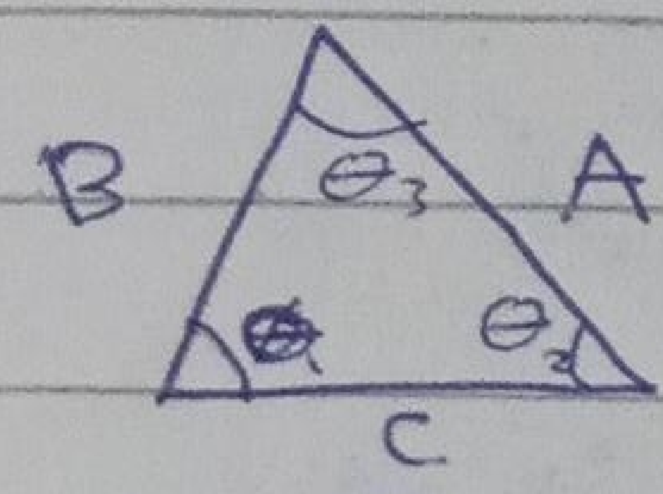
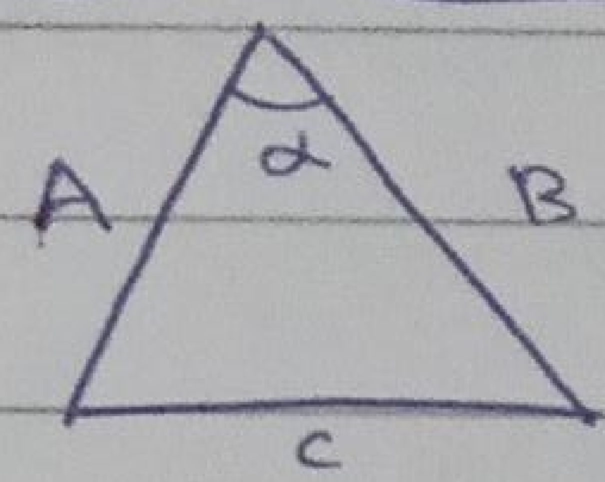
Subject Statics Chapter 2 Date No.

$$\frac{A}{\sin\theta_1} = \frac{B}{\sin\theta_2} = \frac{C}{\sin\theta_3}$$

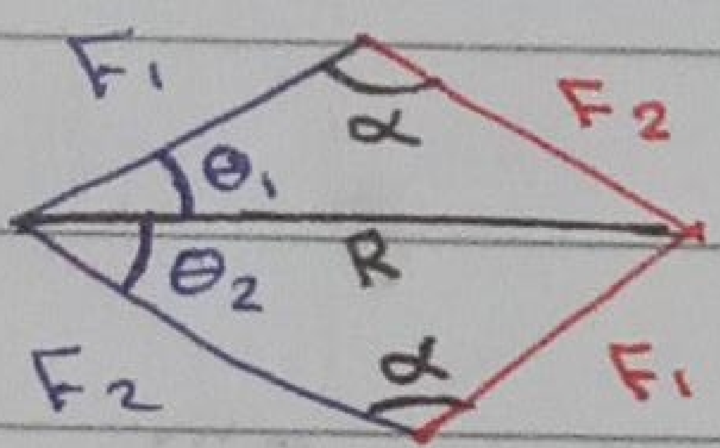


$$C^2 = A^2 + B^2 - (2 . A . B . \cos\alpha)$$

$$C = \sqrt{A^2 + B^2 - 2AB\cos\alpha}$$



⊛ Parallel law :



$$R = \sqrt{(F_1)^2 + (F_2)^2 - 2 . F_1 . F_2 . \cos\alpha}$$

$\theta_1 \neq \theta_2$

$$\frac{F_2}{\sin\theta_1} = \frac{R}{\sin\alpha}$$

$F_x = F\cos\alpha$ $F_y = F\cos B$ $F_z = F\cos\gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

⊛ To find the direction of the resultant vector we have to find it's directional angles after finding the unit vector.

⊛ Force vector $= F \times \hat{u}$

$\hat{u} \langle a, b, c \rangle$

$\cos\alpha = a$ / $\cos\beta = b$ / $\cos\gamma = c$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A||B|}$$

Subject Date

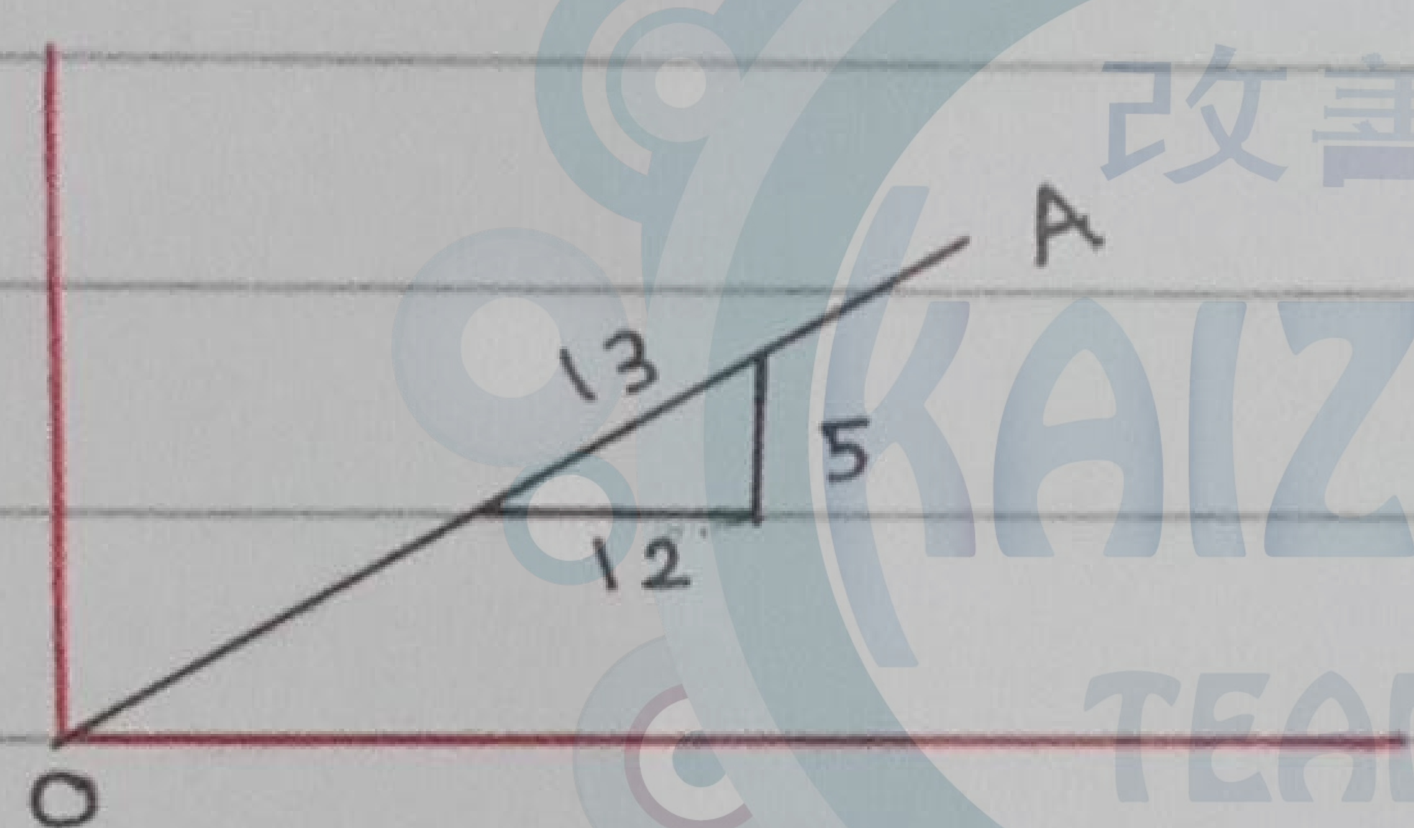
projection :-

- projection $\vec{F_1}$ parallel $\vec{F_2}$:-

$F_{\parallel} = \vec{F_1} \cdot \hat{F_2} \longrightarrow$ Scalar $\qquad \vec{F_{\parallel}} = F_{\parallel}(\hat{F_2}) \longrightarrow$ Vector

- perpendicular :-

$F_{\perp} = \sqrt{F_1^2 - F_{\parallel}^2} \longrightarrow$ Scalar $\qquad \vec{F_{\perp}} = \vec{F_1} - \vec{F_{\parallel}}$



$\longrightarrow u_{OA} = \left\langle \frac{12}{13}, \frac{5}{13} \right\rangle$

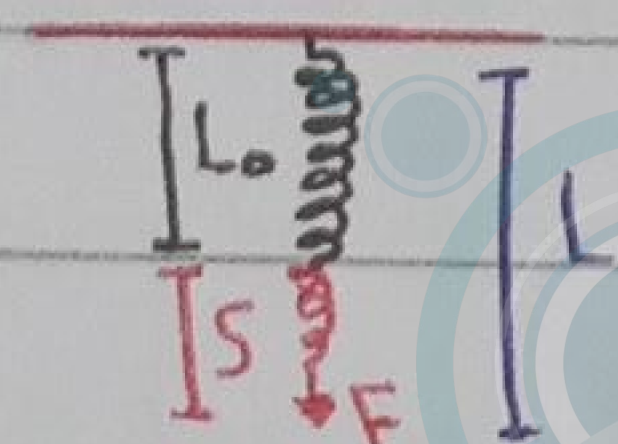
Subject Chapter 3 Date No.

⊛ Equilibrium of particles :-

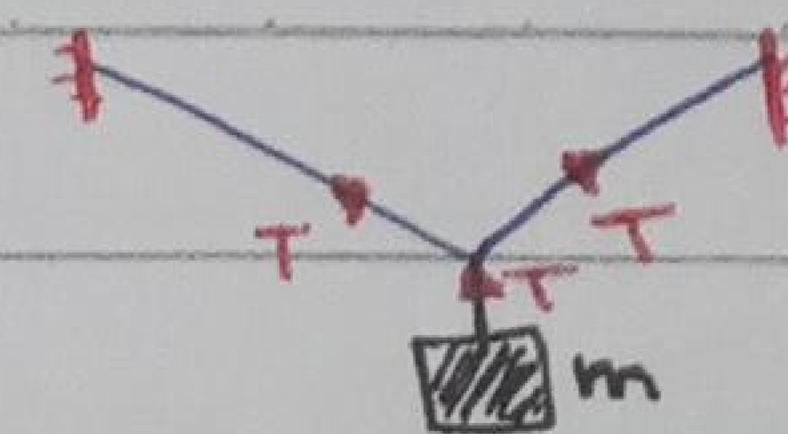
1. Spring

$F = K\,S$

$S = L - L_0$



2. cables



⊛ F.B.D (مخطط الجسم الحر) : يفصل الجسم عن المحيط ويبين جميع القوى الواقعة عليه.

⊛ Equilibrium ⟶ $\sum F = 0$ $\sum F_x = 0$ $\sum F_y = 0$ $\sum F_z = 0$

Subject: Chapter 4

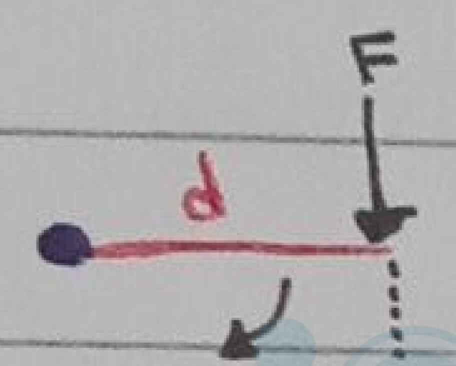
Force - Resultant system :-

Moment in 2D :- (around point)

Moment :- مقدار دوران القوى حول محور.

- $M_o = F . \boxed{d}$ → المسافة العامودية بين [redacted] النقطة وخط عمل القوة

[redacted]



→ clock wise (-z)

- [M] : N.m

- الاصابع مع [redacted] القوة و باطن اليد مع d والابهام لاتجاه العزم

- Several forses :- [redacted]

$M_R =$ [redacted] $\sum F$(same direction) $- \sum F$(same direction)

- (M) can't be negative

- $M_R = \sum M$
  - → (+M) ✓ (c.c.w +)
  - → (− M) نعكس الفرض (c.c.w +) → (c.w +)

- اذا رأينا قوة مائلة نحللها.

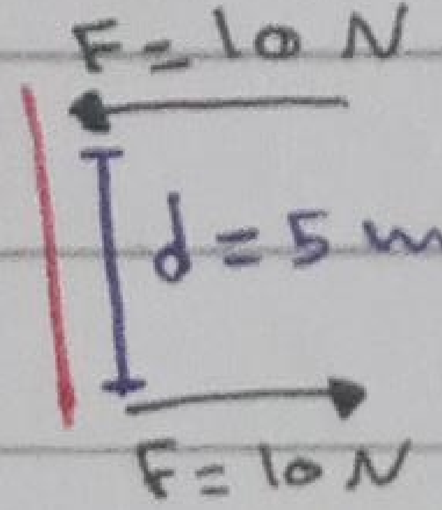
Moment in 3D :- (around point)

- $\vec{M_o} = \boxed{\vec{r}} \times \vec{F}$
  - → position vector between rotation axis and any point [redacted] in Force line.

- لا يلزم اتجاه لأنه متجه اصلاً.

$\vec{F} = \hat{AB} * F$

- Couple moment :-



$M = F \times d$

$= 10 \times 5 = 50$ N.m

⊛ Replacement questions :-

① اول حالة : لما يقلنا Replace ؛ equivalent force/Resultant force + moment

ويقلنا نطلع عند نقطة معينة :

- نجد ΣF_x و ΣF_y و $\sqrt{F_x^2 + F_y^2} = |F|$ (direction must be known)

- نجد زاوية F_R مع محور $x = \tan^{-1}\left(\frac{F_y}{F_x}\right)$ ← لا نفتح F_x او F_y سوالب.

- اوجد ال moment حول النقطة المطلوبة.

② ثاني حالة : لما يقلنا جد x intercept و y intercept :

- $y = \frac{M_R}{F_x}$ ، $x = \frac{M_R}{F_y}$ و نقوم بالخطوات السابقة.

- لا نفتح سوالب لـ F_y / F_x لأن المسافة موجبة.

③ ثالث حالة : لما يقلك عن ال M_R انها تمر بنقطة، يعني M عند هاي النقطة = 0

$M_o = u_o \cdot (\vec{r} \times \vec{F})$

- u_o → the unit vector for the required axis
- M_o → the moment around an axis.

- Ex (around x axis)

$$\begin{vmatrix} 1 & 0 & 0 \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

⊛ Couple moment in 3d : $M = \vec{d} \times \vec{F}$

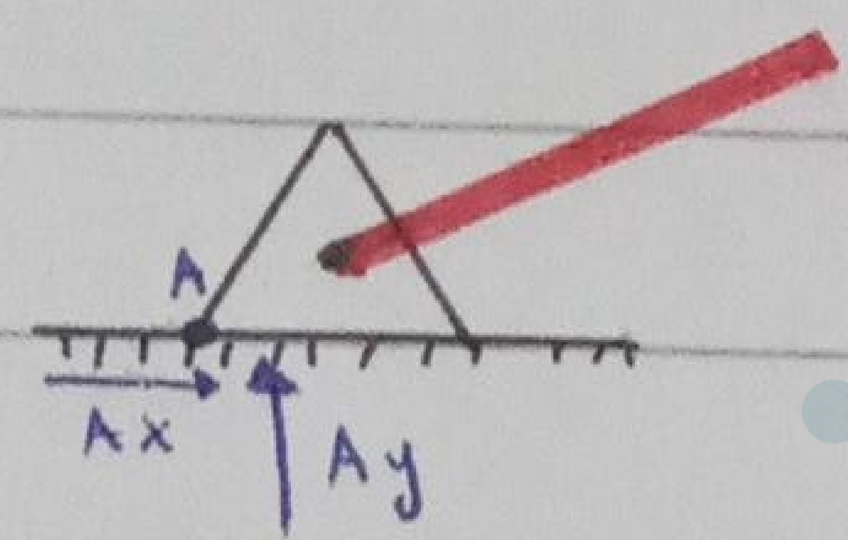
$\vec{d}$ → displacement vector between F_1 and F_2.

