Instructor's Manual to Accompany

FUZZY LOGIC WITH ENGINEERING APPLICATIONS–3rd Edition

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FUZZY LOGIC WITH ENGINEERING APPLICATIONS – 3rd Edition

Solutions Manual

Preface

In the development of a textbook such as this, it has been my desire for years to produce an archival work that could be used by senior undergraduate and first-year graduate students to learn the rudiments of fuzzy set theory and fuzzy logic, and to be able to contrast these theories from other standard uncertainty approaches, such as probability theory. A textbook with numerous examples in the various engineering disciplines and many exercises at the end of each chapter is the pedagogy developed in this text.

This solutions manual is meant for those instructors who adopt this text for use in a traditional class setting. There are numerous worked examples in the text, and over 230 exercises at the ends of the chapters in the book. Instructors are encouraged to work the examples in class and to assign the end-of-chapter exercises for homework assignments to the students. Independent work on the exercises by the students should enhance their learning and produce engineers and scientists knowledgeable in this new and growing field.

The solutions are arranged by chapter, following the table of contents of the associated textbook. Many of the problems have a unique solution. Several, especially those in Chapters 9-15 have typical solutions–many more could be provided by the students. Software needed for problems and methods in Chapter 7 is available to those who purchase the text on the John Wiley & Sons website, www.wileyeurope.com/go/fuzzylogic.

As with any solutions manual, users may come across some mistakes. Should this be the case, I would appreciate a communication to that effect at my e-mail address: ross@unm.edu.

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FUZZY LOGIC WITH ENGINEERING APPLICATIONS

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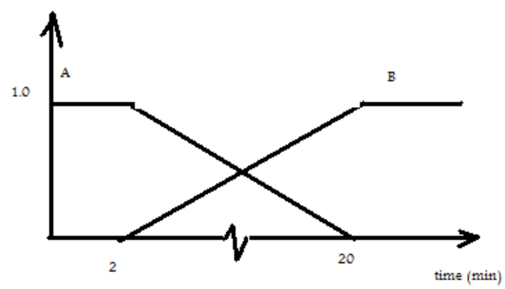
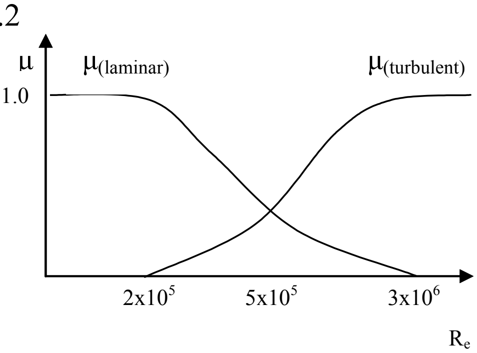
CHAPTER 1

Introduction

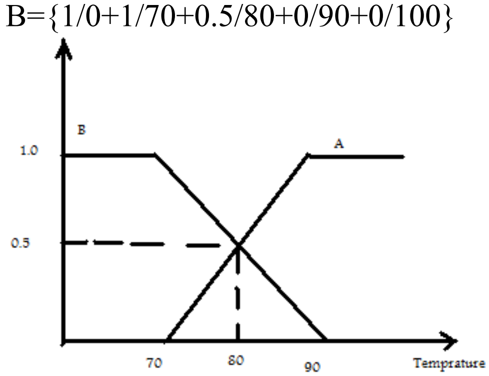
1.1 Assume that Extra Fast takes 2 minutes or less (A) Slow Takes 20 minutes or more (B)
Note that times between 2 min. and 20 min. can be considered as Slow.
A={1/0+1/2+0/20+0/25}
B={0/1+0/2+1/20+1/25}



1.2



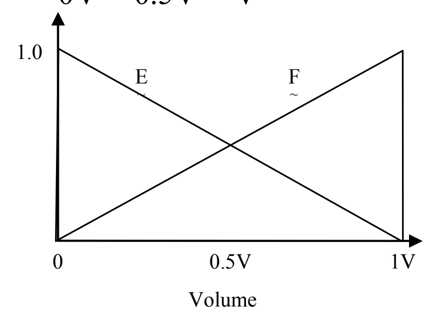
1.3 For T>90° Curling occurs (A)
For T<70° Curling does not occur (B)
A={0/0+0/70+0.5/80+1/90+1/100}
B={1/0+1/70+0.5/80+0/90+0/100}



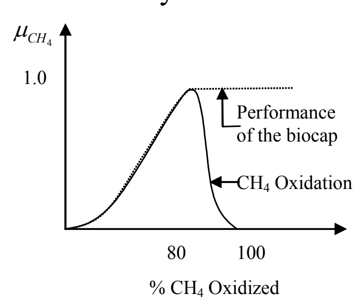
1.4 According to the diameters of base(d) and height (h) ratios, solids can be classified into these types. If $0 < d/h < 1$, it is a rod (its membership is $\mu_{\underset{\sim}{A}(n)}$); if $1 < d/h < \infty$, it is a disk ($\mu_{\underset{\sim}{C}(n)}$): if d/h = 1, it is a right cylinder ($\mu_{\underset{\sim}{B}(n)}$).

1.5 Let V be the volume of the glass, then the condition that glass is full is given by, $\underset{\sim}{F} = \frac{0}{0V} + \frac{0.5}{0.5V} + \frac{1}{V}$ & empty,

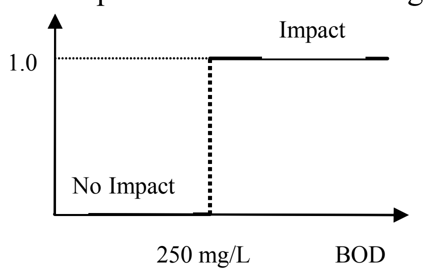
$$\underset{\sim}{E} = \frac{1}{0V} + \frac{0.5}{0.5V} + \frac{0}{V}.$$



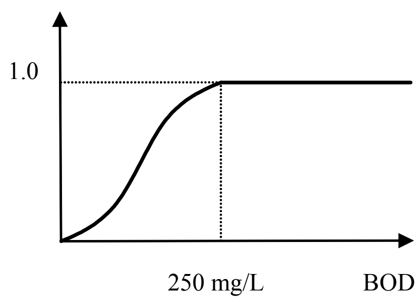
1.6 Since landfills are classified as best if they are capable of oxidizing 80% of the methane that originates, the membership function should achieve a full membership here. A reasonable membership function for the % CH_4 oxidation may be as follows:



1.7 The crisp set Impact membership function, figure (a) above has values of 1 for any BOD greater than or equal to 250mg/L while any value of BOD less than 250mg/L has a value of zero. However the membership function for the fuzzy set, figure (b) above, has some Impact values below 250mg/L.

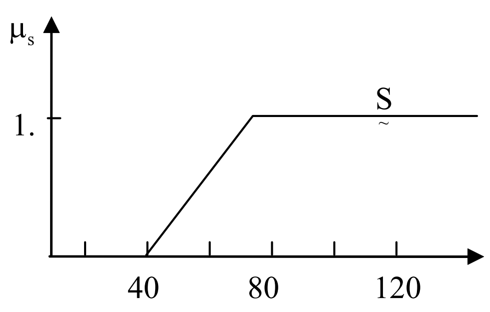


(a)



(b)

1.8 .

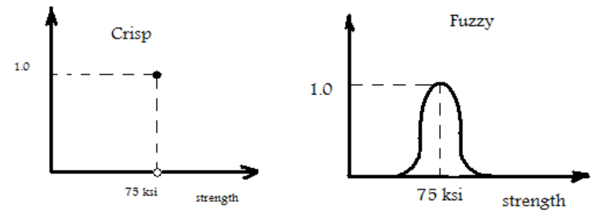


1.9 Crisp:

$\chi_{LD_{50}} = 1$, for $0 < LD_{50} \leq 5000 \ ^{mg}/_{kg}$

$\chi_{LD_{50}} = 0$, for $LD_{50} > 5000 \ ^{mg}/_{kg}$ and $LD_{50} \leq 0$

1.10



1.11 Fuzzy Sets can be represented explicitly by families of parameterized functions, the most common being the following:

a) Triangular Functions

$$A(x)\begin{cases} 0, & x \leq 2 \\ \dfrac{x-a}{m-a}, & x \in [2, m] \\ \dfrac{b-x}{b-m}, & x \in [m, b] \\ 0, & x \geq b \end{cases}$$

Where m , a, and b denote the modal value, the lower and upper bound values, respectively, for all nonzero values of $A(x)$.

b) Trapezoidal Function

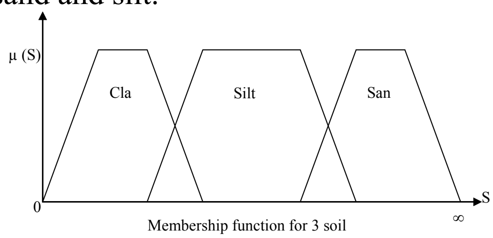
$$A(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{m-a}, & x \in [a, m] \\ 1, & x \in [m, n] \\ \dfrac{b-x}{b-n}, & x \in [n, b] \\ 0, & x > b \end{cases}$$

c) Gaussian Function

$$A(x) = e^{-k(x-m)^2}$$

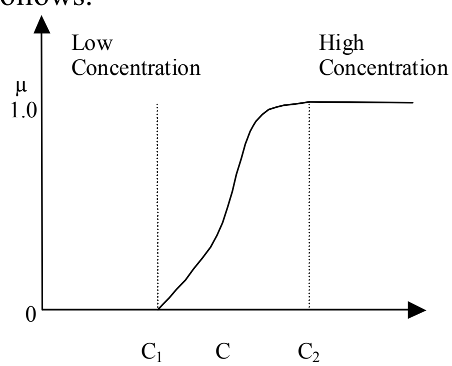
Where $k > 0$

1.12 Fuzzy sets are useful in this situation where there is an inherent overlap among soil types. Clay has a smaller particle size than that of Sand however it is often difficult to distinguish Clay from Silt. This is also true for particle size between sand and silt.



