conveyor during the "worst case" condition and the load on the conveyor will total (167 x 25), or **4,175 lbs**.

**10.94a** Flat belt driven roller conveyor, WBR = 27", RC = 6", tote box length = 24", tote box clearance on first conveyor = 6", average weight of loaded tote box = 30 lbs.

$S_1$ = 200 fpm, $TL_1$ = 200', $TL_2$ = 100', spacing between tote boxes on 2nd conveyor = 8",
$S_2$ = 200 fpm x (24"+8") ÷ (24"+6") = **213.33 fpm**

**10.94b** # full load segments = ⌈(200 x 12)/(24"+6")⌉ = 80
Exactly 80 load segments will be on the conveyor; the average weight of a load segment is 30 lbs. Therefore, the average load on the conveyor is **2,400 lbs.**

It is not recommended that the horsepower calculation be based on average loading conditions. Instead, we recommend using either "worst case" or 99 percentile conditions. Since there are exactly 70 load segments on the conveyor at all times during loading conditions, the load on the conveyor is equal to the sum of 70 load segments.

If the weight in a tote box is statistically independent, based on the Central Limit Theorem the weight of 70 load segments will be normally distributed with an average weight of 2,100 lbs. Since the variance of the sum of independent random variables equals the sum of their variances, the variance of the weight of 70 tote boxes is given by
Var(wt of loaded tote box) = 0.2(100+400+900+1600+2500)-$(30)^2$ =200 $lbs^2$
∴ Var(wt of 70 loaded tote boxes) = 70(200) = 14,000 $lbs^2$

The weight on the conveyor is normally distributed with a mean of 2400 lbs and a standard deviation of 118.32 lbs. Using the NORMINV function from Excel, the following weights should be used in the horsepower calculation in order to provide 95%, 99%, and 99.9% confidence that the horsepower is adequate for the loading condition: 1,694.62 lbs, 1,775.26 lbs, and 1,865.64 lbs, respectively.

**10.95a** RC = 6", WBR = 33", carton weight = 35 lbs., carton length = 30", spacing between cartons on first conveyor = 24"

Roller supported belt conveyor: $TL_1$ = 100' and $S_1$ = 300 fpm
Belt driven roller conveyor: $S_2$ = 300(30 + 12)/(30 + 24) = **233.33 fpm**

**10.95b** HP = [BV + LF(TL) + FF(L)](S)/14,000
$BV_1$ = ⅔(WBR) = ⅔(33) = 22
$LF_1$ = 0.72, $TL_1$ = 100', $FF_1$ = 0.05, $S_1$ = 300 fpm
The number of full load segments: ⌈(100 x 12)/(30 + 24)⌉ = 22; the length of the partial load segment is 1200 - 22(54) = 12"; the weight of the partial segment is 35(12/30) = 14 lbs. Thus, $L_1$ = 22(35) + 14 = 782 lbs.
$HP_1$ = [22 + 0.72(100) + 0.05(782)](300)/14,000 = **2.85 hp**

**10.95c** HP = [BV + LF(TL) + FF(L)](S)/14,000
$BV_2$ = 4.6 + 0.445(WBR) = 4.6 + 0.445(33) = 19.285
$LF_2$ = 1.2, $TL_2$ = 50', $FF_2$ = 0.10, $S_2$ = 233.33 fpm

The number of full load segments: ⌈(50 x 12)/(30 + 12)⌉ = 14; the length of the partial load segment is 50(12) - 14(42) = 12"; the weight of the partial segment is 35(12/30) = 14 lbs. Thus, $L_2$ = 14(35) + 14 = 504 lbs.
$HP_2$ = [19.285 + 1.2(50) + 0.10(504)](233.33)/14,000 = **2.16 hp**

**10.96a** Roller supported belt conveyors, $TL_1$ = 150',$TL_2$ = 100', WBR = 27", RC = 10", $S_1$ = 300 fpm, clearance between tote boxes on 1st conveyor = 12", lightest wt. = 10 lb., avg. wt. = 30 lb., heaviest wt. = 50 lb.

$S_2$ = 300 fpm x (24" + 6") ÷ (24" + 12") = **250 fpm**

**10.96b** Number of full load segments on 1st conveyor = ⌈(150)(12)/(24+12)⌉ = 50
There is no partial segment on 1st conveyor
Lightest load on 1st conveyor = 50(10) = **500 lbs**
Average load on 1st conveyor = 50(30) = **1500 lbs**
Heaviest load on 1st conveyor = 50(50) = **2500 lbs**

Although the problem did not ask for it, by computing the variance of the weight per tote box and utilizing the Central Limit Theorem, one can determine upper limits for conveyor loading for specified confidence levels. A calculation establishes that the variance of the weight of a tote box equals
$[100(0.1)+400(0.2)+900(0.4)+1600(0.2)+2500(0.1)] - (30)^2 = 120 \text{ lbs}^2$

The sum of 50 independently distributed random variables, each having a mean of 30 lbs and a variance of 120 $\text{lbs}^2$, will be normally distributed with a mean of 1500 lbs and a variance of 6000 $\text{lbs}^2$. Using Excel's NORMINV function, the following weights should be used in the horsepower calculation in order to provide 95%, 99%, and 99.9% confidence that the horsepower is adequate for the loading condition: 1627.41 lbs, 1680.20 lbs, and 1739.37, respectively.

**10.97a** $TL_1 = TL_2$ = 100'., $S_1$ = 150 fpm, $S_2$ = 250 fpm, WBR = 33", and RC = 6"

# full load segments on 1st conveyor = ⌈(100 x 12)/(20 + 6)⌉ = 46; the length of the partial load segment is 1200 - 46(26) = 4"; the weight of the partial segment is 40(4/20) = 8 lbs. Thus, $L_1$ = 46(40) + 8 = **1848 lbs**.

**10.97b** For the totes to be processed at the same rates by both conveyors, the clearance between tote boxes on the 2nd conveyor, $c_2$, must be such that $S_1/(20+c_1) = S_2/(20+c_2) = 150/26 = 250/(20+c_2) \therefore c_2 = [250(26)/150] - 20 = 23.33"$
# full load segments on 2nd conveyor = ⌈(100 x 12)/(20 + 23.33)⌉ = 27
Length of the partial load segment is 1200 - 27(43.33) = 30.09", which is longer than a tote box. Hence, there will be 28 tote boxes on the 2nd conveyor. Thus, $L_2$ = 28(40) = **1120 lbs**.

**10.97c** Maximum RC for load stability. Recall, at least 3 rollers should be under a tote box at all times for stability of the load. Therefore, the maximum RC value is the length of a tote box divided by 3, or 20/3 = **6.67"**

**10.98a** Roller-supported belt conveyor: TL = 200', WBR = 27", RC = 6", S = 100 fpm, α = 15□, c = 6"

A sequence of 11 consecutive cartons will be defined as a "train segment." Based on the load distribution, a train segment will consist of the following sequence of cartons: A, A, B, B, C, C, C, D, D, D, E. Including the clearance between cartons, each train segment is 282" long and weighs 310 lbs. There are ⌊(200 x 12)/(282)⌋ = 8 "train segments" on the conveyor, plus a partial segment of length (200 x 12) - (8 x 282) = 144"

To determine the heaviest loading condition on the conveyor, it is necessary to determine the weight of the heaviest 144" section of a train segment, since there will be 8 train segments and a 144" partial segment on the conveyor at all times during loading conditions.

The tote weights (in pounds), in sequence, are 15, 15, 20, 20, 30, 30, 30, 40, 40, 40, 30. Including clearances between cartons, the weight is distributed (in pounds per foot) as follows over a train segment: 10, 10, 10, 10, 15, 15, 15, 16, 16, 16, 10. The heaviest condition will occur when, in addition to the 8 train segments, a partial segment is on the conveyor, with the totes in the partial segment weighing 15, 15, 16, 16, and 16 pounds per foot. The length of a partial segment of C, C, D, D, D, including clearances is 24 + 24 + 30 + 30 + 30, or 138"; it weighs 180 lbs.

Another 6" must be accounted for in the partial segment. The next heaviest 6" over the partial segment will be 6" of C, at a weight of (6/18)(30), or 10 lbs. (You may wish to verify that if the 6" had come from E, it would have weighed only 6 lbs.) Hence, the weight on the conveyor during the heaviest loading condition will total 8(310) + 180 + 10, or **2,670 lbs**.

**10.98b** To determine the heaviest loading condition on the conveyor, it is necessary to determine the weight of the lightest 144" section of a train segment. Based on the weight distribution shown above, the lightest section will be E, A, A, B, B, C, with a length, including clearances, of 144" and a weight of 130 lbs. Therefore, the weight on the conveyor during the lightest loading condition will be 8(310) + 130, or **2,610 lbs**.

**10.99a** Grain is to be conveyed horizontally 150' $S_{max}$ = 200 + 16.667BW, grain density = 50 lb/ft$^3$, design capacity = 250 tph
BW = 18", $S_{max}$ = 200 + 16.667(18) = 500 fpm.
BW = 24", $S_{max}$ = 200 + 16.667(24) = 600 fpm
BW = 30", $S_{max}$ = 200 + 16.667(30) = 700 fpm
BW = 36", $S_{max}$ = 200 + 16.667(36) = 800 fpm.

@ 50 lb/$ft^3$ density and BW = 18", the required speed to deliver 250 tph equals (250/26.5)(100), or 943 fpm > 500 fpm. Thus, BW = 18" is not feasible.

@50 lb/$ft^3$ density and BW = 24", the required speed to deliver 250 tph equals (250/52)(100), or 480.77 fpm < 600 fpm. Thus, BW = 24" is feasible.
HP (BW=24") = (0.7)(480.77/100) to drive empty conveyor = 3.37 hp

@50 lb/$ft^3$ density and BW = 30", the required speed to deliver 250 tph equals (250/81)(100), or 308.64 fpm < 700 fpm. Thus, BW = 30" is feasible.
HP(BW=30") = (0.9)(3.0864) to drive empty conveyor = 2.78 hp

@50 lb/$ft^3$ density and BW = 36", the required speed to deliver 250 tph equals (250/113.5)(100), or 220.26 fpm < 800 fpm. Thus, BW = 36" is feasible.
HP(BW=36") = (1.1)(2.2026) to drive empty conveyor = 2.42 hp.

Thus, the lowest horsepower requirement (2.42 hp) to drive the empty conveyor occurs with the **36**" width conveyor belt traveling at a speed of **220.26 fpm**. (ignoring the safety factor of 1.2).

**10.99b** f(BW,HP) = \$300BW + \$5,000HP
Here, we assume the HP value is for the total horsepower requirement, including safety factor, rather than just the horsepower required to drive the empty conveyor. The horsepower requirements for feasible belt widths of 24", 30", and 36" are, respectively,
HP(BW=24") = (3.37 + 2.3)(1.2) = 6.80 hp
HP(BW=30") = (2.78 + 2.3)(1.2) = 6.10 hp
HP(BW=36") = (2.42 + 2.3)(1.2) = 5.66 hp

f(BW=24,HP=6.80) = \$300(24) + \$5,000(6.80) = \$41,200
f(BW=30,HP=6.10) = \$300(30) + \$5,000(6.10) = \$39,500
f(BW=36,HP=5.66) = \$300(36) + \$5,000(5.66) = \$39,100
The least cost belt width is **36"**

(Note: if the conveyor cost \$366.67 per inch of width, the optimum is BW = 30"; if it cost \$583.34 per inch of width, the optimum is BW = 36")

**10.100a** Dry sand; conveyor length = 75'; elevation = 24'; BW = 30"; S = 600 fpm.
Maximum angle of incline = 15□
Since 75 sin(15□) = 19.41 < 24', the angle of incline is not feasible.
Likewise, since $\sin^{-1}$(24/75) = 18.66□ > 15□, the **angle of incline is not feasible**.

**10.100b** $S_{max}$ = 200 + 16.667BW = 200 + 16.667(30) = 700 fpm > 600 fpm. Therefore, the **speed is feasible**.

**10.100c** 50 lb/$ft^3$, BW = 30", and S = 600 fpm yields a delivered capacity of 81.0(600/100), or **486 tph**.

**10.101a** Lumpy, abrasive steel trimmings; conveyor length = 50'; elevation = 20'; BW = 30"; S = 400 fpm

Maximum angle of incline = 18□. Since 50 sin(18□) = 15.45 < 20', the **angle of incline is not feasible**.

**10.101b** $S_{max}$ = 100 + 8.333BW = 100 + 8.333(30) = 350 fpm < 400 fpm. Therefore, the **speed is not feasible**.

**10.101c** 100 lb/ft$^3$, BW = 30", and S = 400 fpm yields a delivered capacity of 162.0(400/100), or **648 tph**.

**10.102a** Unsized, washed gravel; horizontal distance = 232.55'; elevation = 59.33'; BW = 24"; S = 400 fpm; density = 75 lb/ft$^3$

Maximum angle of incline = 12□. Since 232.55tan(12□) = 49.43 < 59.33', the **angle of incline is feasible**.

**10.102b** $S_{max}$ = 200 + 8.333BW = 200 + 8.333(24) = 400 fpm = 400 fpm. Therefore, the **speed is feasible**.

**10.102c** 75 lb/ft$^3$, BW = 24", and S = 400 fpm yields a delivered capacity of [(52.0 + 104.0)/2](400/100), or **312 tph**.

**10.102d** conveyor length = $[(232.55)^2 + (59.33)^2]^{0.5}$ = 240'. HP to drive empty conveyor = [0.8 + (4/5)(0.9 - 0.8)](400/100) = **3.52 hp**.

**10.103a** horizontal distance = 200'; angle of incline = 10□; rate = 100 tph; density = 60 lb/ft$^3$

BW = 24"; delivered capacity/100 fpm = 52.0 + (1/5)(104.0 - 52.0) = 62.4 tph/100 fpm. Hence, S = (100 tph)/(0.624 tph/fpm) = **160.26 fpm**.

**10.103b** Conv. length = 200cos(10□) = 196.96', S = 160.26 fpm, BW = 24". HP to drive empty conveyor = [0.7 + (10.26/50)(0.8 - 0.7](160.26/100) = **1.155 hp**

**10.103c** 100 tph; conv. length = 196.96'. Δ HP to convey material horizontally = [0.9 + (10.26/50)(1.1 - 0.9)] = **0.94 hp**.