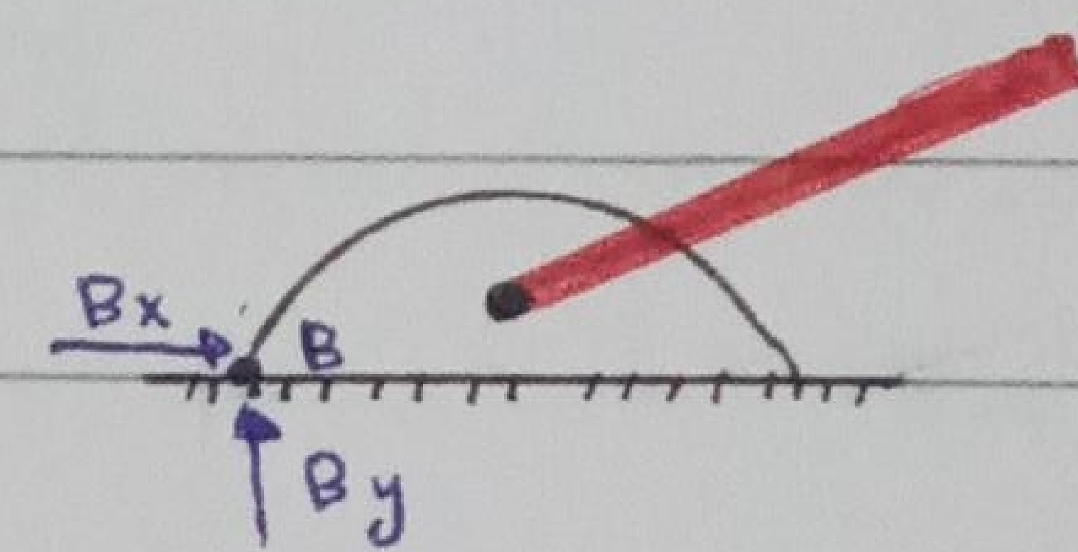
Chapter 5

- Equillibrium of riged bodies :- $\Sigma F = 0$ $\Sigma M = 0$

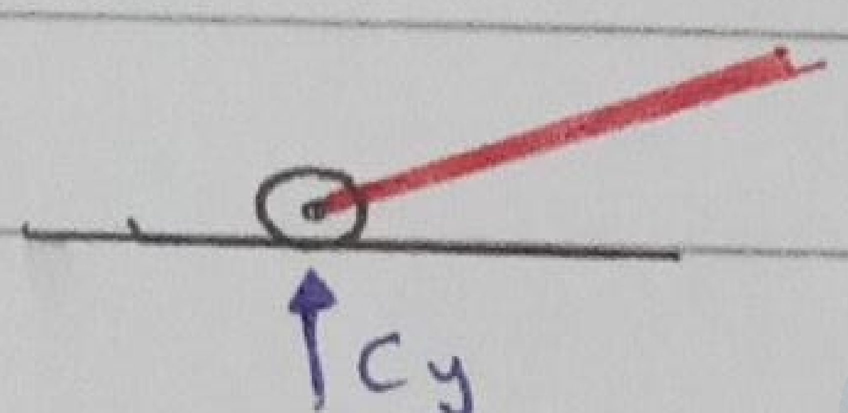
- Support وسيلة تطبق على نظام ما لتحقيق وضع الاتزان

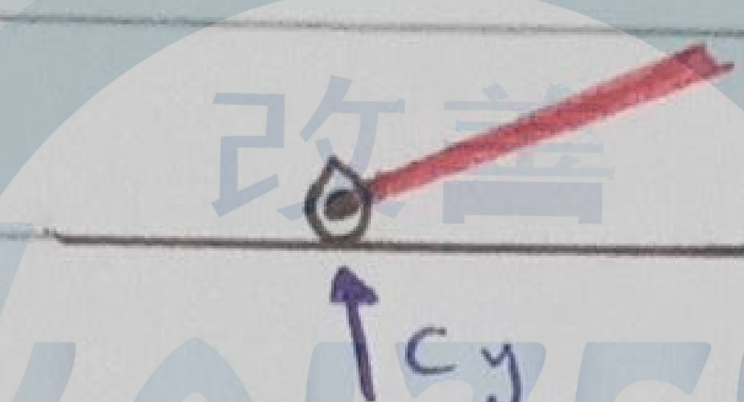
① Pin / Hinge :-

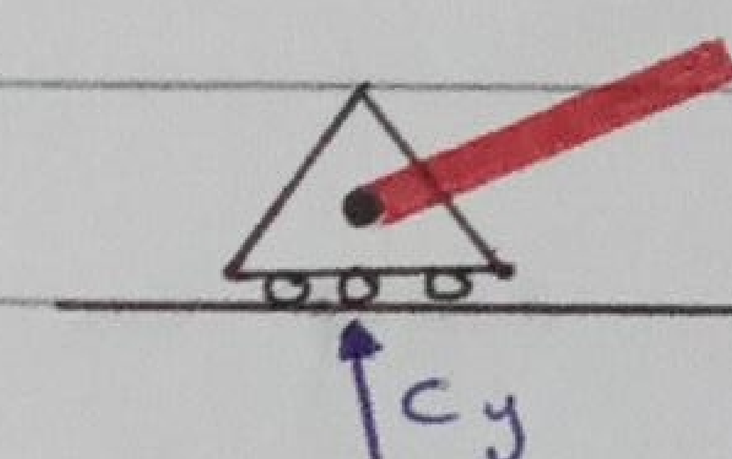




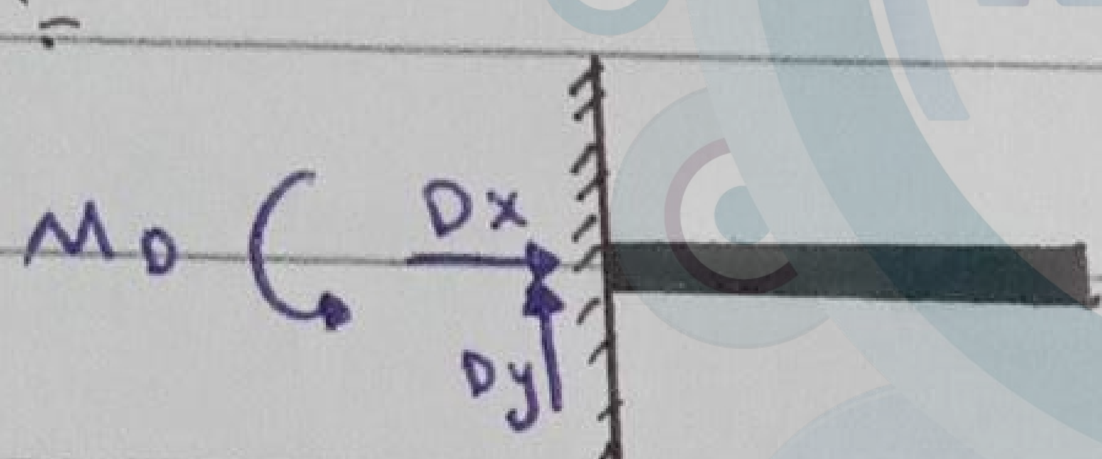
② Roller / Rocker :-







③ Fixed :-



- خطوات الحل :-

① نرسم F.B.D

② نبدل ال support بال reactions تابعتها.

③ نحلل القوى المائلة (ان وجدت).

④ اطبق معادلات الاتزان $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_o = 0$

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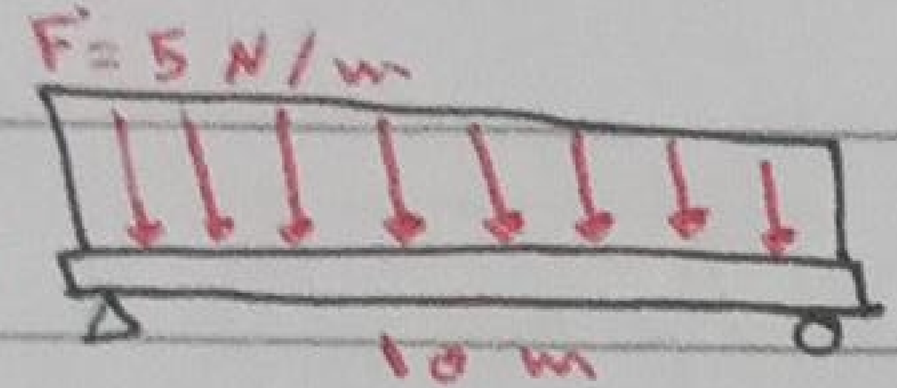
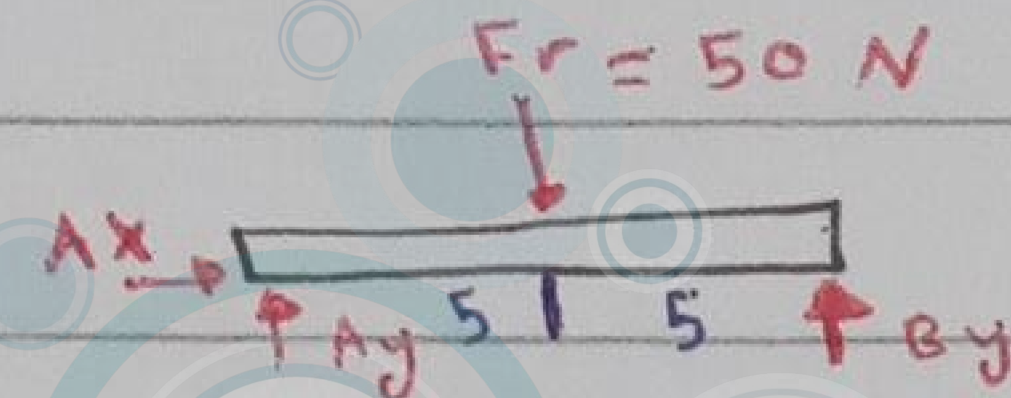
⊛ Distributed load :- الأحمال الموزعة