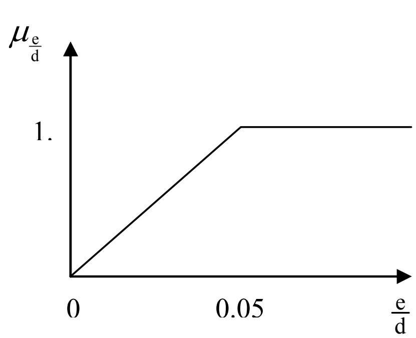
Membership function for 3 soil

1.13 The membership function for the absorber "sales gas" sour gas concentration as a function of concentration C, with C_1 and C_2 is as follows.

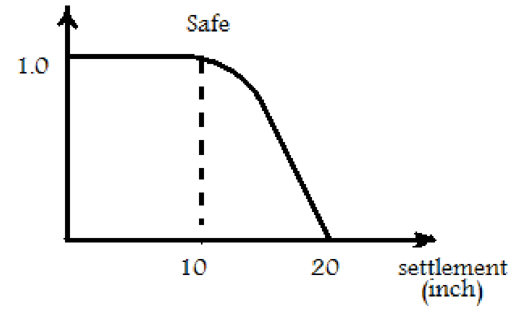


Absorber "Sales Gas" Sour Gas Concentration

1.14 The relation shows that the load becomes more eccentric as it approaches $\frac{e}{d} = 0.05$, it remains eccentric thereafter

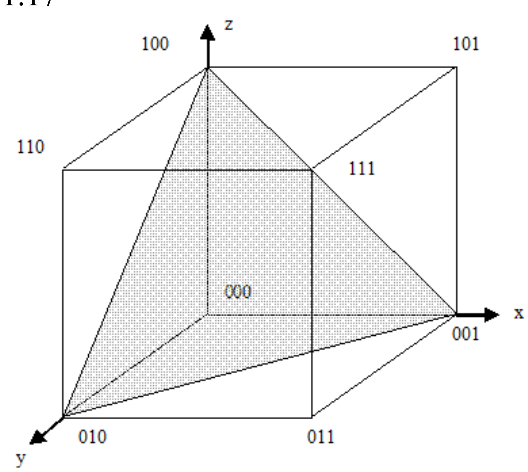


1.15



1.16 The geometric shape can resemble a disk, a cylinder, or a rod depending on the aspect ratio of d/h. For d/h << 1 the shape of the object approaches a long rod; in fact, as d/h approaches 0 the shape approaches a line. For d/h >> 1 the object approaches the shape of a flat disk; as d/h approaches infinity the object approaches a circular area. For other values of this aspect ratio, e.g. for d/h $\approx$ 1, the shape is typical of what we would call a "right circular cylinder."

1.17



CHAPTER 2

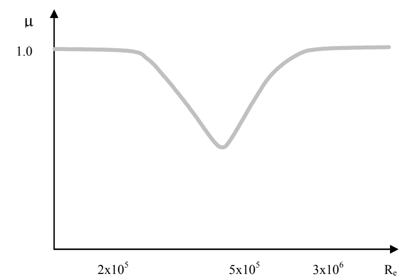
Classical Sets and Fuzzy Sets

2.1

a)

$\underset{\sim}{T} \cup \underset{\sim}{L}$

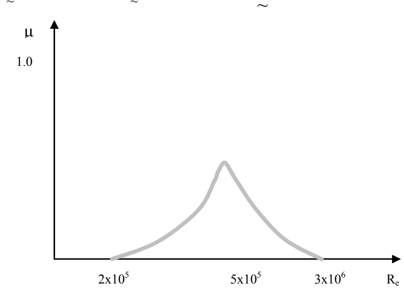
$$\mu_{\underset{\sim}{T} \cup \underset{\sim}{L}}(R_e) = \mu_{\underset{\sim}{T}}(R_e) \vee \mu_{\underset{\sim}{L}}(R_e)$$



b)

$\underset{\sim}{T} \cap \underset{\sim}{L}$

$$\mu_{\underset{\sim}{T} \cup \underset{\sim}{L}}(R_e) = \mu_{\underset{\sim}{T}}(R_e) \vee \mu_{\underset{\sim}{L}}(R_e)$$



c)

$$\underset{\sim}{T} = \left\{ \frac{0}{0} + \frac{0.1}{2x10^5} + \frac{0.75}{5x10^5} + \frac{1}{3x10^6} \right\}$$

$$\underset{\sim}{L} = \left\{ \frac{1.0}{0} + \frac{0.9}{2x10^5} + \frac{0.25}{5x10^5} + \frac{0}{3x10^6} \right\}$$

2.2 (a) Select ponds number 4 and 5

(b) A U B = {0.5/1 + 0.6/2 + 0.8/3 +1/4 +1/5}

2.3

AUB={0.2/1+0.3/2+0.6/3+0.9/4}

A∩B={0.15/1+0.25/2+0.5/3+0.8/4}

A|B=A∩$\bar{B}$, $\bar{B} = \left\{\frac{0.8}{1} + \frac{0.7}{2} + \frac{0.5}{3} + \frac{0.2}{4}\right\}$

A|B={0.15/1+0.85/2+0.5/3+0.2/4}

B|A=B∩$\bar{A}$,

$\bar{A}$={0.85/1+0.75/2+0.4/3+0.1/4}

B|A={0.2/1+0.3/2+0.4/3+0.1/4}

2.4 a)

$$\underset{\sim}{D_1} \cup \underset{\sim}{D_2} = \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1.0}{3.0}$$

b)

$$\underset{\sim}{D_1} \cap \underset{\sim}{D_2} = \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}$$

c) $$\overline{\underset{\sim}{D_1}} = \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1.0}{3.0}$$

d) $$\overline{\underset{\sim}{D_2}} = \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1.0}{3.0}$$

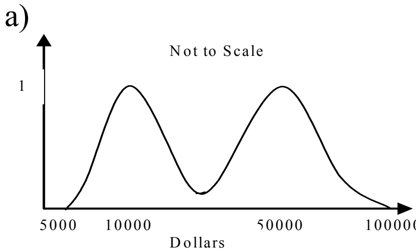
e) $\underset{\sim}{D_1} / \underset{\sim}{D_2} = \underset{\sim}{D_1} \cap \overline{\underset{\sim}{D_2}} =$

$$\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}$$

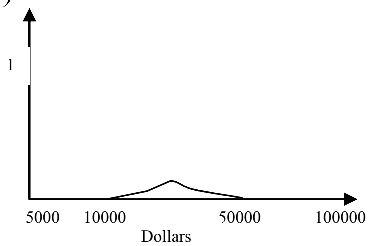
f) $\overline{\underset{\sim}{D_1} \cup \underset{\sim}{D_2}} = \overline{\underset{\sim}{D_1} \cap \underset{\sim}{D_2}} =$

$$\frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0}$$

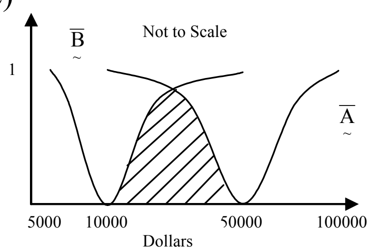
2.5 a)



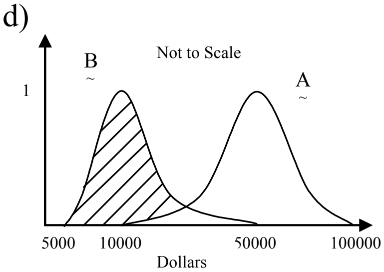
b)



c)



d)



