改善
KAIZEN
TEAM

OR 1

PAST PAPERS

لعلّه من عجائب الحياة .. انّك اذا رفضت
كل ماهو دون مستوى القمة ..
فإنك دائما تصل إليها
سومرست موم

Mid term 2012 summer

Question 1 (10 points)

A product is assembled from three different parts. The parts are manufactured by two departments at different production rates as given in the following table:

Department	Capacity (hr/week)	Part 1	Part 2	Part 3
		Production rate (units/hr)		
1	100	8	5	10
2	80	6	12	4

The objective is to maximize the number of final assembled units that can be produced weekly.

Formulate the problem as LP model. **Define clearly your decision variables! Do not solve!**

$x_{ij} \Rightarrow$ # of parts j $(j=1,2,3)$ produced in dept i $(i=1,2)$

Then obj function

$$\max Z = \min\{(x_{11}+x_{21}),(x_{12}+x_{22}),(x_{13}+x_{23})\}$$

S.t

$$\frac{x_{11}}{8}+\frac{x_{12}}{5}+\frac{x_{13}}{10}\leq 100$$

$$\frac{x_{21}}{6}+\frac{x_{22}}{12}+\frac{x_{23}}{4}\leq 80$$

$$x_{ij}\geq 0 \text{ for } \forall i=1,2 \ \& \ j=1,2,3$$

Let $\min\{(x_{11}+x_{21}),(x_{12}+x_{22}),(x_{13}+x_{23})\}=X_4$

Then

$$\max Z = X_4$$

S.t

$$\frac{x_{11}}{8}+\frac{x_{12}}{5}+\frac{x_{13}}{10}\leq 100 \Rightarrow 5X_{11}+8X_{12}+4X_{13}\leq 4{,}000$$

$$\frac{x_{21}}{6}+\frac{x_{22}}{12}+\frac{x_{23}}{4}\leq 80 \Rightarrow 2X_{21}+X_{22}+3X_{23}\leq 960$$

$$x_{11}+x_{21}\geq x_4$$

$$x_{12}+x_{22}\geq x_4$$

$$x_{13}+x_{23}\geq x_4$$

$$x_{ij}\geq 0 \quad \forall i=1,2 \ \& \ j=1,2,3 \qquad X_4\geq 0$$

Question 2 (16 points)

Use Simplex technique (two phase) to solve the following L.P problem:

Min $Z = X_1 + 4X_2 + X_3$

S.t $X_1 + 4X_2 + 2X_3 \geq 8 \Rightarrow X_1 + 4X_2 + 2X_3 - X_4 + R_1 = 8$

$2X_1 + 3X_2 \geq 6 \Rightarrow 2X_1 + 3X_2 - X_5 + R_2 = 6$

$X_1, X_2, X_3 \geq 0,$

	X_1	↓ X_2	X_3	X_4	X_5	R_1	R_2	
r	3	7	2	-1	-1	0	0	14
← R_1	1	$\boxed{4}$	2	-1	0	1	0	8
R_2	2	3	0	0	-1	0	1	6
r	↓ 5/4	0	-3/2	3/4	-1	-7/4	0	0
X_2	1/4	1	1/2	-1/4	0	1/4	0	2
← R_2	$\boxed{5/4}$	0	-3/2	3/4	-1	-3/4	1	0
	0	0	0	0	0	-1	-1	0
X_2	0	1	4/5	-2/5	1/5	2/5	-1/5	2
X_1	1	0	-6/5	3/5	-4/5	-3/5	4/5	0

	-1	-4	-↓	0	0	
	X_1	X_2	X_3	X_4	X_5	
Z	0	0	1	-1	0	8
← X_2	0	1	$\boxed{4/5}$	-2/5	1/5	2
X_1	1	0	-6/5	3/5	-4/5	0
	0	-5/4	0	-1/2	-1/4	11/2
X_3	0	5/4	1	-1/2	1/4	5/2
X_1	1	3/2	0	0	-1/2	3

Mid term 2012 Summer

Question 2 (16 points)

Use Simplex technique (two phase) to solve the following L.P problem:

Min $Z = 2X_1 + 4X_2 + X_3$

S.t $X_1 + 4X_2 + 2X_3 \geq 8$ $\rightarrow X_1 + 4X_2 + 2X_3 - X_4 + R_1 = 8$

$3X_1 + 2X_2 \geq 6$ $\rightarrow 3X_1 + 2X_2 - X_5 + R_2 = 6$

$X_1, X_2, X_3 \geq 0,$

	C	2	4	1	0	0			
		↓							
		X_1	X_2	X_3	X_4	X_5	R_1	R_2	
	r	4	6	2	−1	−1	0	0	14
4/5	← R_1	1	$\boxed{4}$	2	−1	0	1	0	8
2/5	R_2	3	2	0	0	−1	0	1	6
	r	5/2 ↓	0	−1	1/2	−1	−3/2	0	2
	X_2	1/4	1	1/2	−1/4	0	1/4	0	2
	← R_2	$\boxed{5/2}$	0	−1	1/2	−1	−1/2	1	2
	r	0	0	0	0	0	−1	−1	0
	X_2	0	1	3/5	−3/10	+ 1/10	3/10	−1/10	9/5
− 1/4	X_1	1	0	−2/5	1/5	−2/5	−1/5	2/5	4/5

		−2	−4	−1 ↓	0	0			
		X_1	X_2	X_3	X_4	X_5			
	Z	0	0	3/5	$\boxed{-4/5}$	−2/5			44/5
4	← X_2	0	1	$\boxed{3/5}$	−3/10	1/10	3/10	−1/10	9/5
2	X_1	1	0	−2/5	1/5	−2/5	−1/5	2/5	4/5
	Z	0	−1	0	−1/2	−1/2		−	7
	X_3	0	5/3	1	−1/2	1/6	1/2	−1/6	3
	X_1	1	2/3	0	0	−1/3	0	1/3	2

Q1 (15 points)

A manufacturer produces 3 models, 1, 2, and 3, of a certain product using raw materials A and B. The following table gives the data for the problem:

	Requirements per unit			
Raw material	Model 1	model 2	Model 3	Availability
A	2	3	5	4000
B	4	2	7	6000
Minimum demand	200	200	150	
Price per unit ($)	30	20	50	

The labor time per unit of model 1 is twice that of model 2 and three times that of model 3. The entire labor force of the factory can produce the equivalent of 1500 units of model 1. Market requirements specify ratios 3:2:5 for the production of the three respective models.

Formulate as L.P model. Do not solve! Define decision variables clearly!

15

X_1 = Number of units produced of model 1

X_2 = Number of units produced of model 2

X_3 = Number of units produced of model 3

$$\text{Max } Z = 30X_1 + 20X_2 + 50X_3$$

s.t

$$2X_1 + 3X_2 + 5X_3 \leq 4000 \rightarrow (1)$$

$$4X_1 + 2X_2 + 7X_3 \leq 6000 \rightarrow (2)$$

$$X_1 \geq 200 \rightarrow (3)$$

$$X_2 \geq 200 \rightarrow (4)$$

$$X_3 \geq 150 \rightarrow (5)$$

$$X_1 + \frac{X_2}{2} + \frac{X_3}{3} \leq 1500 \rightarrow (6)$$

$$\frac{X_1}{3} = \frac{X_2}{2} \rightarrow (7)$$

$$\frac{X_2}{2} = \frac{X_3}{5} \rightarrow (8)$$

$$X_1, X_2, X_3 \geq 0$$

Q2 (16 points) Solve the following L.P problem using two phase method:

$Max\ Z = 2X_1 + X_2 + X_3$

Such that: $X_1 + 2X_2 + 4X_3 \geq 20$

$6X_1 + 2X_2 + 3X_3 \leq 24$

$X_1, X_2, X_3 \geq 0$

16

[redacted]

	x1	x2	x3	x4	x5	R	
Z1	1	2	4	-1	0	0	20
R	1	2	4	-1	0	1	20
x5	6	2	3	0	1	0	24

Z1	0	0	0	0	0	-1	0
x3	1/4	1/2	1	-1/4	0	1/4	5
x5	21/4	1/2	0	3/4	1	-3/4	9

R=0

X

Start of phase 2 !

	x1	x2	x3	x4	x5	
Z	-2	-1	-1	0	0	0
x3	1/4	1/2	1	-1/4	0	5
x5	21/4	1/2	0	3/4	1	9

Z	-7/4	-1/2	0	-1/4	0	5
x3	1/4	1/2	1	-1/4	0	5
x5	21/4	1/2	0	3/4	1	9

Z	0	-1/3	0	0	1/3	8
x3	0	10/21	1	-2/7	-1/21	32/7
x1	1	2/21	0	1/7	4/21	12/7

Ent

Z	0	0	7/10	-1/5	3/10	56/5
x2	0	1	21/10	-3/5	-1/10	~~84/5~~
x1	1	0	-1/5	1/5	1/5	4/5

48/5

leav. ← x1

	x1	x2	x3	x4	x5	R.H.S
Z	1	0	1/2	0	1/2	12
x2	3	1	3/2	0	1/2	12
x4	5	0	-1	1	1	4

(1/5)

Z = 12

Q3 (9 points) Briefly define the following terms:

1. A basic solution to a system of linear equations:

obtained by

basic solution of linear Programming Problem is setting by make

ing for variables

(n-m) Variables equal to Zero and solved ~~with~~ remeaning m ~~which~~

Provides, adeterminate of Coefficient of m Variables (non-Zero). all m ③

ed that there is

Variables are basic solutions.