- دائماً عشان اوجد ال reaction لازم احول dis load ← cons load

①

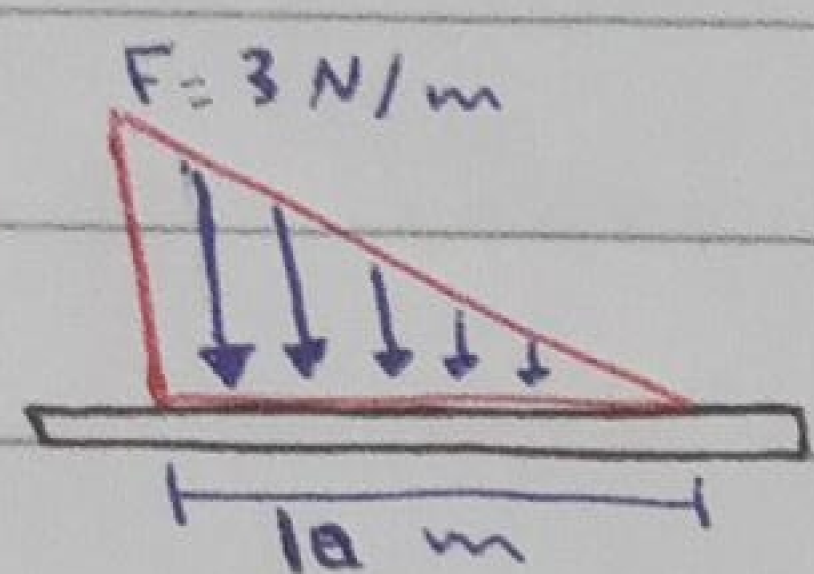


→

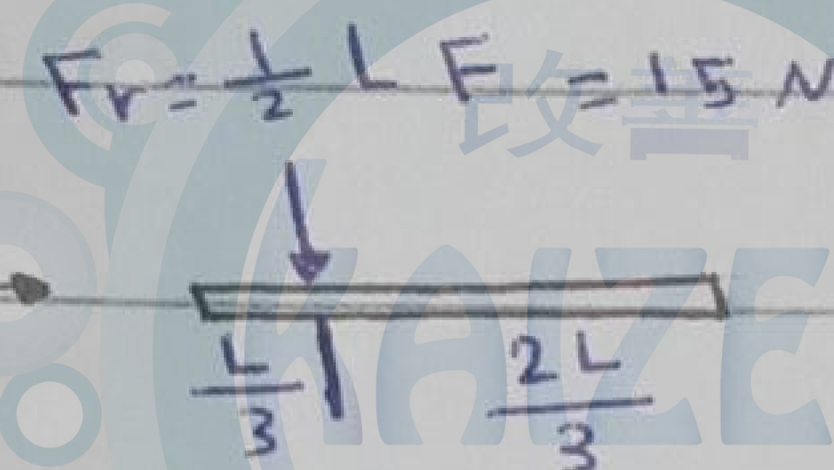


$Fr = F * L = 5 * 10 = 50\ N$

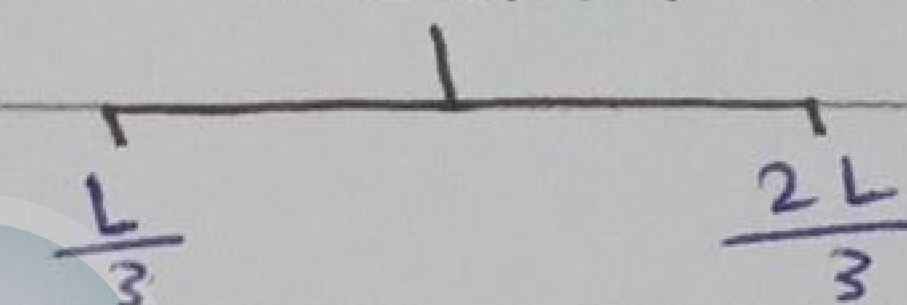
②



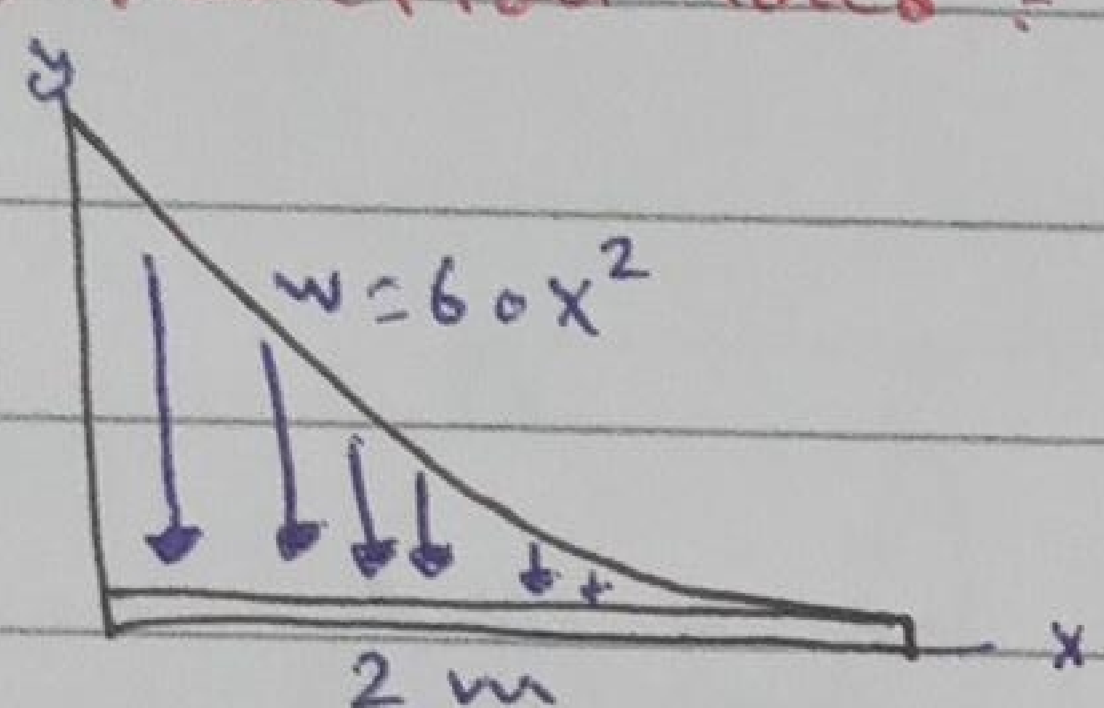
→

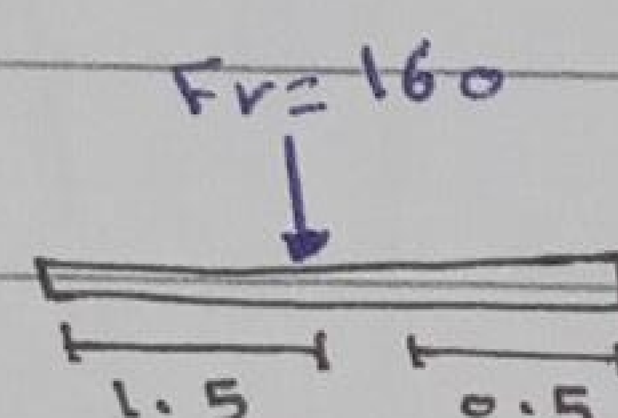


قاعدة ال Location



③ Function load :-





$$Fr = \int_0^2 60x^2\,dx = 60\frac{x^3}{3}\Big|_0^2$$

$Fr = 160$

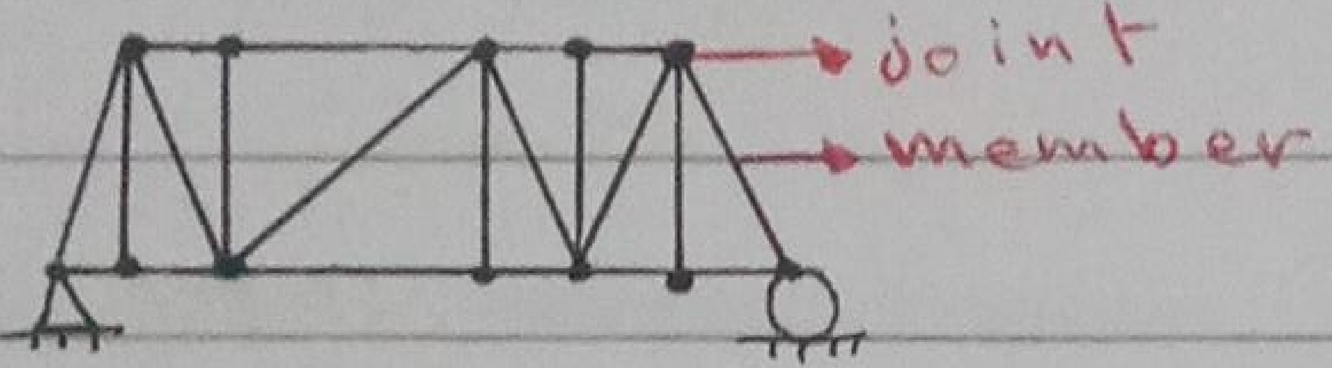
location: ~~[illegible]~~ time x on both side

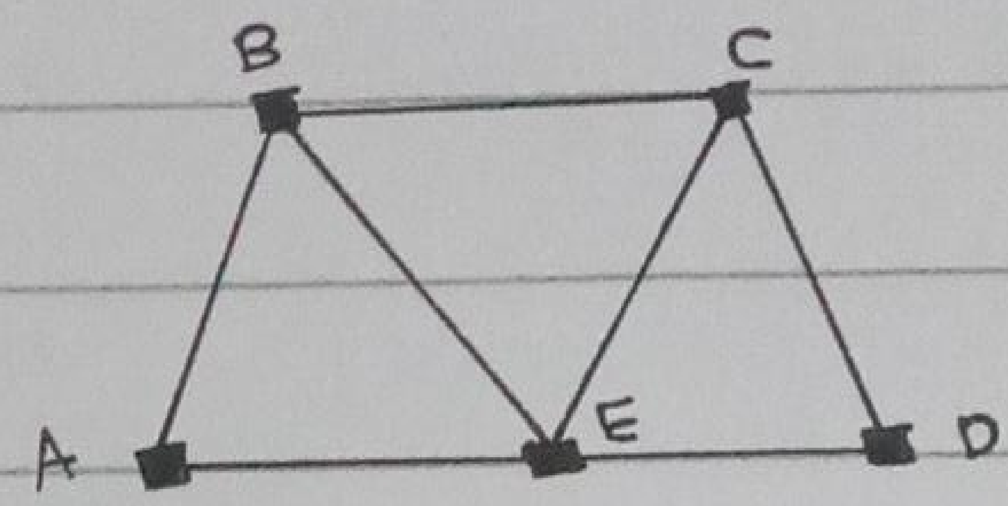
$$160x = \int_0^2 60x^3\,dx$$

$x = 1.5$

⊛ Trusses analysis.

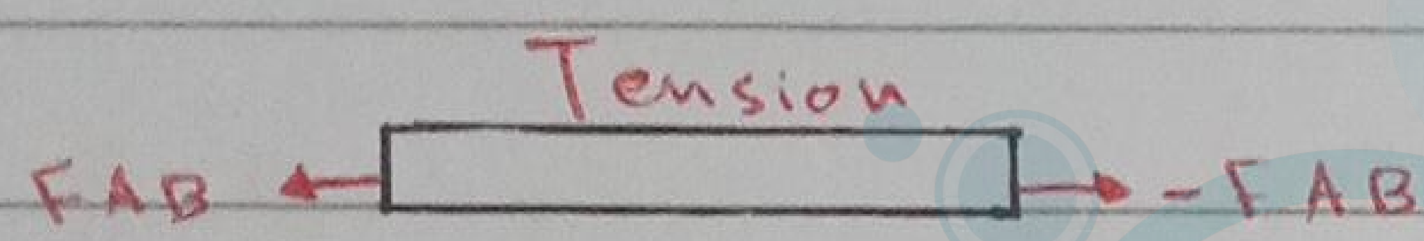
① Method of joints:-

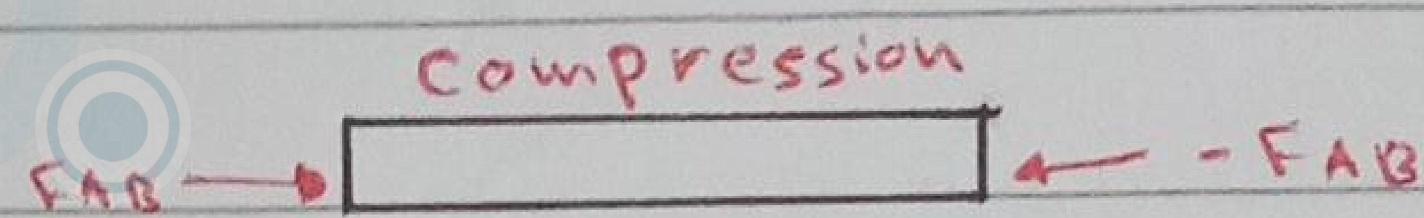


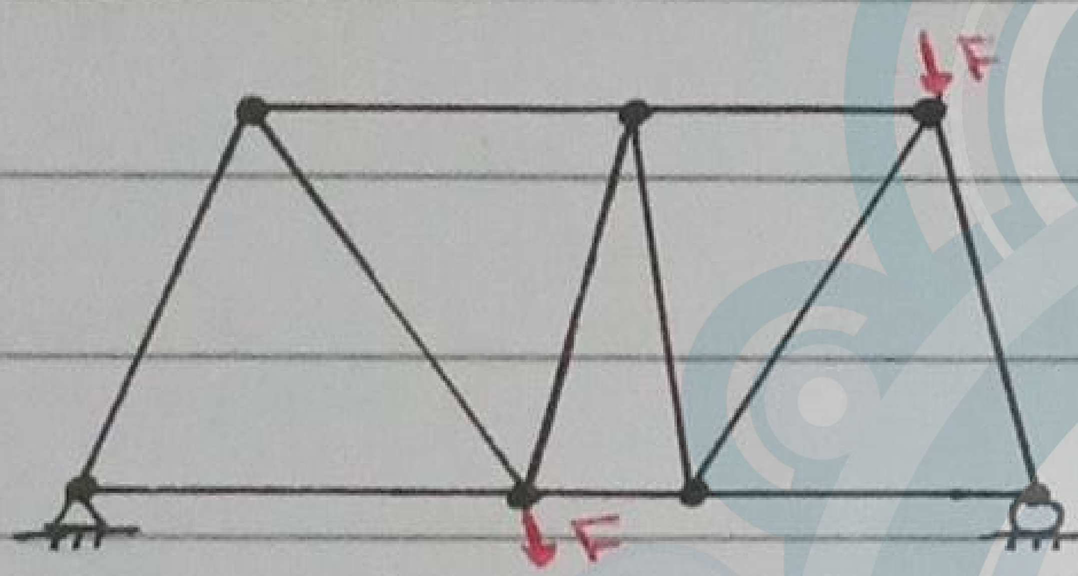


- All forces are applied at joints only.

- Each member are 2 force member.

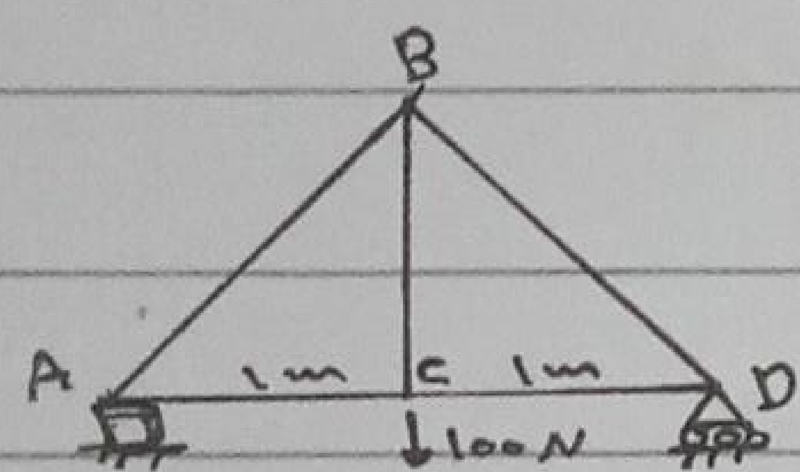




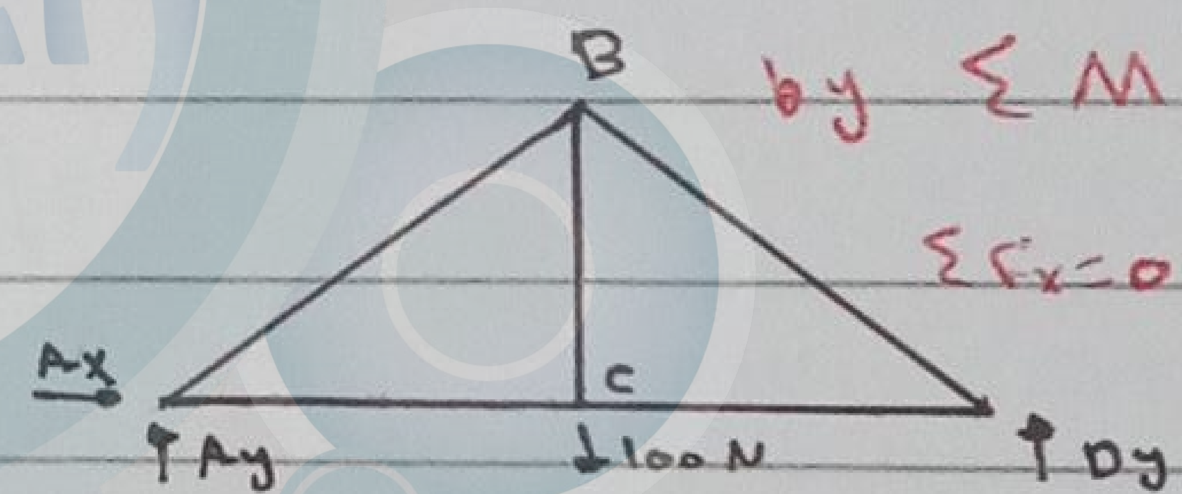


- load and reaction are external.
- Force in members are internal. (T, C)

- How to solve with joint method?

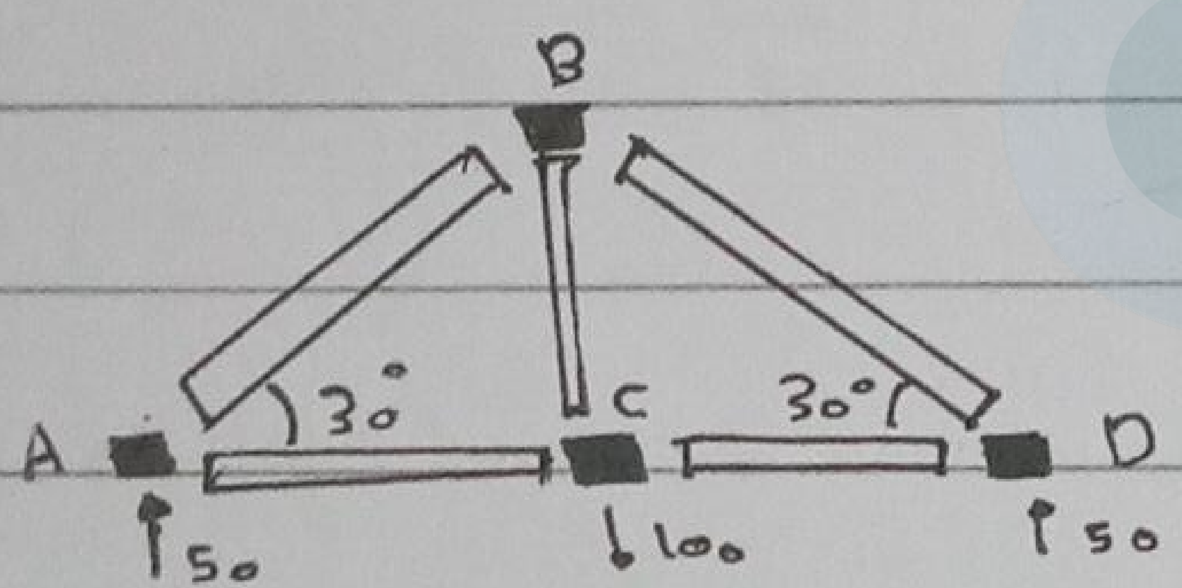


FBD, then find reactions



by $\Sigma M = 0$

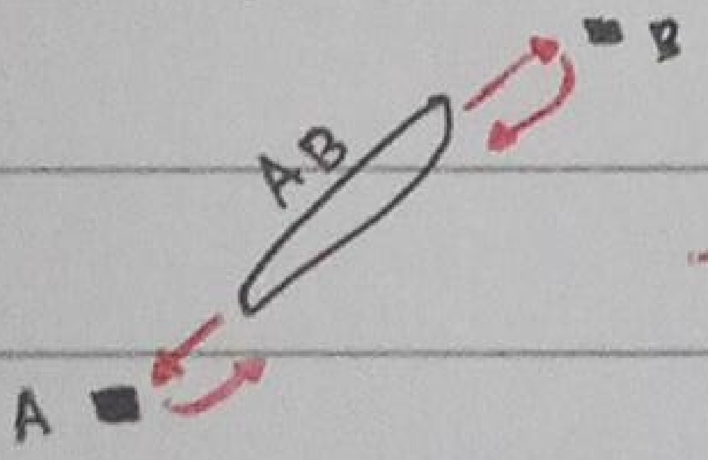
$\Sigma F_x = 0$ $\Sigma F_y = 0$



Joint A:

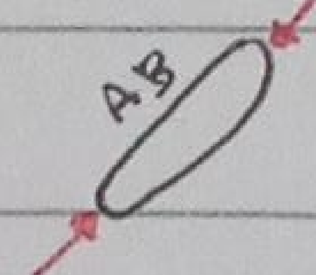
We choosed these direction (logically) to cancel each other (equilibrium)

- To determine the force member by action & reaction, we decide if T or C according to the action.



