Differential Equations:

1 st order:

$\boxed{1}$ Separable : can be written as

$$f(x)\,dx = g(y)\,dy.$$

$\boxed{2}$ Using 'integrating factor : written as.

$$y' + p(x)\,y = f(x) \quad \text{(linear)}$$

then using integrating factor : $e^{\int p(x)\,dx}$.
then multiply by this factor
then simplifly and integrate.

$\boxed{3}$ Bernoulli Equation: written as

$$y' + p(x)\,y = f(x)\,y^n. \quad \text{(non linear)}$$

- take $u = y^{1-n}$

$$u' = (1-n)\,y^{-n}\,y'$$

~~multiply equation by this~~

- then see what is (u) and what is (u')
- its linear! continue using $\boxed{2}$ method.

$$u' + (1-n)\,p(x)\,u = (1-n)\,f(x)$$

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[4] Homogeneous

$$y' = g\left(\frac{y}{x}\right)$$ → دلالة كاملة

- we use $u = \frac{y}{x}$ → $y = xu$
  $y' = xu' + u$.
- then substitute these with all of y, y'
- its separable! continue with method [1].

[5] Exact.

$$M(x,y)\,dx + N(x,y)\,dy = 0.$$

in exact DE: $\frac{dM}{dy} = \frac{dN}{dx}$.

1 - we consider $\frac{du}{dx} = M$, $\frac{du}{dy} = N$

2 - we integrate M or N to get u alone,
if M is integrated, we add $f(y)$ as a consta (with respect to x)

3 - we derivate the M integrated with respect to x now with respect to Y to obtain $f'(y)$

4 - we equal this to $\frac{du}{dy} = N$ and nothing with respect to x should remain.

5 - Now we have $f'(y)$, integrate it and عوضها هون, and equal everything to (C) because $u(x,y) = c$.

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b] Integrating Factor

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

<u>however</u> $\frac{dM}{dy} \neq \frac{dN}{dx}$.

- find integrating Factor as

$$\frac{\frac{dM}{dy} - \frac{dN}{dx}}{N} = R(x) \quad \text{so} \quad \mu(x) = e^{\int R(x)\,dx}.$$

$$\frac{\frac{dM}{dy} - \frac{dN}{dx}}{-M} = R(y) \quad \text{so} \quad \mu(y) = e^{\int R(y)\,dy}.$$

- multiply the whole D.E by $\mu(x)$ or $\mu(y)$
- its exact! continue using the before method.

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2nd order:

$$y'' + p(x)\,y' + q(x)\,y = \underline{0}$$ makes this DE homogeneous

f and g are:

linearly independent if: $\frac{f}{g}$ = answer with respect to x.

linearly dependent if: $\frac{f}{g}$ = constant.

Wronskian:

① $W(f,g) = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = fg' - gf'$

② $W(y_1, y_2)(x) = c\,e^{-\int p(x)dx}$

as $\{y_1, y_2\}$ are solutions of the 2nd order equ,

also, $\{y_1, y_2\}$ are linearly independent iff

$W(y_1, y_2) \neq 0$.

General solution of y: $y = c_1 y_1 + c_2 y_2$

if y_1, y_2 are linearly independent.

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[1] Missing y or x.

Missing y : $F(x, y', y'')$.

Let $u = y'$, $u' = y'' \rightarrow F(x, u, u') = 0$.

Solve as First order, then عوض y' in u after Finding solution, then solve as First order again.

Missing x : $F(y, y', y'')$

Let $u = y'$, $u' = y'' \rightarrow F(y, u, u') = 0$

Here, $u' = u\frac{du}{dy}$, and solve as mentioned before.