2. A feasible solution of an L.P problem:

contains vector $X = (x_1, x_2, x_3 --- x$

It's solution of linear Programming Problem which satisfies ~~Condition~~ condition (2 & 3), which Condition(2) ⟹ $AX = b$

& condition (3) ⟹ $X > 0$. ③

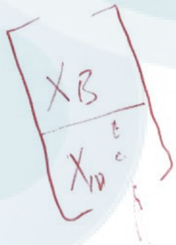
3. Degenerate solution of an L.P problem:

1.5

It's a basic & feasible Solution ~~wich~~ which contain at minimum one (Zero), and we find It in the optimal tableaue (Right hand side)

??

No!



Question 1 (10 points)

Four products are processed successively on two machines. The manufacturing times in hours per unit of each product are tabulated for the two machines:

Time per unit (hr)

Machine	product 1	product 2	product 3	product 4	
1	2 x_{11}	3 x_{12}	4 x_{13}	2 x_{14}	→ 10 $/hr] 500
2	3 x_{21}	2 x_{22}	1 x_{23}	2 x_{24}	→ 15 $/hr] 380

The total cost of producing one unit of each product is based directly on the machine time. Assume that the cost per hour of machine 1 and 2 is $10 and $15, respectively. The total hours budgeted for all the products on machines 1 and 2 are 500 and 380. If the sales price per unit for products 1, 2, 3 and 4 are $65, $70, $55, and $45, formulate the problem as L.P model to maximize total net profit. **Do not solve!** **Define clearly your decision variables!**

→ desision variables: x_{ij}: number of units produced of product j on machine i.

$10 \frac{\$}{hr} \times 2 \frac{hr}{unit} \times unit$

→ ① objective function

$$\max z = 65(2x_{11} + 3x_{21}) + 70(3x_{12} + 2x_{22}) + 55(4x_{13} + x_{23}) + 45(2x_{14} + 2x_{24})$$
$$- 10(2x_{11} + 3x_{12} + 4x_{13} + 2x_{14}) - 15(3x_{21} + 2x_{22} + x_{23} + 2x_{24}).$$

No!

② s.t

$$2x_{11} + 3x_{12} + 4x_{13} + 2x_{14} \leq 500$$
$$3x_{21} + 3x_{22} + x_{23} + 2x_{24} \leq 380$$

③ $x_{ij} \geq 0$

②

Question 2 (15 points)

Use Simplex technique (Two phase) to solve the following L.P problem:

Maximize $Z = X_1 + 3X_2$

S.t $\quad 2X_1 + X_2 \le 5$

$2X_1 - 4X_2 = 10$

$X_1 \ge 0,\quad X_2$ *is unrestricted in sign*

* in standa form

max $Z = x_1 + 3x_2' - 3x_2''$

s.t

$2x_1 + x_2' - x_2'' + x_3 = 5$

$2x_1 - 4x_2' + x_2'' + R_1 = 10$

$x_1 \geq 0$, $x_2 \Rightarrow$ unrestr in sign.

* in standard form

max $Z = x_1 + 3x_2$

s.t

$2x_1 + x_2 + x_3 = 5$

$2x_1 - 4x_2 + R_1 = 10$ $\Rightarrow$ $2x_1 + x_2' - x_2'' + x_3 = 5$

$2x_1 - 4x_2' + x_2'' + R_1 = 10$

- phase 1 → min $Z = R_1$

- phase 2 → max $Z = x_1 + 3(x_2' - x_2'')$

$Z = x_1 + 3x_2' - 3x_2''$

8

		X_1	X_2'	X_2''	X_3	R_1	RHS	ratio
	Z	0	0	0	0	-1	0	
	x_3	2	1	-1	1	0	5	
	R_1	2	-4	1	0	1	10	
	Z	2	-4	1	0	0	10	
leaving variable ←	x_3	② pivot	1	-1	1	0	5	2.5
	R_1	2	-4	1 ↓ entering var	0	1	10	5
	Z	0	-5	2	-1	0	5	
-2	x_1	1	1/2	-1/2	1/2	0	5/2	-
←	R_1	0	-5	②	-1	1	5	5/2
	Z	0	0	0	0	-1	0	
	x_1	1	-3/4	0	1/4	1/4	15/4	
end of phase 1 ← -2	x_2''	0	-5/2	1	-1/2	1/2	5/2	
	Z	-1	-3	3	0		0	
	x_1	1	-3/4	0	1/4		15/4	
	x_2''	0	-5/2	1	-1/2		5/2	
	Z	0	15/4	0	7/4		-15/4	
	x_1	1	-3/4	0	-1/4		15/4	
1/2	x_2''	0	-5/2	1	-1/2		5/22	
	Z	0	-21/4	0	17/4		309/20	
	x_1	1					17/10	
	x_2''						5/2	

$[-1 \quad -3 \quad 3 \quad 0 \mid 0]$

$[1 \quad -3/4 \quad 0 \quad 1/4 \mid 15/4]$

$[0 \quad -5/2 \quad 1 \quad -1/2 \mid 5/2]\,{-3}$

$0 \quad -15/4 \quad 0 \quad 7/4 \quad -15/4$

$1 \quad -5/2 \quad 0$

أخذنا max ← عند ال cell 0

ال Z row يكون non negu

$\boxed{\rightarrow \text{max} \rightarrow -15/4}$ X it is satisfy ② يكون حل ال [System.]

Question 3 (9 Points)

Briefly define the following terms:

1. A basic solution to a system of linear equations:

~~it is the solution that obtained from put n-m variable to be equal to zero and solving for the remaining m variables", the remaining m variables called Basic variables"~~

2. A feasible solution of an L.P problem:

~~it is the solution that satisfy ②, ③~~

~~② → $\underset{m\times n}{A}\ \underset{n\times 1}{X} = \underset{m\times 1}{b}$~~

~~③ $x_j \geq 0$~~

3. Degenerate solution of an L.P problem:

it is a basic and ~~feasib~~ feasible solution with ~~all~~ all X_n are ~~positive~~ ~~which the (X) in Basic solution are strictly positive~~

it ~~means that all the (X) in Basic solution are~~

non negative. → X can assume a value $\begin{cases} 0 \\ +ve \end{cases}$ no!

and in this case → ratios are equal.

---

it is the vector $X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

→ ① basic solution! it is the solution obtained by setting n-m variables to be equal to zero and solving for the remaining m variables, provided that the coefficient of the m variables [the determinant] does not equal to zero, the m variables called Basic Variables.

→ ② feasible solution: it is the solution that ~~satisfies~~ satisfies these conditions: → (لو يكون حل ال system)

→ { ① $AX = b$ → (

② $x_j \geq 0$ → (non negative values (الاهم ال X يأخذو

ما الخطأ؟!

غير مفهوم وغير محدد !!

③ degenerate solution :- it is a basic and feasible solution with all values of X which are non negative → [0, +ve] no!