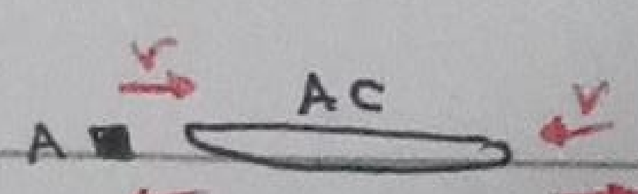
(reaction)

SO



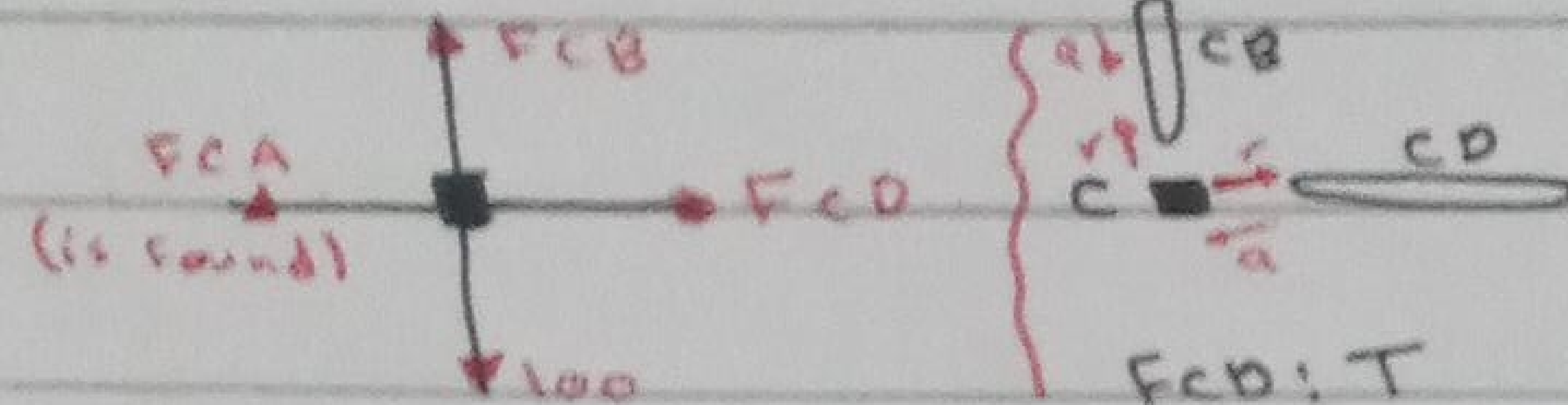
(action)

So, FAB is (C)



(T)

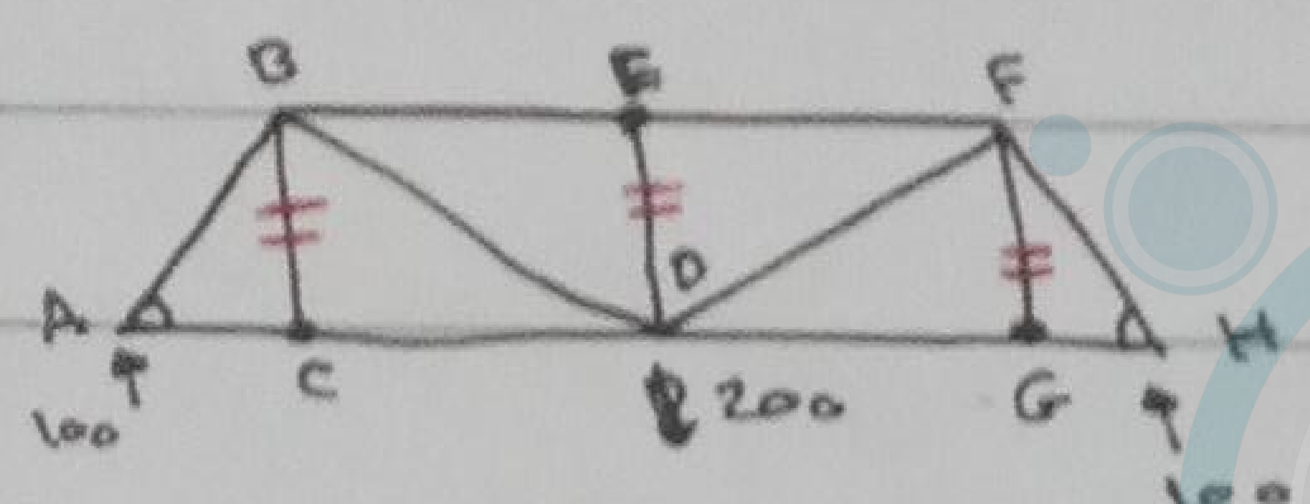
Joint C



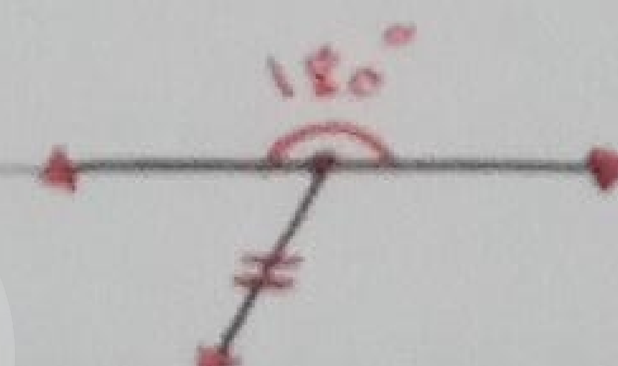
FCD : T   FCB : T

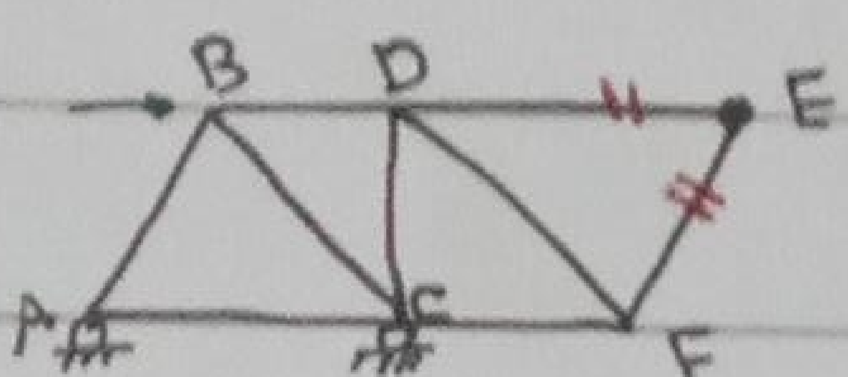
Solve by $\Sigma F_x = 0$, $\Sigma F_y = 0$ E.O.E , Finally solve for joint D.

~~Zero-force method~~ ② Zero-force member :-

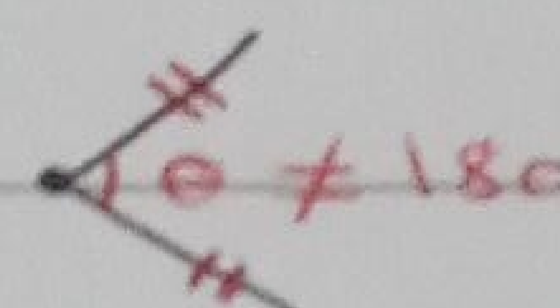


Case 1





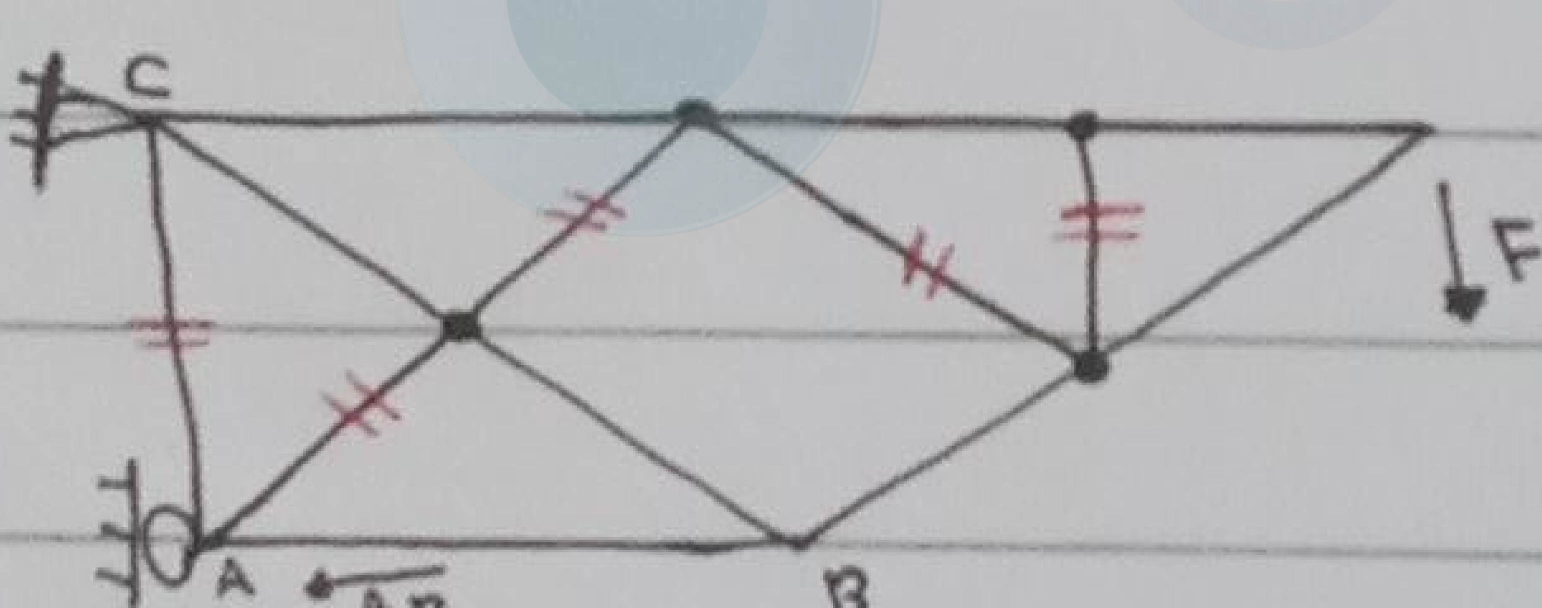
Case 2



Case 3

if I know that on member is Z.F. So the other one as shown in figure is also Z.F.

- Determine Z.F.M :-



AX , So AB will be ⟵ , then AC will be Z.F.

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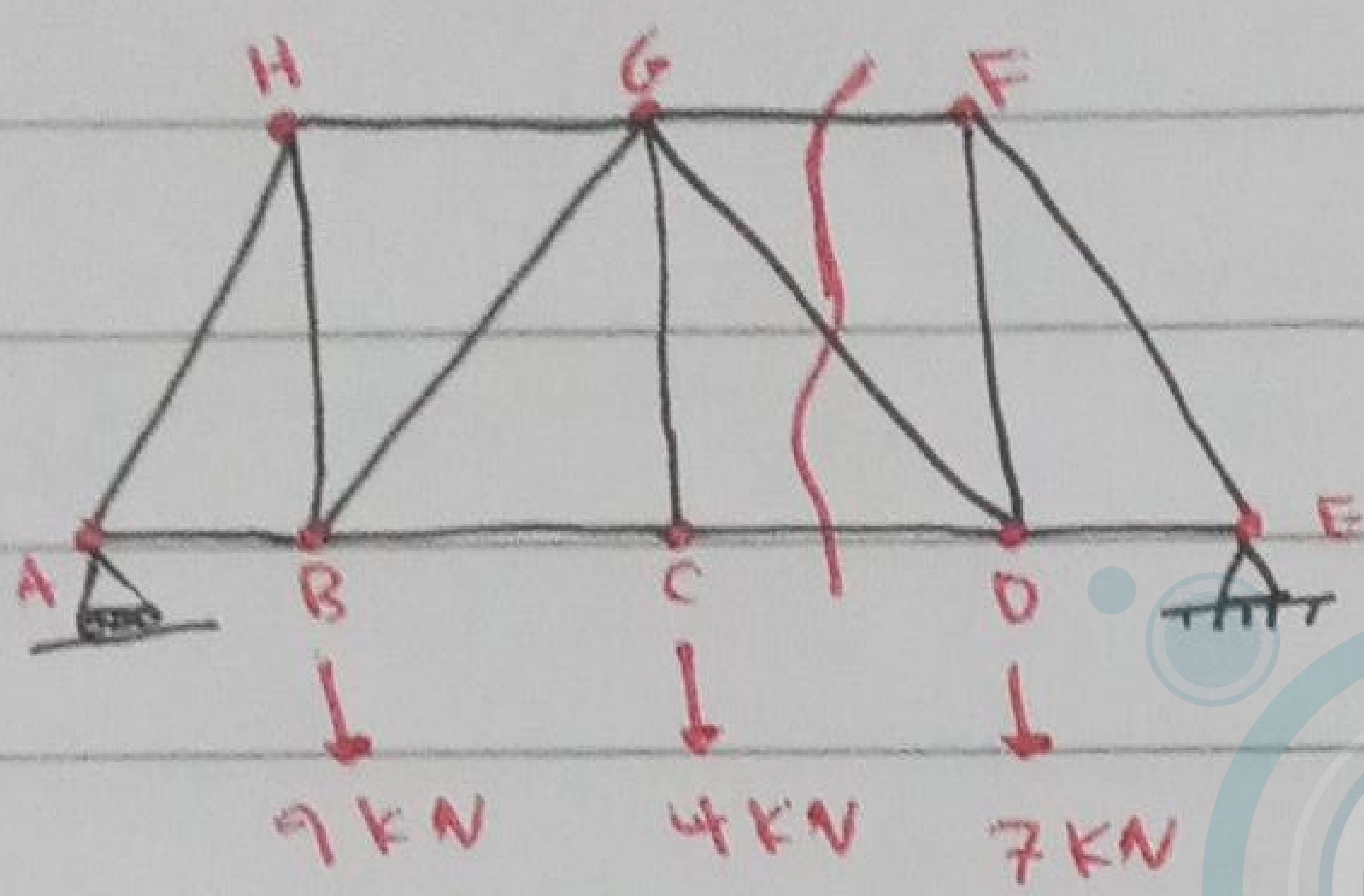
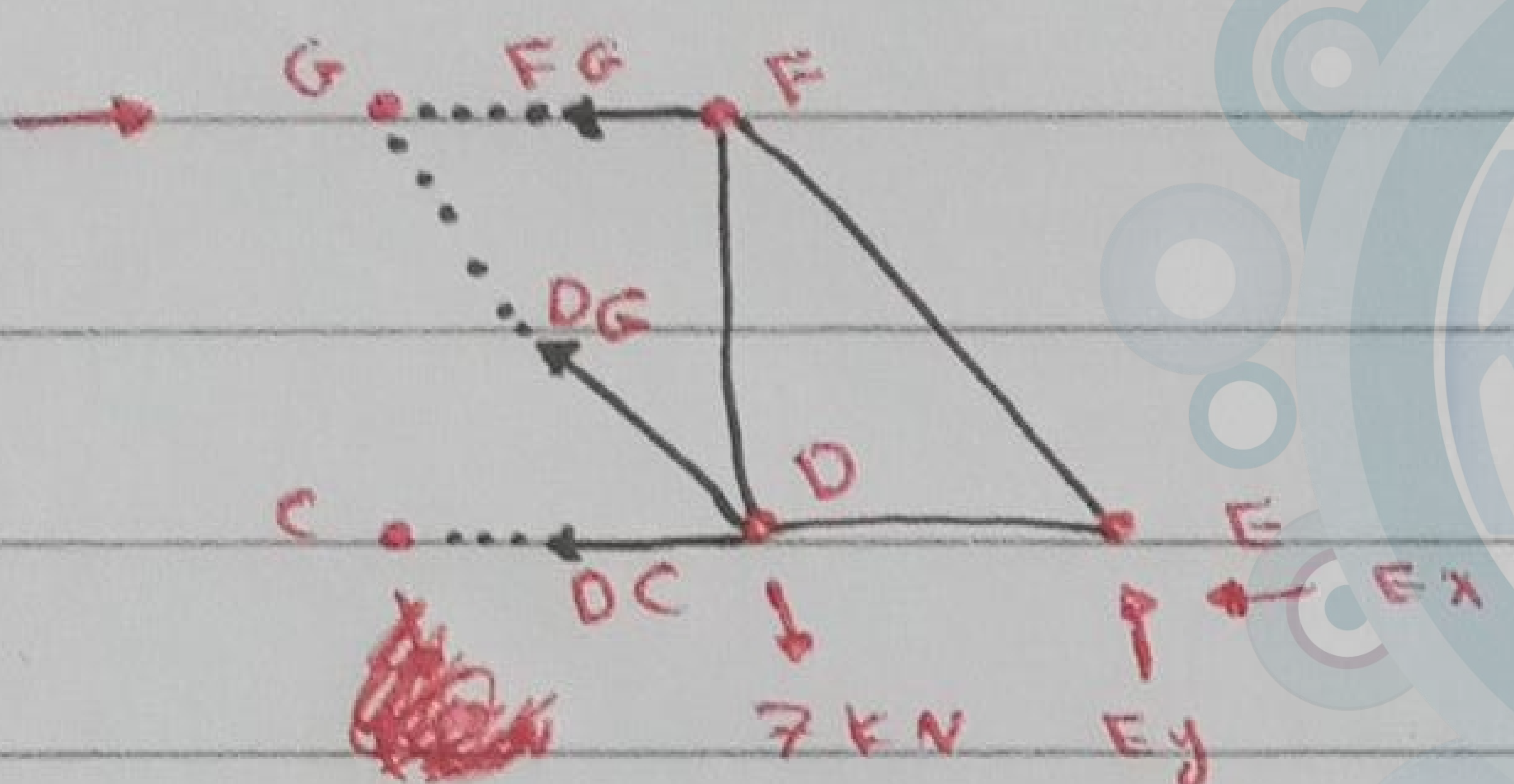
③ Method of section :-



Ex. to find FG, DG, DC, we will cut between them just like:-



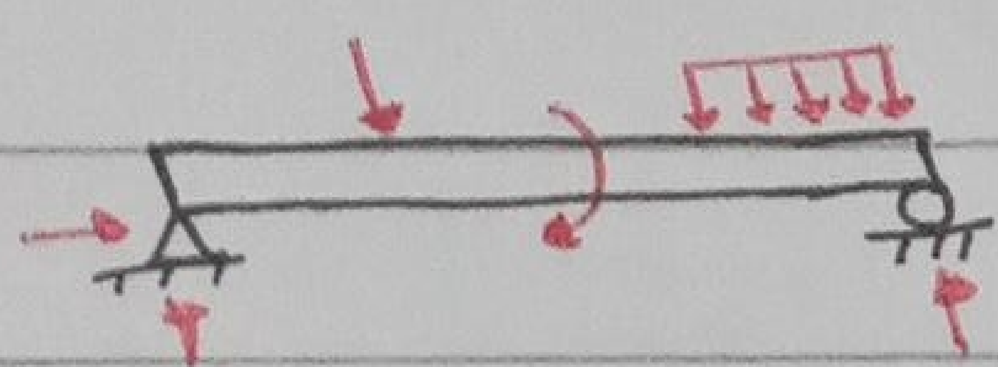
① Find the reactions.