[2] Reduction of Order

$y'' + p(x)y' + q(x)y = 0$. $\leftarrow$ لازم تكون هيك

Given y_1 a solution of this equ.

$$y_2 = y_1 \int \frac{e^{-\int p(x)dx}}{y_1^2} dx.$$

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$\boxed{3}$ Homogeneous Linear ODEs with constants

$$y'' + a y' + by = 0$$

consider :- $y'' = r^2$, $y' = ar$, $y = b$.

so : $r^2 + ar + b = 0$ (auxiliary equation)

→ Distinct real roots $(r-c)(r-d) = 0$

or قانون عام $\quad r = c, d$

$y_1 = e^{cx}$, $y_2 = e^{dx}$. (set of solutions)

$y = c_1 e^{cx} + c_2 e^{dx}$. (general solution)

→ Real double roots : $(r-c)(r-c) = 0$

$r = c, c$.

$y_1 = e^{cx}$, $y_2 = x e^{cx}$.

$y = c_1 e^{cx} + c_2 . x\, e^{cx}$

→ Complex numbers $r = \lambda \pm \mu i$

$y_1 = e^{(\lambda + \mu i)x}$, $y_2 = e^{(\lambda - \mu i)x}$

$y_1 = e^{\lambda x} \cos \mu x$, $y_2 = e^{\lambda x} \sin \mu x$

$y = c_1 e^{\lambda x} \cos \mu x + c_2 e^{\lambda x} \sin \mu x$.

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4 Euler-Cauchy Equation

$x^2 y'' + a x y' + b y = 0 \quad , \quad x > 0.$

consider : $x^2 y'' = r(r-1)$, $x y' = r$, $y =$ nothing.

so: $r(r-1) + ar + b = 0$

Different Real roots $r_1 \neq r_2$

$y_1 = x^{r_1} \quad , \quad y_2 = x^{r_2}.$

$y = c_1 x^{r_1} + c_2 x^{r_2}$

Equal Roots $r_1 = r_2 = r$

$y_1 = x^r \quad , \quad y_2 = x^r \ln x.$

$y = c_1 x^r + c_2 x^r \ln x.$

Complex Roots $\lambda \pm \mu i$

$y_1 = x^{\lambda} \cos(\mu \ln x) \quad , \quad y_2 = x^{\lambda} \sin(\mu \ln x).$

$y = c_1 (x^{\lambda} \cos(\mu \ln x)) + c_2 (x^{\lambda} \sin(\mu \ln x))$

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$\boxed{5}$ Nonhomogeneous undetermined coefficients.

$a y'' + b y' + c y = r(x)$ → makes this nonhomogen.

$r(x)$ could be: polynomial, e^{ax}, $\sin ax$, $\cos ax$...

General Solution: $y = y_h + y_p$

$a y'' + b y' + c y = 0$

solve with $\boxed{3}$

$y_p(x) =$

this should rely on $r(x)$

→ $Ax^2 + Bx + C$ or

$Ax + B$ or

A if polynomial $r(x)$

→ Ae^{kx} if exponen $r(x)$

→ $A \sin x + B \cos x$

if trigo. $r(x)$

if any of these have the same look of y_1, y_2 of y_h, then multiply by x.

then find y', y'' and substitute in the main equ., then using $r(x)$ find A, B, C.

if $r(x) = \cos^2 x$ or $r(x) = \sin x \cos x$ try getting rid of having something you can't deal with like

$\frac{1}{2}(1 + \cos 2x)$ or $\frac{\sin 2x}{2}$

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$\boxed{6}$ Variation of Parameters

$$y'' + p(x)\,y' + q(x)\,y = r(x)$$

General Solution: $y = y_h + y_p$

$$y_h(x) = C_1 y_1 + C_2 y_2$$

you could use $\boxed{3}$ or $\boxed{4}$

بحيث y'' ما في جنبها اشي.

$$y_p(x) = -y_1 \int \frac{y_2}{W(y_1, y_2)} r(x)\,dx + y_2 \int \frac{y_1 r(x)\,dx}{W(y_1, y_2)}$$

$$W(y_1, y_2) = y_1 y_2' - y_2 y_1'$$

Higher order:

1 Homogeneous

$y^{(n)} + a\, y^{(n-1)} + \dots + b y'' + c y' + d y = 0.$

(homogeneous linear with constants)

consider: $y^{(n)} = r^n$, $y^{(n-1)} = r^{n-1}$ ... , $y'' = r^2$, $y' = r$

and solve as before rules.