⑥

Question 2 (15 points)

(a) 4 points Show graphically the feasible space of the following L.P:

$Max\ Z = 5X_1 + 7X_2$

S.t

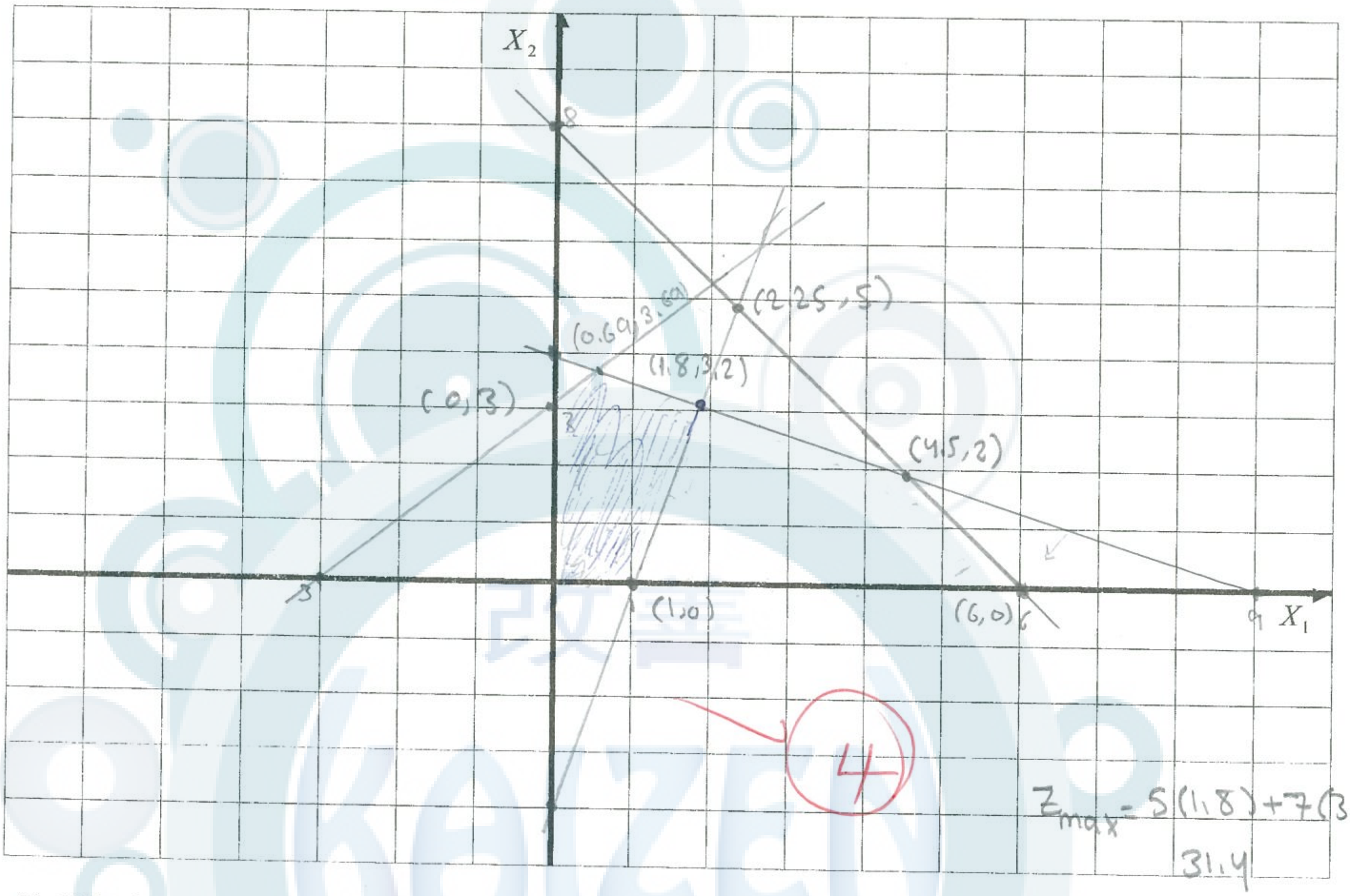
$8X_1 + 6X_2 \leq 48$ ………. **(1)**

$4X_1 + 9X_2 \leq 36$ ………. **(2)**

$4X_1 - X_2 \leq 4$ ………. **(3)**

$-X_1 + X_2 \leq 3$ ………. **(4)**

$X_1, X_2 \geq 0$



4

$Z_{max} = 5(1.8)+7(3.2)$

31.4

(b) 10.5 points

In reference to part (a), Circle one and only one answer in the following multiple answers questions or fill in blanks:

9

1. The binding (active) constraints of the above problem are:
   - a. Constraints (1) & (2).
   - [✓ b.] Constraints (2) & (3).
   - c. Constraints (2) & (4).
   - d. Constraints (3) & (4).
   - e. None of the above is correct.

2. The per unit worth of resource (1) is:
   - a. 0.6541
   - b. 0.4354
   - [✓ c.] Zero.
   - d. None of the above is correct, the answer is ………………… (fill in)

3. The per unit worth of resource (2) is:
   - a. 0.7852.
   - b. 0.4516
   - c. Zero.
   - [✓ d.] None of the above is correct, the answer is ..0.825..... (fill in).

4. The per unit worth of resource (3) is:
   - a. 0.2354
   - b. 0.6250
   - c. Zero.
   - [✓ d.] None of the above is correct, the answer is ……0.425…… (fill in).

5. Which of the following statements concerning the above problem is correct:
   - a. Resource (4) is abundant.
   - b. Constraint (4) is redundant.
   - c. Resource (1) is abundant.
   - d. Constraint (1) is redundant.
   - [✓ e.] Of course none of the above is correct.

   resource(1) and (4) are abundant

6. The coordinates of the optimal point is:

   $X_1 =$ ……1.8…… (fill in)

   $X_2 =$ ……3.2…… (fill in)

7. The value of $Z_{opt} =$ ……31.4…… (fill in)

Question 3 (8 Points)

The following tableau represents a specific simplex iteration. All variables are nonnegative. The tableau is not optimal for a maximization problem. By selecting the entering variable as the one that has the most promising variable that may improve the value of the objective function **conduct only one iteration**.

	X_1	X_2	X_3	X_4	X_5	↓ X_6	X_7	X_8	Solution
Z	0	−5	0	4	−1	−10	0	0	620
X_8	0	3	0	−2	−3	−1	5	1	12
X_3	0	0	1	3	1	0	3	0	6
← X_1	1	−1	0	0	6	+4	0	0	0

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	Solution
Z	~~2.5~~	~~-7.5~~	~~0~~	~~4~~	~~14~~	~~0~~	~~0~~	~~0~~	~~620~~
~~X8~~	~~$\frac{1}{4}$~~	~~$\frac{11}{4}$~~	~~0~~	~~-2~~	~~$\frac{-3}{2}$~~	~~0~~	~~5~~	~~1~~	~~12~~
~~X3~~	~~0~~	~~0~~	~~1~~	~~3~~	~~1~~	~~0~~	~~3~~	~~0~~	~~6~~
~~X6~~	~~$\frac{1}{4}$~~	~~$\frac{-1}{4}$~~	~~0~~	~~0~~	~~$\frac{6}{4}$~~	~~1~~	~~0~~	~~0~~	~~0~~

8

Kaizen Team

The Jordan University
Faculty of Engineering and Technology
Department of Industrial Engineering

Operations Research – 1 0906353
First Semester 2007/2008
Course Instructors: Dr. Mohammad D. Al-Tahat.
Eng. Ahmad Jaradat

Mid-Term Exam
Date: 12.11.2007
Time: 1:15 Minutes

Important Notes:

1. Write your name and number in Arabic language.
2. Pages for this exam are 3 pages.
3. Submitting all the questions pages of the exam is a must

اسم الطالب الرباعي: [redacted] .... رقم الطالب: .... الشعبة ....

Question No. 1: Formulation (10 Points)

(a) Two products are manufactured independently on two machines A and B. The time available on each machine is 8 hours per day and may be increased by up to 4 hours of overtime if necessary, at an additional cost of $10 per hour for machine A and $8 per hour for machine B. The table below gives the production rate on the two machines as well as the profit per unit of the two products. Formulate as L.P model. **(Define clearly your decision variables!), Do not solve!**

	Production rate (hrs/unit)	
	Product 1	Product 2
Machine A	0.15	0.10
Machine B	0.15	0.08
Profit $/Unit	100	75

5 hrs

(b) If the working time of one machine must not exceed that of the other machine by more than 15 minutes, show how this condition is reflected in your L.P model.