② you can choose any of two parts, here if you choose the other part just take the opposite direction.

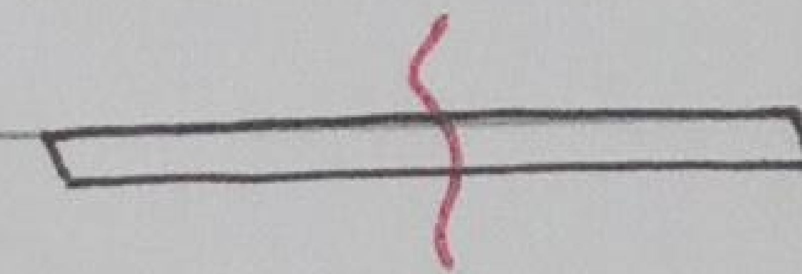
③ Solve by E.O.E, you can here take the moment around D to eliminate DG and DC and 7 KN then you find FG, then by $\Sigma F_y = 0$ find DG then DC by $\Sigma F_x = 0$.

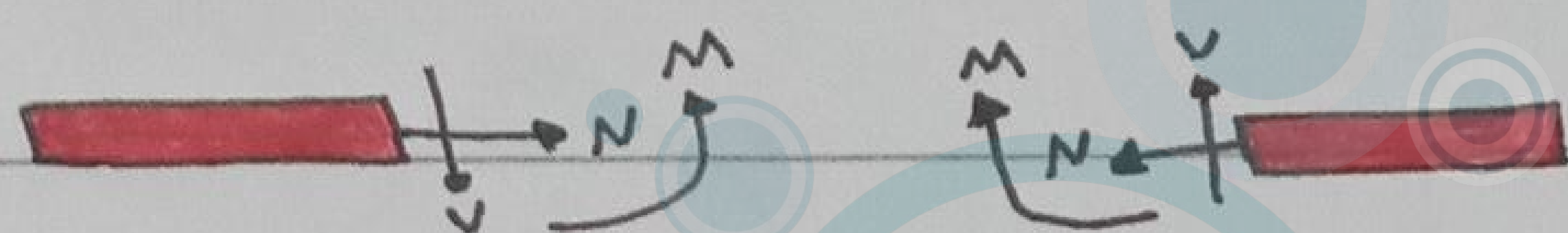
Subject Chapter 7 Date No.

⊛ Internal Forces :-



when we cut the beam





N: Normal force

V: Shear force

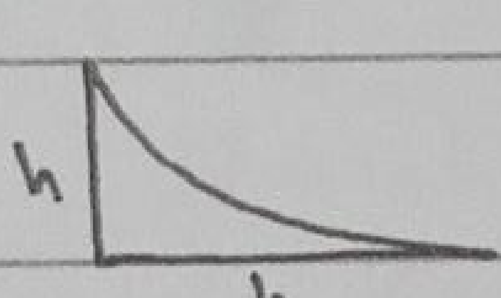
M: B. Moment

- to find internal forces:-

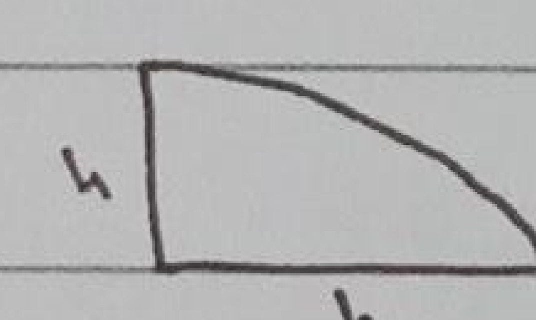
First Draw F.B.D then find the reaction in the supports then take the esiest section and apply EOE.

⊛ Moment Shear diagrams :-

- Integration → Area
- Lines integrates:-

- 

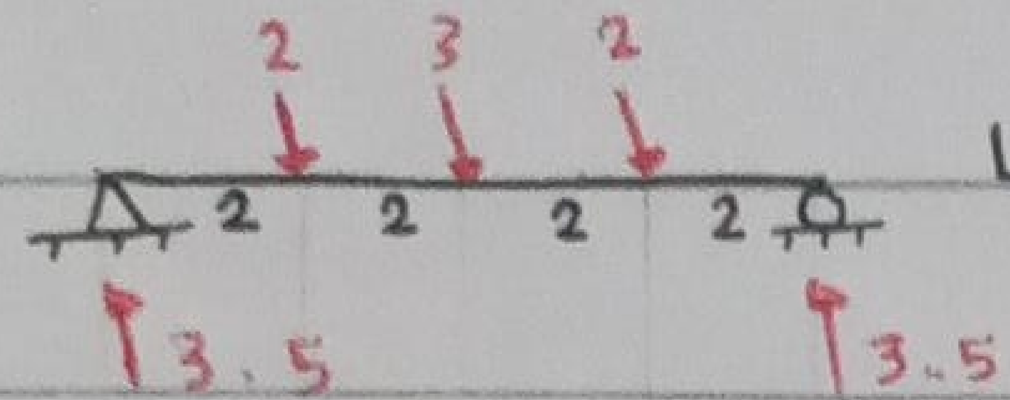
Area $= \frac{1}{3} bh$



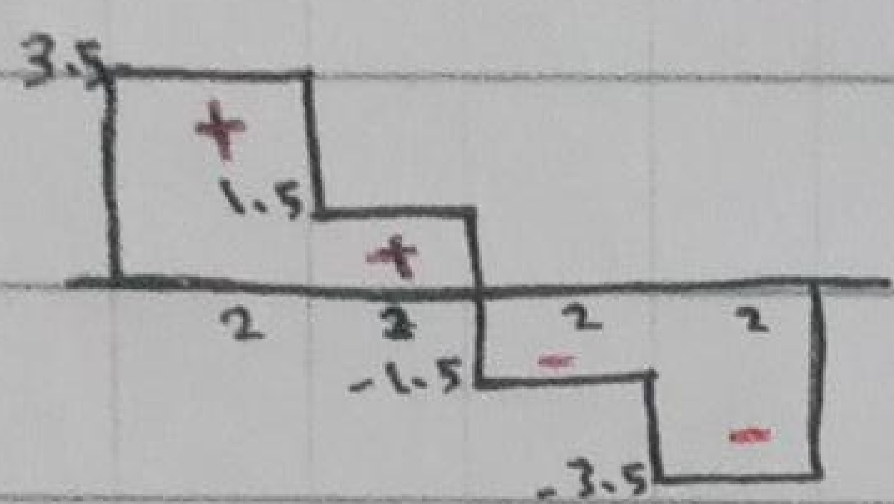
Area $= \frac{2}{3} bh$

- Find the reactions and put them on load dig. then draw Sear dig. under it, then Moment dig. under it.

Ex.