$x^n y^{(n)} + \dots + a x^2 y'' + b x y' + c y = 0.$

(euler cauchy equation).

consider: $x^3 y''' = r(r-1)(r-2)$, $x^2 y'' = r(r-1)$, $x y' = r$

and solve as before rules.

notes *:

* $r^3 - 2r^2 - 5r + 6 = (r-1)(\dots)$

$$\begin{array}{c|cccc} & 1 & -2 & -5 & 6 \\ 1 & \downarrow & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array} \rightarrow (r-1)(r^2-r-6)$$

$(r-1)(r-3)(r+2)$

* take each answer as an individual unique one even if it is repeated, and even if it has a power.

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$\boxed{2}$ Undetermined coefficient.

$$y^{(n)} + \dots + a y'' + b y' + c y = r(x)$$

General solution: $y = y_h + y_p$

solve as we did in second order.

$\boxed{3}$ Variation of parameters

$$y''' + P_2(x)\, y'' + P_1(x)\, y' + P_0(x)\, y = r(x)$$

General solution: $y = y_h + y_p$

we solve as we did before, however:

$$y_p(x) = y_1 \int \frac{W_1}{W} \underset{=}{r} + y_2 \int \frac{W_2}{W} r + y_3 \int \frac{W_3}{W} r$$

In original answer

$$W = W(y_1, y_2, y_3) = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$$

$$W_1 = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ 1 & y_2'' & y_3'' \end{vmatrix} \qquad W_3 = \begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & 1 \end{vmatrix}$$

$$W_2 = \begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & -1 & y_3'' \end{vmatrix}$$

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System of ODEs:

[1] Homogeneous Linear System with Constant Coefficients

$$\acute{y}_1 = a y_1 + b y_2$$
$$\acute{y}_2 = c y_1 + d y_2$$
or
$$\acute{y} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\acute{y} = A y$$

① $$[A - \lambda \underset{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}{\underset{\downarrow}{I}}] = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$$

② $$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$$(a-\lambda)(d-\lambda) - bc = 0.$$

③ $[A - \lambda I]\xi = 0$

at λ_1 or λ_2 $\rightarrow$ $$\begin{bmatrix} a-\lambda_1 \text{ or } \lambda_2 & b \\ c & d-\lambda_1 \text{ or } \lambda_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$K \underline{\underline{y_2}} = J \underline{\underline{y_1}}$$ so $$\xi_{(1) \text{ or } (2)} = \begin{bmatrix} K \\ J \end{bmatrix}$$

④ General solution: $\xi_{(1)} e^{\lambda_1 t} c_1 + \xi_{(2)} e^{\lambda_2 t} c_2$

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* Note: if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $|A| \neq 0$,

then $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ as $\boxed{|A| = ad - cb}$

[2] Homogeneous Systems as Complex or Equal eingen Values.

Complex:

① $[A - \lambda I]$

② $|A - \lambda I| = 0$ and λ = complex $= \mu + i\nu$

③ at λ_1 so $\begin{bmatrix} a-\lambda_1 & b \\ c & d-\lambda_1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0$

$K y_2 = J i\, y_1 \qquad \xi_{(1)} = \begin{bmatrix} K \\ Ji \end{bmatrix}$

④ $y_{(1)} = e^{(\mu + i\nu)t} \begin{bmatrix} K \\ Ji \end{bmatrix}$

$y_{(1)} = e^{\mu t} \cdot \underline{e^{i\nu t}} \begin{bmatrix} K \\ Ji \end{bmatrix}$

$\cos t + i \sin t$

$y_{(1)} = e^{\mu t} \begin{bmatrix} (\cos t + i \sin t) K \\ (\cos t + i \sin t) Ji \end{bmatrix}$

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$$y_{(1)}(t) = e^{\mu t}\left(\begin{bmatrix}\cos t\\ -\sin t\end{bmatrix} + i\begin{bmatrix}\sin t\\ \cos t\end{bmatrix}\right)$$

$$y(t) = e^{\mu t}\begin{bmatrix}\cos t\\ -\sin t\end{bmatrix} + i e^{\mu t}\begin{bmatrix}\sin t\\ \cos t\end{bmatrix}$$