**10.103d** Elevation = 200tan(10□) = 35.27'. Δ HP to lift material 35.27' = [3.0 + (5.27/10)(3.0 - 4.0)] = **3.527 hp**.

**10.103e** From a), the delivered capacity of material of this density is 62.4 tph/100 fpm of belt speed. If the material is classified as unsized material, then $S_{max}$ = 200 + 8.333BW = 200 + 8.333(24) = 400 fpm. At 400 fpm, material will be delivered at a rate of 62.4(4), or **249.6 tph**. If the material is crushed stone, ore, grain, or sand, then the delivered capacity will be greater.

**10.104a** Coffee beans; angle of incline = 10°; elevation = 30'; density = 30 lb/$ft^3$

BW = 24"; $S_{max}$ = 200 + 16.667BW = 200 + 16.667(24) = 600 fpm
Maximum delivered capacity = 31.0(600/100) = **186 tph**

**10.104b** To achieve 200 tph, speed must be 200 tph(0.31 tph/fpm) = **645.16 fpm**.

**10.104c** Conveyor length = 30/sin(10°) = 172.763'; S = 500 fpm
HP to drive empty conveyor = [0.7 + (22.763/50)(0.8 - 0.7)](500/100) = **3.73 hp**

**10.104d** ΔHP to convey material horiz = 1.8 + (22.763/50)(2.1 - 1.8) = **1.94 hp**

**10.104e** 200 tph; 30' elevation. ΔHP to elevate material = **6.1 hp**

**10.105a** oats; angle of incline = 15°; elevation = 50'; density = 30 lb/$ft^3$

BW = 30"; S = 400 fpm
Conveyor length = 50/sin(15°) = 193.185'
HP to drive empty conveyor = [0.9 + (43.185/50)(1.0 - 0.9)](400/100) = **3.945 hp**

**10.105b** Delivered capacity = 48.5(400/100) = 194 tph
@ 150 tph, ΔHP to convey material horiz = 1.4 + (43.185/50)(1.6 - 1.4) = 1.573 hp
@ 200 tph, ΔHP to convey material horiz = 1.8 + (43.185/50)(2.1 - 1.8) = 2.059 hp
Thus, @ 194 tph, ΔHP to convey material horiz = 1.573 +(44/50)(2.059 - 1.573) = 2.00 hp

To elevate 50' @ 194 tph
ΔHP to elevate material = 8.8 + (19/25)(10.1 - 8.8) = 9.788 hp.

To drive empty conveyor, convey material 193.185' horizontally, and to elevate material 50' requires 3.945 hp + 2.000 hp + 9.788 hp = 15.733 hp, not including the 20% safety factor.

**10.105c** BW = 18" delivers oats at rate of 16.0(4) = 64 tph < 200 tph; BW = 24" has rate of 31.0(4) = 124 tph < 200 tph; BW = 30" has rate of 48.5(4) = 194 tph < 200 tph; BW = 31" has rate of [48.5 + (1/6)(68.0 - 48.5)](4) = 207 tph. > 200 tph. **31" belt width required**.

**10.105d**

| BW | Delivered Capacity | Speed | tph |
|---|---|---|---|
| 24 | 31 | 440 | 136.4 |
| 25 | 33.9167 | 450 | 152.625 |
| 26 | 36.8333 | 460 | 169.433 |
| 27 | 39.75 | 470 | 186.825 |
| 28 | 42.6667 | 480 | 204.8 |
| 29 | 45.5833 | 490 | 223.358 |
| 30 | 48.5 | 500 | 242.5 |

A 28" belt width is required.

**10.106** The From-To matrix can be determined from the problem statement. Note that stations 2, 3, and 9 are I/O stations. The From-To Matrix is as follows:

| $f_{ij}$ | 2 | 3 | 4 | 5 | 9 | $\lambda$ |
|---|---|---|---|---|---|---|
| 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 3 | 0 | 0 | 0 | 9 | 12 |
| 4 | 0 | 9 | 0 | 3 | 0 | 12 |
| 5 | 0 | 0 | 9 | 0 | 0 | 9 |
| 9 | 0 | 3 | 3 | 0 | 0 | 6 |
| $\Lambda$ | 3 | 12 | 12 | 3 | 9 | 39 |

Empty travel time can be determined from figure 10.59b where $\sigma_{ij}$ is equal to the rectilinear distance in grids from station i to station j multiplied by 5 seconds/grid.

| $\sigma_{ij}$ | 2 | 3 | 4 | 5 | 9 |
|---|---|---|---|---|---|
| 2 | 0.00 | 0.33 | 0.67 | 1.00 | 0.50 |
| 3 | 0.33 | 0.00 | 0.33 | 0.67 | 0.17 |
| 4 | 0.67 | 0.33 | 0.00 | 0.33 | 0.50 |
| 5 | 1.00 | 0.67 | 0.33 | 0.00 | 0.83 |
| 9 | 0.50 | 0.17 | 0.50 | 0.83 | 0.00 |

Each loaded trip required 2(0.15) = 0.30 min for pick-up and deposit, so the total loaded travel time from station i to station j is $\tau_{ij} = \sigma_{ij} + 0.30$ min/trip. By (10.135):

$\alpha_f = [3(0.63) + 9(0.47) + 9(0.63) + 3(0.63) + 9(0.63) + 3(0.47) + 3(0.80)]/60 = 0.3867$

Assuming FCFS dispatching rule, by (10.136)

$\bar{s} = 3(3(0.96) + 9(0.80) + 9(1.30) + 3(1.30) + 9(1.63) + 3(0.97) + 3(1.30))/39^2$

$+ 12(3(0.63) + 9(0.47) + 9(0.96) + 3(0.96) + 9(1.30) + 3(0.64) + 3(0.97))/39^2$
$+ 12(3(0.96) + 9(0.80) + 9(0.63) + 3(0.63) + 9(0.96) + 3(0.97) + 3(1.30))/39^2$
$+ 3(3(1.30) + 9(1.14) + 9(0.96) + 3(0.96) + 9(0.63) + 3(1.30) + 3(1.63))/39^2$
$+ 9 (3(0.80) + 9(0.64) + 9(1.13) + 3(1.13) + 9(1.46) + 3(0.47) + 3(0.80))/39^2$
$= 0.01555$ hr $= 0.9331$ min

Hence,
$\rho = \lambda_T\bar{s} = 39(0.1555) = 0.6065$
$\alpha_e = \rho - \alpha_f = 0.6065 - 0.3867 = 0.2199$

The AGV can meet throughput since 0.6065 < 1. The fraction of time the AGV travels empty while busy is $\alpha_e/\rho = 0.2199/0.6065 = .3626$ (roughly one-third of the time).

**SECTION 10.9**

**10.107** (M|M|10):(GD|∞|∞)
$\lambda = 10$/hr, $\mu = 2$/hr, $c = 10$, $\rho = 10/20 = 0.5$
$L_q = \rho(c\rho)^c P_0/[c!(1 - \rho)^2]$

$$P_0 = \left[\frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1}\frac{(c\rho)^n}{n!}\right]^{-1}$$

$$P_0 = \left[\frac{5^{10}}{10!(0.5)} + 1 + 5 + \frac{5^2}{2} + \frac{5^3}{6} + \frac{5^4}{24} + \frac{5^5}{120} + \frac{5^6}{720} + \frac{5^7}{5{,}040} + \frac{5^8}{40{,}320} + \frac{5^9}{362{,}880} + \right]^{-1}$$

$P_0 = 0.0067082$
$L_q = [(0.5)(5)^{10}(0.0067082)/10!(0.5)^2] = 0.0361$
$W_q = L_q/\lambda = 0.00361$ hr $= 0.2136$ min
Avg # "busy servers" $= L - L_q = \lambda/\mu = 10/2 = 5$

**10.108** (M|M|1):(GD|N|∞)
$\lambda = 30$/hr, $\mu = 50$/hr, $c = 1$ $\rho = 30/50 = 0.6$
Determine the smallest value of N such that $Pr(n=N) \le 0.05$
$P_0 = (1-\rho)/(1-\rho^{N+1}) = 0.4/[1-(0.4)^{N+1}]$ and
$P_N = (0.6)^N(0.4)/[1-(0.4)^{N+1}] \le 0.05$ which reduces to
$(0.6)^N \le 0.05/(1 - 0.95(0.6)]$ or
$N\log(0.6) \le \log(0.116279)$
$N \ge 4.2123$
Therefore, N = 5 and the accumulation line must have the capacity for four cartons waiting to be processed by the workstation.

**10.109** (M|M|1):(GD|N|∞)
$\lambda = 2$/min, $\mu = 3.333$/min, $c = 1$ $\rho = 0.6$
Determine the smallest value of N such that $Pr(n=N) \le 0.01$
$P_0 = (1-\rho)/(1-\rho^{N+1}) = 0.4/[1-(0.4)^{N+1}]$ and
$P_N = (0.6)^N(0.4)/[1-(0.4)^{N+1}] \le 0.01$ which reduces to
$(0.6)^N \le 0.05/(1 - 0.99(0.6)]$ or

$N\log(0.6) \le \log(0.123153)$
$N \ge 4.09989$
Therefore, N = 5 and there must be at least four waiting spaces at the workstation.

**10.110** (M|M|1):(GD|∞|∞)
$\lambda = 3$/min, $\mu = 4$/min, and $\rho = 0.75$
. $L_q = \rho^2/(1-\rho) = (0.75)^2/(0.25)$, or $L_q = 2.25$ parts

**10.111** (M|G|c):(GD|c|∞)
$\lambda = 30$/hr, $\mu = 20$/hr, $\sigma = 1$ min. Find the smallest value of c such that $P_c \le 0.05$. From the figure in the text, at least 4 packaging stations should be provided in order to have a loss no greater than 5%. Using Excel to compute the vlaue of $P_c$ yields the results shown below, verifying that 4 is the answer.

| c | $P_c$ |
|---|---|
| 1 | 0.600000 |
| 2 | 0.310345 |
| 3 | 0.134328 |
| 4 | 0.047957 |
| 5 | 0.014183 |

**10.112** (M|G|1):(GD|∞|∞)
$\lambda = 10$/hr, $\mu = 12$/hr, $\sigma = 1/60$ hr, and $\rho = 0.83333$.
$W_q = \lambda(\sigma^2 + \mu^{-2})/(2(1-\rho) = 10[(1/3600) + (1/144)]/2(1/6)$, or
$W_q = 0.216667$ hr or 13 min

**10.113** (M|M|1):(GD|∞|∞)
$\lambda = 12$/min, $\mu = 15$/min, and $\rho = 0.8$
. $L_q = \rho^2/(1-\rho) = (0.64)/(0.2)$, or $L_q = 3.2$ cartons

**10.114a** (M|M|2):(GD|∞|∞)
$\lambda = 10$/hr, $\mu = 6$/hr, c = 2, and $\rho = 0.83333$
Pr(both inspectors are idle) = $P_0$

$$P_0 = \left[\frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1}\frac{(c\rho)^n}{n!}\right]^{-1}$$

$$P_0 = \left[\frac{\left(5/3\right)^2}{2\left(1-5/6\right)} + 1 + \frac{5}{3}\right]^{-1}$$

$P_0 = 0.090909$

**10.114b** $L_q = \rho(c\rho)^c P_0/[c!(1 - \rho)^2]$
$L_q = [(5/6)(5/3)^2(0.090909)/2!(1/6)^2] = 3.7879$ unit loads

**10.115** $(M^{(b)}|M|1):(GD|\infty|\infty)$

$E(b) = [1+2+3+4]/4 = 2.5]$, $\lambda = 2/min$, $\mu = 10/min$, $\rho = 2(2.5)/10 = 0.50$

$E(b^2) = [1+4+9+16]/4 = 7.5 \therefore V(b) = 7.5 - (2.5)^2 = 1.25$

$W = [1.25 + (2.5)^2 + 2.5]/\{2(2.5)[10 - 2(2.5)]\} = 0.40$ min.

$W_q = W - (1/\mu) = 0.40 - 0.10 = 0.30$ min

**10.116** $(M^{(b)}|M|1):(GD|\infty|\infty)$

$E(b) = 0.25(1) + 0.5(2) + 0.25(3) = 2.0$, $\lambda = 20/hr$, $\mu = 60/hr$, $\rho = 20(2)/60 = 2/3$

$E(b^2) = 0.25 + 2 + 2.25 = 4.5 \therefore V(b) = 4.5 - (2)^2 = 0.5$

$L = 20[0.5 + (2)^2 + 2]/\{2[60 - 20(2)]\} = 3.25$

$L_q = L - [\lambda\, E(b)/\mu] = 3.25 - [20(2)/60] = 2.5833$

$W_q = L_q/[\lambda\, E(b)] = 2.5833/[20(2)] = 0.064583$ hr = 3.875 min

Pr(busy attendant) = $1 - P_0 = L - L_q = \lambda\, E(b)/\mu = 3.25 - 2.5833 = 2/3$

**10.117a** $c = 1$, $\mu = 12/hr$

$(M|M|1):(GD|\infty|\infty)$

What is the largest value of $\lambda$ such that $L \le 4$?