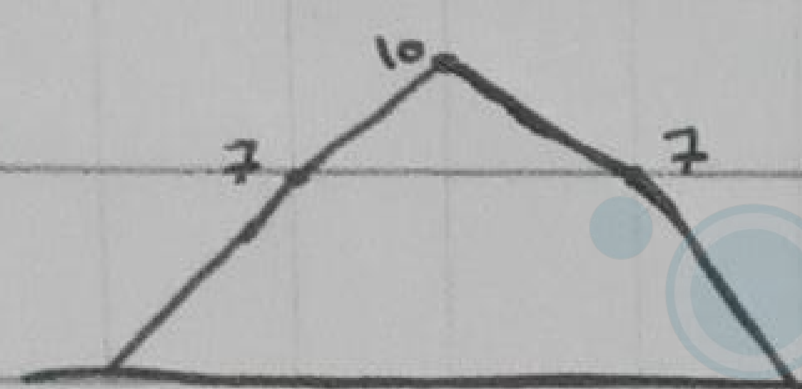


L.D

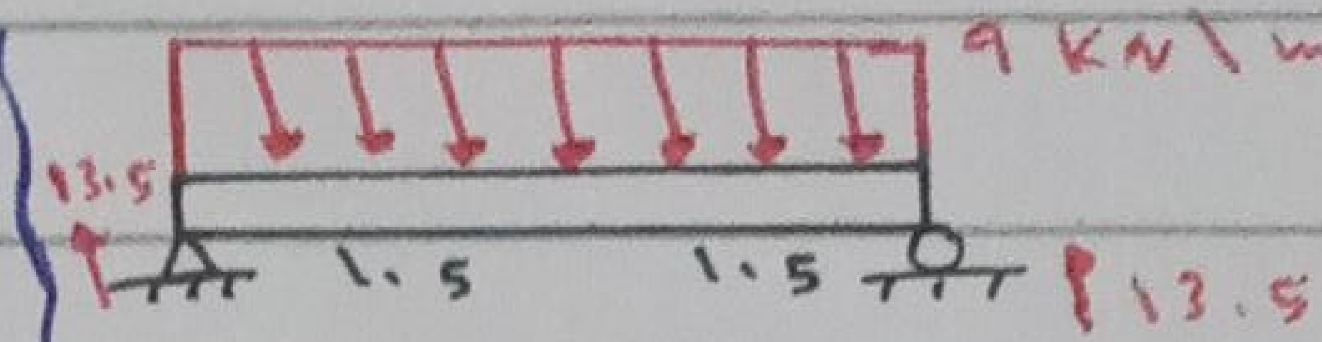


V.D

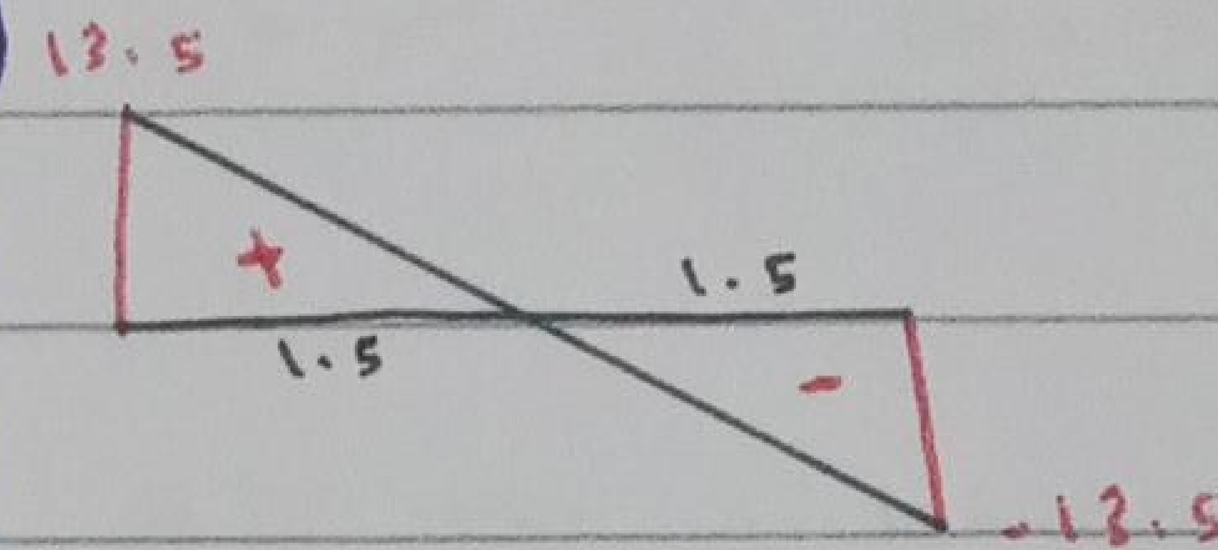
Area of V.D



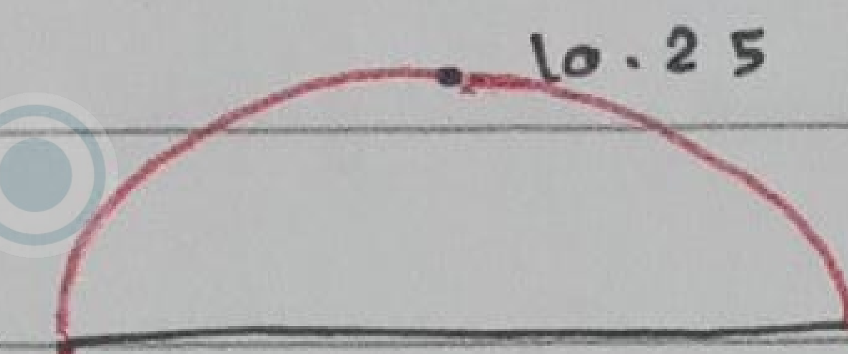
M.D



L.D



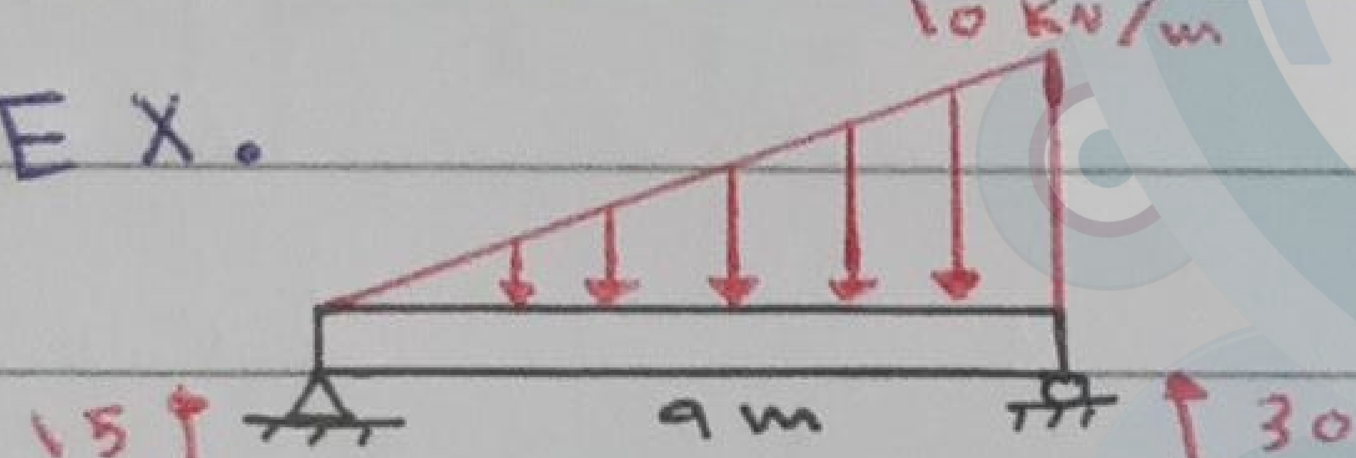
V.D

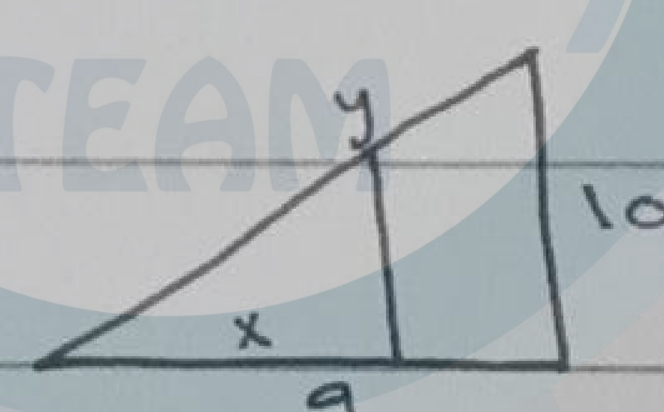


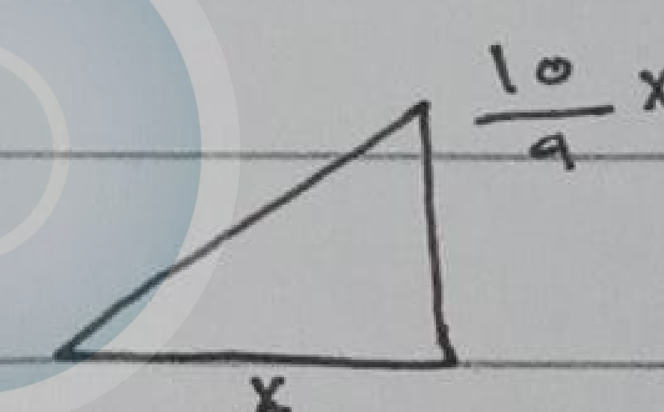
M.D

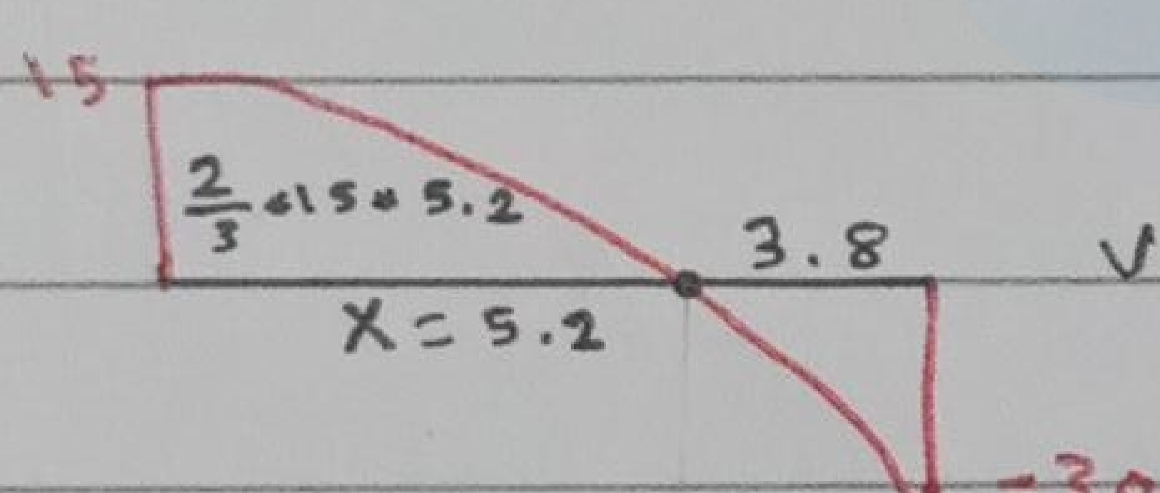
* To ~~[crossed out]~~ draw curves:  if the line is from little to big so from slow to fast  if it from big to small so from fast to slow 

Ex.

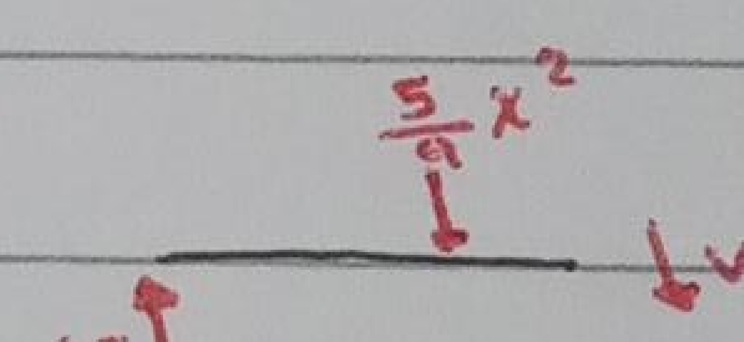








V.D



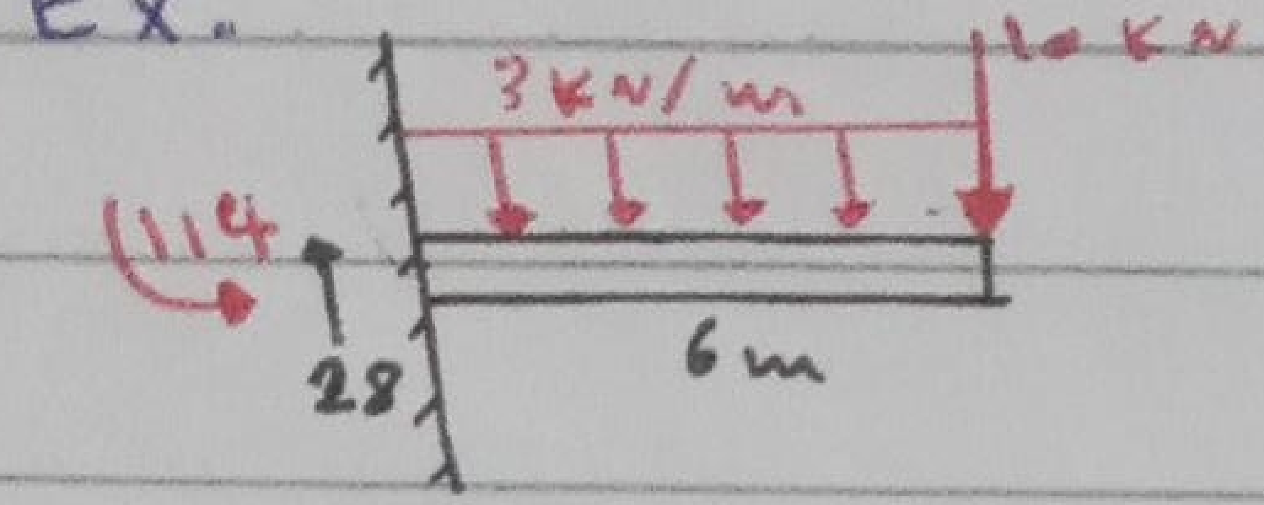
$V = 0$

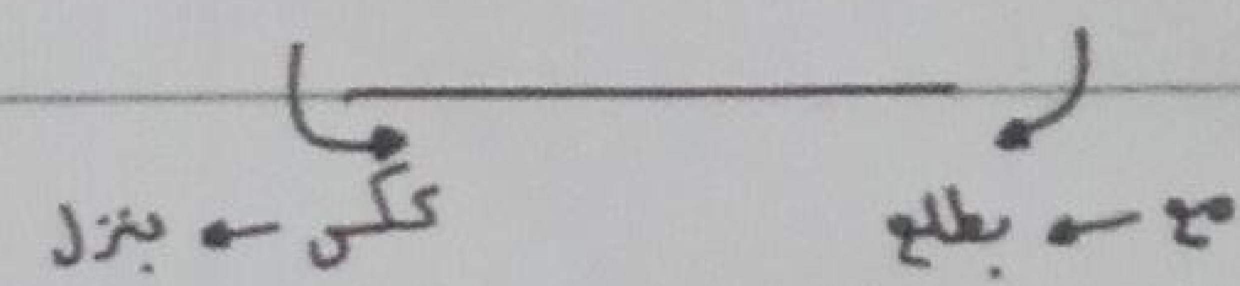
$\sum F_y = 0 \qquad 15 - \frac{5}{9}x^2 - V = 0$

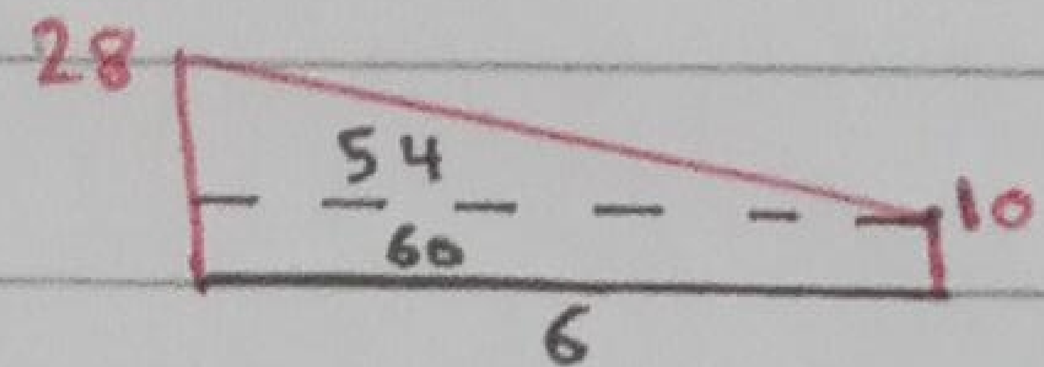
$15 = \frac{5}{9}x^2 \qquad \boxed{X = 5.2}$

M.D

Ex.

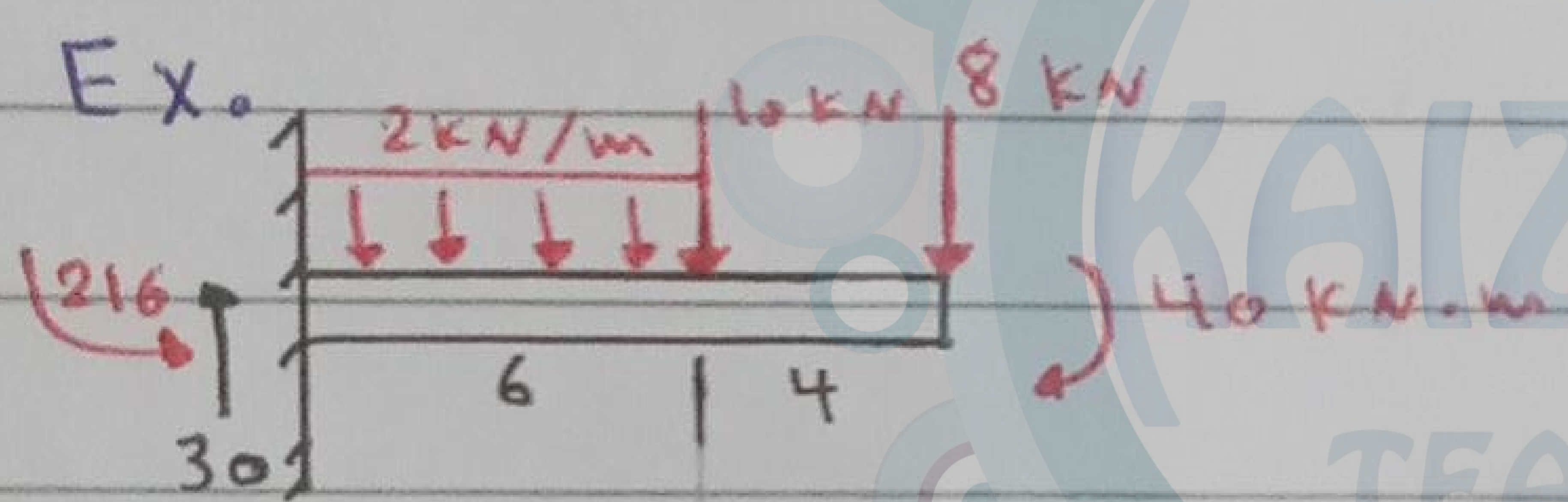


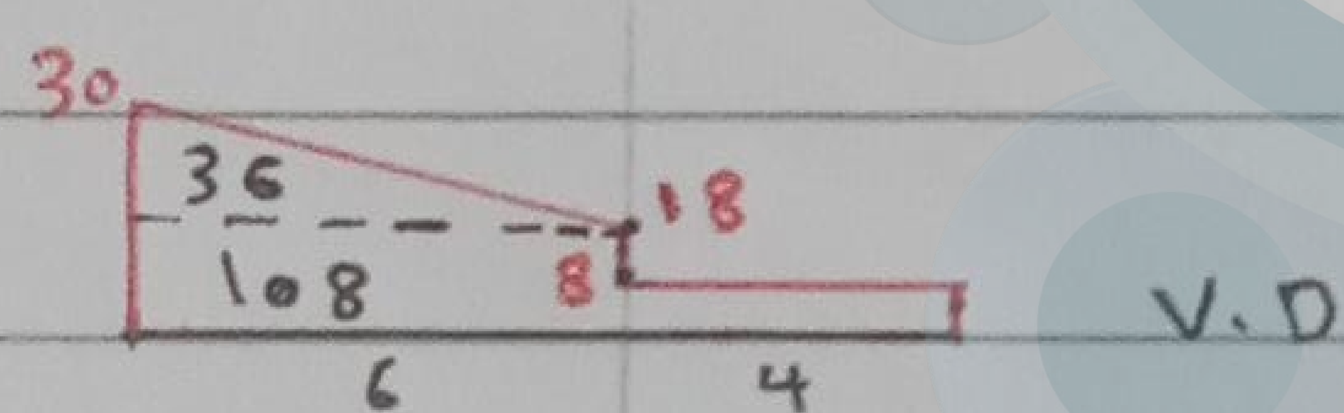


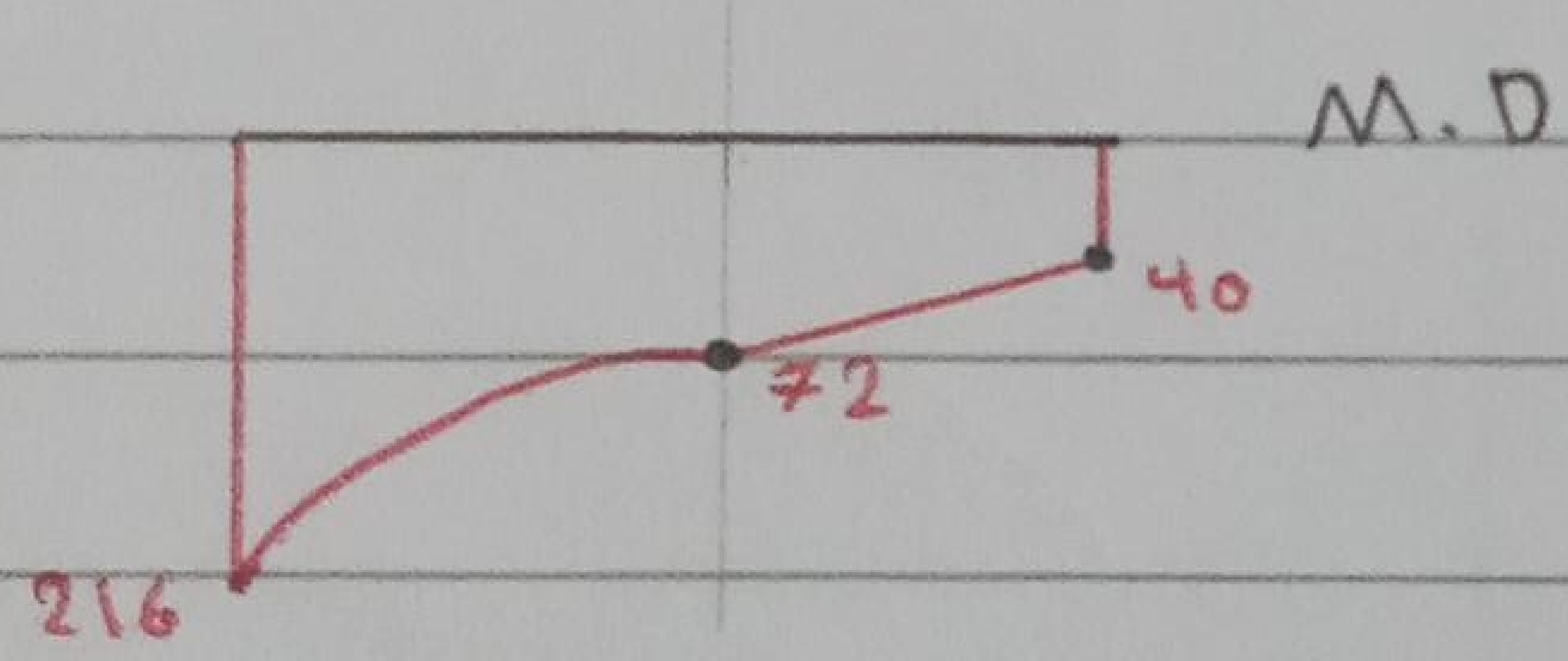


* If the question have an original beam then we apply it in M.D

Ex.