Solution:

Define X_{ij} as the number of daily manufactured units of product j on machine i

Objective function: Max $Z = 100(X_{11}+X_{21}) + 75(X_{12}+X_{22}) - 10X_3'' - 8X_4''$

S.t

$$0.15X_{11} + 0.1X_{12} + X_3' - X_3'' = 8$$

$$0.15X_{21} + 0.08X_{22} + X_4' - X_4'' = 8$$

$$X_3'', X_4'' \le 4$$

$$X_{11}, X_{12}, X_{21}, X_{22}, X_3'', X_4'' \ge 0$$

(b)

$$\|0.15X_{11} + 0.1X_{12} - 0.15X_{21} - 0.08X_{22}\| \le 0.25 \div \frac{15}{60}$$

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Question No. 2: Graphical Solution Method and Sensitivity Analysis (10 Points)

(a) Solve the following L.P problem graphically:

$Max\ Z = 5X_1 + 5X_2$

S.t

$4X_1 + 9X_2 \leq 36$

$8X_1 + 3X_2 \leq 24$

$4X_1 - 3X_2 \leq 12$

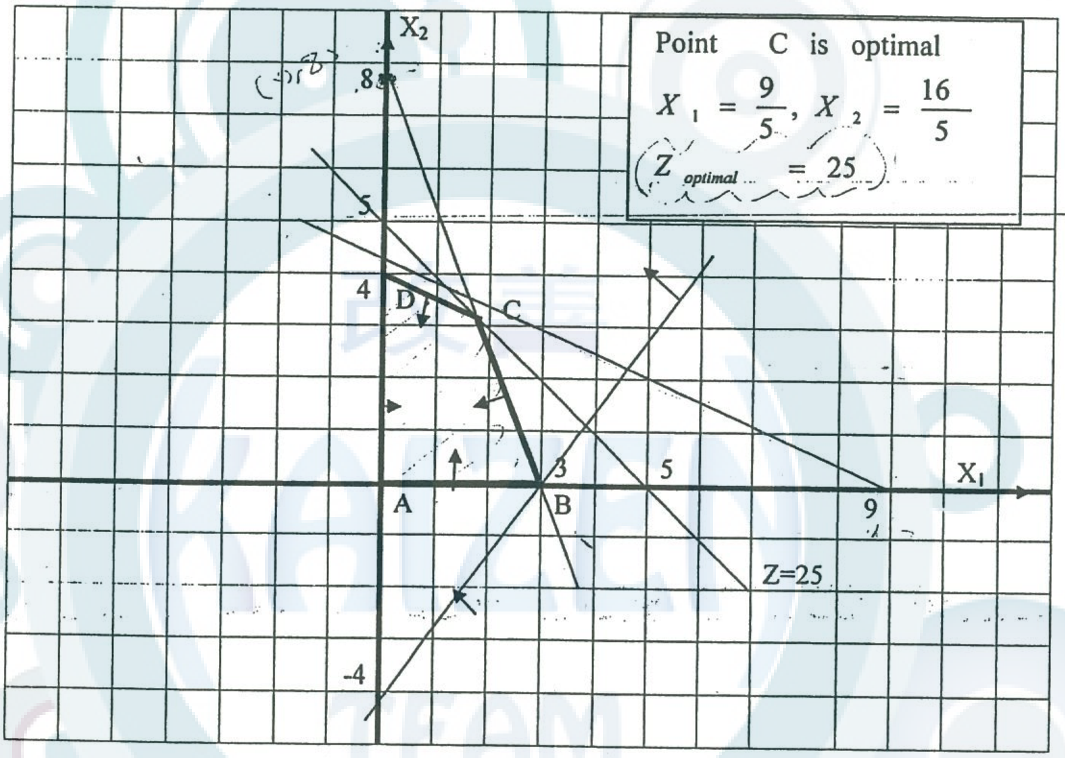
$X_1, X_2 \geq 0$

scarce

abundant

(b)

1. Determine the status of each resource
2. Determine the unit worth of each resource.
3. Based on the unit worth of each resource, which resource should be given priority for an increase in level?



Point C is optimal

$X_1 = \frac{9}{5},\ X_2 = \frac{16}{5}$

$Z_{optimal} = 25$

(b)

1. Resource 1 and resource 2 are scarce resources while resource 3 is abundant.
2. $\Delta b_1 = 9\times8-36=36$ and the corresponding $\Delta Z = 5\times8-25=15$, then per unit worth for resource 1 is: $\frac{\Delta Z}{\Delta b_1}=\frac{15}{36}=\frac{5}{12}$, $\Delta b_2 = 9\times8-24=48$ and the corresponding $\Delta Z = 5\times9-25=20$

   Per unit worth for resource 2 is: $\frac{\Delta Z}{\Delta b_2}=\frac{20}{48}=\frac{5}{12}$, and per unit worth for resource 3 is: $\frac{\Delta Z}{\Delta b_3}=0$.
3. Because $\frac{\Delta Z}{\Delta b_1}=\frac{\Delta Z}{\Delta b_2}=\frac{5}{12}$ Resource 1 and 2 are of equal priority.

scarce or abundant

Kaizen Tea

Question No. 3: Two Phase Method (10 Points)

Solve the following LP problem using two phase method:

Max $Z = 5X_1 + 2X_2$

S.t

$X_1 + X_2 \leq 10$

$X_1 = 5$

$X_1, X_2 \geq 0$

Solution:

Phase I

Min $r = R$

S.t

$X_1 + X_2 + X_3 = 10$

$X_1 + R = 5$

$X_1, X_2, X_3, R \geq 0$

	X_1	X_2	X_3	R	Solution
r	1	0	0	0	5
X_2	1	1	1	0	10
R	1	0	0	1	5
r	0	0	0	−1	0
X_2	0	1	1	−1	5
X_1	1	0	0	1	5

Phase II

	X_1	X_2	X_3	Solution
Z	0	0	2	35
X_2	0	1	1	5
X_1	1	0	0	5

5 2

Kaizen Team

The University of Jordan
Faculty of Engineering and Technology
Industrial Engineering Department
Operations Research I "0906351"
Eng. Jaradat & Dr. Al-Shihabi

Mid Term Exam
1 Hour

Q1) Sketch the following search regions and decide if it is a convex set or not:

a) $S = \{(x, y) : x^2 + y^2 = 25\}$

b) $S = \{(x, y) : x \ + 2y \ \geq 10\}$

c) $S = \{(x, y) : \frac{x^2}{4} + \frac{y^2}{9} \leq 1\}$

d) $S = \{(x, y) : x = 2, |Y| \leq 4\}$

Q2) Solve the following problem using the 2-phase method:

26 28.6

$$\max \quad Z = 5x_1 + 12x_2 + 4x_3$$

s.t.

$$x_1 + 2x_2 + x_3 \leq 10$$
$$2x_1 - x_2 + 3x_3 = 8$$
$$x_1, x_2, x_3 \geq 0$$

Q3) Write a mathematical model for the following real life problem:
Four products are processed successively on two machines. The manufacturing times in hours per unit of each product are tabulated for the two machines

X_{ij} : # of daily produced units of product(j) on machine(i)

Machine	Time per Unit			
	Product 1	Product 2	Product 3	Product 4
1	2 ساعتين	3	4	2
2	3 ساعات	2	1	2

The total cost of producing 1 unit of each product is based directly on the machine time. Assume that the cost per hour for machines 1 and 2 is $10 and $15 respectively. The total hours budgeted for all the products on machines 1 and 2 are 500 and 380. If the sales price per unit for products 1,2,3 and 4 are $65, $70, $55 and $45, formulate the problem as a linear programming model to maximize the total net profit.

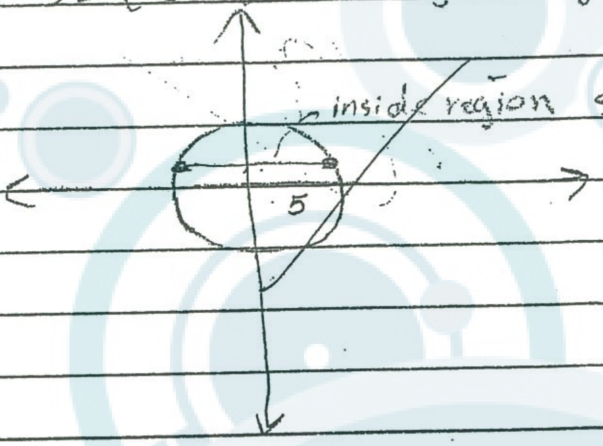
Cost

$$\max Z = 65(X_{11}+X_{21}) + 70(X_{12}+X_{22}) + 55(X_{13}+X_{23}) + 45(X_{14}+X_{24})$$
$$- (20X_{11} + 30X_{12} + 40X_{13} + 20X_{14} + 45X_{21} + 30X_{22} + 15X_{23} + 30X_{24})$$

X (1)

Q1)

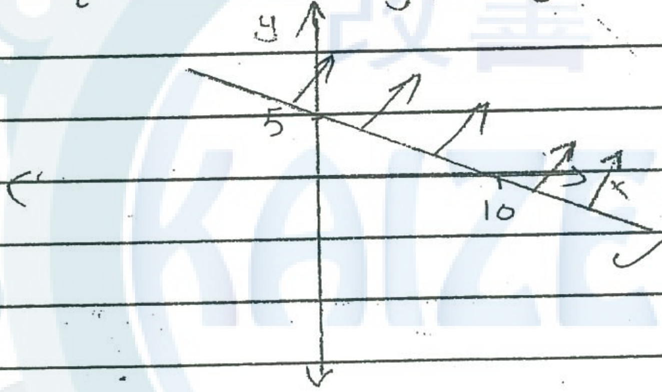
a) $S = \{(x,y) : x^2+y^2 = 25\}$



inside region so Convex.