2.6 a) $\underset{\sim}{A} \cup \underset{\sim}{B} = \frac{0.2}{0} + \frac{0.5}{1} + \frac{1}{2} + \frac{0.7}{3} + \frac{0.2}{4}$

b) $\underset{\sim}{A} \cap \underset{\sim}{B} = \frac{0.1}{0} + \frac{0.5}{1} + \frac{1}{2} + \frac{0.6}{3} + \frac{0.1}{4}$

c) $\overline{\underset{\sim}{A}} = \frac{0.9}{0} + \frac{0.5}{1} + \frac{1}{2} + \frac{0.6}{3} + \frac{0.8}{4}$

d) $\overline{\underset{\sim}{B}} = \frac{0.8}{0} + \frac{0.5}{1} + \frac{0}{2} + \frac{0.3}{3} + \frac{0.9}{4}$

2.7 a)

$$\underset{\sim}{M} \cup \underset{\sim}{F} = \frac{1}{0} + \frac{1}{10} + \frac{1}{20} + \frac{0.7}{40} + \frac{0.9}{80} + \frac{0}{100}$$

b)

$$\underset{\sim}{M} \cap \underset{\sim}{F} = \frac{0.3}{1} + \frac{0.7}{10} + \frac{0.4}{20} + \frac{0}{40} + \frac{0}{80} + \frac{0}{100}$$

c)

$$\overline{\underset{\sim}{M}} = \frac{0}{1} + \frac{0.3}{10} + \frac{0.6}{20} + \frac{1}{40} + \frac{1}{80} + \frac{1}{100}$$

d) $\overline{\underset{\sim}{F}} = \frac{0.7}{0} + \frac{0}{10} + \frac{0}{20} + \frac{0.5}{40} + \frac{0.8}{80} + \frac{1}{100}$

e) $\underset{\sim}{C} \cap \overline{\underset{\sim}{F}} = \frac{0}{1} + \frac{0}{10} + \frac{0}{20} + \frac{0.5}{40} + \frac{0.8}{80} + \frac{1}{100}$

f) $\overline{\underset{\sim}{M} \cap \underset{\sim}{C}} = \overline{\underset{\sim}{M}} \cup \overline{\underset{\sim}{C}}$

$$= \frac{0.3}{1} + \frac{0.7}{10} + \frac{0.4}{20} + \frac{0}{40} + \frac{0}{80} + \frac{0}{100}$$

2.8 $\underset{\sim}{Flow_1} \cap \underset{\sim}{Flow_2}$

$$= \frac{0}{0} + \frac{0.45}{20} + \frac{0.6}{40} + \frac{0.45}{60} + \frac{0.3}{80} + \frac{0.1}{100}$$

$\underset{\sim}{Flow_1} \cup \underset{\sim}{Flow_2}$

$$= \frac{1}{0} + \frac{0.8}{20} + \frac{0.65}{40} + \frac{0.8}{60} + \frac{.95}{80} + \frac{1.0}{100}$$

$\underset{\sim}{Flow_1} | \underset{\sim}{Flow_2} = \underset{\sim}{Flow_1} \cap \overline{\underset{\sim}{Flow_2}}$

$$\overline{\underset{\sim}{Flow_2}} = \{\frac{1.0}{0} + \frac{0.55}{20} + \frac{0.4}{40} + \frac{0.2}{60} + \frac{0.05}{80} + \frac{0}{100}\}$$

$\underset{\sim}{Flow_1} | \underset{\sim}{Flow_2}$

$$= \frac{1.0}{0} + \frac{0.55}{20} + \frac{0.4}{40} + \frac{0.2}{60} + \frac{0.05}{80} + \frac{0}{100}$$

$\underset{\sim}{Flow_2} | Flow_1 = \underset{\sim}{Flow_2} \cap \overline{\underset{\sim}{Flow_1}}$

$$\overline{\underset{\sim}{Flow_1}} = \{\frac{0}{0} + \frac{0.2}{20} + \frac{0.35}{40} + \frac{0.55}{60} + \frac{0.7}{80} + \frac{0.9}{100}\}$$

$\underset{\sim}{Flow_2} | \underset{\sim}{Flow_1}$

$$= \frac{0}{0} + \frac{0.2}{20} + \frac{0.35}{40} + \frac{0.55}{60} + \frac{0.7}{80} + \frac{0.9}{100}$$

2.9 a) $\underset{\sim}{A} \cup \underset{\sim}{B} =$

$$\frac{0}{0.73} + \frac{0.8}{0.735} + \frac{1}{0.74} + \frac{1}{0.745} + \frac{0.6}{0.750}$$

b) $\underset{\sim}{A} \cap \underset{\sim}{B} =$

$$\frac{0}{0.73}+\frac{0.4}{0.735}+\frac{0.8}{0.74}+\frac{0.6}{0.745}+\frac{0}{0.750}$$

c)

$$\overline{\underset{\sim}{A}}=\frac{1}{0.73}+\frac{0.2}{0.735}+\frac{0}{0.74}+\frac{0.4}{0.745}+\frac{1}{0.750}$$

d) $\underset{\sim}{A}\,|\,\underset{\sim}{B}=\underset{\sim}{A}\cap\overline{\underset{\sim}{B}}$

$$\frac{0}{0.73}+\frac{0.6}{0.735}+\frac{0.2}{0.74}+\frac{0}{0.745}+\frac{0}{0.750}$$

f) $\overline{\underset{\sim}{A}\cup\underset{\sim}{B}}=\overline{\underset{\sim}{A}}\cap\overline{\underset{\sim}{B}}=$

$$\frac{1}{0.73}+\frac{0.2}{0.735}+\frac{0}{0.74}+\frac{0}{0.745}+\frac{0.4}{0.75}$$

e) $\overline{\underset{\sim}{A}\cap\underset{\sim}{B}}=\overline{\underset{\sim}{A}}\cup\overline{\underset{\sim}{B}}$

$$\frac{1}{0.73}+\frac{0.6}{0.735}+\frac{0.2}{0.74}+\frac{0.4}{0.745}+\frac{1}{0.75}$$

2.10

$$A\cup B=\left\{\frac{0.15}{winter}+\frac{0.55}{spring}+\frac{0.9}{summer}+\frac{0.25}{fall}\right\}$$

$$A\cap B=\left\{\frac{0.1}{winter}+\frac{0.3}{spring}+\frac{0.52}{summer}+\frac{0.2}{fall}\right\}$$

A|B=$A\cap\bar{B}$,

$$\bar{B}=\left\{\frac{0.9}{winter}+\frac{0.45}{spring}+\frac{0.1}{summer}+\frac{0.8}{fall}\right\}$$

A|B=$\left\{\frac{0.15}{winter}+\frac{0.33}{spring}+\frac{0.1}{summer}+\frac{0.25}{fall}\right\}$

B|A=B∩ $\bar{A}$,

$\bar{A}$=$\left\{\frac{0.85}{winter}+\frac{0.67}{spring}+\frac{0.48}{summer}+\frac{0.75}{fall}\right\}$

B|A=$\left\{\frac{0.1}{winter}+\frac{0.55}{spring}+\frac{0.48}{summer}+\frac{0.2}{fall}\right\}$

2.11

$$\underset{\sim}{MP}=\frac{0}{0}+\frac{0.2}{1}+\frac{0.4}{2}+\frac{0.6}{3}+\frac{0.8}{4}+\frac{1}{5}+\frac{0.8}{6}+\frac{0.6}{7}+\frac{0.4}{8}+\frac{0.2}{9}+\frac{0}{10}$$

$$\overline{\underset{\sim}{MP}}=\frac{1}{0}+\frac{0.8}{1}+\frac{0.6}{2}+\frac{0.4}{3}+\frac{0.2}{4}+\frac{0}{5}+\frac{0.2}{6}+\frac{0.4}{7}+\frac{0.6}{8}+\frac{0.8}{9}+\frac{1}{10}$$

$$\underset{\sim}{HP}=\frac{0}{0}+\frac{0.1}{1}+\frac{0.2}{2}+\frac{0.3}{3}+\frac{0.4}{4}+\frac{0.5}{5}+\frac{0.6}{6}+\frac{0.7}{7}+\frac{0.8}{8}+\frac{0.9}{9}+\frac{1}{10}$$

$$\overline{\underset{\sim}{HP}}=\frac{1}{0}+\frac{0.9}{1}+\frac{0.8}{2}+\frac{0.7}{3}+\frac{0.6}{4}+\frac{0.5}{5}+\frac{0.4}{6}+\frac{0.3}{7}+\frac{0.2}{8}+\frac{0.1}{9}+\frac{0}{10}$$

$$\underset{\sim}{MP}\Big/\underset{\sim}{H}P=\frac{0}{0}+\frac{0.2}{1}+\frac{0.4}{2}+\frac{0.6}{3}+\frac{0.6}{4}+\frac{0.5}{5}+\frac{0.4}{6}+\frac{0.3}{7}+\frac{0.2}{8}+\frac{0.1}{9}+\frac{0}{10}$$

$$\underset{\sim}{MP}\cup\underset{\sim}{HP}=\frac{0}{0}+\frac{0.2}{1}+\frac{0.4}{2}+\frac{0.6}{3}+\frac{0.8}{4}+\frac{1}{5}+\frac{0.8}{6}+\frac{0.7}{7}+\frac{0.8}{8}+\frac{0.9}{9}+\frac{1}{10}$$

$$\overline{\underset{\sim}{MP}}\cap\overline{\underset{\sim}{HP}}=\frac{1}{0}+\frac{0.8}{1}+\frac{0.6}{2}+\frac{0.4}{3}+\frac{0.2}{4}+\frac{0}{5}+\frac{0.2}{6}+\frac{0.3}{7}+\frac{0.2}{8}+\frac{0.1}{9}+\frac{0}{10}$$