$L = \rho/(1-\rho) = (\lambda/12)/[1 - (\lambda/12)] \le 4$

$\lambda/[12 - \lambda] \le 4 \;\therefore\; \lambda \le 9.6/hr$

**10.117b** $(D|M|1):(GD|\infty|\infty)$

$L = \theta/(1-\theta) = 4$. Solving for $\theta$ gives $\theta = 0.8$

$0.8 = \exp(-0.2/(\lambda/12)) = \exp(-2.4/\lambda)$

Taking the natural logarithm of each side of the equality and solving for $\lambda$ gives $\ln(0.8) = -2.4/\lambda = -0.2231425$. Thus, $\lambda = 10.755/hr$

**10.117c** The arrival and service rates behave like an $(M|M|1):(GD|3|3)$ queue with $K = 3$, $\lambda = 5/hr$, $\mu = 12/hr$, and $\rho = 5/12 = 0.41667$. As shown below, $L = 1.244$.

| n | $\lambda_n$ | $\mu_n$ | $P_n$ | $nP_n$ | |
|---|---|---|---|---|---|
| 0 | 15 | 0 | $P_0$ | 0 | |
| 1 | 10 | 12 | $(15/12)P_0$ | $(15/12)P_0$ | |
| 2 | 5 | 12 | $(10/12)(15/12)P_0$ | $(300/144)P_0$ | |
| 3 | 0 | 12 | $(5/12)(10/12)(15/12)P_0$ | $(2250/1728)P_0$ | |
| | | $\Sigma P_n =$ | $(6438/1728)P_0 = 1$ | $(8010/1728)P_0$ | $= L$ |
| | | $P_0 =$ | 0.268406337 | 1.24417521 | $= L$ |

**10.118a** (M|M|1):(GD|5|∞) variation

| n | $\lambda_n$ | $\mu_n$ | $P_n$ | $P_n$ |
|---|---|---|---|---|
| 0 | 20 | 0 | $P_0$ | $P_0$ |
| 1 | 20 | 10 | $(20/10)P_0$ | $2P_0$ |
| 2 | 15 | 12 | $(20/12)P_1$ | $(10/3)P_0$ |
| 3 | 10 | 15 | $(15/15)P_2$ | $(10/3)P_0$ |
| 4 | 5 | 20 | $(10/20)P_3$ | $(5/3)P_0$ |
| 5 | 0 | 30 | $(5/30)P_4$ | $(5/18)P_0$ |
| | | | $\Sigma P_n =$ | $(209/18)P_0$ |
| | | | $P_0 =$ | 0.086124402 |

Pr(idle server) = $P_0$ = 0.0861

**10.118b** (M|M|1):(GD|5|∞) variation. If $P_0$ = 1, what is the value of $P_1$? Since $P_1 = 2P_0$, then $P_1$ = 0.2

**10.119a** (M|M|2):(GD|8|8) λ = 0.1/hr, μ = 0.5/hr, c = 2, K = 8

| n | $\lambda_n$ | $\mu_n$ | $P_n$ | $P_n$ | $P_n$ | # Waiting (m) | $mP_n$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.8 | 0.0 | $P_0$ | $P_0$ | 0.20360004 | 0 | 0.00000000 |
| 1 | 0.7 | 0.5 | $1.6P_0$ | $1.6P_0$ | 0.32576006 | 0 | 0.00000000 |
| 2 | 0.6 | 1.0 | $0.7P_1$ | $1.12P_0$ | 0.22803204 | 0 | 0.00000000 |
| 3 | 0.5 | 1.0 | $0.6P_2$ | $0.672P_0$ | 0.13681923 | 1 | 0.13681923 |
| 4 | 0.4 | 1.0 | $0.5P_3$ | $0.336P_0$ | 0.06840961 | 2 | 0.13681923 |
| 5 | 0.3 | 1.0 | $0.4P_4$ | $0.1344P_0$ | 0.02736385 | 3 | 0.08209154 |
| 6 | 0.2 | 1.0 | $0.3P_5$ | $0.04032P_0$ | 0.00820915 | 4 | 0.03283661 |
| 7 | 0.1 | 1.0 | $0.2P_6$ | $0.008064P_0$ | 0.00164183 | 5 | 0.00820915 |
| 8 | 0.0 | 1.0 | $0.1P_7$ | $0.0008064P_0$ | 0.00016418 | 6 | 0.00098510 |
| | | | $\Sigma P_n =$ | $4.9115904P_0$ | | $L_q =$ | 0.39776086 |
| | | | $P_0 =$ | 0.20360004 | | | |

Pr(at least one idle recharging machine) = $P_0 + P_1$ = 0.52936

**10.119b** (M|M|2):(GD|8|8) λ = 0.1/hr, μ = 0.5/hr, c = 2, K = 8. $L_q$ = 0.39776

**10.120a** (M|M|1):(GD|6|6) λ = 0.5/hr, μ = 2.5/hr, c = 1, K = 6

| n | $\lambda_n$ | $\mu_n$ | $P_n$ | $P_n$ | $P_n$ | # Waiting (m) | $mP_n$ |
|---|---|---|---|---|---|---|---|
| 0 | 3.0 | 0.0 | $P_0$ | $P_0$ | 0.19184726 | 0 | 0.00000000 |
| 1 | 2.5 | 2.5 | $1.2P_0$ | $1.2P_0$ | 0.23021671 | 0 | 0.00000000 |
| 2 | 2.0 | 2.5 | $1.0P_1$ | $1.2P_0$ | 0.23021671 | 1 | 0.23021671 |
| 3 | 1.5 | 2.5 | $0.8P_2$ | $0.96P_0$ | 0.18417337 | 2 | 0.36834674 |
| 4 | 1.0 | 2.5 | $0.6P_3$ | $0.576P_0$ | 0.11050402 | 3 | 0.33151206 |
| 5 | 0.5 | 2.5 | $0.4P_4$ | $0.2304P_0$ | 0.04420161 | 4 | 0.17680643 |
| 6 | 0.0 | 2.5 | $0.2P_5$ | $0.04608P_0$ | 0.00884032 | 5 | 0.04420161 |
| | | | $\Sigma P_n =$ | $5.21248P_0$ | | $L_q =$ | 1.15108355 |
| | | | $P_0 =$ | 0.19184726 | | | |

**10.120b** (M|M|1):(GD|6|6) $\lambda = 0.5$/hr, $\mu = 2.5$/hr, c = 1, K = 6. $L_q = 1.1511$.

**10.121** (M|D|1):(GD|∞|∞)
$\lambda = 30$/hr, $\mu = 240$/hr, $\rho = 0.125$
$L_q = \rho^2/2(1-\rho) = (0.125)^2/2(1 - 0.125) = 0.006836$
Basically, none will have to wait.

**10.122** (D|M|1):(GD|∞|∞)
$\lambda = 4$/min, $\mu = 5$/min, $\rho = 0.80$
$L_q = \theta^2/(1-\theta)$, where $\theta = \exp\text{-}(1-\theta)/\rho$
As shown on a following page, using the iterative solution procedure until
$|\theta_k - \theta_{k+1}| \le 0.00000001$ yields an estimate for $\theta$ of 0.62862974, which yields an estimate for $L_q$ of 1.0641.

**10.123** (M|G|50):(GD|50|∞)
$\lambda = 60$/hr, $\mu = 2$/hr, $\sigma = 1/6$ hr, c = 50

$$P_c = \frac{\left(\lambda/\mu\right)^c / c!}{\sum_{n=0}^{c}\left(\lambda/\mu\right)^n / n!} \qquad L = \frac{\lambda}{\mu}\left(1-P_c\right)$$

As shown on a following page, $P_{50} = 0.00022$ and $L = 29.99$.

**10.124** Operator 1: (M|M|1):(GD|∞|∞)
$\lambda = 5$/hr, $\mu = 6$/hr, $\rho = 5/6$, $C_o = \$10$/hr, $C_w = \$10$/hr
TC = \$10/hr + \$10L
$L = \rho/(1 - \rho) = (5/6)/[1 - 5/6] = 5$
TC = \$60/hr
Operator 2: (M|G|1):(GD|∞|∞)
$\lambda = 5$/hr, $\mu = 7.5$/hr, $\sigma = 2/60$ hr, $\rho = 2/3$, $C_o = \$16$/hr, $C_w = \$10$/hr

TC = \$10/hr + \$10L
$L = \rho + [(\lambda\sigma)^2 + \rho^2]/[2(1 - \rho)] = (2/3) + [(1/6)^2 + (2/3)^2]/[2(1/3)] = 1.375$
TC = \$23.75/hr
Choose the more expensive, but faster operator.

**10.125** Errors were introduced in this problem during the publication process. Service time is supposed to be ***exponentially distributed*** with a mean of ***3*** minutes.
(M|M|c):(GD|∞|∞)
$\lambda = 15/\text{hr}$, $\mu = 20/\text{hr}$, $\rho = 0.75$, $C_1 = \$27/\text{hr}$, $C_3 = \$18/\text{hr}$
$VC = \$27L_q + \$18c$
c = 1
$L_q = \rho^2/(1 - \rho) = (0.75)^2/(0.25) = 2.25$
VC = \$27(2.25) + \$18(1) = \$78.75
c = 2

$$P_0 = \left[\frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1}\frac{(c\rho)^n}{n!}\right]^{-1}$$

$$P_0 = \left[\frac{(0.75)^2}{2(1-0.375)} + 1 + 0.75\right]^{-1} = 0.454545$$

$$L_q = \frac{\rho(c\rho)^c P_0}{c!(1-\rho)^2} = \frac{0.375(0.75)^2(0.454545)}{2(1-0.375)^2} = 0.122727$$

VC = \$27(0.122727) + \$18(2) = \$39.31
c = 3

$$P_0 = \left[\frac{(0.75)^3}{6(1-0.25)} + 1 + 0.75 + \frac{(0.75)^2}{2}\right]^{-1} = 0.470588$$

$$L_q = \frac{0.25(0.75)^3(0.470588)}{6(1-0.25)^2} = 0.01470588$$

$$VC = \$27(0.122727) + \$18(3) = \$54.40$$

Therefore, the optimum number of servers is 2.

**10.126** (M|M|1):(GD|K|K)

Since the number of machines assigned to an individual operator is to be determined, we will seek to minimize the expected total cost per machine. The objective function to be minimized can be written as

Minimize $TC(K) = [C_1L_q + C_2(L - L_q) + C_3]/K$

$\lambda = 4/3/\text{hr}$, $\mu = 12/\text{hr}$, $C_1 = \$90/\text{hr}$, $C_2 = \$45/\text{hr}$, and $C_3 = \$18/\text{hr}$. Let K = 3. Using the approach illustrated in Table 10.34, we obtain the following results.

| $n$ | $\lambda_n$ | $\mu_n$ | $P_0$ Multipliers | $P_n$ | $nP_n$ | $(n-1)P_n$ |
|---|---|---|---|---|---|---|
| 0 | 4.0000 | 0 | 1.000000000 | 0.7063953 | 0.000000 | -- |
| 1 | 2.6667 | 12 | 0.333333333 | 0.2354651 | 0.235465 | 0 |
| 2 | 1.3333 | 12 | 0.074074074 | 0.0523256 | 0.104651 | 0.052326 |
| 3 | 0 | 12 | 0.008230453 | 0.0058140 | 0.017442 | 0.01163 |
| | | Sum = | 1.41563786 | | | L = 0.35756 |
| | | $P_0$ = | 0.706395349 | | | $L_q$ = 0.06395 |

TC(3) = [\$90(0.06395) + \$45(0.35756 - 0.06395) + \$18]/3 = \$12.32
Next, let's consider K = 4. As shown below,

| $n$ | $\lambda_n$ | $\mu_n$ | $P_0$ Multipliers | $P_n$ | $nP_n$ | $(n-1)P_n$ |
|---|---|---|---|---|---|---|
| 0 | 5.3333 | 0 | 1.000000000 | 0.6138086 | 0.000000 | -- |
| 1 | 4.0000 | 12 | 0.444444444 | 0.2728038 | 0.272804 | 0 |
| 2 | 2.6667 | 12 | 0.148148148 | 0.0909346 | 0.181869 | 0.090935 |
| 3 | 1.3333 | 12 | 0.032921811 | 0.0202077 | 0.060623 | 0.04042 |
| 4 | 0 | 12 | 0.003657979 | 0.0022453 | 0.008981 | 0.00674 |
| | | Sum = | 1.62917238 | | | L = 0.52428 |
| | | $P_0$ = | 0.613808588 | | | $L_q$ = 0.13809 |

*TC*(4) = [\$90(0.13809) + \$45(0.52428 - 0.13809) + \$18]/4 = \$11.95
Next, let's consider K = 5. As shown below,

| $n$ | $\lambda_n$ | $\mu_n$ | $P_0$ Multipliers | $P_n$ | $nP_n$ | $(n-1)P_n$ |
|---|---|---|---|---|---|---|
| 0 | 6.6667 | 0 | 1.000000000 | 0.5204664 | 0.000000 | -- |
| 1 | 5.3333 | 12 | 0.555555556 | 0.2891480 | 0.289148 | 0 |
| 2 | 4.0000 | 12 | 0.246913580 | 0.1285102 | 0.257020 | 0.12851 |
| 3 | 2.6667 | 12 | 0.082304527 | 0.0428367 | 0.128510 | 0.08567 |
| 4 | 1.3333 | 12 | 0.018289895 | 0.0095193 | 0.038077 | 0.02856 |
| 5 | 0 | 12 | 0.018289895 | 0.0095193 | 0.047596 | 0.03808 |
| | | Sum = | 1.92135345 | | | L = 0.76035 |
| | | $P_0$ = | 0.520466445 | | | $L_q$ = 0.28082 |

*TC*(5) = [\$90(0.28082) + \$45(0.76035 - 0.28082) + \$18]/5 = \$12.97

The optimum number of machines to assign an operator is 4.

**10.127** (M|M|c):(GD|30|30)
$\lambda$ = 4/3/hr, $\mu$ = 12/hr, $C_1$ =\$90/hr, $C_2$ = \$45/hr, and $C_3$ = \$18/hr. From Equation 10.194, our objective is to

Minimize $TC(c) = C_1L_q + C_2(L - L_q) + C_3c$

As shown on a following page, VC(4) = $538.09, VC(5) = $511.83, and VC(6) = $517.55. Therefore, the optimum number of operators to assign to a pool of 30 machines is 5.