⑤ General Solution: $c_1 e^{\mu t}\begin{bmatrix}\cos t\\ -\sin t\end{bmatrix} + c_2 e^{\mu t}\begin{bmatrix}\sin t\\ \cos t\end{bmatrix}$

Equal :

① $[A - \lambda I]$

② $|A - \lambda I| = 0$ and $\lambda_1 = \lambda_2$

③ at $\lambda \rightsquigarrow \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0$

$K y_2 = J y_1 \qquad \xi = \begin{bmatrix} K \\ J \end{bmatrix}$

④ $y_{(1)} = \xi e^{\lambda t}$ ✓

$y_{(2)} = \xi t e^{\lambda t} + \eta e^{\lambda t}.$

So: $[A - \lambda I]\,\eta = \xi$

$$\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} K \\ J \end{bmatrix}$$

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$$L y_2 = P y_1 \qquad w = \begin{bmatrix} L \\ P \end{bmatrix}$$

⑤ General Solution: $c_1 e^{\lambda t} \begin{bmatrix} K \\ J \end{bmatrix} + c_2 \left(\begin{bmatrix} K \\ J \end{bmatrix} t + \begin{bmatrix} L \\ P \end{bmatrix} \right) e^{\lambda t}$

3 Nonhomogeneous Systems.

1... Undetermined Coefficients.

$$y' = Ay + F(t)$$

① First we find $y_{(h)}$ of $y' = Ay$ alone using methods from before.

② To find $y_{(p)}$ we assume $y_{(p)}$ relatively to $F(t)$

so: $F(t) = \begin{bmatrix} \cdots \\ \cdots \end{bmatrix}$ $\qquad y_{(p)} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

$F(t) = \begin{bmatrix} \cdots \\ \cdots \end{bmatrix} e^t$ $\qquad y_{(p)} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} e^t$ if the same as one of $y_{(h)}$ solutions then:

$$y_{(p)} = \left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} t + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \right) e^t$$

③ We substitute $y_{(p)}$ instead of y in $y' = Ay + F(t)$ and solve, (if there are too many unknows or equaling to zero too hard we use next method)

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2... Variation of Parameters

$'y = Ay + F(t)$

① Find $y_{(h)}$ using $'y = Ay$.

② $y_{(P)} = \underline{\phi(t)} \int \underline{\phi^{-1}(t)}\, \underline{F(t)}\,dt$.

Full $y_{(h)}$ solution

inverse as said before.

$F(t)$ as $\begin{bmatrix} R \\ M \end{bmatrix}$

so:

$y_{(h)} = c_1 \begin{bmatrix} A \\ B \end{bmatrix} e^{t} + c_2 \begin{bmatrix} C \\ D \end{bmatrix} e^{2t}$

$$\underbrace{\begin{bmatrix} Ae^{t} & Ce^{2t} \\ Be^{t} & De^{2t} \end{bmatrix}}_{\phi(t)} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\phi^{-1} F = \begin{bmatrix} Ae^{t} & Ce^{2t} \\ Be^{t} & De^{2t} \end{bmatrix} \times \begin{bmatrix} R \\ M \end{bmatrix} = \begin{bmatrix} L \\ E \end{bmatrix}$$

③ $y = y_{(h)} + y_{(P)}$

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Series.

Analytic: A Function is analytic if at x_0 its Taylor series exists

Analytic functions: ① polynomial ② e^x, $\cos x$, $\sin x$
③ Rational Functions (except zero denominator)

• A point is called an ordinary point x_0 if in

$$A(x)y'' + B(x)y' + C(x)y = 0$$

$\frac{B(x)}{A(x)}$ and $\frac{C(x)}{A(x)}$ are analytic at x_0.

