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Faculty of Engineering & Technology  
Industrial Engineering Department  
System Dynamics & Control Lab

**EXPERIMENT 4**

**INTRODUCTION TO PID CONTROLLER**

## Introduction to PID controller

Consider the following unity feedback system:

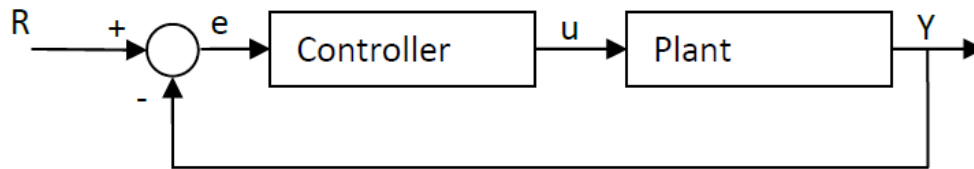


Figure 1: Block diagram of closed loop control system.

Plant: A system to be controlled.

Controller: Provides excitation for the plant; Designed to control the overall system behavior. The three-term controller: The transfer function of the PID controller looks like the following:

$$K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s}$$

KP = Proportional gain

KI = Integral gain

KD = Derivative gain

First, let's take a look at how the PID controller works in a closed-loop system using the schematic shown above. The variable (e) represents the tracking error, the difference between the desired input value (R) and the actual output (Y). This error signal (e) will be sent to the PID controller, and the controller computes both the derivative and the integral of this error signal. The signal (u) just past the controller is now equal to the proportional gain (KP) times the magnitude of the error plus the integral gain (KI) times the integral of the error plus the derivative gain (KD) times the derivative of the error.

$$u = K_P e(t) + K_I \int e(t) dt + K_D \frac{de(t)}{dt}$$

This signal (u) will be sent to the plant, and the new output (Y) will be obtained. This new output (Y) will be sent back to the sensor again to find the new error signal (e). The controller takes this new error signal and computes its derivatives and its internal again. The process goes on and on.

Example:

Suppose we have a simple mass, spring, and damper problem.

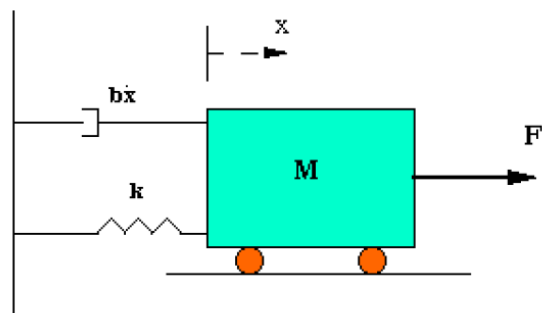


Figure 2: Spring Mass Damper System.

The transfer function between the displacement  $X(s)$  and the input  $F(s)$  then becomes:

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + bs + k}$$

Let

$$M = 1\text{kg}, b = 10 \text{ N.s/m}, k = 20 \text{ N/m}, F(s) = 1$$

Plug these values into the above transfer function

The goal of this problem is to show you how each of  $K_p$ ,  $K_i$  and  $K_d$  contributes to obtain

- Fast rise time
- Minimum overshoot
- No steady-state error

## Open-loop step response:

Let's first view the open-loop step response.

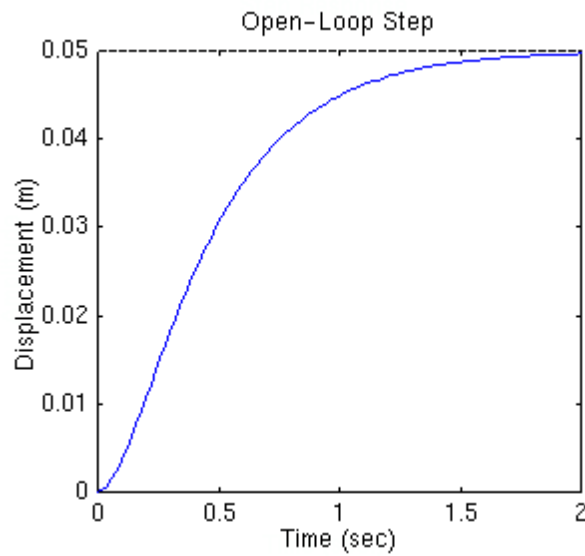
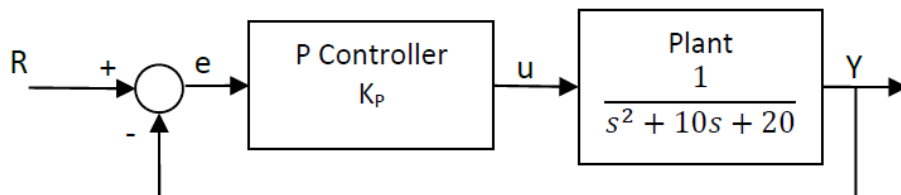