**10.122** (Continued)

| $\theta_k$ | $\theta_{k+1}$ | $\lvert\theta_k - \theta_{k+1}\rvert$ |
|---|---|---|
| 0.55000000 | 0.54881164 | 0.00118836 |
| 0.54881164 | 0.54794274 | 0.00086890 |
| 0.54794274 | 0.54730830 | 0.00063444 |
| 0.54730830 | 0.54684552 | 0.00046278 |
| 0.54684552 | 0.54650820 | 0.00033732 |
| 0.54650820 | 0.54626245 | 0.00024574 |
| 0.54626245 | 0.54608349 | 0.00017896 |
| 0.54608349 | 0.54595321 | 0.00013029 |
| 0.54595321 | 0.54585837 | 0.00009483 |
| 0.54585837 | 0.54578936 | 0.00006902 |
| 0.54578936 | 0.54573914 | 0.00005022 |
| 0.54573914 | 0.54570259 | 0.00003654 |
| 0.54570259 | 0.54567601 | 0.00002659 |
| 0.54567601 | 0.54565666 | 0.00001934 |
| 0.54565666 | 0.54564259 | 0.00001407 |
| 0.54564259 | 0.54563235 | 0.00001024 |
| 0.54563235 | 0.54562490 | 0.00000745 |
| 0.54562490 | 0.54561948 | 0.00000542 |
| 0.54561948 | 0.54561554 | 0.00000394 |
| 0.54561554 | 0.54561267 | 0.00000287 |
| 0.54561267 | 0.54561059 | 0.00000209 |
| 0.54561059 | 0.54560907 | 0.00000152 |
| 0.54560907 | 0.54560796 | 0.00000110 |
| 0.54560796 | 0.54560716 | 0.00000080 |
| 0.54560716 | 0.54560658 | 0.00000058 |
| 0.54560658 | 0.54560615 | 0.00000043 |
| 0.54560615 | 0.54560584 | 0.00000031 |
| 0.54560584 | 0.54560562 | 0.00000022 |
| 0.54560562 | 0.54560545 | 0.00000016 |
| 0.54560545 | 0.54560533 | 0.00000012 |
| 0.54560533 | 0.54560525 | 0.00000009 |
| 0.54560525 | 0.54560518 | 0.00000006 |
| 0.54560518 | 0.54560514 | 0.00000005 |
| 0.54560514 | 0.54560511 | 0.00000003 |
| 0.54560511 | 0.54560508 | 0.00000002 |
| 0.54560508 | 0.54560506 | 0.00000002 |
| 0.54560506 | 0.54560505 | 0.00000001 |
| 0.54560505 | 0.54560504 | 0.00000001 |
| 0.54560504 | 0.54560503 | 0.00000001 |
| 0.54560503 | 0.54560503 | 0.00000000 |

$L_q =$ 0.6551

**10.123** (Continued)

| k | Individual Terms in Sum | $P_k$ |
|---|---|---|
| 0 | 1.00000 | 9.36041E-14 |
| 1 | 30.00000 | 2.80812E-12 |
| 2 | 450.00000 | 4.21219E-11 |
| 3 | 4500.00000 | 4.21219E-10 |
| 4 | 33750.00000 | 3.15914E-09 |
| 5 | 202500.00000 | 1.89548E-08 |
| 6 | 1012500.00000 | 9.47742E-08 |
| 7 | 4339285.71429 | 4.06175E-07 |
| 8 | 16272321.42857 | 1.52316E-06 |
| 9 | 54241071.42857 | 5.07719E-06 |
| 10 | 162723214.28571 | 1.52316E-05 |
| 11 | 443790584.41558 | 4.15406E-05 |
| 12 | 1109476461.03896 | 0.000103852 |
| 13 | 2560330294.70529 | 0.000239657 |
| 14 | 5486422060.08277 | 0.000513552 |
| 15 | 10972844120.16550 | 0.00102710 |
| 16 | 20574082725.31040 | 0.001925819 |
| 17 | 36307204809.37130 | 0.00339850 |
| 18 | 60512008015.61880 | 0.005664174 |
| 19 | 95545275814.13500 | 0.008943432 |
| 20 | 143317913721.20300 | 0.013415148 |
| 21 | 204739876744.57500 | 0.01916450 |
| 22 | 279190741015.33000 | 0.026133405 |
| 23 | 364161836106.95200 | 0.03408705 |
| 24 | 455202295133.69000 | 0.042608813 |
| 25 | 546242754160.42700 | 0.051130575 |
| 26 | 630280100954.33900 | 0.058996817 |
| 27 | 700311223282.59900 | 0.065552019 |
| 28 | 750333453517.07100 | 0.070234306 |
| 29 | 776207020879.72800 | 0.072656179 |
| 30 | 776207020879.72800 | 0.072656179 |
| 31 | 751168084722.31800 | 0.070312431 |
| 32 | 704220079427.17300 | 0.06591790 |
| 33 | 640200072206.52100 | 0.059925368 |
| 34 | 564882416652.81200 | 0.052875324 |
| 35 | 484184928559.55400 | 0.045321707 |
| 36 | 403487440466.29400 | 0.037768089 |
| 37 | 327151978756.45500 | 0.030622775 |
| 38 | 258277877965.62200 | 0.024175875 |
| 39 | 198675290742.78600 | 0.018596827 |
| 40 | 149006468057.09000 | 0.01394762 |
| 41 | 109029122968.60200 | 0.010205576 |
| 42 | 77877944977.57310 | 0.00728970 |
| 43 | 54333449984.35330 | 0.005085835 |
| 44 | 37045534080.24090 | 0.003467615 |
| 45 | 24697022720.16060 | 0.002311743 |
| 46 | 16106753947.93080 | 0.001507659 |
| 47 | 10280906775.27500 | 0.000962335 |
| 48 | 6425566734.54687 | 0.00060146 |
| 49 | 3934020449.72257 | 0.000368241 |

**10.127** (Continued)

$\lambda$ = 1.33333 $\mu$ = 12 c= 4

| $n$ | $\lambda_n$ | $\mu_n$ | $P_0$ *Multipliers* | $P_n$ | $nP_n$ | $(n-c)P_n$ |
|---|---|---|---|---|---|---|
| 0 | 40.0000 | 0 | 1.000000000 | 0.0356268 | 0.000000 | -- |
| 1 | 38.6667 | 12 | 3.333333333 | 0.1187561 | 0.118756 | 0 |
| 2 | 37.3333 | 24 | 5.370370370 | 0.1913293 | 0.382659 | 0 |
| 3 | 36.0000 | 36 | 5.569272977 | 0.1984156 | 0.595247 | 0 |
| 4 | 34.6667 | 48 | 4.176954733 | 0.1488117 | 0.595247 | 0 |
| 5 | 33.3333 | 48 | 3.016689529 | 0.1074751 | 0.537375 | 0.107475 |
| 6 | 32.0000 | 48 | 2.094923284 | 0.0746355 | 0.447813 | 0.149271 |
| 7 | 30.6667 | 48 | 1.396615523 | 0.0497570 | 0.348299 | 0.149271 |
| 8 | 29.3333 | 48 | 0.89228214 | 0.0317892 | 0.254313 | 0.12716 |
| 9 | 28.0000 | 48 | 0.54528353 | 0.0194267 | 0.174841 | 0.097134 |
| 10 | 26.6667 | 48 | 0.318082059 | 0.0113323 | 0.113323 | 0.067994 |
| 11 | 25.3333 | 48 | 0.176712255 | 0.0062957 | 0.069253 | 0.04407 |
| 12 | 24.0000 | 48 | 0.093264801 | 0.0033227 | 0.039873 | 0.026582 |
| 13 | 22.6667 | 48 | 0.046632401 | 0.0016614 | 0.021598 | 0.014952 |
| 14 | 21.3333 | 48 | 0.022020856 | 0.0007845 | 0.010983 | 0.007845 |
| 15 | 20.0000 | 48 | 0.009787047 | 0.0003487 | 0.005230 | 0.003835 |
| 16 | 18.6667 | 48 | 0.004077936 | 0.0001453 | 0.002325 | 0.001743 |
| 17 | 17.3333 | 48 | 0.001585864 | 0.0000565 | 0.000960 | 0.000734 |
| 18 | 16.0000 | 48 | 0.000572673 | 0.0000204 | 0.000367 | 0.000286 |
| 19 | 14.6667 | 48 | 0.000190891 | 0.0000068 | 0.000129 | 0.000102 |
| 20 | 13.3333 | 48 | 5.83278E-05 | 0.0000021 | 0.000042 | 3.32E-05 |
| 21 | 12.0000 | 48 | 1.62022E-05 | 0.0000006 | 0.000012 | 9.81E-06 |
| 22 | 10.6667 | 48 | 4.05054E-06 | 0.0000001 | 0.000003 | 2.6E-06 |
| 23 | 9.3333 | 48 | 9.00121E-07 | 0.0000000 | 0.000001 | 6.09E-07 |
| 24 | 8.0000 | 48 | 1.75023E-07 | 0.0000000 | 0.000000 | 1.25E-07 |
| 25 | 6.6667 | 48 | 2.91706E-08 | 0.0000000 | 0.000000 | 2.18E-08 |
| 26 | 5.3333 | 48 | 4.05147E-09 | 0.0000000 | 0.000000 | 3.18E-09 |
| 27 | 4.0000 | 48 | 4.50163E-10 | 0.0000000 | 0.000000 | 3.69E-10 |
| 28 | 2.6667 | 48 | 3.75136E-11 | 0.0000000 | 0.000000 | 3.21E-11 |
| 29 | 1.3333 | 48 | 2.08409E-12 | 0.0000000 | 0.000000 | 1.86E-12 |
| 30 | 0.0000 | 48 | 5.78914E-14 | 0.0000000 | 0.000000 | 5.36E-14 |

Sum = 28.06873189 L = 3.71865

$P_0$ = 0.035626832 $L_q$ = 0.79850

VC(4) = $538.09

VC(5) = $511.83

VC(6) = $517.55

**10.128** (M|M|1):(GD|K|K)

We modify Problem 10.126 and seek to minimize the expected total cost per running machine. The objective function to be minimized can be written as

Minimize $TC(K) = [C_0(K - L) + C_1L_q + C_2(L - L_q) + C_3]/(K - L)$

$C_0$ is the cost per unit time of a running machine. Eliminating nonvariable costs

Minimize $VC(K) = [C_1L_q + C_2(L - L_q) + C_3]/(K - L)$

$\lambda = 4/3$/hr, $\mu = 12$/hr, $C_1$ =\$90/hr, $C_2$ = \$45/hr, and $C_3$ = \$18/hr. Let's begin with K = 3. The same probability distribution and operating characteristics are obtained as in the solution to Problem 10.126. The only change is in the calculation of VC(K).

VC(3) = [\$90(0.06395) + \$45(0.35756 - 0.06395) + \$18]/(3 - 0.35756) = \$13.99

Given the results for K = 4 in the solution to Problem 10.126,

VC(4) = [\$90(0.13809) + \$45(0.52428 - 0.13809) + \$18]/(4 - 0.52428) = \$13.75

Using the results for K = 5 in the solution to Problem 10.126, we obtain

VC(5) = [\$90(0.28082) + \$45(0.76035 - 0.28082) + \$18]/(5 - 0.76035) = \$15.30

The optimum number of machines to assign an operator is 4, the same as for Problem 10.126.

**10.129** (M|M|1):(GD|K|K)

We modify Problem 10.128 and let $\lambda = 4/3$/hr, $\mu = 12$/hr, $C_1$ =\$75/hr, $C_2$ = \$75/hr, and $C_3$ = \$18/hr. As before, we use the results obtained in solving Problem 10.126.

VC(3) = [\$75(0.35756) + \$18]/(3 - 0.35756) = \$16.96

VC(4) = [\$75(0.52428) + \$18]/(4 - 0.52428) = \$16.49

VC(5) = [\$75(0.76035) + \$18]/(5 - 0.76035) = \$17.70

The optimum number of machines to assign an operator, 4, is the same as for Problem 10.126 and Problem 10.128.

Palm (see reference 57) provides a figure that can be used to determine the number of machines to assign to an operator in a (M|M|1):(GD|K|K) queue, given the ratio of $\lambda$ and $\mu$ and the ratio of $C_1$ and $C_3$. For our example, $\lambda/\mu = 0.1111$ and $C_1/C_3 = 4.1667$. From his figure, the optimum number is 4, which we obtained.

Ashcroft (see reference 2) provides a figure that can be used to determine the number of machines to assign to an operator in a (M|D|1):(GD|K|K) queue, given the ratio of $\lambda$ and $\mu$ and the ratio of $C_1$ and $C_3$. For our example, $\lambda/\mu = 0.1111$ and $C_1/C_3 = 4.1667$. From his figure, the optimum number is 5.

**10.130** (M|G|c):(GD|c|∞)
We modify Example 10.73 and let $\lambda = 100$/hr, $\mu = 4$/hr, R = \$6/hr, $C_5$ =\$1/hr, and $C_6$ = \$20/hr. The optimum solution is 41 spaces.

| c | $P_c$ | TC(c ) |
|---|---|---|
| 35 | 0.011645823929 | \$60.04 |
| 36 | 0.008022496767 | \$53.25 |
| 37 | 0.005391381370 | \$48.59 |
| 38 | 0.003534424958 | \$45.60 |
| 39 | 0.002260535427 | \$43.86 |
| 40 | 0.001410841356 | \$43.03 |
| 41 | 0.000859529693 | \$42.85 |
| 42 | 0.000511363191 | \$43.10 |
| 43 | 0.000297215817 | \$43.64 |
| 44 | 0.000168844110 | \$44.36 |
| 45 | 0.000093793485 | \$45.20 |

**10.131** (M|G|c):(GD|c|∞)
The total expected revenue is given by

$TR(c) = \$10\lambda - \$5c - \$15\lambda P_c$

As shown below for a selected number of values for c, expected revenue is maximized when 54 spaces are contracted for in the parking garage. To break-even, either contract for 21 spaces or 120 spaces. The expected total revenue is a concave function, passing through zero at the two values indicated.

| c | $P_c$ | TC(c ) |
|---|---|---|
| 21 | 0.550400236816 | -$0.36 |
| 22 | 0.529592991212 | $13.37 |
| 30 | 0.367358992936 | $119.38 |
| 40 | 0.184559445550 | $233.90 |
| 50 | 0.054104472165 | $301.31 |
| 51 | 0.045564047156 | $303.99 |
| 52 | 0.037934646186 | $305.86 |
| 53 | 0.031203634542 | $306.92 |
| 54 | 0.025344007821 | $307.19 |
| 55 | 0.020314759417 | $306.72 |
| 56 | 0.016062155828 | $305.54 |
| 57 | 0.012521863972 | $303.73 |
| 58 | 0.009621761572 | $301.34 |
| 59 | 0.007285168539 | $298.44 |
| 60 | 0.005434184691 | $295.11 |
| 70 | 0.000126855516 | $249.89 |
| 80 | 0.000000722807 | $200.00 |
| 90 | 0.000000001186 | $150.00 |
| 100 | 0.000000000001 | $100.00 |
| 110 | 0.000000000000 | $50.00 |
| 120 | 0.000000000000 | $0.00 |

**10.132** (M|M|c):(GD|∞|∞)
$\lambda$ = 4/hr, $\mu$ = 0.5/hr, $C_1$ =$10,000/hr, and $C_3$ = $2,500/hr. Based on Equation 10.187),
.