• If x_0 is an ordinary point in:

$$A(x)y'' + B(x)y' + C(x)y = 0$$

Then the D.E has two solutions in the form

$$y = \sum_{n=0} a_n (x - x_0)^n.$$

• If $\sum_{n=0} a_n x^n = \sum_{n=0} b_n x^n$ then $a_n = b_n$

If $\sum_{n=0} a_n x^n = 0$ then $a_n = 0$.

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1 Finding a power series solution.

① We make sure that we can use $y = \sum_{n=0} a_n(x^* - x_0)^n$.

by making sure x_0 is ordinary.

② We use $y = \sum_{n=0} a_n(x^* - x_0)^n$.

$$y' = \sum_{n=1} n\, a_n (x - x_0)^{n-1}$$

$$y'' = \sum_{n=2} n(n-1)\, a_n (x - x_0)^{n-2}.$$

③ Substitute in the original equ.

④ Anything "out" of the substitution try to get it inside by making it the same as inside the series.

⑤ We try making the inside powers equal by adding and subtracting from it while taking $\sum_{n=a}$ into consideration. all n

⑥ We try to make all $\sum_{n=a} + \sum_{n=b} + \sum_{n=c}$ equal

by نعوض the $\sum_{n=\bigcirc}$ inside and making it as variable until you get to the highest n of them

⑦ As is equal to zero (without x's) so variables equal zero and series equal zero, and we take a [illegible] from variables.

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⑧ When equaling series to zero you will have

For example: $(n+2)(n+1)\,\underset{\text{highest}}{a_{n+2}} - a_{n-1} - 2a_n = 0.$ $\quad \underline{\underline{n \geq 1}}$

$\downarrow$

n of step 6

Recurrence Relation

$$a_{n+2} = \frac{a_{n-1} + 2a_n}{(n+2)(n+1)} \qquad n \geq 1$$

⑨ start عوض $\boxed{n=1}$ $\quad a_3 = \dfrac{a_0 + 2a_1}{6} =$ From عندك

$\boxed{n=2}$ $\quad a_4 = \dfrac{a_1 + 2a_2}{(4)(3)} =$ From عندك

until you use a_3 or a_4

⑩ Solution: $\quad y = \sum_{n=0} a_n (x-2)^n$

$\approx a_0 + a_1 (x-2)^1 + a_2 (x-2)^2 \cdots$

$a_0 \left(1 + (x-2)^2 + \dfrac{(x-2)^3}{6} \cdots \right) + a_1 \left((x-2) + \dfrac{(x-2)^3}{3} \cdots \right)$

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- If $A(x)y'' + B(x)y' + C(x)y = 0$

$\frac{B(x)}{A(x)}$ or $\frac{C(x)}{A(x)}$ are not ordinary or have not ordinary points like zero denominator, they are called singular points.

They can be → regular

① $\lim_{x \to x_0} \frac{B(x)}{A(x)}(x-x_0) = p_0 < \infty$

② $\lim_{x \to x_0} \frac{C(x)}{A(x)}(x-x_0)^2 = q_0 < \infty$

→ irregular (else of)

with indicial ~~eq~~ equation:

$$r(r-1) + p_0 r + q_0 = 0.$$

- In case of regular singular, let $x_0 = 0$ be one, and indicial equation solutions are r_1, r_2 then:

$\boxed{1}$ $r_1 > r_2$, $r_1 - r_2$ is a decimal

$$y_1 = \sum_{n=0}^{\infty} a_n x^{n+r_1}$$

$$y_2 = \sum_{n=0}^{\infty} b_n x^{n+r_2}$$

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$r_1 > r_2$, $r_1 - r_2$ is a nonzero integer

$$y_1 = \sum_{n=0}^{\infty} a_n x^{n+r_1}$$

$$y_2 = C\, y_1 \ln x + \sum_{n=0} b_n x^{n+r_2} \qquad a_0 \neq 0.$$

$\boxed{3}$ $r_1 = r_2$, $r_1 - r_2 = 0$.