2.12

$$\underset{\sim}{Q}\cup\underset{\sim}{C}=\frac{1}{0}+\frac{1}{1}+\frac{0.8}{2}+\frac{0.4}{5}+\frac{0.6}{7}+\frac{0.8}{9}+\frac{1}{10}$$

$$\underset{\sim}{Q}\cap\underset{\sim}{C}=\frac{0}{0}+\frac{0}{1}+\frac{0}{2}+\frac{0.3}{5}+\frac{0.1}{7}+\frac{0}{9}+\frac{0}{10}$$

$$\overline{\underset{\sim}{Q}}=\frac{0}{0}+\frac{0}{1}+\frac{0.2}{2}+\frac{0.7}{5}+\frac{0.9}{7}+\frac{1}{9}+\frac{1}{10}$$

$$\overline{\underset{\sim}{C}}=\frac{1}{0}+\frac{1}{1}+\frac{1}{2}+\frac{0.6}{5}+\frac{0.4}{7}+\frac{0.2}{9}+\frac{0}{10}$$

$$\underset{\sim}{Q}\Big/\underset{\sim}{C}=\frac{1}{0}+\frac{1}{1}+\frac{0.8}{2}+\frac{0.3}{5}+\frac{0.1}{7}+\frac{0}{9}+\frac{0}{10}$$

$$\overline{\underset{\sim}{Q}\cup\underset{\sim}{C}}=\frac{0}{0}+\frac{0}{1}+\frac{0.2}{2}+\frac{0.6}{5}+\frac{0.4}{7}+\frac{0.2}{9}+\frac{0}{10}$$

$$\overline{\underset{\sim}{Q}\cap\underset{\sim}{C}}=\frac{1}{0}+\frac{1}{1}+\frac{1}{2}+\frac{0.7}{5}+\frac{0.9}{7}+\frac{1}{9}+\frac{1}{10}$$

$$\underset{\sim}{Q}\cup\overline{\underset{\sim}{Q}}=\frac{1}{0}+\frac{1}{1}+\frac{0.8}{2}+\frac{0.7}{5}+\frac{0.9}{7}+\frac{1}{9}+\frac{1}{10}$$

$$\underset{\sim}{Q} \cap \overline{\underset{\sim}{Q}} = \frac{0}{0} + \frac{0}{1} + \frac{0.2}{2} + \frac{0.3}{5} + \frac{0.1}{7} + \frac{0}{9} + \frac{0}{10}$$

$$\underset{\sim}{C} \cup \overline{\underset{\sim}{C}} = \frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \frac{0.6}{5} + \frac{0.6}{7} + \frac{0.8}{9} + \frac{1}{10}$$

$$\underset{\sim}{C} \cap \overline{\underset{\sim}{C}} = \frac{0}{0} + \frac{0}{1} + \frac{0}{2} + \frac{0.4}{5} + \frac{0.4}{7} + \frac{0.2}{9} + \frac{0}{10}$$

2.13

$$\underset{\sim}{A} = \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.62}{2} + \frac{0.88}{3} + \frac{1}{4} + \frac{0.88}{5} + \frac{0.61}{6} + \frac{0.32}{7} + \frac{0.14}{8} + \frac{0.04}{9} + \frac{0}{10}$$

$$\underset{\sim}{B} = \frac{0.003}{0} + \frac{0.0022}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0.14}{4} + \frac{0.32}{5} + \frac{0.61}{6} + \frac{0.88}{7} + \frac{1}{8} + \frac{0.88}{9} + \frac{0.61}{10}$$

$$\overline{\underset{\sim}{A}} = \frac{0.86}{0} + \frac{0.68}{1} + \frac{0.38}{2} + \frac{0.12}{3} + \frac{0}{4} + \frac{0.12}{5} + \frac{0.39}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.96}{9} + \frac{1}{10}$$

$$\overline{\underset{\sim}{B}} = \frac{0.997}{0} + \frac{0.998}{1} + \frac{0.99}{2} + \frac{0.96}{3} + \frac{0.86}{4} + \frac{0.68}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.12}{9} + \frac{0.39}{10}$$

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.61}{2} + \frac{0.88}{3} + \frac{1}{4} + \frac{0.88}{5} + \frac{0.61}{6} + \frac{0.88}{7} + \frac{1}{8} + \frac{0.88}{9} + \frac{0.61}{10}$$

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \frac{0.00}{0} + \frac{0.002}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0.14}{4} + \frac{0.32}{5} + \frac{0.61}{6} + \frac{0.32}{7} + \frac{0.14}{8} + \frac{0.04}{9} + \frac{0.01}{10}$$

$$\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}} = \frac{0.99}{0} + \frac{0.998}{1} + \frac{0.99}{2} + \frac{0.96}{3} + \frac{0.86}{4} + \frac{0.68}{5} + \frac{0.39}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.96}{9} + \frac{0.99}{10}$$

$$\overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}} = \frac{0.86}{0} + \frac{0.68}{1} + \frac{0.39}{2} + \frac{0.12}{3} + \frac{0}{4} + \frac{0.12}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.12}{9} + \frac{0.39}{10}$$

$$\underset{\sim}{A} \big/ \underset{\sim}{B} = \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.61}{2} + \frac{0.88}{3} + \frac{0.86}{4} + \frac{0.68}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.04}{9} + \frac{0.01}{10}$$

$$\overline{\underset{\sim}{A}} \cap \underset{\sim}{B} = \frac{0.00}{0} + \frac{0.002}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0}{4} + \frac{0.12}{5} + \frac{0.39}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.88}{9} + \frac{0.61}{10}$$

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} = \frac{0.86}{0} + \frac{0.68}{1} + \frac{0.61}{2} + \frac{0.88}{3} + \frac{1}{4} + \frac{0.88}{5} + \frac{0.61}{6} + \frac{0.68}{7} + \frac{0.86}{8} + \frac{0.96}{9} + \frac{0.99}{10}$$

$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} = \frac{0.14}{0} + \frac{0.32}{1} + \frac{0.39}{2} + \frac{0.12}{3} + \frac{0}{4} + \frac{0.12}{5} + \frac{0.39}{6} + \frac{0.32}{7} + \frac{0.14}{8} + \frac{0.04}{9} + \frac{0.01}{10}$$

$$\underset{\sim}{B} \cup \overline{\underset{\sim}{B}} = \frac{0.9997}{0} + \frac{0.998}{1} + \frac{0.99}{2} + \frac{0.96}{3} + \frac{0.86}{4} + \frac{0.68}{5} + \frac{0.61}{6} + \frac{0.88}{7} + \frac{1}{8} + \frac{0.88}{9} + \frac{0.61}{10}$$

$$\underset{\sim}{B} \cap \overline{\underset{\sim}{B}} = \frac{0.0003}{0} + \frac{0.002}{1} + \frac{0.01}{2} + \frac{0.04}{3} + \frac{0.14}{4} + \frac{0.32}{5} + \frac{0.39}{6} + \frac{0.12}{7} + \frac{0}{8} + \frac{0.12}{9} + \frac{0.39}{10}$$

2.14 $\underset{\sim}{A}$ = "fast" chips

$$\underset{\sim}{A} = \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.2}{5} + \frac{0.6}{6} + \frac{1}{7} + \frac{1}{8}$$

$\underset{\sim}{D}$ = " hot" chips

$$\underset{\sim}{D} = \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0.5}{4} + \frac{0.1}{5} + \frac{1}{6} + \frac{0.5}{7} + \frac{1}{8}$$

$$\underset{\sim}{A} \cup \underset{\sim}{D} = \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0.5}{4} + \frac{0.2}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$$

$$\underset{\sim}{A} \cap \underset{\sim}{D} = \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.1}{5} + \frac{0.6}{6} + \frac{0.5}{7} + \frac{1}{8}$$

$$\overline{\underset{\sim}{A}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{0.8}{5} + \frac{0.4}{6} + \frac{0}{7} + \frac{0}{8}$$

$$\underset{\sim}{A} \cap \overline{\underset{\sim}{D}} = \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.2}{5} + \frac{0}{6} + \frac{0.5}{7} + \frac{0}{8}$$

$$\overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{D}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{0.5}{4} + \frac{0.8}{5} + \frac{0}{6} + \frac{0}{7} + \frac{0}{8}$$

$$\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{D}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{0.9}{5} + \frac{0.4}{6} + \frac{0.5}{7} + \frac{0}{8}$$