Figure 3: open-loop step response

## Proportional control:

The closed-loop transfer function of the above system with a proportional controller is:



$$\frac{X(s)}{F(s)} = \frac{K_P}{s^2 + 10s + (20 + K_P)}$$

Let the proportional gain ( $K_P$ ) equal 300:

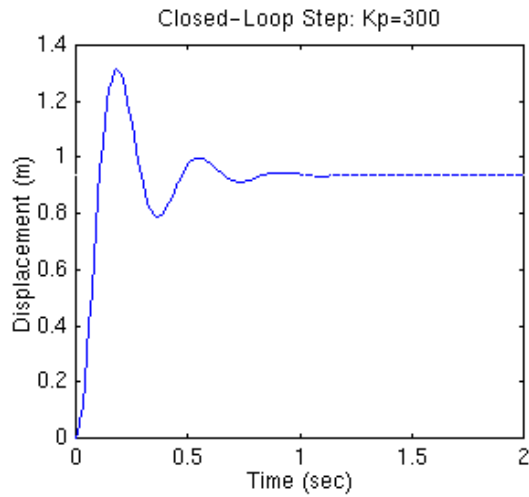
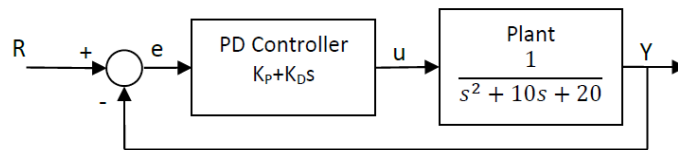


Figure 4: step response with a proportional controller at  $K_p=300$ .

### **Proportional-Derivative control:**

The closed-loop transfer function of the given system with a PD controller is:



$$\frac{X(s)}{F(s)} = \frac{K_D s + K_P}{s^2 + (10 + K_D)s + (20 + K_P)}$$

Let  $K_P$  equal 300 as before and let  $K_D$  equal 10.

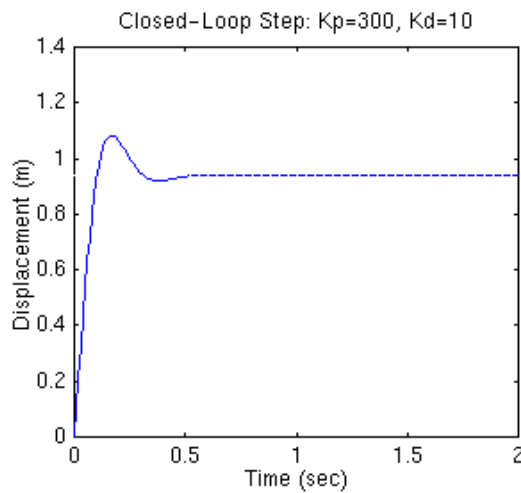
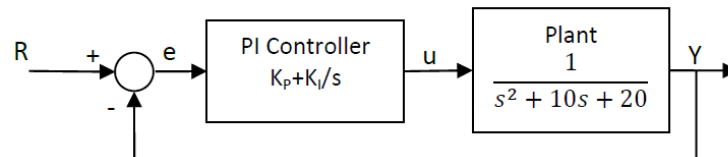


Figure 5: step response with a PD controller at  $K_p=300$ ,  $K_d=10$ .

This plot shows that the derivative controller reduced both the overshoot and the settling time, and had a small effect on the rise time and the steady-state error.

### **Proportional-Integral control:**

Before going into a PID control, let's take a look at a PI control. For the given system, the closed-loop transfer function with a PI control is:



$$\frac{X(s)}{F(s)} = \frac{K_p s + K_I}{s^3 + 10s^2 + (20 + K_p)s + K_I}$$

Let's reduce the KP to 30, and let KI equal 70.