Minimize $VC(c) = C_1L_q + C_3c$

Minimize VC(c) = $10,000$L_q$ + $2,500c

As shown below, 12 unloading spaces should be provided.

| *c* | $L_q$ | VC(*c* ) |
|---|---|---|
| 9 | 5.226615 | $74,766.15 |
| 10 | 1.636721 | $41,367.21 |
| 11 | 0.653221 | $34,032.21 |
| 12 | 0.279683 | $32,796.83 |
| 13 | 0.085183 | $33,351.83 |
| 14 | 0.036603 | $35,366.03 |
| 15 | 0.015305 | $37,653.05 |

**10.133** (M|M|c):(GD|50|50) As shown, 7 unloading spaces are to be provided.

$\lambda$ = 0.05 $\mu$ = 0.5 c= 6

| *n* | $\lambda_n$ | $\mu_n$ | $P_0$ *Multipliers* | $P_n$ | $nP_n$ | $(n-c)P_n$ |
|---|---|---|---|---|---|---|
| 0 | 2.50 | 0 | 1 | 0.0074129 | 0.000000 | -- |
| 1 | 2.45 | 0.5 | 5 | 0.0370647 | 0.037065 | 0 |
| 2 | 2.40 | 1.0 | 12 | 0.0908085 | 0.181617 | 0 |
| 3 | 2.35 | 1.5 | 20 | 0.1452936 | 0.435881 | 0 |
| 4 | 2.30 | 2.0 | 23 | 0.1707199 | 0.682880 | 0 |
| 5 | 2.25 | 2.5 | 21 | 0.1570623 | 0.785312 | 0 |
| 6 | 2.20 | 3.0 | 16 | 0.1177968 | 0.706781 | 0 |
| 7 | 2.15 | 3.0 | 12 | 0.0863843 | 0.604690 | 0.086384 |
| 8 | 2.10 | 3.0 | 8 | 0.0619087 | 0.495270 | 0.12382 |
| 9 | 2.05 | 3.0 | 6 | 0.0433361 | 0.390025 | 0.130008 |
| 10 | 2.00 | 3.0 | 4 | 0.0296130 | 0.296130 | 0.118452 |
| 11 | 1.95 | 3.0 | 3 | 0.0197420 | 0.217162 | 0.09871 |
| 12 | 1.90 | 3.0 | 2 | 0.0128323 | 0.153988 | 0.076994 |
| 13 | 1.85 | 3.0 | 1 | 0.0081271 | 0.105653 | 0.05689 |
| 14 | 1.80 | 3.0 | 1 | 0.0050117 | 0.070164 | 0.040094 |
| 15 | 1.75 | 3.0 | 0 | 0.0030070 | 0.045106 | 0.027063 |
| 16 | 1.70 | 3.0 | 0 | 0.0017541 | 0.028066 | 0.017541 |
| 17 | 1.65 | 3.0 | 0 | 0.0009940 | 0.016898 | 0.010934 |
| 18 | 1.60 | 3.0 | 0 | 0.0005467 | 0.009841 | 0.00656 |
| 19 | 1.55 | 3.0 | 0 | 0.0002916 | 0.005540 | 0.00379 |
| 20 | 1.50 | 3.0 | 0 | 0.0001506 | 0.003013 | 0.002109 |
| 21 | 1.45 | 3.0 | 0 | 0.0000753 | 0.001582 | 0.00113 |
| 22 | 1.40 | 3.0 | 0 | 0.0000364 | 0.000801 | 0.000582 |
| 23 | 1.35 | 3.0 | 0 | 0.0000170 | 0.000391 | 0.000289 |
| 24 | 1.30 | 3.0 | 0 | 0.0000076 | 0.000183 | 0.000138 |
| 25 | 1.25 | 3.0 | 0 | 0.0000033 | 0.000083 | 6.29E-05 |
| 26 | 1.20 | 3.0 | 0 | 0.0000014 | 0.000036 | 2.76E-05 |
| 27 | 1.15 | 3.0 | 0 | 0.0000006 | 0.000015 | 1.16E-05 |
| 28 | 1.10 | 3.0 | 0 | 0.0000002 | 0.000006 | 4.66E-06 |
| 29 | 1.05 | 3.0 | 0 | 0.0000001 | 0.000002 | 1.78E-06 |
| 30 | 1.00 | 3.0 | 0 | 0.0000000 | 0.000001 | 6.52E-07 |
| 31 | 0.95 | 3.0 | 0 | 0.0000000 | 0.000000 | 2.26E-07 |
| 32 | 0.90 | 3.0 | 0 | 0.0000000 | 0.000000 | 7.45E-08 |
| 33 | 0.85 | 3.0 | 0 | 0.0000000 | 0.000000 | 2.32E-08 |
| 34 | 0.80 | 3.0 | 0 | 0.0000000 | 0.000000 | 6.82E-09 |
| 35 | 0.75 | 3.0 | 0 | 0.0000000 | 0.000000 | 1.88E-09 |
| | | Sum = | 134.89930182 | | | L = 5.27418 |
| | | $P_0$ = | 0.007412937 | | | $L_q$ = 0.80160 |

VC(6) = $23,015.96
VC(7) = $20,248.83
VC(8) = $20,929.21

**10.134** $(M^{(b)}|M|1):(GD|\infty|\infty)$

P = parts to be provided/hr = 125/hr

b = unit load size $\lambda = P/b$ $\mu = 360$/hr

TC(b) = \$40L + \$25P/b

$L = [b\lambda(1 + b)]/[2(\mu - b\lambda)]$

VC(b) = \$40(125)(1 + b)/[2(360 - 125)] + \$25(125)/b

Taking the first derivative of VC(b) with respect to b, setting the derivative equal to zero and solving for b gives b = 17.14. Since b is integer valued, we enumerate b from 16 to 18, as shown below, and determine that the unit load should consist of 17 parts.

| b | VC(B) |
|---|---|
| 16 | \$376.16 |
| 17 | \$375.31 |
| 18 | \$375.74 |

**10.135** $(M|M|c):(GD|N|\infty)$

$\lambda = 15$/mo, $\mu = 10$/mo, $C_1$ =\$300, $C_3$ = \$15,000, $C_4$ = \$25,000, $C_5$ = \$5,000, and $C_6$ = \$30,000. As shown below, c* = 2 and N* = 7.

| c | Calculations for $P_0$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 0.188235294 | 0.162436548 | 0.147295742 | 0.137671417 | 0.131239987 | 0.126797406 | 0.123657963 | 0.121403543 | 0.119765945 | 0.118566448 | 0.117682476 |
| 3 | | 0.210526316 | 0.198757764 | 0.193353474 | 0.190760060 | 0.189489267 | 0.188860199 | 0.188547229 | 0.188391132 | 0.188313181 | 0.188274230 |
| 4 | | | 0.227353464 | 0.223336968 | 0.221867129 | 0.221320915 | 0.221116777 | 0.221040322 | 0.221011666 | 0.221000921 | 0.220996892 |
| 5 | | | | 0.224128874 | 0.223179258 | 0.222895941 | 0.222811086 | 0.222785642 | 0.222778010 | 0.222775720 | 0.222775033 |
| 6 | | | | | 0.223336968 | 0.223139866 | 0.223090644 | 0.223078342 | 0.223075267 | 0.223074498 | 0.223074306 |
| 7 | | | | | | 0.223168002 | 0.223131828 | 0.223124078 | 0.223122417 | 0.223122061 | 0.223121985 |
| 8 | | | | | | | 0.223136349 | 0.223130415 | 0.223129303 | 0.223129094 | 0.223129055 |
| 9 | | | | | | | | 0.223131074 | 0.223130195 | 0.223130049 | 0.223130024 |
| 10 | | | | | | | | | 0.223130283 | 0.223130165 | 0.223130147 |
| 11 | | | | | | | | | | 0.223130967 | 0.223130952 |
| 12 | | | | | | | | | | | 0.223131061 |

| c | Calculations for $L_q$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 0.000000000 | 0.001631617 | 0.003698833 | 0.069488726 | 0.092937245 | 0.113970401 | 0.132370993 | 0.148188819 | 0.161605783 | 0.172864112 | 0.182226042 |
| 3 | | 0.000000000 | 0.004658385 | 0.036253776 | 0.049180328 | 0.057735011 | 0.063076356 | 0.066286135 | 0.068163002 | 0.069238196 | 0.069844407 |
| 4 | | | 0.000000000 | 0.008030138 | 0.013960256 | 0.017283021 | 0.018944099 | 0.019723379 | 0.020074400 | 0.020228107 | 0.020294030 |
| 5 | | | | 0.000000000 | 0.002281418 | 0.003645635 | 0.004259213 | 0.004504686 | 0.004596762 | 0.004629918 | 0.004641525 |
| 6 | | | | | 0.000000000 | 0.000529521 | 0.000794107 | 0.000893321 | 0.000926394 | 0.000936730 | 0.000939831 |
| 7 | | | | | | 0.000000000 | 0.000104883 | 0.000149828 | 0.000164275 | 0.000168402 | 0.000169508 |
| 8 | | | | | | | 0.000000000 | 0.000018195 | 0.000025018 | 0.000026937 | 0.000027417 |
| 9 | | | | | | | | 0.000000000 | 0.000002814 | 0.000003752 | 0.000003987 |
| 10 | | | | | | | | | 0.000000000 | 0.000000393 | 0.000000511 |
| 11 | | | | | | | | | | 0.000000000 | 0.000000050 |
| 12 | | | | | | | | | | | 0.000000000 |

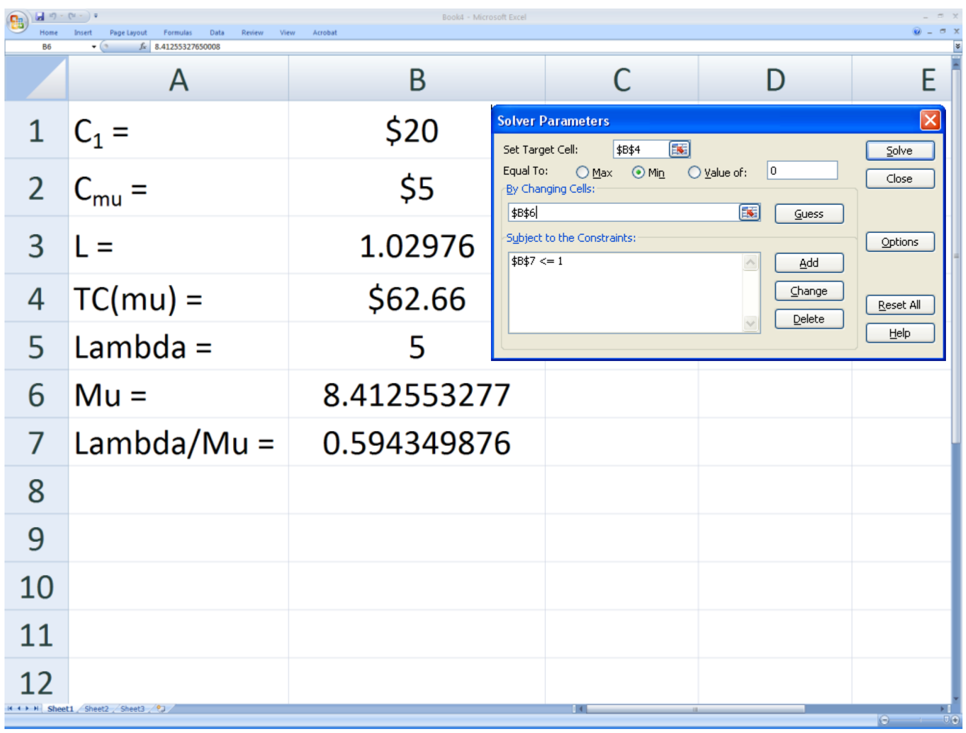
| c | Calculations for L | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 1.023529412 | 1.158992023 | 1.239602170 | 1.355318250 | 1.412129812 | 1.456208858 | 1.490895309 | 1.518407939 | 1.540319943 | 1.557800661 | 1.571748199 |
| 3 | | 1.026315789 | 1.141304348 | 1.223564955 | 1.260804769 | 1.281273131 | 1.292511988 | 1.298655864 | 1.301996137 | 1.303802124 | 1.304773506 |
| 4 | | | 0.916518650 | 0.979022504 | 1.004887321 | 1.015618116 | 1.020047810 | 1.021864005 | 1.022603684 | 1.022903112 | 1.023023676 |
| 5 | | | | 0.832603747 | 0.852542084 | 0.859174238 | 0.861365590 | 0.862084160 | 0.862318146 | 0.862393872 | 0.862418250 |
| 6 | | | | | 0.794329335 | 0.799453046 | 0.800864912 | 0.801250867 | 0.801355621 | 0.801383876 | 0.801391457 |
| 7 | | | | | | 0.781371325 | 0.782484202 | 0.782745103 | 0.782805823 | 0.782819867 | 0.782823097 |
| 8 | | | | | | | 0.777856488 | 0.778066743 | 0.778109576 | 0.778118247 | 0.778119992 |
| 9 | | | | | | | | 0.777058033 | 0.777093243 | 0.777099580 | 0.777100715 |
| 10 | | | | | | | | | 0.776901628 | 0.776906927 | 0.776907781 |
| 11 | | | | | | | | | | 0.776877414 | 0.776878139 |
| 12 | | | | | | | | | | | 0.776873633 |

| c | N | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | \$145,529.41 | \$118,249.22 | \$104,304.91 | \$97,282.21 | \$94,241.92 | \$93,689.45 | \$94,766.74 | \$96,950.64 | \$99,905.62 | \$103,408.13 | \$107,304.87 |
| 3 | | \$123,552.63 | *\$101,523.14* | *\$94,119.64* | *\$93,166.77* | *\$95,250.48* | *\$98,807.19* | *\$103,089.30* | *\$107,731.33* | *\$112,552.61* | \$117,463.32 |
| 4 | | | *\$110,746.00* | *\$102,662.17* | *\$102,875.03* | *\$106,096.39* | *\$110,431.78* | *\$115,182.92* | *\$120,089.66* | *\$125,054.70* | \$130,041.59 |
| 5 | | | | *\$114,708.46* | *\$115,409.90* | *\$119,127.64* | *\$123,743.65* | *\$128,628.53* | *\$133,594.01* | *\$138,583.65* | \$143,580.55 |
| 6 | | | | | *\$129,533.26* | *\$133,386.54* | *\$138,100.21* | *\$143,028.66* | *\$148,010.77* | *\$153,006.30* | \$158,005.19 |
| 7 | | | | | | *\$148,154.16* | *\$152,896.77* | *\$157,841.63* | *\$162,829.81* | *\$167,827.28* | \$172,826.74 |
| 8 | | | | | | | *\$167,842.39* | *\$172,792.46* | *\$177,783.10* | *\$182,781.34* | \$187,781.01 |
| 9 | | | | | | | | *\$187,781.22* | *\$192,772.68* | *\$197,771.25* | \$202,771.02 |
| 10 | | | | | | | | | *\$207,770.61* | *\$212,769.30* | \$217,769.11 |
| 11 | | | | | | | | | | *\$227,768.99* | \$232,768.81 |
| 12 | | | | | | | | | | | \$247,768.76 |

**10.136a** (M|D|1):(GD|∞|∞). In the production process, the infinity symbols were changed to 4. This is supposed to be an (M|D|1):(GD|∞|∞) queue.

$$TC(\mu) = C_1\left[\frac{\lambda}{\mu} + \frac{\left(\lambda/\mu\right)^2}{2\left(1-\lambda/\mu\right)}\right] + C_\mu \mu$$

Let λ = 5/hr, $C_1$ = \$20, and $C_\mu$ = \$5. Using Excel's SOLVER tool, a value of μ = 8.41255 was obtained, as shown below.