CHAPTER 3

Classical Relations and Fuzzy Relations

3.1

$$\underset{\sim}{T} = \underset{\sim}{P} \times \underset{\sim}{D} = \begin{bmatrix} 0.1 & 0.1 \\ 0.4 & 0.5 \\ 0.4 & 0.8 \end{bmatrix}$$

3.2

$$\underset{\sim}{R} = \underset{\sim}{B} \times \underset{\sim}{T} = \begin{bmatrix} 0.5 & 0.5 & 0.4 \\ 0.7 & 0.6 & 0.4 \\ 0.9 & 0.6 & 0.4 \end{bmatrix}$$

$$\underset{\sim}{S} = \underset{\sim}{T} \times \underset{\sim}{U} = \begin{bmatrix} 0.9 & 0.8 & 0.6 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix}$$

Using max-min composition

$$\underset{\sim}{W} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.4 \\ 0.7 & 0.7 & 0.6 & 0.4 \\ 0.9 & 0.8 & 0.6 & 0.4 \end{bmatrix}$$

Using max-product composition

$$\underset{\sim}{W} = \underset{\sim}{R} \circ \underset{\sim}{S} = \begin{bmatrix} 0.45 & 0.4 & 0.3 & 0.20 \\ 0.63 & 0.56 & 0.42 & 0.28 \\ 0.81 & 0.54 & 0.54 & 0.36 \end{bmatrix}$$

3.3

a)

$$\underset{\sim}{G} = \begin{array}{c|cc} & t_1 & t_2 \\ h_1 & 0.1 & 0.4 \\ h_2 & 0.1 & 1 \\ h_3 & 0.1 & 0.6 \end{array}$$

b)

$$\mu_{\underset{\sim}{C}}(\mathrm{t}) = \vee\left(\mu_{\underset{\sim}{E}}(\mathrm{h}) \wedge \mu_{\underset{\sim}{G}}(\mathrm{h,t})\right)$$

$$\underset{\sim}{C} = \begin{bmatrix} 0.2 & 1 & 0.3 \end{bmatrix} \begin{bmatrix} 0.1 & 0.4 \\ 0.1 & 1 \\ 0.1 & 0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & 1 \end{bmatrix}$$

3.4 a) $\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \begin{Bmatrix} 1 \\ 0.5 \\ 0.2 \end{Bmatrix} \times \{1 \quad 0.5 \quad 0.3\}$

$$\begin{array}{c|ccc} & w1 & w2 & w3 \\ s1 & 1 & 0.5 & 0.3 \\ s2 & 0.5 & 0.5 & 0.3 \\ s3 & 0.2 & 0.2 & 0.2 \end{array}$$

b) Using Cartesian product

$$\underset{\sim}{S} = \underset{\sim}{C} \times \underset{\sim}{B} = \begin{Bmatrix} 0.1 \\ 0.6 \\ 1 \end{Bmatrix} \begin{bmatrix} 1 & 0.5 & 0.3 \end{bmatrix}$$

$$\begin{array}{c|ccc} & w1 & w2 & w3 \\ s1 & 0.1 & 0.1 & 0.1 \\ s2 & 0.6 & 0.5 & 0.3 \\ s3 & 1 & 0.5 & 0.3 \end{array}$$

c) Using max-min composition

$$\underset{\sim}{C} \circ \underset{\sim}{R} =$$

$$= \begin{bmatrix} 0.1 & 0.6 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.5 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.2 \end{bmatrix}$$

$$= \frac{0.5}{w1} + \frac{0.5}{w2} + \frac{0.3}{w3}$$

d) Using max-product composition

$$\underset{\sim}{C} \circ \underset{\sim}{R} = \frac{0.3}{w1} + \frac{0.3}{w2} + \frac{0.2}{w3}$$

3.5 a)

$$\underset{\sim}{P}=\underset{\sim}{V}\times\underset{\sim}{I}=\begin{Bmatrix}0.2\\0.6\\1\\0.9\\0.7\end{Bmatrix}\times\{0.4\quad 0.7\quad 1\quad 0.8\quad 0.6\}$$

	0.8	0.9	1.0	1.1	1.2
30	0.2	0.2	0.2	0.2	0.2
45	0.4	0.6	0.6	0.6	0.6
60	0.4	0.7	1	0.8	0.6
75	0.4	0.7	0.9	0.8	0.6
90	0.4	0.7	0.7	0.7	0.6

b) $\underset{\sim}{T}=\underset{\sim}{I}\times\underset{\sim}{C}=\begin{Bmatrix}0.4\\0.7\\1\\0.8\\0.6\end{Bmatrix}\times\{0.4\quad 1\quad 0.8\}$

	0.5	0.6	0.7
0.8	0.4	0.4	0.4
0.9	0.4	0.7	0.7
= 1	0.4	1	0.8
1.1	0.4	0.8	0.8
1.2	0.4	0.6	0.6

c)

$$\underset{\sim}{E}=\underset{\sim}{P}\circ\underset{\sim}{T}=\begin{Bmatrix}0.2&0.2&0.2&0.2&0.2\\0.4&0.6&0.6&0.6&0.6\\0.4&0.7&1&0.8&0.6\\0.4&0.7&0.9&0.8&0.6\\0.4&0.7&0.7&0.7&0.6\end{Bmatrix}\cdot$$

$$\begin{bmatrix}0.4&0.4&0.4\\0.4&0.7&0.7\\0.4&1&0.8\\0.4&0.8&0.8\\0.4&0.6&0.6\end{bmatrix}=\begin{bmatrix}0.2&0.2&0.2\\0.4&0.6&0.6\\0.4&1&0.8\\0.4&0.9&0.8\\0.4&0.7&0.7\end{bmatrix}$$

d) In max-product composition each value in the matrix is calculated by taking the product of each pair and then taking the max of all the products over a column.

$$E=\begin{bmatrix}0.08&0.2&0.16\\0.24&0.6&0.48\\0.40&1.0&0.80\\0.36&0.9&0.72\\0.28&0.7&0.56\end{bmatrix}$$

3.6 Let $\underset{\sim}{U}$ = Relationship matrix between Temperature and reliability index.

a) Using max-min composition

$$\underset{\sim}{U}=\begin{Bmatrix}0.2&0.5&0.7&1&0.9\\0.3&0.5&0.7&1&0.8\\0.4&0.6&0.8&0.9&0.4\\0.9&1&0.8&0.6&0.4\end{Bmatrix}\cdot$$

$$\begin{bmatrix}1&0.8&0.6&0.3&0.1\\0.7&1&0.7&0.5&0.4\\0.5&0.6&1&0.8&0.8\\0.3&0.4&0.6&1&0.9\\0.9&0.3&0.5&0.7&1\end{bmatrix}=$$

	1.0	2.0	4.0	8.0	16.0
8	0.9	0.6	0.7	1.0	0.9
16	0.8	0.6	0.7	1.0	0.9
25	0.6	0.6	0.8	0.9	0.9
33	0.9	1.0	0.8	0.8	0.8

sample calculation

$$\underset{\sim}{U}_{35}=\max\begin{Bmatrix}(0.4\wedge 0.1),(0.6\wedge 0.4),\\(0.8\wedge 0.8),(0.9\wedge 0.9),\\(0.4\wedge 1)\end{Bmatrix}$$

$$\underset{\sim}{U}_{35}=0.9$$

3.6 Cont.

b) Using max-product composition:

$$\underset{\sim}{U} = \begin{Bmatrix} 0.81 & 0.5 & 0.7 & 1 & 0.9 \\ 0.72 & 0.5 & 0.7 & 1 & 0.9 \\ 0.42 & 0.6 & 0.8 & 0.9 & 0.81 \\ 0.9 & 1 & 0.8 & 0.64 & 0.64 \end{Bmatrix}$$

sample calculation:

$$\underset{\sim}{U}_{35} = \max\begin{Bmatrix} (0.4)(0.1), (0.6)(0.4), \\ (0.8)(0.8), (0.9)(0.9), \\ (0.4)(1) \end{Bmatrix}$$

$$\underset{\sim}{U}_{35} = \max\{0.4, 0.24, 0.64, 0.81, 0.4\}$$

$$\underset{\sim}{U}_{35} = 0.81$$

3.7 a) $\underset{\sim}{S}_{\text{max-min}} = \underset{\sim}{I}_T \circ (\underset{\sim}{T} \times \underset{\sim}{O})_{\text{max-min}}$

$$= [0.5 \quad 1.0 \quad 0.7] \cdot \begin{Bmatrix} 0.1 & 0.2 & 0.9 \\ 0.1 & 1 & 0.7 \\ 0.8 & 0.7 & 0.1 \end{Bmatrix}$$

$$= \left\{ \frac{0.7}{1} + \frac{1}{2} + \frac{0.7}{6} \right\}$$

b) $\underset{\sim}{S}_{\text{max-}product} = \underset{\sim}{I}_T \circ \underset{\sim}{R}$

$$= \left\{ \frac{0.56}{1} + \frac{1}{2} + \frac{0.7}{6} \right\}$$

sample calculation max-min:

