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Q1) a) Find the solution of the following differential equation using inverse laplace transform (2 points):

$\ddot{X} + 2\dot{X} = u(t)$

$u(t)$ is the unity step function

The Laplace Transform

Transform Pairs:

f(t)	F(s)
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-st}	$\frac{1}{s+a}$

f(t)	F(s)
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$

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$$s^2 X(s) + 2X(s) = \frac{1}{s}$$

$$X(s) (s^2 + 2) = \frac{1}{s}$$

1/2

$$X(s) = \frac{1}{s(s^2 + 2)}$$

B) write the laplace code to inverse laplace $y(s) = \frac{1}{s^2 + 2s + 3}$ (2 points)

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syms s, t
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ilaplace ( 1 / ( s^2 + 2*s + 3 ) )
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c) what is the final value for $y(s)$ in part $y(s) = \frac{1}{s^2 + 2s + 3}$ (1 points)

lim $s y(s)$
 $s \rightarrow 0$

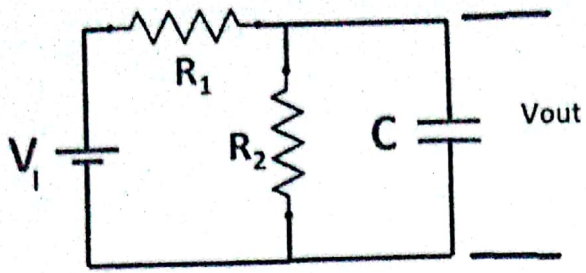
0

$$\frac{s}{s^2 + 2s + 3}$$

$$\frac{0}{3} = 0$$

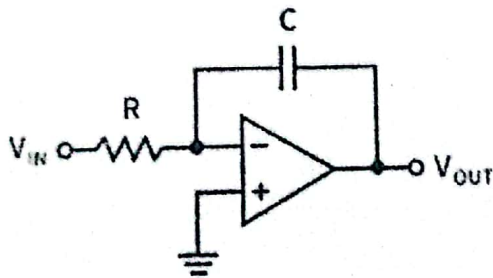
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Q2) find the transfer function for the following



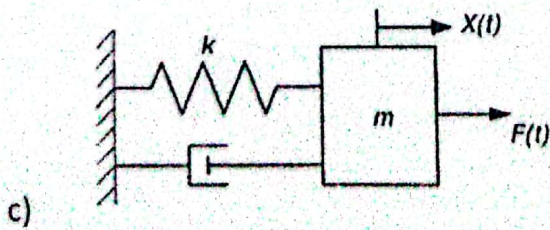
a)

V_{out}/V_i (1 points)



b)

V_{out}/V_{in} (2 points)

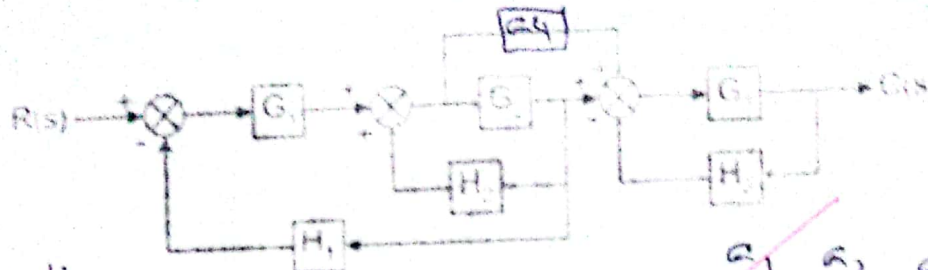


c)

(damper coefficient = C) X/F (2 points)

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Q3) Find the equivalent transfer function for (5 points)



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Loop $G_2 H_2$ $-G_3 H_3$ $-G_1 G_2 H_1$

G_1 G_2 G_3
 G_1 G_2 G_3

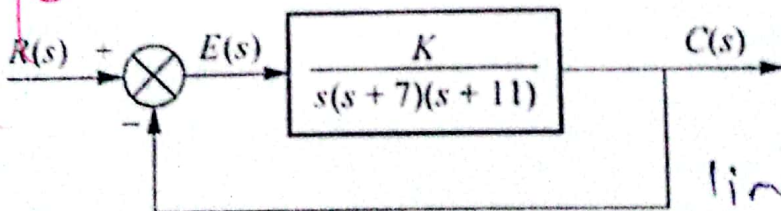
Non touching $-G_2 G_3 H_2 H_3$

$G_1 G_2 G_3 H_1 H_3$

$G_1 G_2 G_3 + G_1 G_3$

$1 - G_2 H_2 + G_3 H_3 + G_1 G_2 H_1 - G_2 G_3 H_2 H_3 + G_1 G_2 G_3 H_1 H_3$

Q4) Find the steady state error for a step input (5 points)



$R = \frac{1}{s}$

$\lim_{s \rightarrow 0} s E(s)$

$E(s) = \frac{R}{1+G(s)}$

$\lim_{s \rightarrow 0}$

$1 + \frac{K}{s(s+7)(s+11)}$

$= \frac{1}{8} =$

Final answer

Q5) a) For the following pole what is the ζ and ω_n associated with this pole (2.5 points)