Convex — 1/4

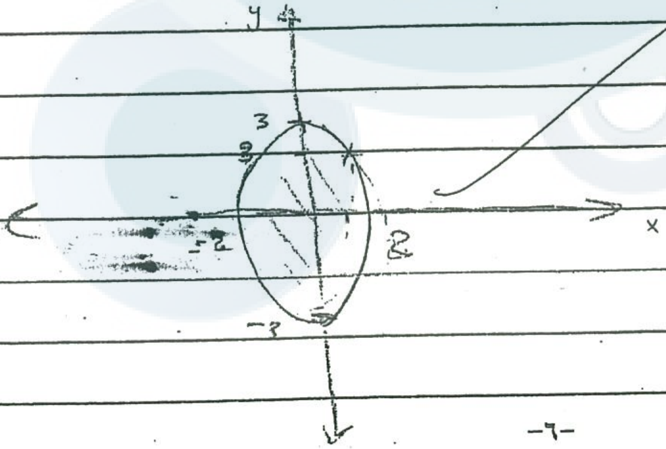
b) $S = \{(x,y) : x+2y \geq 10\}$



Convex

2/4

c) $S = \{(x,y) : \frac{x^2}{4} + \frac{y^2}{9} \leq 1\}$.



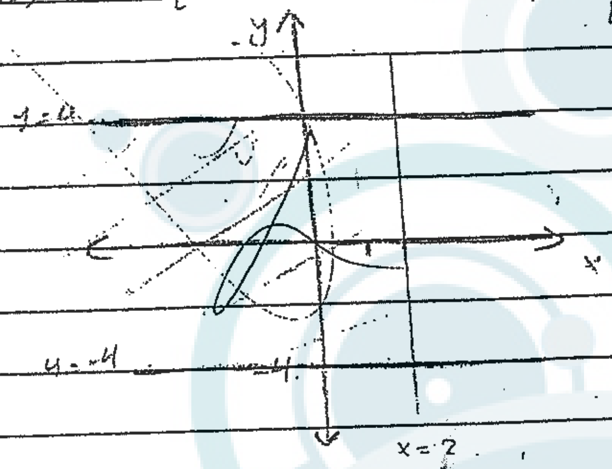
convex 2/4

d) $S = \{(x, y) : x = 2 \quad |Y| \leq 4\}$.

$\to y$

$\to -y$

$-4 \leq y \leq 4$



Convex

Q2: max $Z = 5X_1 + 12X_2 + 4X_3 \Rightarrow Z - 5X_1 - 12X_2 - 4X_3 = 0$

st $X_1 + 2X_2 + X_3 + S_1 = 10$

$2X_1 - X_2 + 3X_3 + R_1 = 8$

min	$A_1 x_1$	x_2	x_3	S_1	R_1	RHS
	0	0	0	0	-1	0
	1	2	1	1	0	10
	2	-1	3	0	1	8

	2	-1	3	0	0	8
B_1	1	2	1	1	0	10
R_1	2	-1	3	0	1	8

0	0	0	0	-1	0	$-R_2 * 3 + R_0$	
$\frac{+1}{3}$	$\frac{+7}{3}$	0	+1	$-\frac{1}{3}$	$+\frac{22}{3}$	$-R_2 + R_1$	$-\frac{1}{3} - \frac{2*3}{3}$
$\frac{2}{3}$	$-\frac{1}{3}$	1	0	$\frac{1}{3}$	$\frac{8}{3}$	$\frac{8}{3} - \frac{10*3}{3}$	

(3)

	x_1	x_2	x_3	s_1	
	-5	-12	-4	0	0
s_1	$+\frac{1}{3}$	$+\frac{7}{3}$	0	+1	$+\frac{22}{3}$
X_3	$\frac{2}{3}$	$\frac{-1}{3}$	1	0	$\frac{8}{3}$

14

	$\frac{-7}{3}$	$\frac{-40}{3}$	0	0	$\frac{32}{3}$
s_1	$\frac{1}{3}$	$\frac{7}{3}$	0	1	$\frac{22}{3}$
X_3	$\frac{2}{3}$	$\frac{-1}{3}$	1	0	$\frac{8}{3}$

$-\frac{4}{3} - \frac{12}{3}$

$\boxed{R_2 * 4 + R_0}$

$\frac{8}{3} - \frac{5 * 2}{3}$

$\frac{32}{3} + 0$

	x_1	x_2	x_3		
	$\frac{-9}{21}$	0	0	$\frac{40}{7}$	$\frac{1104}{21}$
X_2	$\frac{1}{7}$	1	0	$\frac{3}{7}$	$\frac{22}{7}$
X_3	$\frac{15}{21}$	0	1	$\frac{1}{7}$	$\frac{78}{21}$

5.2

$\boxed{R_1 * \frac{40}{3} + R_0}$

$\frac{40}{21} - \frac{7 \cdot 7}{3 \cdot 7}$

$\frac{22}{7} * \frac{40}{3} + \frac{32}{3}$

$\boxed{\frac{1}{3} R_1 + R_2}$

$\frac{1}{21} + \frac{2}{3}$

	0	0	$\frac{9}{15}$	$\frac{609}{105}$	54.8
X_2	0	1	$\frac{-21}{105}$	$\frac{14}{35}$	$\frac{252}{105}$
X_1	1	0	$\frac{21}{15}$	$\frac{1}{5}$	$\frac{78}{15}$

$\boxed{R_2 * \frac{9}{21} + R_0}$

$\boxed{-R_2 * \frac{1}{7} + R_1}$

$-\frac{21}{105} +$

$-\frac{4}{35} + \frac{3}{7}$

$-\frac{78}{105} + \frac{22}{7}$

Point $\left(\frac{78}{15}, \frac{252}{105}, 0\right)$ $\boxed{Z = 54.8}$

(5.2, 2.4, 0)

Kaizen Team

بسم الله الرحمن الرحيم

UNIVERSITY OF JORDAN **Instructor: Eng. Ahmad Jaradat**
Faculty of Engineering & Technology
Industrial Engineering Department
Operations research 0906353

Question 1

One of the most important problems in the field of *statistics* is the *linear regression problem.* Roughly speaking, this problem involves fitting a straight line to statistical data represented by points: $(x_1, y_1), (x_2, y_2), \ldots\ldots, (x_n, y_n)$ on a graph. If we denote the line by $y = a + bx$, the objective is to choose the constants a and b to provide the "best" fit according to some criterion. The criterion usually used is the *method of least squares*, but there are other interesting criteria where linear programming can be used to solve for the optimal values of a and b.

(a) For the following criteria, formulate the linear programming model for this problem:
Minimize the sum of the absolute deviations of the data from the line; that is,

$$\text{Minimize } \sum_{i=1}^{n} | y_i - (a + bx_i) |$$

(b) Apply the above criteria to the following set of points: (2, 8), (3, 10), (4, 12), (5, 14): and **solve the model using *computer program (Tora)***

Question 2

A knitwear manufacturer has weekly orders for three styles of cardigan, winter wooly (40 dozens), Chic style (50 dozens), and classic (20 dozens). Each style requires two operations. It must first be knitted and then made up. Each style can be knitted on either flat or circular machinery and made up either manually or on machines. The table gives the times and costs for each style on each operation and the available weekly capacity for each operation.

	Flat Knitting	Circular Knitting	Manual Make-up	Machine Make-up	
Winter wooly	2.5	2	3	4	Hours per dozen
High Style	1	3	2	3	Hours per dozen
Standard	4	3	4	5	Hours per dozen
Cost	5	3	1	2	$ per hour
Capacity	240	200	200	400	Hours

Manual knitting-up is done by outworkers and management wishes to guarantee a minimum of 50 hours per week for all outworkers combined. There is an additional transport cost of 1 $ for each dozen for cardigans made up manually, irrespective of type. Cardigans which are flat knitted and made up by machine require an additional seam binding process which costs $0.75 per dozen. The company needs to produce a weekly schedule which minimizes its costs.

Ignoring any other considerations, formulate as a linear programming problem (LP). In order for the production control department to allocate the appropriate quantities for each production route for each style, the formulation should enable these quantities to be determined easily. **Solve the model using *computer program (Tora)*!**

Sample Final Exam

Question No. 1 Formulation (10 Points)

The village Butcher shop traditionally makes its meat loaf from a combination of lean ground beef and ground lamp. The ground beef contains 80 percent meat and 20 percent fat, and costs the shop 2.5 JD per kilogram; the ground lamp contains 65 percent meat and 35 percent fat and costs 4.5 JD per kilogram. How much of each kind of meat should the shop use in each kilogram of meat if it wants to minimize its cost and to keep the fat content of the meat loaf to no more than 30 percent?

Solution:

Decision Variables:

$X_1 \Rightarrow$ Percentage of ground beef in one kilogram of meat.

