26.5/30

System dynamics and contro… …D Exam

Student Na… …umber:

Q1)a) (10 p… …equa…

$$F(s) = \frac{5.2}{s^2 + 2s + 5}$$

1

$$\frac{5.2}{S^2+2S+1+4} = \frac{5.2}{(S+1)(S+1)+4} = \frac{5.2}{(S+1)^2+2^2} \times \frac{2}{2} \rightarrow F(s) = \frac{5.2}{2} \cdot \frac{2}{(S+1)^2+2^2}$$

$$f(t) = \frac{5.2}{2} e^{-t} \sin 2t$$

$$F(s) = \frac{8(s+1)}{(s+2)^2} = \frac{K_1}{(S+2)} + \frac{K_2}{(S+2)^2}$$

1.5/1

$$8(S+1) = K_1(S+2) + K_2$$

$S = -2$

$-8 = K_2$

$S = 0 \rightarrow 8 = -8(2) + K_2$

$24 = K_2$

$$F(s) = \frac{-8}{S+2} + \frac{24}{(S+2)^2}$$

$$f(t) = -8e^{-2t} + 24t\, e^{-2t}$$

b)Find the final value for the following function

$$F(s) = \frac{(s+2)^2 - 3^2}{[(s+2)^2 + 3^2]}$$

3/

using Final value theorem:-

$$\lim_{s \to 0} s F(s) = \lim_{s \to 0} s\left(\frac{(S+2)^2 - 3^2}{(S+2)^2 + 3^2}\right)$$

$$= \lim_{s \to 0} 0 \left(\frac{2^2 - 3^2}{2^2 + 3}\right)$$

$$= 0$$

c) Write the MATLAB code for the to find laplace inverse for

$$F(s) = \frac{(s+2)^2 - 3^2}{[(s+2)^2 + 3^2]}$$

0/3

$$F(s) = \frac{(S+2)^2}{(S+2)^2+3} - \frac{3}{(S+2)^2+3}$$

$$f(t) = e^{-2t}\cos\sqrt{3}t - (e^{-2t}\sin\sqrt{3}t)\frac{3}{\sqrt{3}}$$

Laplace^{-1} «F(s)»

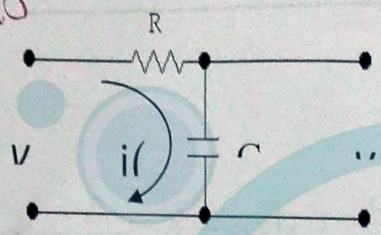
inverse Laplace «F(s)»

Input = t.f

16/25

Q2)(15 points) a) Find the transfer function Vout/ Vin for if R =1 Mohms and C= 1microFarad then the location of the pole is.

8/6



$V_{in} = \left(R + \frac{1}{CS}\right) I$

$= \left(\frac{RCS + 1}{CS}\right) I$

$V_{out} = \frac{1}{CS} I$

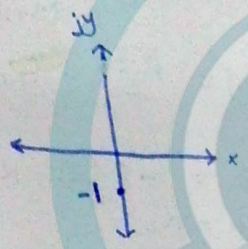
$\frac{V_{out}}{V_{in}} = \frac{I(1/CS)}{I\left(\frac{RCS+1}{CS}\right)}$