Subject Chapter 12 Date No.

Curvilinear ~~[illegible]~~ motion.

1. Rectangular motion components :-

$S = \sqrt{x^2 + y^2}$

$V = \sqrt{V_x^2 + V_y^2}$

$a = \sqrt{a_x^2 + a_y^2}$

$V_x = \dot{x} = \frac{dx}{dt}$

$a_x = \ddot{x} = \frac{d^2x}{dt^2}$

Same to V_y / a_y.

Projectile motion.

X - dir :

$a_x = 0$

$V_x = \frac{\Delta x}{\Delta t}$

$V_x = V_{x_0}$

$R = V_{x_0} t$ $\quad \Delta x = V_{ix} t$

y - dir :

$V_{fy} = V_{iy} + gt$

$V_{fy}^2 = V_{iy}^2 + 2g\Delta y$

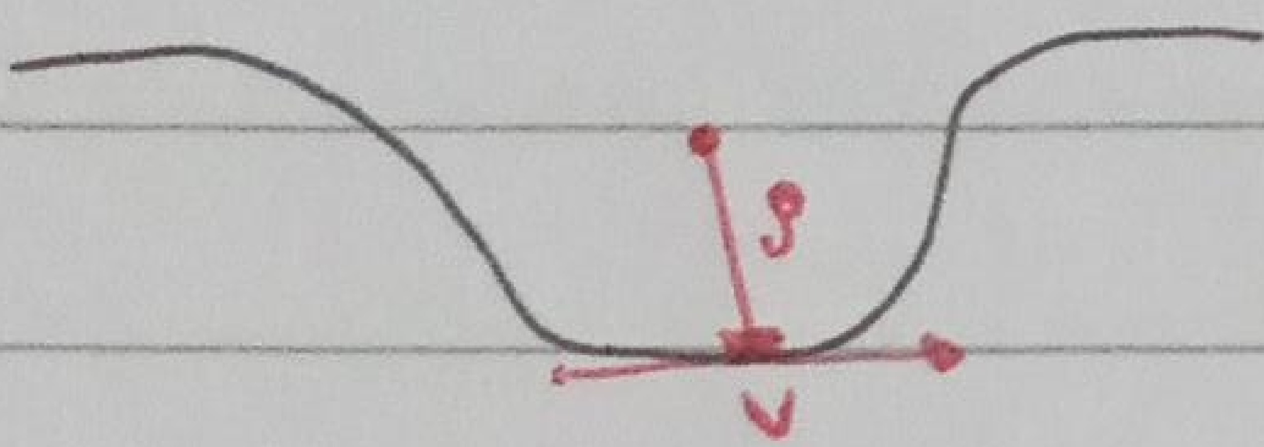
$\Delta y = V_{iy} t + \frac{1}{2} g t^2$

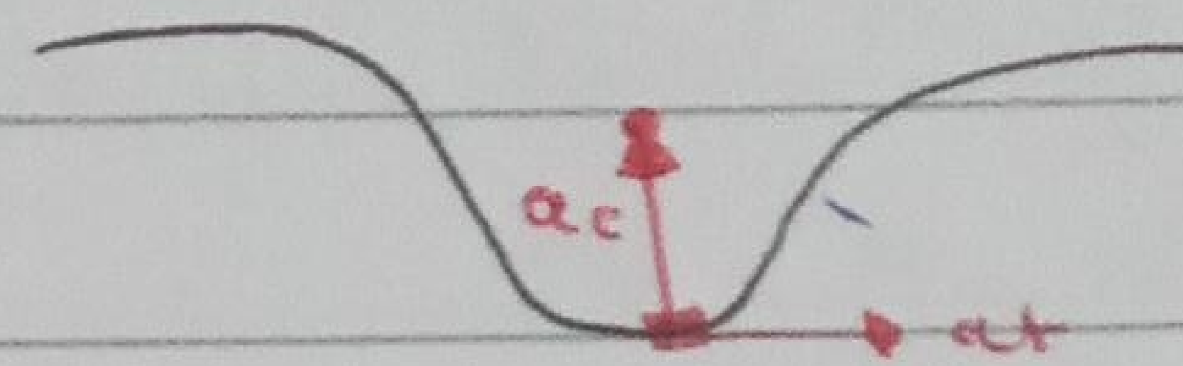
- $g = -9.81$
- max height : $V_{fy} = 0$
- with constant acc.

- acceleration with function of position :- (variable acc)

$a\,ds = v\,dv$

Curvalinear motion, Normal-Tangent Component.





- ρ: radius
- a_t: as the same direction of velocity

$$a_t = \frac{dV}{dt} \quad \Big\{ \quad a_t\, ds = V\, dV \quad \Big\{ \quad a_c = \frac{V^2}{\rho} \quad \Big\{ \quad a = \sqrt{a_t^2 + a_c^2}$$

for a_t we can use Eqs of motion:

- a_t is constant.

$$\Delta S = V_0 t + \frac{1}{2} a_t t^2$$

$$V_f = V_i + a_t t$$

$$V_f^2 = V_i^2 + 2 a_t \Delta S$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$\rho = \frac{\left[1 + (y')^2\right]^{\frac{3}{2}}}{|y''|}$$

- θ between $a, a_c = \tan^{-1} \frac{a_t}{a_c}$

θ between $a, a_t = \tan^{-1} \frac{a_c}{a_t}$

Subject # Chapter 13 Date No.

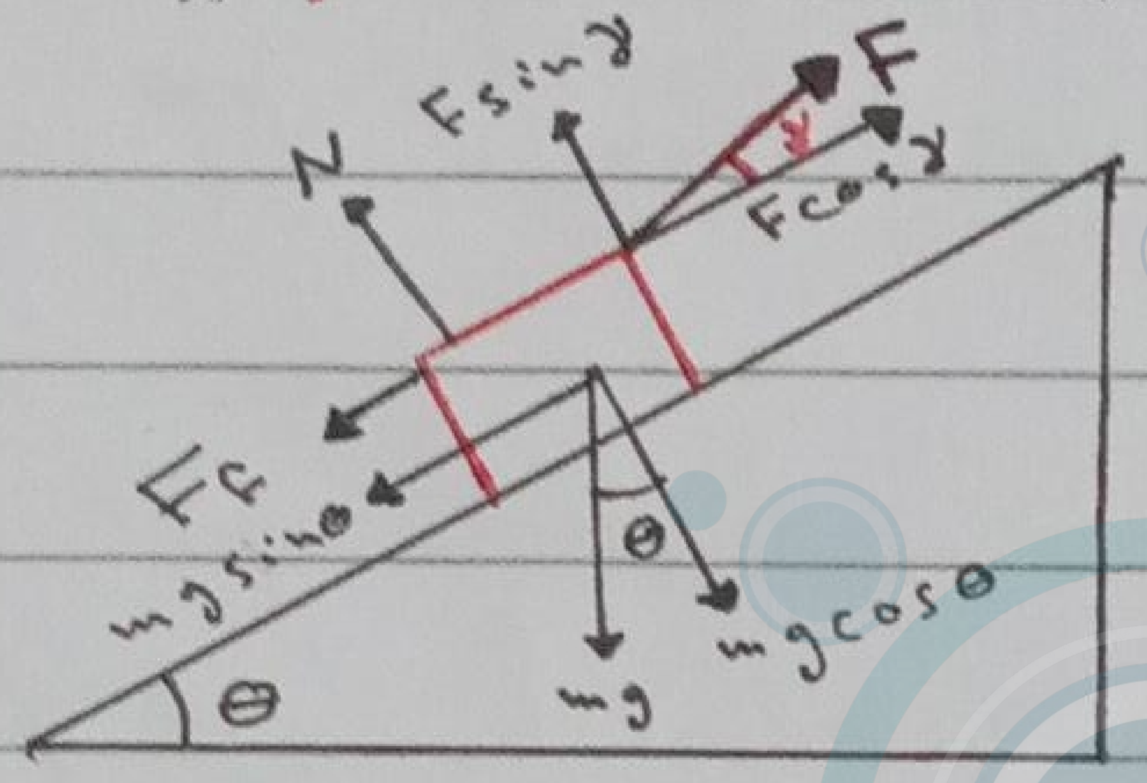
⊛ Newton's second law of motion.

$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_t = ma_t \qquad \Sigma F_c = ma_c$

- Friction force: → Static → $F_f = \mu_s N$
  → dynamic → $F_f = \mu_k N$

- $\mu_s > \mu_k$, and both < 1

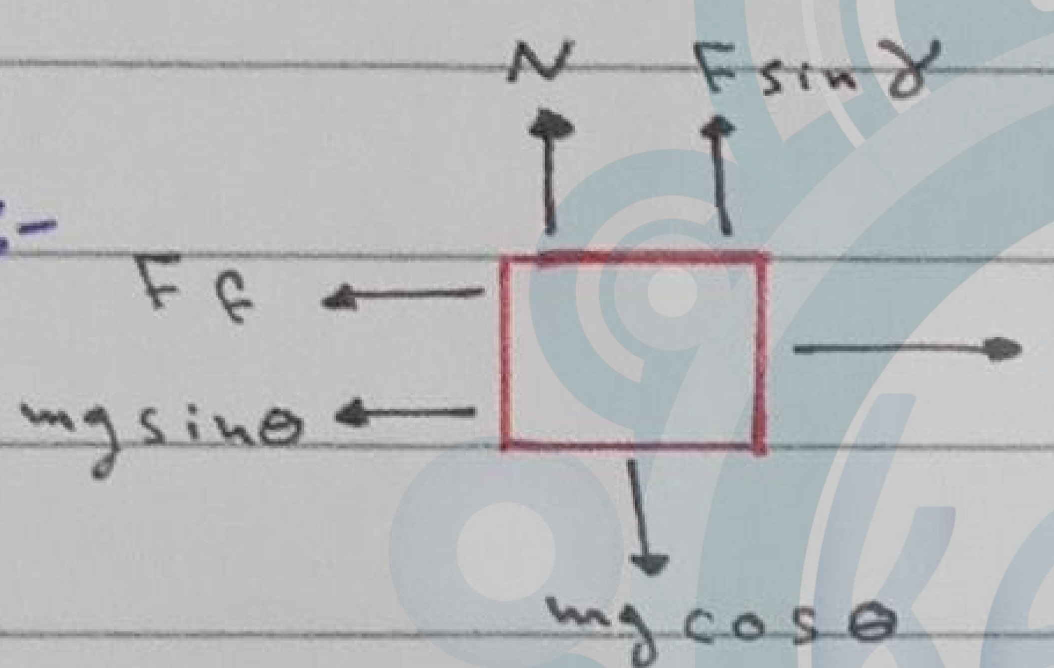
Ex.



N: ⊥ to the surface

Ff: opposite to motion direction

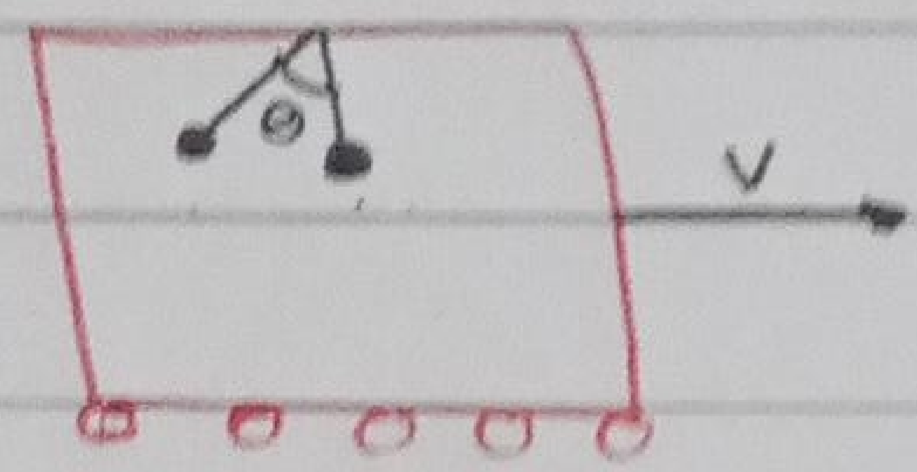
FBD :-

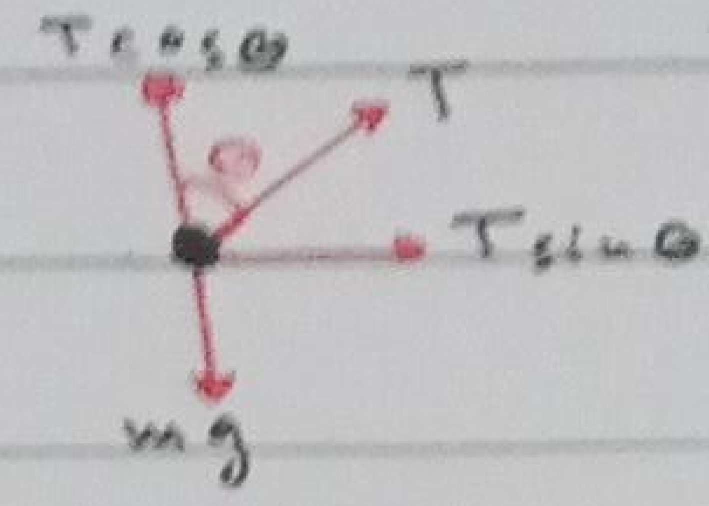


- if there is no motion on y direction then $\Sigma F_y = 0$ because $a_y = 0$
- you can ~~use~~ use Eq.s of motion because (a) is constant if the force is constant.
- if the body isn't moving then $\Sigma F_x = 0$ and use μ_s
- if the body is moving then $\Sigma F_x = ma_x$ and use μ_k
- the body will start moving when $(F_f = \mu_s N) \leq F_x$ then F_f will become $\mu_k N$
- if the acceleration isn't constant then :-

$$a_x = \frac{dV}{dt}, \quad \int_0^V dV = \int_{t_1(\text{start moving})}^{t_2(\text{required})} a_x \, dt$$

- Ex.





$\Sigma F_y = ma_y \; (a_y = 0)$ $\qquad T\cos\theta = mg$

$\Sigma F_x = ma_x$ $\qquad T\sin\theta = ma_x$

$\tan\theta = \frac{a_x}{g}$. a_x is for the car also because both are one body.