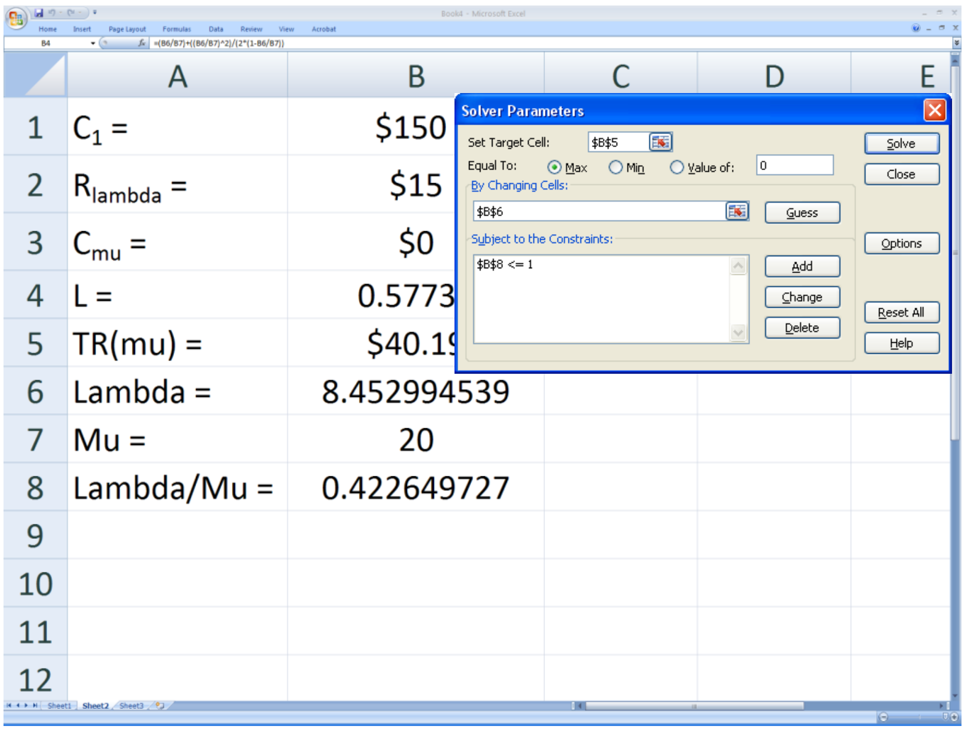



**10.136b** (M|D|1):(GD|∞|∞). In the production process, the infinity symbols were changed to 4. This is supposed to be an (M|D|1):(GD|∞|∞) queue.

If we associate a cost with each incremental increase in λ the optimum value will be λ = 0. Hence, instead of having demand create costs, we let demand generate revenue and develop an expected total revenue model, as follows:

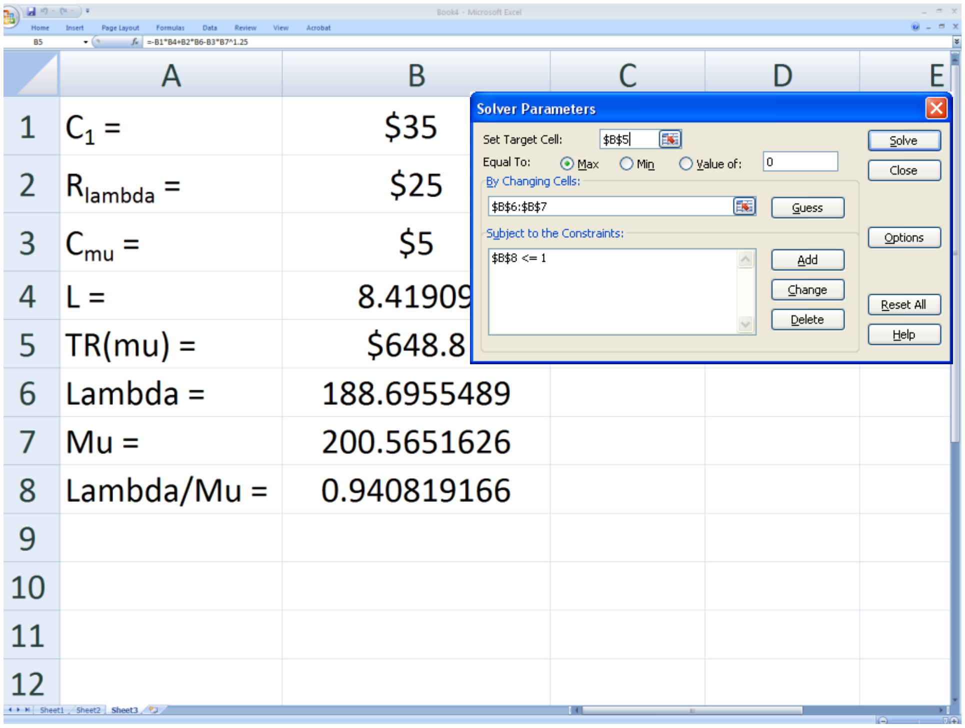
$TR(\lambda) = R_\lambda \lambda - C_1 L$

where L is given as in part a). Letting $\mu$ = 20/hr, $R_\lambda$ = \$15, and $C_1$ = \$150 and using Excel's SOLVER tool, a value of $\lambda$ = 8.453/hr was obtained, as shown below.




**10.136c** (M|D|1):(GD|∞|∞). In the production process, the infinity symbols were changed to 4. This is supposed to be an (M|D|1):(GD|∞|∞) queue.

A combined model for determining an optimum $\lambda$ and $\mu$ is
$TR(\lambda,\mu) = R_\lambda\lambda - C_\mu\mu^{1.25} - C_1L$
Notice, we introduced a nonlinear component associated with increases in $\mu$. We let $C_1$ = \$35, $R_\lambda$ = \$25, and $C_\mu$ = \$5. Also, we required that $\lambda/\mu < 1$. Using Excel's SOLVER tool, values of $\lambda$ = 188.7/hr and $\mu$ = 200.57/hr were obtained, as shown below.




## SECTION 10.10

**10.137** The primary challenge in developing such a simulation model is to keep track of the position of each crane so that "blocking" (i.e., one crane cannot move because it is blocked by the neighboring crane) is accurately captured. Also, one would have to develop (and incorporate into the simulation model) a crane dispatching logic; i.e., deciding which crane is dispatched to move which load and when. Without a reasonably good crane dispatching logic, the cranes may block each other too often. In terms of data, one would at least need to identify each pickup/deposit point in the layout, the rate at which move requests arrive from each pickup point, and the destination of the loads picked up. Lastly, one needs the crane travel times and the load pickup/deposit times.

**10.138** Simulation has extensive applications in facility location and layout. A full treatment of the topic is beyond the scope of this book. The primary question is one of purpose and scope. That is, what is the purpose of the simulation experiment (what answers are we trying to obtain from the model) and how much detail should the model capture. Typical sources of randomness in such simulation experiments are those associated with demand, parts flow, and resource availability.

**10.139** The primary source of variation is the arrival pattern of the storage and retrieval requests. Simulation may be used to study the dynamic interactions in the system as well as the *distribution* of certain variables (as opposed to just their average values). For example, one may determine the distribution of the space utilization (including minimum and maximum values) as well as the material handling capacity needed to perform all the necessary storage and retrieval operations.

**10.140** Simulation may play an important role in the design of AS/R systems and order picking systems. For example, the results shown in sections 10.6 and 10.7 are based on certain simplifying assumptions such as randomized storage and first-come-first-served (FCFS) storage and retrieval requests. One may use simulation to keep track of the location of the items in the rack and thus use actual S/R machine travel times (rather than assuming each opening is equally likely to be visited). One may also model S/R machine travel more accurately, including acceleration/deceleration effects and multiple travel speeds that are distance-dependent.

**10.141** Simulation may be used to model cases where the load attempts (or the number of parts loaded per attempt) is randomly distributed and not all parts are successfully unloaded at the unloading stations; i.e., some items may recirculate. Using simulation, one may also obtain more detailed results on how long particular parts circulate on the conveyor before they are unloaded. In some manufacturing applications, the time a part spends on the conveyor may have a lower limit (say, for cooling the part down) or an upper limit (if the next process on the part must be performed within a certain time period). If the conveyor is subject to random stoppages (due to breakdowns or the workers who need additional time), again one may need simulation to study its performance.

**10.142** Left as an exercise for the reader, who may wish to try the following reference as a starting point: Rajan, S., Derleth, D., and Tomsicek, M., "Modeling manufacturing systems using MANUPLAN and SIMSTARTER - A tutorial," 1990 Winter Simulation Conference Proceedings, New Orleans, LA, USA.

**10.143** There are many papers on this topic. The following represents only a sample list identified through a computer-based keyword search:

Chew, E.P. and Johnson, L.A., "Service levels in distribution systems with random customer order size," *Naval Research Logistics,* Vol. 42, No. 1, pp. 39-56, 1995.

Gray, A. E., Karmarkar, U. S., and Seidmann, A., "Design and operation of an order-consolidation warehouse: models and application," *European Journal of Operational Reseach,* Vol. 58, No. 1, pp.14-36, 1992.

Linn, R.J. and Xie, X., "Simulation analysis of sequencing rules for ASRS in a pull-based assembly facility," *International Journal of Production Research,* Vol. 31, No. 10, pp.2355-2367, 1993.

Randhawa, S. U. and Shroff, R., "Simulation-based design evaluation of unit load automated storage/retrieval systems," *Computers & Industrial Engineering,* Vol. 28, No.1, pp. 71-79, 1995.

Takakuwa, S., "Precise modeling and analysis of large-scale AS/RS," Proceedings of the 1994 Winter Simulation Conference, Buena Vista, FL, USA.

**10.144** There are many papers on this topic. The following represents only a sample list identified through a computer-based keyword search:

Cardarelli, G. and Pelagagge, P. M., “Simulation tool for design and management optimization of automated interbay material handling and storage systems for large wafer fab,” *IEEE Transactions on Semiconductor Manufacturing,* Vol. 8, No. 1, pp. 44-49, 1995.

Chan, F. T.S., Jayaprakash, B., and Tang, N. K.H., “Design of automated cellular manufacturing systems with simulation modelling: a case study,” *International Journal of Computer Applications in Technology,* Vol. 8, No. 1-2, pp. 1-11, 1995.

Haddock, J., “Automated simulation modelling and analysis for manufacturing systems,” *Production Planning and Control,* Vol. 6, No. 4, pp. 352-357, 1995.

Huq, Z., “Process oriented manufacturing system simulation to measure the effect of shop control factors,” *Simulation,* Vol. 62, No. 4, pp. 218-228, 1994.

Prakash, A. and Chen, M., “Simulation study of flexible manufacturing systems,” *Computers & Industrial Engineering,* Vol. 28, No.1, pp. 191-199, 1995.

**10.145** Solution depends on example choice.

**10.146** Solution depends on problem choice.

# Chapter 11

# *Evaluating and Selecting the Facilities Plan*

## SECTION 11.2.1

**11.1** If equipment need change sooner than anticipated, leasing can provide more flexibility than purchasing equipment. Depending on the lease conditions, lease payments can be treated as annual expenses and, thus, yield tax advantages over purchased equipment. (Lease payments can be expensed; depreciation deductions are allowed with purchased equipment.) Lease payments might be "affordable" in a manager's annual operating budget; capital investments might require higher level approval and, thus, introduce more uncertainty in the selection process. Leases minimize the uncertainty in maintenance cash flows resulting from the use of capital equipment.

When equipment is purchased, risks of escalating lease charges in the future are avoided. Purchasing has the advantage of ownership of the asset; if owned, the equipment can be modified. Because of different types of flexibility, purchasing the equipment can provide certain types of flexibility not provided by leasing; for example, if the equipment is purchased it can be relocated to another plant location outside the service region of the leasing company) if changes in equipment needs occur.

**11.2** Manufacturing space is usually more expensive than storage space. As manufacturing needs expand, storage space is often converted to manufacturing space. If sufficient land is not available for expansion of manufacturing and storage, then storage space is often sought off-site. Using either leased space, public warehousing, or another option, off-site storage is provided.

On-site storage of materials and supplies has the advantage of proximity. As a result, the time required to respond to demands for materials and supplies is minimized; likewise, the amount of inventory required is generally less than that required with off-site storage. On-site storage has the advantage of ensuring that storage personnel are made to feel a part of the overall team. Often when storage is assigned to off-site space, morale problems arise; if storage personnel are split between on-site and off-site locations, the quality of management often suffers. When off-site storage is used, delivery trucks are required; likewise, material must be loaded and unloaded several times in the process of satisfying the ultimate needs of manufacturing.

Off-site storage has the advantage of being dedicated to storage activities. When located on-site, manufacturing pressures can result in a reactive mode of operation, instead of a planned, responsive mode that can be provided by off-site storage. Since on-site storage is close to manufacturing, the discipline needed to ensure a carefully managed activity is often displaced by a brushfire, crisis management style. Because material is stored off-site, it necessitates that manufacturing managers plan daily activities.

**11.3** High-rise warehouses can be significantly less expensive to construct in terms of cubic footage of storage space than conventional storage facilities; likewise, when land is expensive, less land is needed for high-rise storage. Conventional storage has the advantage of flexibility. In a conventional facility, conventional and automated storage and retrieval equipment can be used; whereas, in a high-rise facility it is less likely that conventional equipment can be used. Likewise, if the need for the storage facility declines over time, alternative uses of a conventional facility might arise; such is not likely with high-rise storage. If the facility is to be sold, fewer buyers are likely for a high-rise facility than a conventional facility.