$X_2 \Rightarrow$ Percentage of ground lamp in one kilogram of meat.

Objective function:

$$\max\ Z = 2.5X_1 + 4.5X_2$$

S.t

$$X_1 + X_2 = 1$$

$$0.2X_1 + 0.35X_2 \leq 0.3$$

$$X_1, X_2 \geq 0$$

Kaizen Team

Question No. 2: Two Phase Method (10 Points)

Solve the following LP problem using two phase method:

$$Max\ Z = X_1 + 2X_2 + 5X_3$$

Such that:

$$X_1 + 2X_2 + X_3 \leq 40 \quad \Rightarrow \text{ Std Form} \quad X_1 + 2X_2 + X_3 + X_4 = 40$$

$$2X_1 + 2X_2 - X_3 = 20 \quad \Rightarrow \quad 2X_1 + 2X_2 - X_3 + R = 20$$

$$X_1, X_2, X_3 \geq 0 \quad \Rightarrow \quad X_1, X_2, X_3, R \geq 0$$

Solution: **Phase 1**

Min r = R

S.t $X_1 + 2X_2 + X_3 + X_4 = 40$

$2X_1 + 2X_2 - X_3 + R = 20$

$X_1, X_2, X_3, R \geq 0$

	X1	X2	X3	X4	R	Sol
r	2	2	-1	0	0	20
X4	1	2	1	1	0	40
R	2	2	-1	0	1	20

End of phase 1 ⇒

	X1	X2	X3	X4	R	Sol
r	0	0	0	0	-1	0
X4	-1	0	2	1	-1	20
X2	1	1	-1/2	0	1/2	10

Start of phase 2 ⇒

	X1	X2	X3	X4	Sol
Z	1	0	-6	0	20
X4	-1	0	2	1	20
X2	1	1	-1/2	0	10

	X1	X2	X3	X4	Sol
Z	-2	0	0	3	80
X3	-1/2	0	1	1/2	10
X2	3/4	1	0	1/4	15

Optimal Solution ⇒

	X1	X2	X3	X4	Sol
Z	0	8/3	0	11/3	120
X3	0	0	1	2/3	20
X1	1	4/3	0	1/3	20

Kaizen Team

Question No. 3 Sensitivity Analysis (10 Points)

Consider the following L.P problem: max $Z = 3X_1 + 2X_2 + 5X_3$

Such that:

$$X_1 + 2X_2 + X_3 \le 430$$
$$3X_1 + \quad +2X_3 \le 460$$
$$X_1 \quad +4X_2 \quad \le 420$$
$$X_1, X_2, X_3 \ge 0$$

The corresponding optimal tableau is shown below:

	X1	X2	X3	S1	S2	S3	Sol
Z	4	0	0	1	2	0	1350
X2	-1/4	1	0	1/2	-1/4	0	100
X3	3/2	0	1	0	1/2	0	230
S3	2	0	0	-2	1	1	20

Now suppose that the following changes occurred, find the new feasible and optimal solution if exist:

(a) The objective function changes to $\max imize\ Z = 2X_1 + X_2 + 4X_3$

(b) The following constraint is added: $X_1 \ge 10$

Solution:

(a) Because the objective function is changed, only optimality **might** be changed.

The new dual variables are $\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 0 \end{bmatrix} \times \begin{bmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 7/4 & 0 \end{bmatrix}$

And the new coefficients in Z row:

$X_1 \Rightarrow 1/2 \times 1 + 7/4 \times 3 + 0 \times 1 - 2 = 15/4$

$X_2 \Rightarrow 1/2 \times 2 + 7/4 \times 0 + 0 \times 4 - 1 = 0$

$X_3 \Rightarrow (1/2) \times 1 + (7/4) \times 2 + 0 \times 0 - 4 = 0$

$S_1 \Rightarrow 1/2 \times 1 + 7/4 \times 0 + 0 \times 0 = 1/2$

$S_2 \Rightarrow 1/2 \times 0 + 7/4 \times 1 + 0 \times 0 = 7/4$

$S_3 \Rightarrow 1/2 \times 0 + 7/4 \times 0 + 0 \times 1 = 0$; Which implies that the optimality is not affected.

	X1	X2	X3	S1	S2	S3	Sol
Z	15/4	0	0	1/2	7/4	0	1020
X2	-1/4	1	0	1/2	-1/4	0	100
X3	3/2	0	1	0	1/2	0	230
S3	2	0	0	-2	1	1	20

New Tableau

(b) Because the inequality $X_1 \ge 10$ is not satisfied; so feasibility is affected.

$X_1 - S_4 = 10$

$-X_1 + S_4 = -10$

S_4 is leaving and X_1 is entering.

	X1	X2	X3	S1	S2	S3	S4	Sol
Z	4	0	0	1	2	0	0	1350
X2	-1/4	1	0	1/2	-1/4	0	0	100
X3	3/2	0	1	0	1/2	0	0	230
S3	2	0	0	-2	1	1	0	20
S4	-1	0	0	0	0	0	1	-10

The new optimal and feasible tableau $\Longrightarrow$

	X1	X2	X3	S1	S2	S3	S4	Sol
Z	0	0	0	1	2	0	4	1310
X2	0	1	0	1/2	-1/4	0	-1/4	102.5
X3	0	0	1	0	1/2	0	3/2	215
S3	0	0	0	-2	1	1	2	0
X1	1	0	0	0	0	0	-1	10

Kaizen Team

Question No. 4 Transportation Model (10 Points)

Consider the following transportation problem:

- (a) Use North-West (Corner) method to find the starting solution.
- (b) Continue from part (a) to find the optimal solution.

TO From	Destination				
	1	2	3	4	Supply
1	3	7	6	4	50
2	2	4	3	2	20
3	4	3	8	5	30
Demand	30	30	20	20	

Solution:

3 30	7 20	6 0	4 -1	50	U_i 0
2 -2	4 10	3 10	2 -2	20	-3
4 +1	3 +6	8 10	5 20	30	2
30	30	20	20	$\theta = 10$	
$V_j = 3$	7	6	3		

1

3 30	7 20	6 0	4 +5	50	U_i 0
2 -2	4 0	3 20	2 +4	20	-3
4 -5	3 10	8 -6	5 20	30	-4
30	30	20	20	$\theta = 20$	
$V_j = 3$	7	6	9		

2

3 30	7 0	6 0	4 20	50	U_i 0
2 -2	4 0	3 20	2 -1	20	-3
4 -5	3 30	8 -6	5 -5	30	-4
30	30	20	20	*Optimal*	
$V_j = 3$	7	6	4	$Z = 320$	

Alternative Optima

$Z = 320$

3

3 30	7 0	6 0	4 20	50	U_i 0
2 -2	4 0	3 20	2 -1	50	-3
4 -5	3 30	8 -6	5 -5	30	-4
30	30	20	20	*Optimal*	
$V_j = 3$	7	6	4	$Z = 320$	

4

Question No. 5 (10 Points) **Solve either part (a) or part (b) but not both!**

(a) A backing firm can produce a specialty bread in either of its two plants as shown in table below:

Plant	Production Capacity Loaves	Production Cost c/loaf
A	4500	13
B	3500	15

Four restaurant chains are willing to purchase this bread; their demands and the prices they are willing to pay are shown in table below:

:

Chain	Maximum Demand Loaves	Price offered C/loaf
1	1800	55
2	2200	50
3	800	60
4	1400	48

The cost (in cents) of shipping a loaf from a plant to a restaurant chain is given in the table below

	Cost of shipping (C/loaf)			
	Chain 1	Chain 2	Chain 3	Chain 4
Plant A	6	8	11	9
Plant B	12	6	8	5

Formulate as transportation model (tableau) to maximize the total profit. **Do not solve!**

(b) Solve the following assignment problem:

	Worker				
	1	2	3	4	5
1	3	8	2	10	3
2	8	7	2	9	7
3	6	4	2	7	7
4	8	4	2	3	5
5	9	10	6	9	10

(a) Solution:

	Chain 1	Chain 2	Chain3	Chain4	Dummy Chain	Supply
Plant A	36	29	36	26	0	4500
Plant B	28	29	37	28	0	3500
Demand	1800	2200	800	1400	1800	Sum = 8000

(b) Solution:

$Z_{\min} = 21$

	1	2	3	4	5
1	3	8	2	10	$\boxed{3}$
2	8	7	$\boxed{2}$	9	7
3	6	$\boxed{4}$	2	7	7
4	8	4	2	$\boxed{3}$	5
5	$\boxed{9}$	10	6	9	10

Kaizen Team

SAMPLE FINAL EXAM

Time: 120 min

Question1 *15 Points*

Consider the following L.P problem:

$$Max\ Z = X_1 + 2X_2 + 5X_3$$

Such that:

$$X_1 + 2X_2 + X_3 \le 40$$
$$2X_1 + 2X_2 - X_3 = 20$$
$$X_1, X_2, X_3 \ge 0$$

(a) Solve using two phase method.
(b) Write the dual problem.
(c) Solve the dual problem.