⊛ Spring force:-

$F_s = k\,S \longrightarrow S = L_f - L_i$ (elongation)

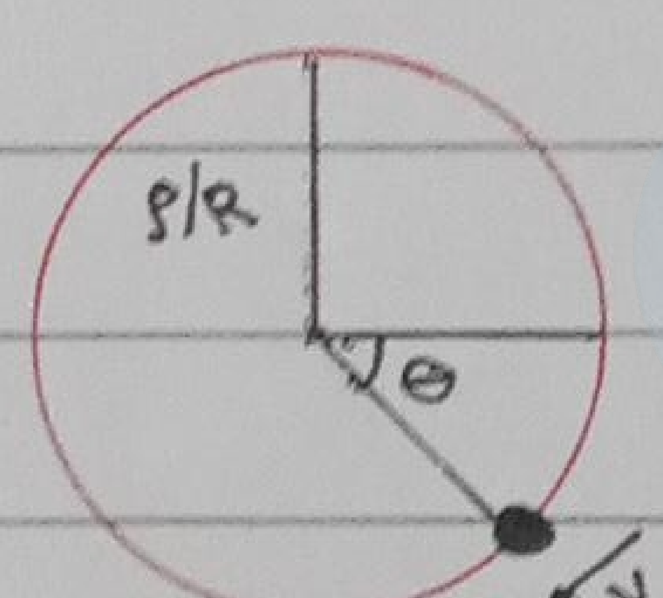
⊛ Kinetic diagram:-

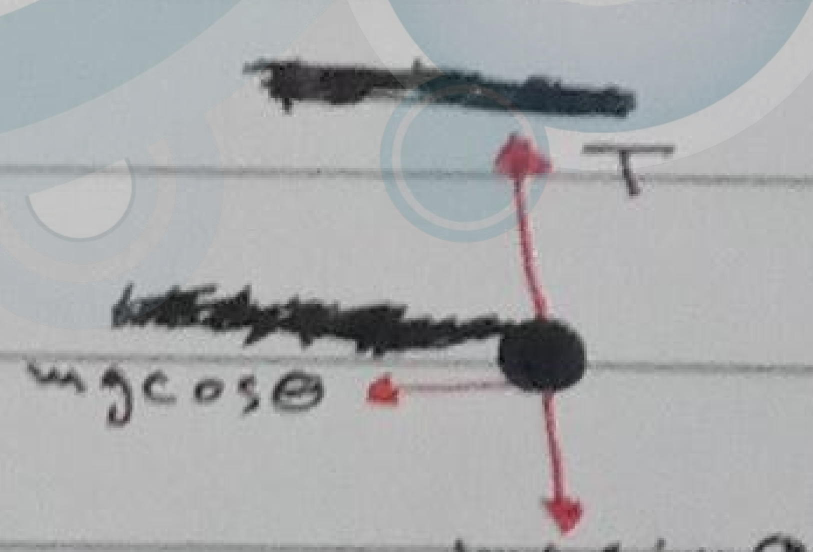
⊛ Normal-Tangent components:-

$\rho = \frac{[1+(y')^2]^{\frac{3}{2}}}{|y''|}$

$\Sigma F_t = ma_t \qquad \Sigma F_c = ma_c$

Ex.



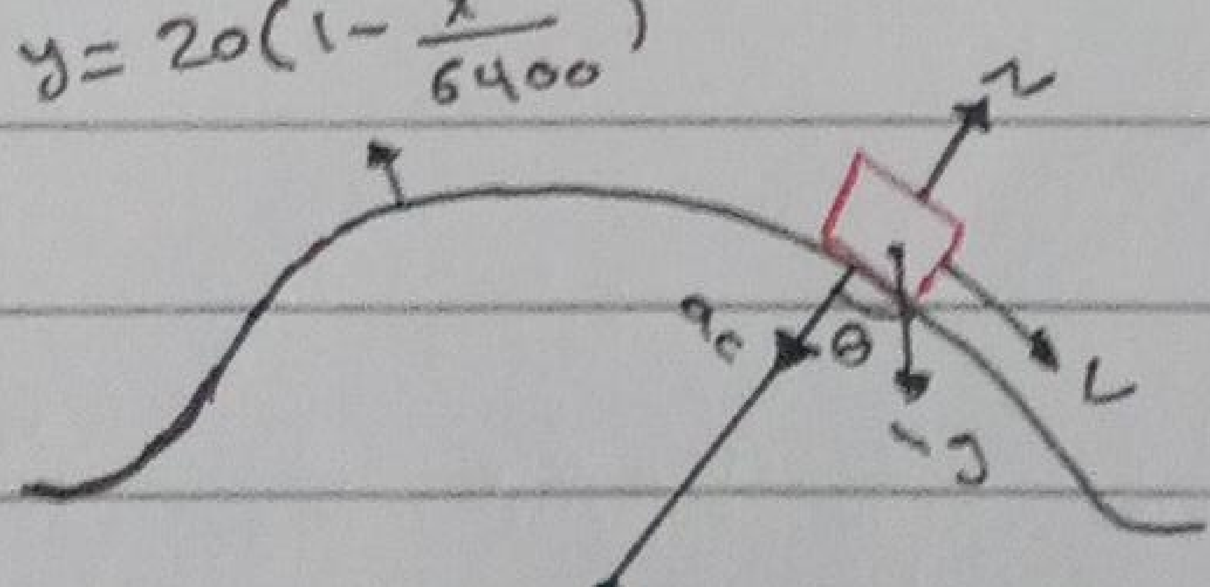


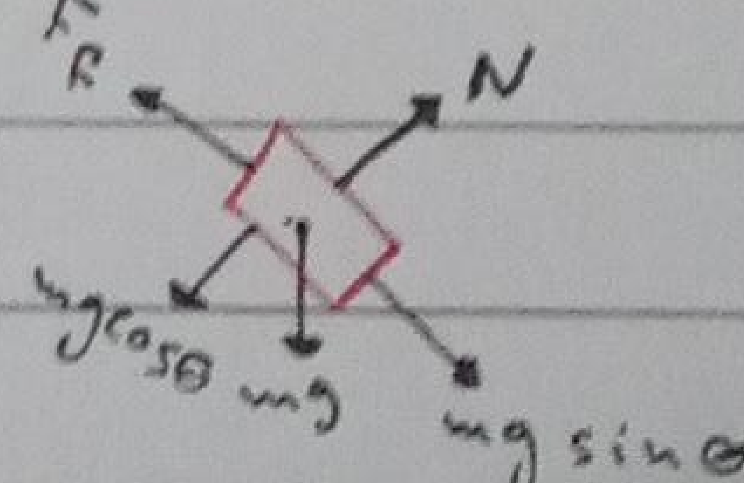
$mg\cos\theta = ma_t$

$T - mg\sin\theta = ma_c$

Ex.

$y = 20\left(1 - \frac{x^2}{6400}\right)$





$N - mg\cos\theta = ma_c$

$mg\sin\theta - F_f = ma_t$

Subject Date

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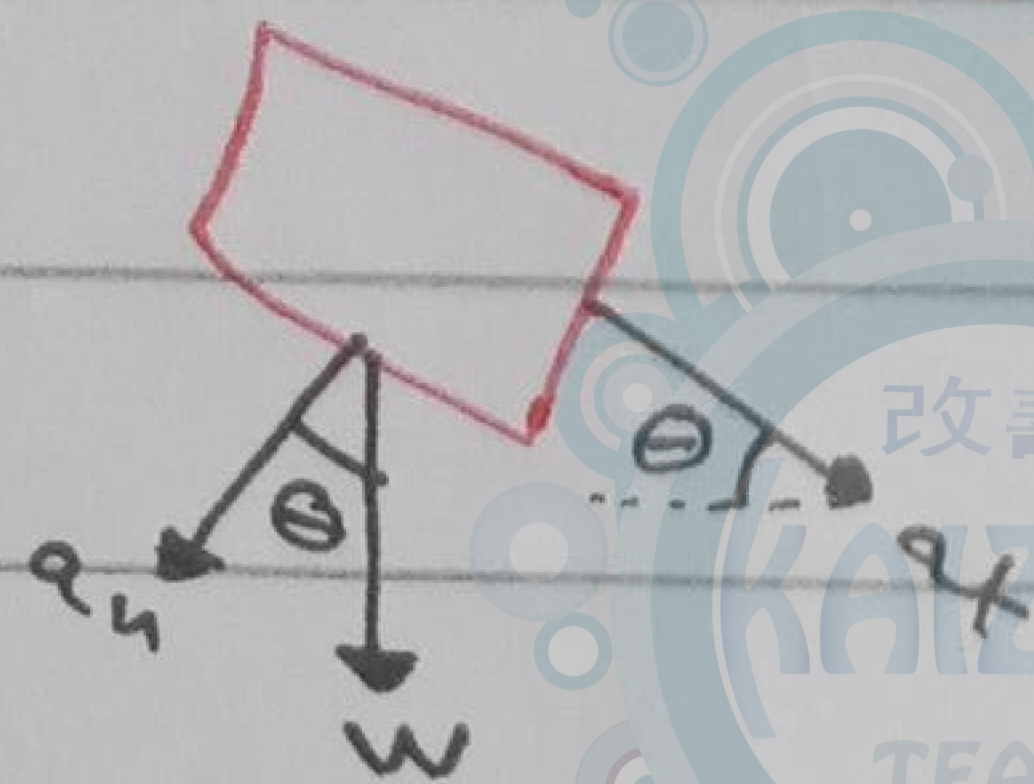
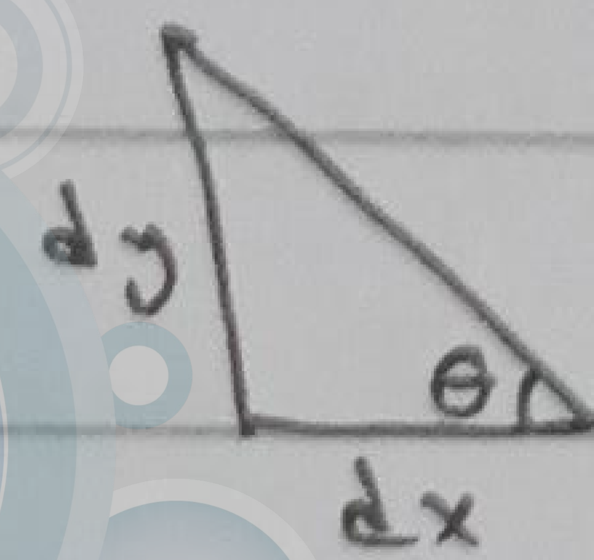




$$\tan\theta = \frac{dy}{dx}$$

$$y' = \tan\theta$$

Subject Chapter 14 Date No.

* Work and energy. ($W \equiv u$)

① Work from external forces:-

$W_n = \pm F_n\, d$
- + when F is at same direction of motion
- − when F is at opposite direction of motion

- When F_n is normal to motion direction then $W_n = 0$

② Work from Friction force:-

$W = -F_f d = -\mu N d$ → always negative

③ Work from Weight (potential energy):-

$W = \pm mg\,h$
- + down ward moving
- − upward moving

h: difference between first height and final height.

④ Work from Spring:-

$W_{(\text{from } A \text{ to } B)} = \pm \frac{1}{2} K\, \Delta S^2$ → $\Delta S^2 = (S_A - S_o)^2 - (S_B - S_o)^2$

- S_o: original length. - K for nested springs $= K_1 + K_2 + \dots$

⑤ Kinetic energy:-

$T = \frac{1}{2} m v^2$

* principle of work and energy:-

$T_1 + u_{12} = T_2$
- T_1: T at point (1)
- T_2: T at point (2)
- u_{12}: Work to move object from (1) to (2)

- Note for external force:-

$$u = \int_{S_1}^{S_2} F\, ds$$

Subject Date

① power and efficiency.

$p = \frac{du}{dt}$ [p] : watt

$P = F \cdot \frac{dr}{dt}$ So, $p = F \cdot V$ (the output power)

- Efficiency = $\frac{\text{output power}}{\text{input power}} = \varepsilon$

- 1 Megagram $\longrightarrow$ 1000 kg

- $P_{avg} = F \cdot V_{avg}$, $V_{avg} = \frac{V_f + V_i}{2}$

Subject Date No.

④ Conservation of energy :-

- $T_1 + V_1 = T_2 + V_2$

V_1: potential energy at position 1

T_1: Kinetic energy at position 1

- $V_{total} = V_{gravity} + V_{spring}$

$V_g = \pm mgh$

$V_{spring} = \frac{1}{2} k s^2$

- Assume a datum line any where you want then solve.
- Remember: $a_c = \frac{v^2}{\rho}$

Chapter 16

⊛ Rotation around specific (fixed) axis:-

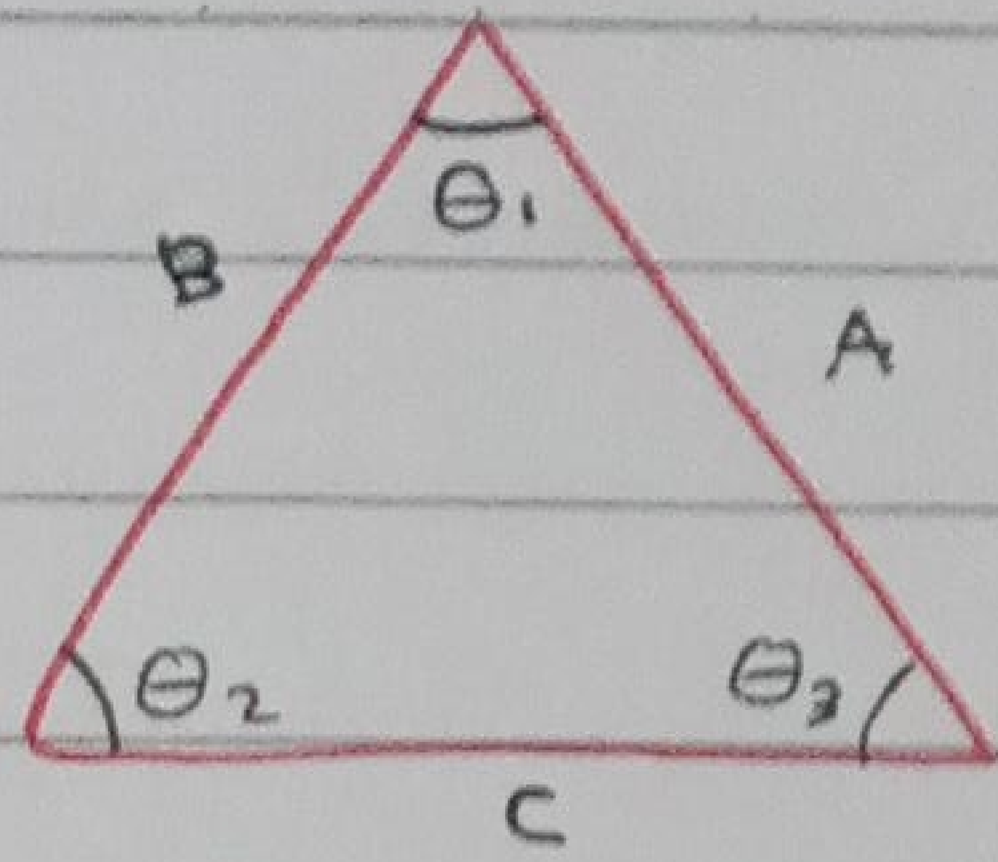
- 1 revolution = 2π rad
- angular velocity :- $\omega = \frac{d\theta}{dt}$ (rad/s) ↺+
- angular acceleration:- $\alpha = \frac{d\omega}{dt}$ (rad/s^2) ↺+
- when α is constant, then we can use Eq. of motion:-

$\omega_f = \omega_i + \alpha t$ $\Big\{$ $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$ $\Big\{$ $\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$

- The velocity at point in a circular path ($V_p = \omega r_p$), r_p is the radius at this point.
- The tangential acceleration at any point ($a_t = \alpha r$), r is the radius at this point, and ($a_c = \omega^2 r$)
- Remember $a = \sqrt{(a_t)^2 + (a_c)^2}$.
- Important:- $\alpha\, d\theta = \omega\, d\omega$.
- When gear A rotate gear B, then they have the same velocity and a_t, so ($\alpha_A r_A = \alpha_B r_B$) and ($\omega_A r_A = \omega_B r_B$) and ($\theta_A r_A = \theta_B r_B$).
- But when they are assumed as one body then ($\alpha_A = \alpha_B$) and ($\omega_A = \omega_B$)

Absolute motion analysis :-

- Remember :-



$$C^2 = A^2 + B^2 - 2 * A * B * \cos\theta_1$$

$$\frac{A}{\sin\theta_2} = \frac{B}{\sin\theta_3} = \frac{C}{\sin\theta_1}$$

- Rotational motion makes transition motion.
  The change in θ makes an object move a transition motion.
- When the rotation is C.W and we got a positive ω, then ω is C.W, but if we got a negative α, so that it will be C.C.W
- To solve: find a relation between θ and X or y then differentiate.

⊛ Relative velocity.

- If we have link AB and assume we need the velocity at point B then:

$\boxed{V_B = V_A + w_{AB} \times r_{B/A}}$ ⊛ if A is fixed point then $V_A = 0$.

- always w in K direction.
- if we say $r_{B/A}$ then its the position vector for link AB from $A \rightarrow B$.
- all of elements in Eq. ⊛ are vectors.
- $\hat{k} \times \hat{i} = \hat{j}$, $\hat{k} \times \hat{j} = -\hat{i}$

⊛ Relative acceleration.

- for link AB, then: $\boxed{a_B = a_A + \alpha \times r_{B/A} - w^2 r_{B/A}}$ and same for a_A.
- you should find w by relative velocity method, then plug it as a scalar quantity.
- always α in K direction.
- w and α
  - positive $\rightarrow$ C.C.W
  - Negative $\rightarrow$ C.W