**SECTION 11.2.2**

**11.4** The rank order of factors identified is dependent on the student's judgment.

Depending on the application, many factors should be considered in evaluating horizontal handling alternatives. Among the various factors to be considered are the following: discounted present worth; installation time; discounted payback period; maintenance requirements; flexibility in coping with changes in throughput requirements; flexibility in accommodating changes in load sizes and weights; labor skills required; in-process inventory buffer sizes required; attitude of plant floor personnel toward the alternatives; previous experience in using the equipment alternatives.

**11.5** The rank order of factors identified is dependent on the student's judgment.

Depending on the application, many factors should be considered in evaluating alternate site locations for corporate headquarters. Among the various factors to be considered are the following: proximity to major customers; proximity to major operating locations; proximity to major capital sources; ease of access by customers, suppliers, members of the Board of Directors; location preferences of upper management; tax incentives and economic consequences; potential for expansion; access to major airport; compatibility with image to be projected by the firm; quality of educational facilities in the city or area; availability of accommodations for visitors; quality and magnitude labor supply; attractiveness in recruiting mid to upper management; and quality of life for employees.

**11.6** The rank order of factors identified is dependent on the student's judgment.

Depending on the application, many factors should be considered in evaluating alternate site locations for a distribution center. Among the various factors to be considered are the following: proximity to major supply locations (time required to replenish from suppliers); proximity to major customer locations (time required to ship to customers); access to major transportation facilities (rail, air freight, interstate highways, river and ocean terminals); number of distribution centers and service regions of each; availability of existing facilities; cost to have a leased facility built to specifications; labor availability, labor cost, and attitude of organized labor; locations of competitors (ability to meet or exceed competition in terms of service to customers); and overall economic impact of the location decision, including taxes.

## SECTION 11.2.3

**11.7** From the table below, Alternative B is preferred.

| | | A | | B | | C | |
|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score |
| Net Present Value | 40 | 10 | 400 | 8 | 320 | 6 | 240 |
| Time to Fill Customer's Order | 35 | 6 | 210 | 10 | 350 | 8 | 280 |
| Flexibility | 25 | 5 | 125 | 7 | 175 | 10 | 250 |
| Totals | 100 | | 735 | | 845 | | 770 |

**11.8a** From the table below, Alternatives B and C are equally preferred. Alternative A has a relatively close weighted factor total to B and C. To make a decision a "coin toss" may be necessary.

| | | A | | B | | C | |
|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score |
| Net Present Value | 50 | 10 | 500 | 8 | 400 | 6 | 300 |
| Time to Fill Customer's Order | 10 | 6 | 60 | 10 | 100 | 8 | 80 |
| Flexibility | 40 | 5 | 200 | 7 | 280 | 10 | 400 |
| Totals | 100 | | 760 | | 780 | | 780 |

**11.8b** From the table below, Alternative C is preferred.

| | | A | | B | | C | |
|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score |
| Net Present Value | 40 | 10 | 400 | 8 | 320 | 6 | 240 |
| Time to Fill Customer's Order | 35 | 5 | 175 | 7 | 245 | 10 | 350 |
| Flexibility | 25 | 5 | 125 | 7 | 175 | 10 | 250 |
| Totals | 100 | | 700 | | 740 | | 840 |

**11.8c** From the table below, Alternative A is preferred.

| | | A | | B | | C | |
|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score |
| Net Present Value | 30 | 10 | 300 | 8 | 240 | 6 | 180 |
| Time to Fill Customer's Order | 20 | 6 | 120 | 10 | 200 | 8 | 160 |
| Flexibility | 10 | 5 | 50 | 7 | 70 | 10 | 100 |
| Safety | 40 | 10 | 400 | 8 | 320 | 9 | 360 |
| Totals | 100 | | 870 | | 830 | | 800 |

**11.9** From the table below, Alternative A is preferred.

| | | A | | B | | C | | D | |
|---|---|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score | Rating | Score |
| Annual Mat'l Handling Cost | 20 | 10 | 200 | 7 | 140 | 5 | 100 | 2 | 40 |
| Construction Cost | 25 | 8 | 200 | 4 | 100 | 10 | 250 | 9 | 225 |
| Ease of Expansion | 15 | 7 | 105 | 8 | 120 | 10 | 150 | 9 | 135 |
| Employee's Preference | 20 | 6 | 120 | 8 | 160 | 7 | 140 | 10 | 200 |
| Rail Siding Access | 20 | 10 | 200 | 6 | 120 | 5 | 100 | 7 | 140 |
| Totals | 100 | | 825 | | 640 | | 740 | | 740 |

**11.10** From the table below, Alternative D is preferred.

| | | A | | B | | C | | D | |
|---|---|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score | Rating | Score |
| Annual Mat'l Handling Cost | 15 | 10 | 150 | 7 | 105 | 5 | 75 | 2 | 30 |
| Construction Cost | 25 | 8 | 200 | 4 | 100 | 10 | 250 | 9 | 225 |
| Ease of Expansion | 15 | 7 | 105 | 8 | 120 | 10 | 150 | 9 | 135 |
| Employee's Preference | 30 | 6 | 180 | 8 | 240 | 7 | 210 | 10 | 300 |
| Rail Siding Access | 15 | 10 | 150 | 6 | 90 | 5 | 75 | 7 | 105 |
| Totals | 100 | | 785 | | 655 | | 760 | | 795 |

**11.11** From the table below, using the weights from Problem 9, Alternative A is still preferred.

| | | A | | B | | C | | D | |
|---|---|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score | Rating | Score |
| Annual Mat'l Handling Cost | 20 | 8 | 160 | 10 | 200 | 2 | 40 | 5 | 100 |
| Construction Cost | 25 | 8 | 200 | 4 | 100 | 9 | 225 | 10 | 250 |
| Ease of Expansion | 15 | 7 | 105 | 8 | 120 | 10 | 150 | 9 | 135 |
| Employee's Preference | 20 | 10 | 200 | 8 | 160 | 7 | 140 | 6 | 120 |
| Rail Siding Access | 20 | 10 | 200 | 6 | 120 | 5 | 100 | 8 | 160 |
| Totals | 100 | | 865 | | 700 | | 655 | | 765 |

From the table below, using the weights from Problem 10, Alternative A is preferred.

| | | A | | B | | C | | D | |
|---|---|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score | Rating | Score |
| Annual Mat'l Handling Cost | 15 | 8 | 120 | 10 | 150 | 2 | 30 | 5 | 75 |
| Construction Cost | 25 | 8 | 200 | 4 | 100 | 9 | 225 | 10 | 250 |
| Ease of Expansion | 15 | 7 | 105 | 8 | 120 | 10 | 150 | 9 | 135 |
| Employee's Preference | 30 | 10 | 300 | 8 | 240 | 7 | 210 | 6 | 180 |
| Rail Siding Access | 15 | 10 | 150 | 6 | 90 | 5 | 75 | 8 | 120 |
| Totals | 100 | | 875 | | 700 | | 690 | | 760 |

**11.12** From the table below, Alternative A would be preferred.

| | | A | | B | | C | | D | |
|---|---|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score | Rating | Score |
| Annual Mat'l Handling Cost | 15 | 7 | 105 | 10 | 150 | 6 | 90 | 5 | 75 |
| Construction Cost | 25 | 8 | 200 | 7 | 175 | 9 | 225 | 10 | 250 |
| Ease of Expansion | 15 | 7 | 105 | 8 | 120 | 10 | 150 | 9 | 135 |
| Employee's Preference | 30 | 10 | 300 | 8 | 240 | 7 | 210 | 9 | 270 |
| Rail Siding Access | 15 | 10 | 150 | 6 | 90 | 7 | 105 | 8 | 120 |
| Totals | 100 | | 860 | | 775 | | 780 | | 850 |

**11.13** From the table below, Alternative A would be preferred.

| | | A | | B | | C | | D | |
|---|---|---|---|---|---|---|---|---|---|
| Factor | Weight | Rating | Score | Rating | Score | Rating | Score | Rating | Score |
| Annual Mat'l Handling Cost | 20 | 7 | 140 | 10 | 200 | 6 | 120 | 5 | 100 |
| Construction Cost | 25 | 8 | 200 | 7 | 175 | 9 | 225 | 10 | 250 |
| Ease of Expansion | 15 | 7 | 105 | 8 | 120 | 10 | 150 | 9 | 135 |
| Employee's Preference | 20 | 10 | 200 | 8 | 160 | 7 | 140 | 9 | 180 |
| Rail Siding Access | 20 | 10 | 200 | 6 | 120 | 7 | 140 | 8 | 160 |
| Totals | 100 | | 845 | | 775 | | 775 | | 825 |

**11.14** The weighted factor comparison method is easy to use. Sensitivity analyses can be performed quite easily, particularly if a computer spreadsheet is used to perform the calculations. However, its qualitative and subjective aspects are a source of concern. Scaling issues exist with the method. Ordinal data cannot be treated as ratio data without the potential for errors. Hence, caution should be used when applying the method.

**SECTION 11.2.4**

**11.15** From the problem statement the WACC = 9%. Using the EASTMAN hurdle rate calculation method we have the following:

I. No, 0%
II. C, 2%
III. F, 1%
IV. B, 0%
V. B, 0%
VI. E, 2.4%
VII. A, 0%
VIII. H, 3%
IX. A, 0%

Thus, the total calculated hurdle rate (discount rate) is 17.4%.

**11.16** The change in ownership would change the answer to part II of the calculation from 2% to 0.5%. Thus, the discount rate decreases from 17.4% to 15.9%.

**11.17** Case 1: Part VIII of the calculation changes from 3% to 4%, thus the discount rate becomes 16.9%.
Case 2: Part VIII of the calculation changes from 3% to 3.5%, thus the discount rate becomes 16.4%.

**11.18a** From the table below, Alternative B is preferred.

| Year | A | B | Δ(B-A) | Net CF(B-A) | Cum NPV |
|---|---|---|---|---|---|
| 0 | -$250,000 | -$1,000,000 | -$750,000 | -$750,000 | -$750,000 |
| 1 | -$600,000 | -$300,000 | $300,000 | -$450,000 | -$477,272.73 |
| 2 | -$600,000 | -$300,000 | $300,000 | -$150,000 | -$229,338.84 |
| 3 | -$600,000 | -$300,000 | $300,000 | $150,000 | -$3,944.40 |
| 4 | -$600,000 | -$300,000 | $300,000 | $450,000 | $200,959.63 |
| 5 | -$600,000 | -$300,000 | $300,000 | $750,000 | $387,236.03 |
| 6 | -$600,000 | -$300,000 | $300,000 | $1,050,000 | $556,578.21 |
| 7 | -$600,000 | -$300,000 | $300,000 | $1,350,000 | $710,525.65 |
| 8 | -$600,000 | -$300,000 | $300,000 | $1,650,000 | $850,477.86 |
| 9 | -$600,000 | -$300,000 | $300,000 | $1,950,000 | $977,707.14 |
| 10 | -$600,000 | -$300,000 | $300,000 | $2,250,000 | $1,093,370.13 |
| NPV | -$3,936,740.26 | -$2,843,370.13 | $1,093,370.13 | | |
| AW | -$640,686.35 | -$462,745.39 | $177,940.95 | | |
| FW | -$10,210,890.38 | -$7,374,969.84 | $2,835,920.54 | | |
| IRR | | | 38.45% | | |
| PBP | | | | 3 years | |
| DPBP | | | | | 4 years |

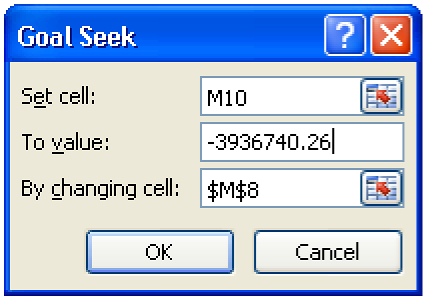
**11.18b** Referring to the table in the solution for Problem 11.18a, the $IRR_{(B\text{-}A)}$ = 38.45%.

**11.18c** There is a error in the problem statement. The incremental investment should be **$750,000**.

Referring to the table in the solution for Problem 11.18a, the payback period is 3 years, and the discounted payback period is 4 years.

**11.19** We can obtain the value by setting the NPV of Alternative A to the NPV of Alternative B, where the operating expenses of Alternative B are treated as a variable. This analysis can be performed in MS Excel using Goal Seek, illustrated below:

| | A | B |
|---|---|---|
| Initial Investment | -$250,000.00 | -$1,000,000.00 |
| Operating Expenses | -$600,000.00 | $0.00 |
| Planning Horizon | 10 | 10 |
| NPV | -$3,936,740.26 | -$1,000,000.00 |




The result is given as follows:

| | A | B |
|---|---|---|
| Initial Investment | -$250,000.00 | -$1,000,000.00 |
| Operating Expenses | -$600,000.00 | -$477,940.95 |
| Planning Horizon | 10 | 10 |
| NPV | -$3,936,740.26 | -$3,936,740.26 |

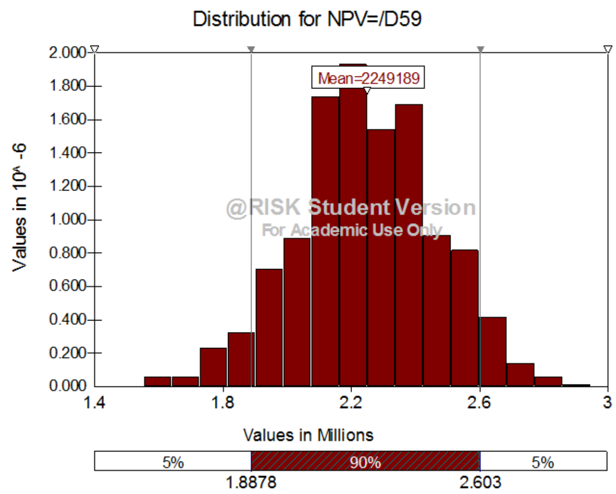
Therefore, the operating expenses could increase by $177,940.95, or 59.3%, before you would be indifferent as to the alternative to select; any further increase would result in a change of decision.