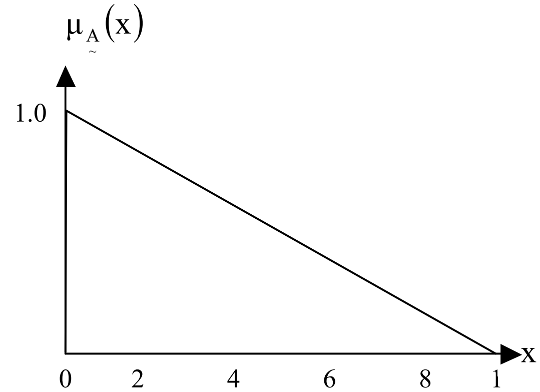
$$S_2 = \vee[\wedge(0.5, 0.2), \wedge(1, 1), \wedge(0.7, 0.1)] = 0.7$$

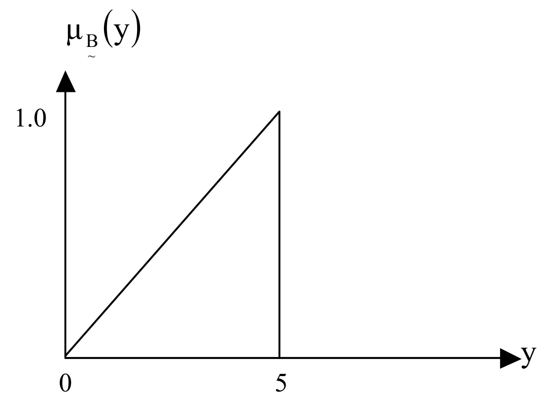
3.8 Using Max-Min Composition

$\underset{\sim}{A} = \underset{\sim}{I}_7 \circ \underset{\sim}{R}$

$$= \left\{ \frac{0.5}{0.2} + \frac{0.6}{0.4} + \frac{0.8}{0.6} + \frac{1.0}{0.8} + \frac{0.8}{1.0} + \frac{0.6}{1.2} \right\}$$

3.9





Discretizing:

$$\underset{\sim}{A} = \left\{ \frac{1}{0} + \frac{0.8}{2} + \frac{0.6}{4} + \frac{0.4}{6} + \frac{0.2}{8} + \frac{0}{10} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5} \right\}$$

a) Fuzzy Relation: $\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B}$

	0	1	2	3	4	5
0	0	0.2	0.4	0.6	0.8	1
2	0	0.2	0.4	0.6	0.8	0.8
= 4	0	0.2	0.4	0.6	0.6	0.6
6	0	0.2	0.4	0.4	0.4	0.4
8	0	0.2	0.2	0.2	0.2	0.2
10	0	0	0	0	0	0

3.9 Cont.

b)

$$\underset{\sim}{A}' = \left\{ \frac{0}{0} + \frac{0}{2} + \frac{1}{3} + \frac{0}{4} + \frac{0}{6} + \frac{0}{8} + \frac{0}{10} \right\}$$

$$\underset{\sim}{B}' = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 \\ 0 & 0.2 & 0.4 & 0.6 & 0.8 & 0.8 \\ 0 & 0.2 & 0.4 & 0.6 & 0.6 & 0.6 \\ 0 & 0.2 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underset{\sim}{B}' = \left\{ \frac{0}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.7}{4} + \frac{0.7}{5} \right\}$$

Sample calculation:

$$\left(\underset{\sim}{B}'\right) = \vee \begin{bmatrix} (0 \wedge 0.8), (0 \wedge 0.8), (1 \wedge .7), \\ (0 \wedge 0.6), (0 \wedge 0) \end{bmatrix} = 0.7$$

3.10 Risk = Consequence $\circ$ Possibility

$= \max[\min(\mu_{\underset{\sim}{c}}, \mu_{\underset{\sim}{p}})]$

Consequence = Mitigation $\cap$ Secure Team

$= \max[\min(\mu_{\underset{\sim}{M}}(x,y), \mu_{\underset{\sim}{ST}}(x,y))]$

Possibility = Human factors $\cap$ System Reliability

$= \max[\min(\mu_{\underset{\sim}{H}}(x,y), \mu_{\underset{\sim}{S}}(x,y))]$

3.11a) $\underset{\sim}{R} = \underset{\sim}{V} \times \underset{\sim}{T} = \min\left(\mu_{\underset{\sim}{V}}(v), \mu_{\underset{\sim}{T}}(s)\right)$

$$= \begin{Bmatrix} 0.1 \\ 0.3 \\ 0.7 \\ 0.4 \\ 0.2 \end{Bmatrix} \times \begin{bmatrix} 0.1 & 0.3 & 0.3 & 0.4 & 0.5 & 0.2 \end{bmatrix}$$

	.05	.06	.07	.08	.09	0.1
2.98	0.1	0.1	0.1	0.1	0.1	0.1
2.99	0.1	0.3	0.3	0.3	0.3	0.2
3.00	0.1	0.3	0.3	0.4	0.5	0.2
3.01	0.1	0.3	0.3	0.4	0.4	0.2
3.02	0.1	0.2	0.2	0.2	0.2	0.2

b) $\underset{\sim}{S} = \underset{\sim}{T} \times \underset{\sim}{Z} = \min\left(\mu_{\underset{\sim}{T}}(S), \mu_{\underset{\sim}{Z}}(L)\right)$

$$= \begin{Bmatrix} 0.1 \\ 0.3 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.2 \end{Bmatrix} \times \begin{bmatrix} 0.1 & 0.7 & 0.3 \end{bmatrix}$$

	0	0.5	1.0
0.05	0.1	0.1	0.1
0.06	0.1	0.3	0.3
0.07	0.1	0.3	0.3
0.08	0.1	0.4	0.3
0.09	0.1	0.5	0.3
0.1	0.1	0.2	0.2

c) $\underset{\sim}{M} = \underset{\sim}{R} \circ \underset{\sim}{S}$

$$\begin{bmatrix} .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .3 & .3 & .3 & .3 & .2 \\ .1 & .3 & .3 & .4 & .5 & .2 \\ .1 & .3 & .3 & .4 & .4 & .2 \\ .1 & .2 & .2 & .2 & .2 & .2 \end{bmatrix} \circ \begin{bmatrix} .1 & .1 & .1 \\ .1 & .3 & .3 \\ .1 & .3 & .3 \\ .1 & .4 & .3 \\ .1 & .5 & .3 \\ .1 & .2 & .2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.5 & 0.3 \\ 0.1 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.2 \end{bmatrix}$$

d) $\underset{\sim}{M} = \begin{bmatrix} 0.01 & 0.05 & 0.03 \\ 0.03 & 0.15 & 0.09 \\ 0.05 & 0.25 & 0.15 \\ 0.04 & 0.2 & 0.12 \\ 0.02 & 0.1 & 0.06 \end{bmatrix}$

3.12 a)

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{G} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.2 & 0 & 0 & 0 & 0.2 \\ 0.2 & 0.2 & 0 & 0 & 0 & 0.2 \\ 0.2 & 0.2 & 0 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b) $\underset{\sim}{B} = \underset{\sim}{C} \circ \underset{\sim}{R}$

$$\underset{\sim}{B} = \begin{bmatrix} 0.06 \\ 0.06 \\ 0 \\ 0 \\ 0 \\ 0.06 \end{bmatrix}$$

3.13 a) $\underset{\sim}{R} = I_{\underset{\sim}{C}} \times D = \begin{Bmatrix} 0.5 \\ 1 \\ 0.6 \end{Bmatrix} \times \begin{bmatrix} 1 & 0.7 & 0.3 \end{bmatrix}$

$$= \begin{bmatrix} 0.5 & 0.5 & 0.3 \\ 1 & 0.7 & 0.3 \\ 0.6 & 0.6 & 0.3 \end{bmatrix}$$

b) $\underset{\sim}{R} = \underset{\sim}{IM} \circ \underset{\sim}{R}$, using max min

$$\underset{\sim}{Q} = \begin{bmatrix} 0.7 & 0.9 & 0.4 \end{bmatrix} \circ \begin{bmatrix} 0.5 & 0.5 & 0.3 \\ 1 & 0.7 & 0.3 \\ 0.6 & 0.6 & 0.3 \end{bmatrix}$$

$$\underset{\sim}{Q} = \left\{ \frac{0.9}{0} \quad \frac{0.7}{1} \quad \frac{0.3}{2} \right\}$$

$$\underset{\sim}{Q}_2 = \vee\{(0.7 \wedge 0.5), (0.9 \wedge 0.7), (0.4 \wedge 0.6)\}$$