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$$\frac{s}{\sqrt{1-\zeta^2}} = \frac{1}{2}$$

$$\frac{s^2}{1-\zeta^2} = \frac{1}{4} \rightarrow \frac{1}{4} - \frac{1}{4}\zeta^2 = -\zeta^2$$

$$s^2 = \frac{1}{5} \leftarrow \frac{5}{4}\zeta^2 = \frac{1}{4}$$

$\zeta = 0.447$ damping ratio
 $\omega_n = 2.23$
 (2.5 points)

$0.447 \times \omega_n = 1$
 $\omega_n = 2.23$

$MP = e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}} \times 100 \%$

$MP = 20.7\%$

$\omega_n = 2.23$
 $\zeta = \frac{\sigma}{\omega_n} = \frac{2}{2 \times 2.23} = 0.447$

Q6) What is the) a) For the following system $G = \frac{s+12}{(s+10)(s+5)}$, $H=1$ design the feedback controller k to achieve a damping ratio of > 0.97 . (you must use the root locus figure below) (5 points)

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$\cos^{-1} 0.97 = 14 \text{ degree}$

$G = \frac{s+12}{(s+10)(s+5)}$

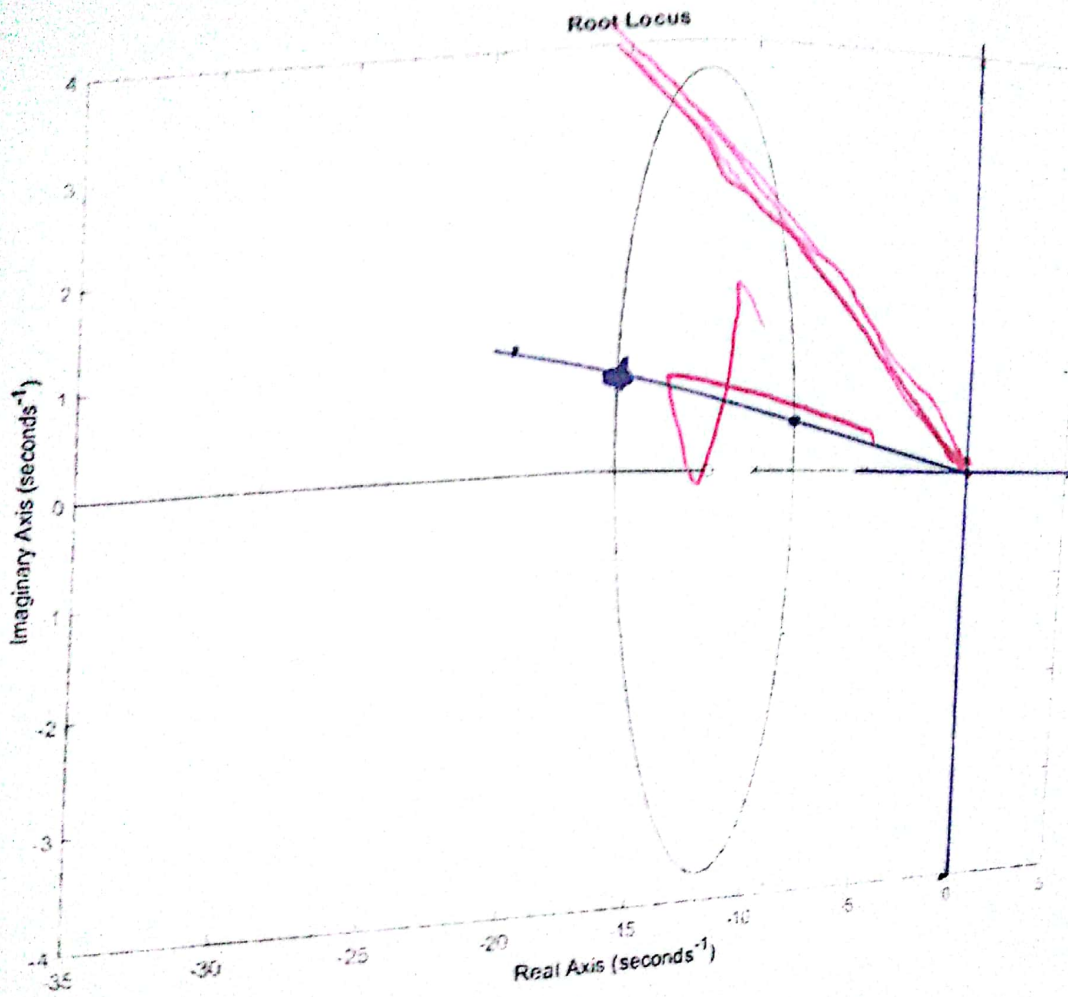
$s_F = -9 + 0.5j$

$0.5k_1, 1.48j$

$1.62k_2, 8j$
 \downarrow
 i/p

$s_2 = -15 + j$

5/4



$$s = -9 + 0.5j$$

$$s_2 = -15 + j$$