Question2 *10 Points*

Consider the following L.P problem:

$$\max Z = 2X_1 + 3X_2 + 5X_3$$

St

$$X_1 + 2X_2 + X_3 \le 20$$
$$2X_1 - 3X_2 \ge 4$$
$$X_1, X_2, X_3 \ge 0$$

Using two phase method, the obtained optimal solution is:

	X_1	X_2	X_3	X_4	X_5	R.H.S
Z	0	23/2	0	5	3/2	94
X_3	0	7/2	1	1	1/2	18
X_1	1	-3/2	0	0	-1/2	2

Now suppose that the following constraint is added: $X_1 + X_2 \ge 5$

Find the new feasible and optimal solution if exist.

Question 3 *5 Points*

A company has two plants producing a certain product that is to be shipped to three distribution centers. The unit production costs are the same at the two plants, and the shipping cost (in hundreds of dollars) per unit of the product is shown for each combination of plant and distribution center as follows:

	Destination center		
	1	*2*	*3*
Plant 1	4	6	3
2	6	5	2

A total of 60 units is to be produced and shipped per week. Each plant can produce and ship any amount up to a maximum of 50 units per week, so there is considerable flexibility on how to divide the total production between the two plants so as to reduce the shipping costs.
Management's objective is to determine how much should be produced at each plant, and what the overall shipping pattern should be in order to minimize total shipping cost. Assume that each distribution center must receive exactly 20 units per week. Formulate as transportation problem! *Do not solve!*

Question 4 10 Points

In the transportation problem below, the total demand exceeds the total supply. Suppose that the penalty costs per unit of unsatisfied demand are 6, 4 and 3 for destinations 1, 2, 3 respectively

		Destination			Supply
From	TO	1	2	3	
	1	6	2	8	20
Source	2	7	5	6	100
	3	4	3	6	25
Demand		95	40	60	

(a) Use Northwest-corner method to find the starting solution.
(b) Continue from part (a) to find the optimal solution.

Question 5 10 Points

Joshop needs to assign 4 jobs to 4 workers. The cost of performing a job is a function of the skills of the workers. The table below summarizes the cost of assignments. Worker 1 cannot do job 3, and worker 3 cannot do job 4. Determine the optimal assignment using the Hungarian method typically as discussed in class!

		Job			
		1	2	3	4
Worker	1	5	5	-	2
	2	7	4	2	3
	3	9	3	5	-
	4	7	2	6	7
	5	7	3	7	8

Kaizen Team

SAMPLE FINAL EXAM

Q1 (14 points)

(a) **(10 points)** Solve the following L.P problem using two phase method:

$$Min\ Z = -X_1 + 2X_2 + 3X_3$$

S.t

$$-X_1 + X_2 + X_3 \geq 3$$
$$X_1 + 2X_2 + X_3 \leq 10$$
$$X_1,\ X_2,\ X_3 \geq 0$$

(b) **(4 points)** In applying simplex technique, the following substitution is used for the unrestricted variable y_i:

$$y_i = y_i' - y_i'' \quad where\ y_i'\ and\ y_i''\ \text{are nonnegative}$$

The question is why the variables y_i' and y_i'' can't assume positive values simultaneously?

Solution

(a) Phase 1

$$Min\ r = R$$

S.t

$$-X_1 + X_2 + X_3 - X_4 + R_1 = 3$$
$$X_1 + 2X_2 + X_3 + X_5 = 10$$
$$X_1,\ X_2,\ X_3,\ X_4,\ X_5,\ R \geq 0$$

	X_1	X_2 ↓	X_3	X	X_5	R	Sol
r	-1	1	1	-1	0	0	3
← R	-1	$\boxed{1}$	1	-1	0	1	3
X_5	1	2	1	0	1	0	10

End of phase 1 ⇨

	X_1	X_2	X_3	X_4	X_5	R	Sol
r	0	0	0	0	0	-1	0
X_2	-1	$\boxed{1}$	1	-1	0	1	3
X_5	3	0	-1	2	1	-2	4

Phase 2 start of phase 2 ⇨

	X_1	X_2	X_3	X_4	X_5	Sol
Z	-1	0	-1	-2	0	6
X_2	-1	1	1	-1	0	3
X_5	3	0	-1	2	1	4

Optimal

(b)

To explain this: In simplex technique **only** basic variables can assume positive values and the nonbasic variables are at zero level; now the coefficients of one variable say y_i' in the matrix A (AX=b) is equal to $-1\times$coefficients of the other variable; and since this phenomena makes the matrix containing both variables has a determinant equal to zero, so it is impossible that both variables be a basic variables simultaneously because the matrix containing the coefficients of the basic variables is invertible (has an inverse B^{-1}).

Kaizen Team

OR1

Question No. 2 Sensitivity Analysis (15 Points).

Consider the following L.P problem: $Max\ Z = 3X_1 + 2X_2 + 5X_3$

S.t

$$
\begin{array}{llll}
X_1 + 2X_2 + X_3 + X_4 & & & = b_1 \\
2X_1 \quad + 2X_3 & + X5 & & = 460 \\
X_1 + 4X_2 & & + X_6 & = 420 \\
\quad X_3 & & + X_7 & = b_4
\end{array}
$$

The following optimal tableau corresponds to specific values of b_1 and b_4

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Sol
Z	0	0	0	0	a	1/2	c	k
X_4	0	0	0	1	−1/4	−1/2	−1/2	5
X_1	1	0	0	0	1/2	0	−1	30
X_2	0	1	0	0	−1/8	e	1/4	97.5
X_3	0	0	1	0	0	0	d	f

Determine the following:

(a) The right-hand side values b_1 and b_4

(b) The elements a, c, d, e, f, k

(c) The optimal duel solution

Show your work (calculations) and be advised that no credit will be given for guessing! A full credit will be given for correct calculations or explanations!

Solution:

$$B^{-1} \times b = X_{Basic} \quad \Rightarrow \quad \begin{bmatrix} 1 & -1/4 & -1/2 & -1/2 \\ 0 & 1/2 & 0 & -1 \\ 0 & -1/8 & e & 1/4 \\ 0 & 0 & 0 & d \end{bmatrix} \times \begin{bmatrix} b_1 \\ 460 \\ 420 \\ b_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 30 \\ 97.5 \\ f \end{bmatrix} \Rightarrow \begin{array}{ll} b_1 - \frac{1}{2} b_4 = 330 \ \ldots\ldots & (1) \\ 230 - b_4 = 30 \ \ldots\ldots & (2) \\ 420\,e - 7.5 = 97.5 \ \ldots & (3) \\ b_4\, d = f \ \ldots\ldots\ldots\ldots & (4) \end{array}$$

Solving Equations (1), (2), and (3) yields:

$b_1 = 430$, $b_4 = 200$, $e = 1/4$ & Equation (4) becomes $200\ d = f$ .....(4)

$$Y = C_B \times B^{-1} \quad [0 \ \ 3 \ \ 2 \ \ 5] \times \begin{bmatrix} 1 & -1/4 & -1/2 & -1/2 \\ 0 & 1/2 & 0 & -1 \\ 0 & -1/8 & 1/4 & 1/4 \\ 0 & 0 & 0 & d \end{bmatrix} = \begin{bmatrix} 0 & \frac{5}{4} & \frac{1}{2} & \frac{-5}{2} + 5d \end{bmatrix} \Rightarrow \begin{array}{l} y_1 = 0 \\ y_2 = 5/4 = a \\ y_3 = 1/2 \\ y_4 = -5/2 + 5d = c \end{array}$$

Coefficient of X_3 in the Z row is equal to zero because X_3 is a basic variable, so the left – the right hand side of the corresponding duel constraint must equal zero.

Then $y_1 + 2y_2 + 0y_3 + y_4 - 5 = 0$ and by substitution of the values of the duel variables in this equation yields $y_4 = 5/2 = C$ and $d = 1$, and by substituting the value of $d = 1$ in equation (4) gives $f = 200$

And the value of $k = Z_{opt}$ is ready to be calculated:

$$k = Z_{opt} = C_B \times X_B = 3 \times 30 + 2 \times 97.5 + 5 \times 200 = 1285$$

Summary of previous calculations:

(a) $b_1 = 430$
$b_4 = 200$

(b) $a = 5/4$ $\quad d = 1$ $\quad f = 200$
$C = 5/2$ $\quad e = 1/4$ $\quad k = 1285$

(c) $y_1 = 0$, $y_2 = 5/4$, $y_3 = 1/2$, $y_4 = 5/2$, $W_{opt} = 1285$

Question No. 4 Transportation Model (13 Points)

Consider the following transportation problem:

(a) Use least cost method to find the starting solution.