=0.7

c) $\underset{\sim}{R} = \underset{\sim}{IM} \circ \underset{\sim}{R}$, using max product composition $\underset{\sim}{Q} = \left\{ \frac{0.9}{0} \quad \frac{0.63}{1} \quad \frac{0.27}{2} \right\}$

3.14 a) $\underset{\sim}{R} = \underset{\sim}{X} \times \underset{\sim}{Y}$

$$= \begin{Bmatrix} 1.0 \\ 0.8 \\ 0.6 \\ 0.5 \\ 0.3 \\ 0.1 \end{Bmatrix} \times \begin{bmatrix} 0.2 & 0.4 & 0.5 & 1.0 & 0.6 & 0.3 \end{bmatrix}$$

$\underset{\sim}{R} =$

	20	25	32	50	90	105
1500	0.2	0.4	0.5	1	0.6	0.3
2175	0.2	0.4	0.5	0.8	0.6	0.3
7000	0.2	0.4	0.5	0.6	0.6	0.3
12750	0.2	0.4	0.5	0.5	0.5	0.3
16500	0.2	0.3	0.3	0.3	0.3	0.3
20000	0.1	0.1	0.1	0.1	0.1	0.1

b) $\underset{\sim}{S} = \underset{\sim}{Z} \circ \underset{\sim}{R}$ using max min

$$\underset{\sim}{S} = \frac{0.2}{20} + \frac{0.4}{25} + \frac{0.5}{32} + \frac{0.6}{50} + \frac{0.6}{90} + \frac{0.3}{105}$$

c) $\underset{\sim}{S} = \underset{\sim}{Z} \circ \underset{\sim}{R}$ using max product

$$\underset{\sim}{S} = \frac{0.2}{20} + \frac{0.36}{25} + \frac{0.45}{32} + \frac{0.48}{50} + \frac{0.45}{90} + \frac{0.3}{105}$$

3.15 $\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S}$

$\underset{\sim}{R}$ can be written as

	$\underset{\sim}{A}_1$	$\underset{\sim}{A}_2$	$\underset{\sim}{A}_3$	$\underset{\sim}{A}_4$
$\underset{\sim}{D}_1$	1	0.2	0	0
$\underset{\sim}{D}_2$	0.3	1	0.7	0.1
$\underset{\sim}{D}_3$	0.1	0.3	1	0.4
$\underset{\sim}{D}_4$	0	0.1	0.2	1

Use max-min composition

$$\underset{\sim}{T} = \begin{bmatrix} 1 & 0.2 & 0 & 0 \\ 0.3 & 1 & 0.7 & 0.1 \\ 0.1 & 0.3 & 1 & 0.4 \\ 0 & 0.1 & 0.2 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 0.4 \\ 0.5 & 1 \\ 0.15 & 0.3 \\ 0.06 & 0.1 \end{bmatrix}$$

$$\quad\quad\quad\quad\quad\quad\quad\quad \underset{\sim}{W_1} \quad \underset{\sim}{W_2}$$

$$= \begin{matrix} \underset{\sim}{D_1} \\ \underset{\sim}{D_2} \\ \underset{\sim}{D_3} \\ \underset{\sim}{D_4} \end{matrix} \begin{bmatrix} 1 & 0.4 \\ 0.5 & 1 \\ 0.3 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}$$

Sample calculation: $\underset{\sim}{T}_{12} = \max\{(1 \wedge 0.4), (0.2 \wedge 1), (0 \wedge 0.1), (0 \wedge 0)\} = 0.4$

b) Use max-product composition:

$$T = \begin{bmatrix} 1 & 0.4 \\ 0.5 & 1 \\ 0.15 & 0.3 \\ 0.06 & 0.1 \end{bmatrix}$$

Sample calculation: $\underset{\sim}{T}_{31} = \max\{(0.1 \wedge 1), (0.3 \wedge 0.5), (1.0 \wedge 0.3), (0.4 \wedge 0.2)\} = 0.15$

3.16 a) $\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B}$

$$= \begin{Bmatrix} 0.9 \\ 0.4 \\ 0 \end{Bmatrix} \times \begin{bmatrix} 0.1 & 0.7 & 1 \end{bmatrix} =$$

$$\begin{matrix} & y_1 & y_2 & y_3 \\ x_1 & 0.1 & 0.7 & 0.9 \\ x_2 & 0.1 & 0.4 & 0.4 \\ x_3 & 0 & 0 & 0 \end{matrix}$$

b) $\underset{\sim}{S} = \underset{\sim}{C} \times \underset{\sim}{R}$ using max-min composition, $\underset{\sim}{S} =$

$$\{0.3 \quad 1 \quad 0.2\} \times \begin{bmatrix} 0.1 & 0.7 & 0.9 \\ 0.1 & 0.4 & 0.4 \\ 0 & 0 & 0 \end{bmatrix} = \{0.1 \quad 0.4 \quad 0.4\}$$

3.17

If we have the fuzzy sets

$$\underset{\sim}{F} = \begin{matrix} \text{High Flow} \\ \text{Med Flow} \\ \text{Low Flow} \end{matrix} \begin{bmatrix} 0.1 \\ 0.7 \\ 0.4 \end{bmatrix}$$

$$\underset{\sim}{P} = \begin{matrix} \textit{High C3} & \textit{Low C3} \\ [\ 0.3 & 0.8\ ] \end{matrix}$$

A Cartesian product can give the relationship between $\underset{\sim}{F}$ and $\underset{\sim}{P}$.

$\underset{\sim}{R} = \underset{\sim}{F} \times \underset{\sim}{P} =$

	HighC3	*LowC3*
HighFlow	0.1	0.1
MedFlow	0.3	0.7
LowFlow	0.3	0.4

Another Relationship that can be found is the one between P and T, because they are highly related. If we have,

$\underset{\sim}{T} =$

Tray14	Tray15	Tray16	Tray17
0.1	0.3	0.8	0.7

Then the relationship $\underset{\sim}{S}$ can be

$\underset{\sim}{S} = \underset{\sim}{P}^T \times \underset{\sim}{T} =$

	Tray14	*Tray15*	*Tray16*	*Tray17*
High C3	0.1	0.3	0.3	0.3
Low C3	0.1	0.3	0.8	0.7

But, as mentioned before, composition is not easily measured, so it is not important to know, how F and T relate to each other. In this case, it can be done with a max-min composition,

$\underset{\sim}{C} = \underset{\sim}{R} \circ \underset{\sim}{S} =$

	Tray14	*Tray15*	*Tray16*	*Tray17*
High Flow	0.1	0.1	0.1	0.1
Med Flow	0.1	0.3	0.7	0.7
Low Flow	0.1	0.3	0.4	0.4

3.18

The results are as follows:

	X_1	X_2	X_3	X_4	X_5	X_6
HighDeposition	0.05	0.01	0.6	0.03	0	0.5
Med – highDeposition	0.8	0.5	0.3	0.9	0	0.2
MedDeposition	0.1	0.4	0.1	0.04	0.2	0.2
ModerateDeposition	0.05	0.09	0	0.03	0.8	0.1

Now it is desired to find the relationship with a similarity method. This is the result obtained by using the Max-min method.

	$X1$	$X2$	$X3$	$X4$	$X5$	$X6$
$X1$	1.00	0.49	0.29	0.82	0.08	0.25
$X2$	0.49	1.00	0.26	0.41	0.17	0.33
$X3$	0.29	0.26	1.00	0.23	0.05	0.67
$X4$	0.82	0.41	0.23	1.00	0.04	0.18
$X5$	0.08	0.17	0.05	0.04	1.00	0.18
$X6$	0.25	0.33	0.67	0.18	0.18	1.00

There are the results if the Cosine Amplitude method is used:

	$X1$	$X2$	$X3$	$X4$	$X5$	$X6$
$X1$	1.00	0.85	0.51	0.996	0.90	0.46
$X2$	0.51	1.00	0.45	0.80	0.29	0.51
$X3$	0.51	0.45	1.00	0.48	0.04	0.96
$X4$	0.996	0.80	0.80	1.00	0.04	0.39
$X5$	0.90	0.29	0.04	0.04	1.00	0.25
$X6$	0.46	0.51	0.96	0.39	0.25	1.00

3.19

Using the cosine amplitude method determine the similarity of the four beam types.