$$y_1 = \sum_{n=0} a_n x^{n+r}$$

$$y_2 = y_1 \ln x + \sum_{n=1} b_n x^{n+r}$$

* The steps in which you solve the series is exactly as before

Just be careful when you have $\ddot{y} = \sum_{n=1} n(n+1)\, a_n\, x^{n-1}$

Frobenius:

$$y = \sum_{n=0}^{\infty} a_n (x - x_0)^{n+r}$$ if x_0 was not 0 (or even if it was!)

where r is to be determined.

$$y = \sum^{\infty}$$

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$$y = \sum_{n=0}^{\infty}$$

Laplace Transform -

* Originally, the laplace of a function is:

$$F(s) = L\{f(t)\} = \int_0^{\infty} f(t)\, e^{-st}\, dt.$$

$$\lim_{p \to \infty} \int_0^{p} f(t)\, e^{-st}\, dt$$

* $L\{a f(t) + b g(t)\} = a L\{f(t)\} + b L\{g(t)\}$

* $L^{-1}\{F(s)\} = f(t)$

* $L^{-1}\{aF(s) + b G(s)\} = a L^{-1}\{F(s)\} + b L^{-1}\{G(s)\}$

Basically:

$f(t)$	$F(s)$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$

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Finding L | Finding L^{-1}

$$L\{e^{at} f(t)\} = F(s-a) \qquad L^{-1}\{F(s-a)\} = e^{at} L^{-1}\{F(s)\}$$

دائما في مقابل

$$L\{u(t-a)\, f(t)\} = e^{-as} L\{f(t+a)\} \qquad L^{-1}\{e^{-as} F(s)\} = u(t-a)\, L^{-1}\{F(s)\}\Big|_{t-a} = u(t-a)\, f(t-a)$$

note.

$$L\{u(t-a)\} = \frac{e^{-as}}{s}.$$

$$L\{y'(t)\} = s\,Y(s) - y(0)$$

$$L\{y''(t)\} = s^2\, Y(s) - s\,\underline{\underline{y(0)}} - \underline{\underline{y'(0)}}$$

$$L\{y(t)\} = Y(s)$$

after which we use L^{-1} on $Y(s)$ leaving it alone for solving D.E.

$$L\{\delta(t-a)\} = e^{-as}.$$

$$L\{\underline{\underline{g}}(t)\, \delta(t-a)\} = e^{-as}\, \underline{\underline{g(a)}}$$

$$L\{t\, f(t)\} = -F'(s) = \frac{-d}{ds}(F(s)) \qquad L^{-1}\{F'(s)\} = -t\, f(t)$$

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(u)\,du$$

$$L\left\{\int_0^t f(u)\,du\right\} = \frac{F(s)}{s} \qquad L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(u)\,du.$$

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note: you may use:

$\sin(\pi + t) = -\sin t$

$\sin(\pi - t) = \sin t.$

$\cos(\pi \mp t) = -\cos t$

$\sin(\frac{\pi}{2} \mp t) = \cos t.$

$\cos(\frac{\pi}{2} + t) = -\sin t$

$\cos(\frac{\pi}{2} - t) = \sin t.$

① If $f(t) = \begin{cases} f_1(t) & 0 \le t < a. \\ f_2(t) & a \leq t < b \\ f_3(t) & t \geq b. \end{cases}$

then $f(t) = f_1(t)\,(u(t) - u(t-a)) +$
$f_2(t)\,(u(t-a) - u(t-b)) +$
$f_3(t)\,(u(t-b))$

② There are ways to solve such as:

$$L^{-1}\left\{ \frac{1}{s(s+1)} \right\} \quad \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{A(s+1)+Bs}{s(s+1)}$$

$$1 = A(s+1) + Bs$$

when $s = 0$ $A = 1$

when $s = -1$ $B = -1$

So: $\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$

and you can take L^{-1} for both.

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Also, when there is a function you are not familiar with in L^{-1} such as ln, cot and so on, you can find L^{-1} for its derivation

so $L^{-1}\{F'_{(s)}\}$ and then it should be

$-t f(t)$ so divide by $(-t)$ in the end.

③ Dirac Delta Function:

$$\int_{-\infty}^{\infty} \delta(t-a)\, g(t)\, dt = g(a).$$

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