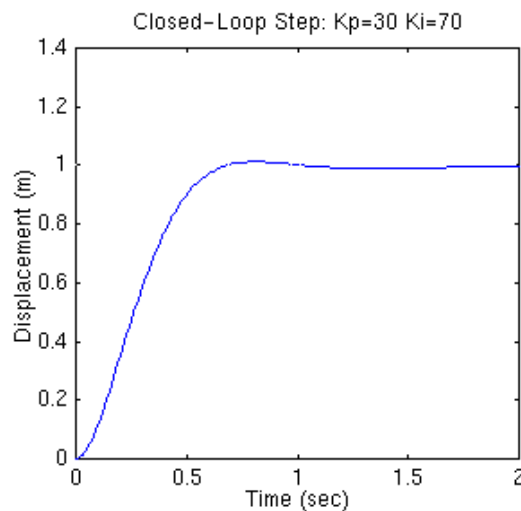
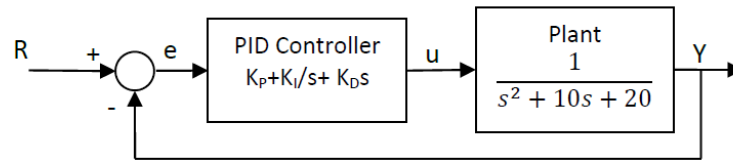


Figure 6: step response with a PI controller at Kp=30, Ki=70.

We have reduced the proportional gain (Kp) because the integral controller also reduces the rise time and increases the overshoot as the proportional controller does (double effect). The above response shows that the integral controller eliminated the steady-state error.

### **Proportional-Integral-Derivative control:**

Now, let's take a look at a PID controller. The closed-loop transfer function of the given system with a PID controller is:



$$\frac{X(s)}{F(s)} = \frac{K_D s^2 + K_p s + K_I}{s^3 + (10 + K_D)s^2 + (20 + K_p)s + K_I}$$

After several trial and error runs, the gains  $K_p=350$ ,  $K_i=300$ , and  $K_d=50$  provided the desired response. To confirm, enter the following commands to an m-file and run it in the command window. You should get the following step response.

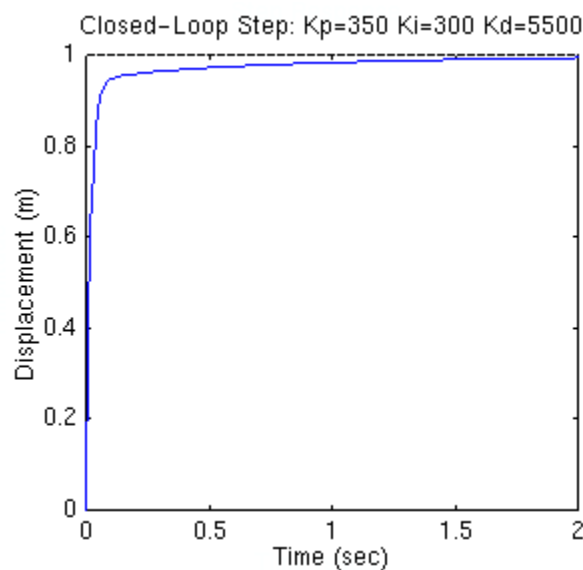


Figure 7: step response with a PID controller at  $K_p=350$ ,  $K_d=5500$ ,  $K_i=300$ .

Now, we have obtained a closed-loop system with no overshoot, fast rise time, and no steady state error.

### **The characteristics of P, I, and D controllers:**

The proportional controller (KP) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady state error. An integral controller (KI) will have the effect of eliminating the steady state error, but it may make the transient response worse. A derivative control (KD) will have the effect of increasing the stability of the system, reducing the overshoot and improving the transient response.

Effect of each controller  $K_P$ ,  $K_I$  and  $K_D$  on the closed-loop system are summarized below

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CL Response	Rise Time	Overshoot	Settling Time	S-S Error
$K_P$	Decrease	Increase	Small Change	Decrease
$K_I$	Decrease	Increase	Increases	Eliminate
$K_D$	Small Change	Decreases	Decreases	Small Change

Note that these corrections may not be accurate, because  $K_P$ ,  $K_I$ , and  $K_D$  are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason the table should only be used as a reference when you are determining the values for  $K_P$ ,  $K_I$ , and  $K_D$ .

**Exersice:**

Consider a process given below to be controlled by a PID controller,

$$G_p(s) = \frac{400}{s(s + 48.5)}$$

- Obtain the unit step response of  $G_p(s)$  Try PI controllers with ( $K_p=2, 10, 100$ ), and  $K_i=K_p/10$ . Investigate the unit step response in each case, compare the results and comment.
- Let  $K_p=100$ ,  $K_i=10$ , and add a derivative term with ( $K_d=0.1, 0.9, 2$ ). Investigate the unit step response in each case, compare the results and comment.
- Based on your results in parts above what do you conclude as a suitable PID controller for this process and give your justification.