(b) Continue from part (a) to find the optimal solution.

TO From	Destination 1	 2	 3	 4	Supply
1	3	7	6	4	60
2	2	4	3	2	10
3	4	3	8	5	30
Demand	20	30	25	25	

Solution:

3 20	7 0	6 25	4 15	60	U_i 0
2 -1	4 +1	3	2 10	10	-2
4 -5	3 30	8 -6	5 -5	30	-4
20	30	25	25	$\theta = 10$	
$V_j = 3$	7	6	4		

$\Rightarrow$

3 20	7 0	6 15	4 25	60	U_i 0
2 -2	4 0	3 10	2 -1	10	-3
4 -5	3 30	8 -6	5 -5	30	-4
30	20	25	25	$Z_{opt.} = 370$	
$V_j = 3$	7	6	4		

Optimal

Kaizen Team

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OPERATIONS RESEARCH
96451
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		X_1	X_2	X_3	X_4	X_5	X_6	R_1	R_2
9	X_4	1	1	1	1	0	0	0	0
5	R_1	1	0	0	1	-1	0	1	0
25	R_2	1	1	5	0	0	-1	0	1
30 ~~0~~	Z	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	-1	-1
		2	1	5	1	-1	-1	0	0

Question 1: 30 Points

Solve the following problem using any method:

Maximize $z = x^1 + 3x^2 - 4x^3 + 5x^4$

s.t. $x^1+x^2+x^3+x^4 = 9$ $\qquad X_1 + X_2 + X_3 + X_4 = 9$

$x^1+x^4 > 5$ $\qquad X_1 + X_4 - X_5 + R_1 = 5$

$x^1+x^2+5x^3 < 25$ $\qquad X_1 + X_2 + 5X_3 - X_6 + R_2 = 25$

Question 2: 30 Points

min $Z = R_1 + R_2$

$R_1 + R_2 + Z$

Find the best linear programming model for the following problem only; do not solve.

A company wishes to plan its production of two items with seasonal demands over a 12-month period. The monthly demand of item 1 is 100,000 units during the months of October, November, and December; 10,000 units during the months of January, February, March, and April; and 30,000 units during the remaining months. The demand of item 2 is 50,000 during the months of October through February and 15,000 during the remaining months. Suppose that the unit product cost of items 1 and 2 is \$5.00 and \$8.00 respectively, provided that these were manufactured prior to June. After June, the unit costs are reduced to \$4.50 and \$7.00 because of the installation of an improved manufacturing system. The total units of items 1 and 2 that can be manufactured during any particular month cannot exceed 120,000 for Jan–Sept, and 150,000 for Oct–Dec. Furthermore, each unit of item 1 occupies 2 cubic feet and each unit of item 2 occupies 4 cubic feet of inventory. Suppose that the maximum inventory space allocated to these items is 150,000 cubic feet and that the holding cost per cubic foot during any month is \$0.10. Formulate the production scheduling problem so that the total production and inventory costs are minimized.

feet (1)

$+ 0.1 (2 x_1 + 4 \ldots) \le 150$

min C = prod. + inv.

= X1 + X2 + X1 +

5 0.00

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Question 3: 40 Points

Determine if each of the following statements is False or True by putting F or T in the empty column:

✗	1.	T	Degeneracy and redundancy are always together, one can not happen without the other.
✗	2.	T	The more variables in a problem the better the chance for producing more products, but that does not mean the value of the objective function is going to be better.
✗	3.	T	The dual variable is the actual rate of change of the objective function with respect to the availability of the resource.
✗	4.	T	The reduced cost is only useful to determine feasibility of the solution.
✗	5.	F	If the feasible region is unbounded it is not necessarily that the optimal solution is unbounded.
✓	6.	T	The reason for using the minimum ratio test is to maintain feasibility by determining the leaving variable.
✓	7.	T	In pivoting, the row operations maintain the same information in the system of linear equations.
✓	8.	T	The M value, in the big M method, can be set to be equal to the largest coefficient in the objective function plus 1.
✗	9.	T	A problem has a multiple, alternate, optimal if one of the variables is equal to 0 at optimality.
✗	10	T	A problem is degenerate if the reduced cost of one of the variables is equal to 0 at optimality.

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10/20

Question 4: 40 Points

Formulate the following problem as a linear program but do not solve:

A manufacturing company is considering diversifying its products to gain more market share. Currently they have 2 products and are considering producing a total of 5 products. The following table provides the data for the existing products and potential new products:

	Product	Type of Product	Material 1 Req.	Material 2 Req.	Labor Hours Req.	Profit per Unit	Expected Demand
x_1	1.	Old	2	1	.5	5	100
x_2	2.	Old	1	2	.4	6	200
x_3	3.	New	2	3	.6	5	100
x_4	4.	New	3	2	.3	4	150
x_5	5.	New	1	4	.7	3	300
	Availability		1000	1500	300		

i number of the product 1, 2, 3, ..5.

a) Formulate a linear program to maximize the profit from all existing and new products.

max $Z = 5x_1 + 6x_2 + 5x_3 + 4x_4 + 3x_5$

$(2X_1 + X_2 + 2X_3 + 3X_4 + x_5) \leq 1000$

$x_1 + 2x_2 + 3x_3 + 2x_4 + 4x_5 \leq 1500$

$.5X_1 + .4X_2 + .6X_3 + .3X_4 + .7X_5 \leq 300$

$X_1 \leq 100$

$X_2 \leq 200$

$X_3 \leq 100$

$X_4 \leq 150$

$X_5 \leq 300$

$x_i \geq 0 \ \forall i$

b) The marketing manager insists that for the old products at least 75% of the demand have to be met, add constraints to reflect that.

$\geq$ 0.75

$X_1 + X_2 \geq \frac{75}{100}(X_1 + X_2 + X_3 + X_4 + X_5)$

c) The production manager, however, argues that not all products (the 5 products) can be produced together due to technical reasons. But he likes to see at least 1 new product. Add constraints to express his desire.

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Question 2: 30 Points

Find an initial feasible solution for the following problem using either the Big M or 2 Phase method. After finding the initial feasible solution set up the first tableau only.

Maximize $Z = 2x_1 + 5x_2 + 4x_3$

Subject to : $x_1 + x_2 + x_3 = 10$

$1.5x_1 + 2x_2 + 2x_3 \leqslant 20$

$x_1 + x_2 - 2x_3 \geqslant 0$

$x_1, x_2, x_3 \geqslant 0$

$Z = 2x_1 + 5x_2 + 4x_3 - M1$

by 2 phase min $r = R_1 - R_2$

$x_1 + x_2 + x_3 + R_1 = 10$

$1.5x_1 + 2x_2 + 2x_3 + S_1 = 20$

$x_1 + x_2 - 2x_3 - S_2 + R_2 = 0$

$x_1, x_2, x_3, S_1, S_2, R_1, R_2 \geqslant 0$

$\times -M + R$

$\times -M + 2$

	x_1	x_2	x_3	R_1	R_2	S_1	S_2	
	-2	-5	-4	M	M	0	0	0
S_1	1.5	2	2	0	0	1	0	20
R_1	1	1	1	1	0	0	0	10
R_2	1	1	-2	0	1	0	1	0
	$(-M-2+M)$	$(-M-5+M)$	$(-M-4+2M)$	0	0	0	$-M$	0
S_1	1.5	2	2	0	0	1	0	20
R_1	1	1	1	1	0	0	0	10
R_2	1	1	-2	0	1	0	1	0
	$-2M-2$	$-2M-5$	$M-4$	0	0	0	$-M$	0
S_1	1.5	2	2	0	0	1	0	20
R_1	1	1	1	1	0	0	0	10
R_2	1	1	-2	0	1	0	1	0

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Question No. 3 Transportation Model (8 Points)

The demand for a perishable item over the next 4 months is 400, 300, 420, 380 tons respectively. The supply capacities for the same months are 500, 600, 200 and 300 tons. The purchase price per ton varies from month to month and is estimated at $100, $140, $120, $150, respectively. Because the item is perishable, a current month's supply must be consumed within 3 months (starting with the current month). The storage cost per ton per month is $3. The nature of the item does not allow back-ordering. Set up the problem as transportation model. **Do not solve!**

Solution

	1	2	3	4	5Dummy	supply
1	100	103	106	-	0	500
2	-	140	143	146	0	600
3	-	-	120	123	0	200
4	-	-	-	150	0	300
Demand	400	300	420	380	100	Balanced