$= \frac{1}{RCS+1}$

$= \frac{1}{1\times10^{6}\times1\times10^{-6}S+1}$

$= \frac{1}{s+1}$

$\rightarrow f(t) = e^{-t}$



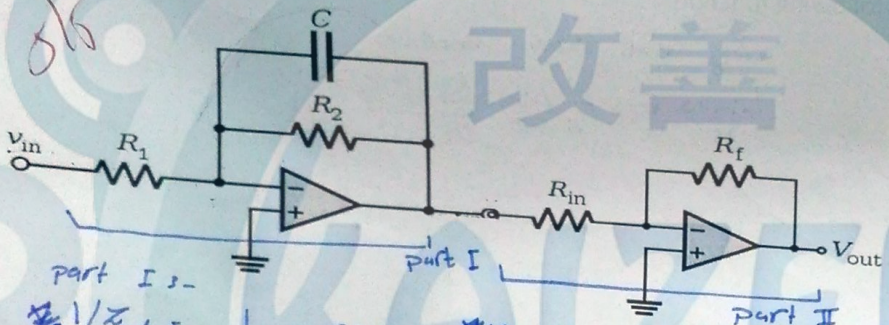
* Pole :-

$s + 1 = 0$

$s = -1$

b)Find the transfer function for

8/6



part I :-

$1/Z_{out} = \frac{1}{R_2} + CS$

$= \frac{1 + R_2CS}{R_2}$

$Z_{out} = \frac{R_2}{1 + R_2CS}$

$Z_{in} = R_1$

$\frac{V_{out_1}}{V_{in_1}} = \frac{-Z_2}{Z_1} = \frac{-R_2}{(1+R_2CS)R_1}$

part II :-

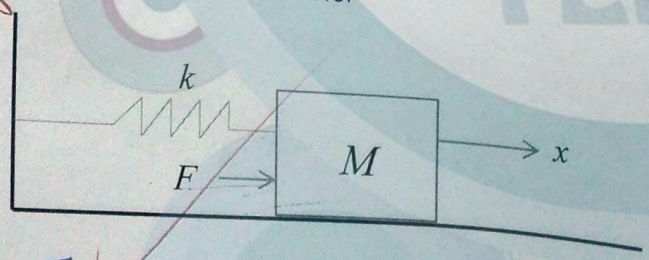
$\frac{V_{out_2}}{V_{in_2}} = -\frac{R_f}{R_{in}}$

For the system :-

$\frac{V_{out_1}}{V_{in_1}} \cdot \frac{V_{out_2}}{V_{in_2}} = \frac{-R_2}{R_1(1+R_fCS)} \div \frac{R_f}{R_{in}}$

c)Find X/F transfer function for

8/7



$F = M\ddot{x} + Kx$

$F(s) = Ms^2X(s) + KX(s)$

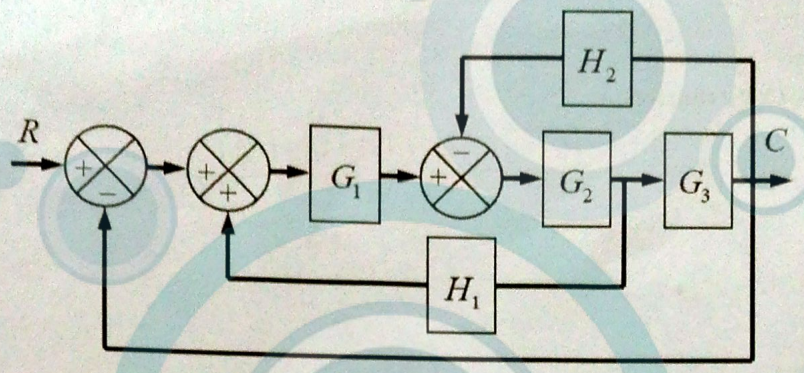
$F(s) = X(s)(Ms^2 + K)$

$\frac{X(s)}{F(s)} = \frac{1}{Ms^2+K} \rightarrow \frac{X(s)}{F(s)} = \frac{1/M}{s^2 + \frac{K}{M}}$

$\frac{X(s)}{F(s)} = \frac{1}{M\sqrt{K}/\sqrt{M}} \cdot \frac{\sqrt{K}/\sqrt{M}}{(s^2 + \frac{K}{M})}$

$\frac{x(t)}{F(s)} = \frac{\sqrt{M}}{M\sqrt{K}} \cdot \sin\frac{\sqrt{K}}{\sqrt{M}}t$

Q3) (5 points) Find the equivalent transfer function for



Using Mason's Rule

$$* \quad \frac{C}{R} = \frac{P \Delta_i}{\Delta}$$

$P = G_1 G_2 G_3$

$\Delta_i = 1$

$\Delta = 1 - \Sigma Loops$

$= 1 - [-G_1 G_2 G_3 + G_1 G_2 H_1 - G_2 G_3 H_2]$

$= 1 + G_1 G_2 G_3 - G_1 G_2 H_1 + G_2 G_3 H_2$

$$\frac{C}{R} = \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3 - G_1 G_2 H_1 + G_2 G_3 H_2}$$

Table of Laplace Transforms

$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. e^{at}	$\frac{1}{s-a}$
3. t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4. t^p, $p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. $\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}}$, $n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10. $t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12. $\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14. $\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$	18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20. $e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22. $e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n e^{at}$, $n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25. $u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26. $\delta(t-c)$ Dirac Delta Function	e^{-cs}
27. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	28. $u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29. $e^{ct}f(t)$	$F(s-c)$	30. $t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t}f(t)$	$\int_s^\infty F(u)du$	32. $\int_0^t f(v)dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$	34. $f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)dt}{1-e^{-sT}}$
35. $f'(t)$	$sF(s)-f(0)$	36. $f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37. $f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$		