**11.20a** This problem is asking for the condition $P(NPV_B(10\%) \geq NPV_A(10\%))$, which is equivalent to $P(NPV_{B-A}(10\%) \geq 0)$. To solve we use the following relationships: $E(NPV) = \sum_{k=0}^{N} c_k E(F_k)$ and $Var(NPV) = \sum_{k=0}^{N} c_k{}^2 Var(F_k)$ where $c_k$ is the interest factor for the present value given a future value corresponding to year k; $F_k$ is the cash flow value for year k; and N is the length of the planning horizon. In addition, $E(NPV_{B-A}(10\%)) = E(NPV_B(10\%)) - E(NPV_A(10\%))$ and $Var(NPV_{B-A}(10\%)) = Var(NPV_B(10\%)) + Var(NPV_A(10\%))$.

Therefore, $E(NPV_{B-A}(10\%))$ = $1,093,370.13. Using the above Var(NPV) equation we find that $Var(NPV_B(10\%)) = Var(NPV_A(10\%))$ = $10,135,194,904.47. Therefore, $Var(NPV_{B-A}(10\%))$ = $20,270,389,808,95, and $SD(NPV_{B-A}(10\%))$ = $142,374.12.

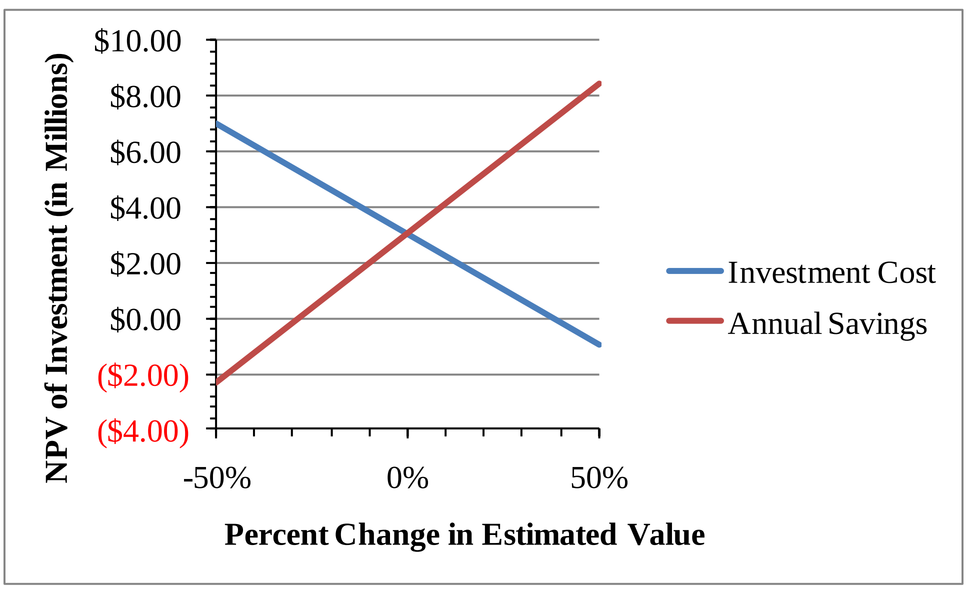
Letting $P(NPV_{B-A}(10\%) \geq 0) = P(Z \geq z)$, where
$z = (NPV - NPV_{B-A}(10\%))/ SD(NPV_{B-A}(10\%))$, we have the following:
$z = (0 - \$1{,}093{,}370.13)/ \$142{,}374.12 = -7.6796$.
$P(Z \geq -7.6796) = 1- P(Z \geq -7.6796) = 1$. So, Alternative B will always be the better alternative.

**11.20b** The results of the Monte Carlo simulation are illustrated in the following histogram. The simulation was run with 1,000 iterations and a fixed random number seed.




The mean NPV was $2,249,189, while the standard deviation was $219,809.3. The minimum and maximum NPV were $1,554,860 and $2,946,856, respectively. Using the mean and standard deviation, z = -10.2324.
Thus, $P(Z \geq -10.2324) = 1- P(Z \geq -10.2324) = 1$.

**11.21a** The sensitivity analysis performed may depend on the student. The following is a plot of the NPV of the investment when value of interest was changed while all other inputs were held constant (also known as a spiderplot).




The investment is relatively insensitive to changes inaccuracies in the estimated investment cost and annual savings.

The breakeven point for the investment cost is $11,042,150 (or, a 38% increase in the current estimate). The breakeven point for the annual savings is $1,254,904 (or, a 28.3% decrease in the current estimate).

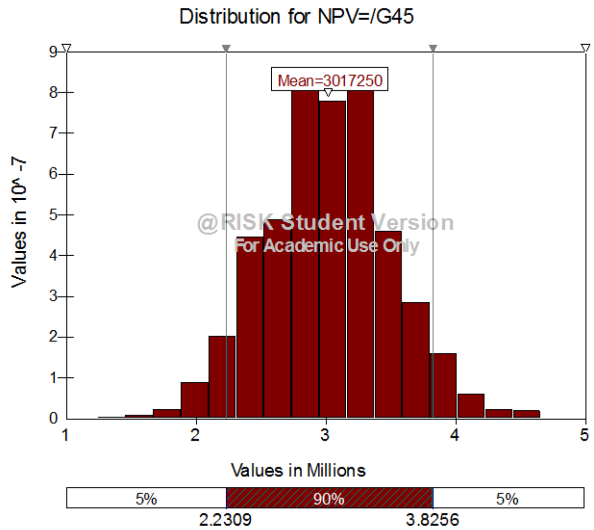
**11.21b** This problem is asking for the condition P(NPV (10%) ≥ 0). To solve we use the following relationships: $E(NPV) = \sum_{k=0}^{N} c_k E(F_k)$ and $Var(NPV) = \sum_{k=0}^{N} c_k{}^2 Var(F_k)$ where $c_k$ is the interest factor for the present value given a future value corresponding to year k; $F_k$ is the cash flow value for year k; and N is the length of the planning horizon.

The analysis is illustrated in the following table:

| Year (k) | $E(F_k)$ | SD | $(P/F, 10\%, k)^2$ | $V(F_k)$ |
|---|---|---|---|---|
| 0 | -$8,000,000 | | | |
| 1 | $1,750,000 | $250,000 | 0.8264 | $51,652,892,561.98 |
| 2 | $1,750,000 | $250,000 | 0.6830 | $42,688,340,960.32 |
| 3 | $1,750,000 | $250,000 | 0.5645 | $35,279,620,628.36 |
| 4 | $1,750,000 | $250,000 | 0.4665 | $29,156,711,263.11 |
| 5 | $1,750,000 | $250,000 | 0.3855 | $24,096,455,589.35 |
| 6 | $1,750,000 | $250,000 | 0.3186 | $19,914,426,106.90 |
| 7 | $1,750,000 | $250,000 | 0.2633 | $16,458,203,394.13 |
| 8 | $1,750,000 | $250,000 | 0.2176 | $13,601,820,986.88 |
| 9 | $1,750,000 | $250,000 | 0.1799 | $11,241,174,369.33 |
| 10 | $2,500,000 | $250,000 | 0.1486 | $9,290,226,751.51 |
| E(NPV) | $3,042,149.90 | | | |
| V(NPV) | $253,379,872,611.86 | | | |
| SD(NPV) | $503,368.53 | | | |
| z | -6.0436 | | | |

Letting P(NPV(10%) ≥ 0) = P(Z ≥ z), we have P(Z ≥ -6.0436) = 1 - P(Z ≥ -6.0436) = 1. Therefore, the investment will always be profitable.

**11.21c** The results of the Monte Carlo simulation are illustrated in the following histogram. The simulation was run with 1,000 iterations and a fixed random number seed.




The mean NPV was \$3,017,250, while the standard deviation was \$487,265.9. The minimum and maximum NPV were \$1,245,764 and \$4,656,046, respectively. Using the mean and standard deviation, z = -6.1922. Thus, $P(Z \geq -6.1922) = 1- P(Z \geq -6.1922) = 1$.

**11.22** True

**11.23** False; the alternative preferred when using the payback period method is seldom the one that is preferred when using discounted cash flow methods.

**11.24** True

**11.25** False; it is not always the lowest cost alternative

# Chapter 12

# *Preparing, Presenting, Implementing, and Maintaining the Facilities Plan*

Due to the nature of the questions at the end of Chapter 12, answers will not be provided for all. In those cases where responses are provided, it is likely that other, equally appropriate, answers can be devised. The purpose in providing responses is not to provide ***the*** answer, but to provide ***an*** answer that will stimulate class discussion.

**SECTION 12.2**

**12.1** This is an open-ended question; response depends on course location.

**12.2** Aesthetics are important in facilities planning, since users' attitudes will be influenced greatly by aesthetic aspects of the facility. Just as it would be inappropriate to plan a facility solely on the basis of aesthetics, so is it inappropriate to ignore aesthetics in facilities planning.

**12.3** Computer-aided layout models can be used in regional and city planning. Computer-aided layout models can be used to determine the locations of the facility relative to the locations of the airport, rail station, transportation hubs, distribution centers, manufacturing plans, and customers. Although some computer-aided layout models are more amenable than others to performing this type analysis, they are not restricted to use within a facility.

**12.4** 2-D layouts are superior to 3-D layouts when considering horizontal handling alternatives, since the vertical dimension is not a factor. 3-D layouts can be difficult to evaluate when mezzanines and other vertical obstructions are in place.

In assigning bins along an aisle, only the length of the aisle and the height of the storage face need to be represented; the depth of the storage faces will not be a factor, since they will be uniform.

**12.5** Three alternatives will be considered: sketches, templates, and computer graphics. Sketches can be generated spontaneously on-site or in meetings with users, planners, and managers; they are easy to generate, requiring no special equipment. In general, sketches are best used in preliminary discussions and lend themselves to rough-cut analyses.

Templates are useful in representing draft or final layout plans. They can be reproduced and circulated to appropriate parties for feedback; they can be marked up by operating and other personnel who might not have access to or familiarity with computer graphics.

Computer-generated layouts require both hardware and software capability. In general, computer graphics would be the preferred method of generating draft and final layout plans, since changes can be made to the layout easily, solutions can be transferred electronically to other locations, and there is no need to store large Mylar or paper representations of the layout.

**12.6** 3-D approaches are useful when the third dimension plays a role in the facility plan. For example, when comparing an overhead monorail with an AGV, it is important to visualize the vertical dimension. Likewise, in comparing conventional storage with high-rise storage, having a 3-D representation of the storage facility can prove beneficial in communicating the benefits (and limitations) of high-rise storage.

In terms of the various options that can be used to generate 3-D representations of the facility plan (block models, scale models, and computer models), block models are useful in preliminary design, particularly when no decision has been made as to the specific model of a machine or even type of machine that will be assigned to a particular location; block models, in this case, might be used to indicate the general location of a machine.

Scale models would be used at the point that machine selections have been made and final locations are being determined. The use of scale models has tended to decline in the past decade, due to the cost of acquiring and maintaining the 3-D scale model.

Computer models are gaining in popularity because of the ease of modifications of both the layout and the 3-D representation of the individual machines. Of course, 3-D computer models are still being seen on a 2-D monitor. The power of the 3-D computer model is that it can be rotated and viewed at any angle of interest to the facility planner. Although currently the cost of entering the computer model arena can be prohibitive for relatively small and infrequent facilities planning activities, it is anticipated that the power of 3-D computer models will increase and prices will decrease in the future to the point that any serious facilities planning will occur using computer models.

**12.7** Currently, the integration of computer graphics with computer-aided layout techniques is accomplished by a human interface. It would be useful to have a software package that combines the best features of descriptive and prescriptive computer layout software. As noted in Chapter 6, efforts are under way to develop software packages that incorporate expert models with computer graphics capabilities.

Until such time as the integration occurs in a software package, the human designer will be required to take the output from a computer-aided layout technique and input it into a computer graphics package. The CAL model can stimulate the human designer to represent via computer graphics various alternative layouts.

**12.8** Block templates or models should be used when the time and expense of generating contour templates and models is not justified. Regardless of which approach is taken, it is important for the facilities plan to be viewed as professional. Don’t be penny-wise and dollar-foolish in choosing the method of depicting the facilities plan. At the same time, the investment required to “sell” the facilities plan should be justified in the same way other investments are justified. If the perceived benefits of using more refined models do not off-set the costs of using them, then “keep it simple!” The choice depends on the application and the audience.

**12.9** Research question. The response will depend on the results of the student's web search.

**12.10** The response depends on the people interviewed.

**12.11** The response depends on the people interviewed.

**SECTION 12.3**

**12.12** A multimedia presentation can embody animation with simulation, as well as film clips of the various equipment that will be used in the facility. Depending on the particular application, alternative paths can be provided to the viewer in addressing question regarding the facilities plan. We can visualize the use of multimedia in incorporating computer-aided layout techniques with computer graphics models of the facility.

**12.13** Improvements in virtual reality and holographic technologies are expected to transform the process of facilities planning. Improvements in computing power and speeds will be such that massive optimization problems will be solved in the facilities planning process.

In the future, rather than plan a large, complex facility, it is likely that the plan will be for a large, complex virtual facility. With improved communication, proximity is likely to be less of a factor in facilities planning; functions will be distributed throughout the world.

**SECTION 12.4**

**12.14** Five additional reasons for people resisting change.

a. previous changes did not produce the promised benefits
b. a history of management by the "fad of the month," causing skepticism regarding any changes
c. WIIFM factor ("what's in it for me?") has not been addressed
d. those being asked to change were the authors of the current methods
e. a short-term reward system is being used, and the proposed change will not produce measurable benefits for several years

**12.15** Five additional ways to overcome resistance to change.

a. show video tapes and films of applications of the proposed change in other locations
b. take key employees on tours of facilities where the changes have been implemented successfully
c. implement the change in a pilot plant setting to gain experience in a limited setting
d. begin selling change at the start of the project, rather than at the end
e. involve at the outset of the planning process those whom you believe will be most resistant to the change; it is particularly important to involve hourly personnel and first- and second-level supervisors in the early stages of the planning process.

Also, if an environment of trust is created such that employees are not afraid they will lose their jobs or employment, then resistance will be much less. If at all possible, ensure that the changes will be made to improve the quality of life of the personnel affected by the change.

**12.16** The response depends on the manufacturing facility chosen.

**12.17** The response depends on the non-manufacturing facility chosen.

**12.18** The response depends on the university (and, likely, the specific library on that campus).

**12.19** The response depends on the university.

**12.20** The response depends on the university.