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.80 & 0.94 & 0.64 \\ 0.80 & 1 & 0.94 & 0.97 \\ 0.94 & 0.94 & 1 & 0.86 \\ 0.64 & 0.97 & 0.86 & 1 \end{bmatrix}$$

3.20 $\underset{\sim}{R} = \begin{bmatrix} 1 & 0.333 & 0.25 \\ 0.333 & 1 & 0.75 \\ 0.25 & 0.75 & 1 \end{bmatrix}$

3.21

$$\underset{\sim}{R} = \begin{bmatrix} 1.000 & 0.538 & 0.250 & 0.538 & 0.538 \\ 0.538 & 1.000 & 0.429 & 0.250 & 0.333 \\ 0.250 & 0.429 & 1.000 & 0.111 & 0.176 \\ 0.538 & 0.250 & 0.111 & 1.000 & 0.818 \\ 0.538 & 0.333 & 0.176 & 0.818 & 1.000 \end{bmatrix}$$

3.22 a)

$$\underset{\sim}{R} = \begin{bmatrix} 1.000 & & & & \\ 0.913 & 1.000 & & & \\ 0.694 & 0.837 & 1.000 & & \\ 0.933 & 0.981 & 0.895 & 1.000 & \\ 0.711 & 0.533 & 0.685 & 0.685 & 1.000 \end{bmatrix}$$

b)

$$\underset{\sim}{R} = \begin{bmatrix} 1.000 & & & & \\ 0.500 & 1.000 & & & \\ 0.429 & 0.538 & 1.000 & & \\ 0.667 & 0.818 & 0.667 & 1.000 & \\ 0.429 & 0.333 & 0.429 & 0.429 & 1.000 \end{bmatrix}$$

c) The above relation is not an equivalence relation $r_{51} = 0.429$ and $r_{12} = 0.5$ and $r_{52} = 0.33$ where $r_{52} < 0.5, 0.429$.

3.23

Use the cosine amplitude method to express these data as a fuzzy relation

$$r_{ij} = \frac{\left|\sum_{k=1}^{3} x_{ik} x_{jk}\right|}{\sqrt{\left(\sum_{k=1}^{3} x_{ik}^2\right)\left(\sum_{k=1}^{3} x_{jk}^2\right)}}$$

$$r_{12} = \frac{0.3*0.2 + 0.6*0.4 + 0.1*0.4}{\left[\left(0.3^2 + 0.6^2 + 0.1^2\right)\left(0.2^2 + 0.4^2 + 0.4^2\right)\right]^{.5}} = 0.836$$

	NW	NE	SW	SE
NW	1	0.836	0.914	0.682
NE	0.836	1	0.934	0.6
SW	0.914	0.934	1	0.441
SE	0.682	0.6	0.441	1

From the above noted table it can be seen that not all areas of ponds perform similarly. Upon further investigation it may be possible to find reasonable explanations or there may be guidelines that have to be modified.

3.24 a)

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}, \quad \begin{array}{l} x_{12} = 1, x_{23} = 0 \\ x_{13} = \min(0,1) = 0 < 1 \end{array}$$

b)

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}, \quad \begin{array}{l} x_{12} = 1, x_{23} = 0 \\ x_{13} = 0 < 1 \end{array}$$

It is not an equivalence relation.

3.26

i) $\underset{\sim}{T} = \begin{bmatrix} 0.5 & 0.7 & 0.5 \\ 0.4 & 0.7 & 0.5 \end{bmatrix}$

ii) $\underset{\sim}{T} = \begin{bmatrix} 0.9 & 0.7 & 0.7 \\ 0.9 & 0.8 & 0.8 \end{bmatrix}$

iii) $\underset{\sim}{T} = \begin{bmatrix} 0.1 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.2 \end{bmatrix}$

iv) $\underset{\sim}{T} = \begin{bmatrix} 0.8 & 0.65 & 0.5 \\ 0.85 & 0.7 & 0.5 \end{bmatrix}$

v) $\underset{\sim}{T} = \begin{bmatrix} 0.493 & 0.537 & 0.323 \\ 0.532 & 0.532 & 0.302 \end{bmatrix}$

3.27

Using min-max composition

$A = [0.1 \quad 0.5 \quad 1] \rightarrow B = [0.1 \quad 0.2 \quad 0.3]$
$A = [1 \quad 0.6 \quad 0.1] \rightarrow B = [0.6 \quad 0.6 \quad 0.6]$
$A = [0.2 \quad 0.6 \quad 0.4] \rightarrow B = [0.2 \quad 0.2 \quad 0.3]$
$A = [0.7 \quad 0.9 \quad 0.8] \rightarrow B = [0.7 \quad 0.7 \quad 0.7]$

Using max-max composition

$A = [0.1 \quad 0.5 \quad 1] \rightarrow B = [1 \quad 1 \quad 1]$
$A = [1 \quad 0.6 \quad 0.1] \rightarrow B = [1 \quad 1 \quad 1]$
$A = [0.2 \quad 0.6 \quad 0.4] \rightarrow B = [0.7 \quad 0.8 \quad 0.9]$
$A = [0.7 \quad 0.9 \quad 0.8] \rightarrow B = [0.9 \quad 0.9 \quad 0.9]$

Using min-min composition

$A = [0.1 \quad 0.5 \quad 1] \rightarrow B = [0.1 \quad 0.1 \quad 0.1]$
$A = [1 \quad 0.6 \quad 0.1] \rightarrow B = [0.1 \quad 0.1 \quad 0.1]$
$A = [0.2 \quad 0.6 \quad 0.4] \rightarrow B = [0.1 \quad 0.2 \quad 0.2]$
$A = [0.7 \quad 0.9 \quad 0.8] \rightarrow B = [0.1 \quad 0.2 \quad 0.3]$

Using min-average composition

$A = [0.1 \quad 0.5 \quad 1] \rightarrow B = [0.85 \quad 0.9 \quad 0.95]$

$A = [1 \quad 0.6 \quad 0.1] \rightarrow B = [0.55 \quad 0.6 \quad 0.65]$

$A = [0.2 \quad 0.6 \quad 0.4] \rightarrow B = [0.55 \quad 0.6 \quad 0.65]$

$A = [0.7 \quad 0.9 \quad 0.8] \rightarrow B = [0.75 \quad 0.8 \quad 0.85]$

CHAPTER 4

Properties of Membership Functions, Fuzzification, and Defuzzification

4.1

$(A) = \{\frac{0.1}{x_1} + \frac{0.7}{x_2} + \frac{0.8}{x_3} + \frac{1}{x_4} + \frac{0.7}{x_5} + \frac{0.1}{x_6}\}$

$(B) = \{\frac{1}{x_1} + \frac{0.9}{x_2} + \frac{0.5}{x_3} + \frac{0.2}{x_4} + \frac{0.1}{x_5} + \frac{0}{x_6}\}$

a) $(\bar{A})_{0.6} = \{x_1, x_6\}$
b) $(B)_{0.5} = \{x_1, x_2, x_3\}$
c) $(A \cup B)_{0.8} = \{x_1, x_2, x_3, x_4\}$
d) $(A \cap B)_{0.6} = \{x_2\}$
e) $(A \cup \bar{A})_{0.7} = X$
f) $(B \cap \bar{B})_{0.6} = \Phi$
g) $(\overline{A \cap B})_{0.8} = \{x_1, x_4, x_5, x_6\}$
h) $(\bar{A} \cup \bar{B})_{0.7} = \{x_1, x_4, x_5, x_6\}$

4.2 4.2 λ – cut of a fuzzy set $A \subset R^n$ is a crisp set whose elements are those $x \in A$ that $\mu_A(n) > \lambda$ where $\lambda \in [0, 1]$. We are asked to prove that A is a convex fuzzy set $\Leftrightarrow \forall \lambda \in [0, 1]$ for all λ is a convex crisp set. A fuzzy set $\underset{\sim}{A}$

$$\mu_{\underset{\sim}{A}} = [\lambda r + (1-\lambda)s] \geq \min[\mu_{\underset{\sim}{A}}(r), \mu_{\underset{\sim}{A}}(s)]$$

for all $r, s \in B$ and $\lambda \in [0,1]$. A crisp set B is convex if for all

$r, s \in R^m$ and $\lambda \in [0,1], \lambda r + (1-\lambda)s \in B$

Let us show that $\underset{\sim}{A}$ is a convex set $\Leftrightarrow \forall \lambda \in [1, 0]$, A is a convex crisp set. Let $\lambda \in [0,1]$ and $\forall \lambda$ be any λ – cut of a convex fuzzy set and let $r, s \in A\lambda, \mu_{\underset{\sim}{A}}(r) \geq \lambda$ and $\mu_{\underset{\sim}{A}}(s) \geq \lambda$ by definition of a lambda cut. Hence $\min[\mu_{\underset{\sim}{A}}(r), \mu_{\underset{\sim}{A}}(s)] \geq \lambda$.

Let $\lambda \in [0,1]$; then

$$\mu_{\underset{\sim}{A}}[\lambda r + (1-\lambda)s] \geq \min[\mu_{\underset{\sim}{A}}(r), \mu_{\underset{\sim}{A}}(s)] \geq \lambda$$

and $\lambda r + (1-\lambda)s \in A\lambda$. For the converse, let $r, s \in A$ where we now assume $A\lambda$ is convex crisp set. i.e., $\lambda r + (1-\lambda)s \in A\lambda$. Now every λ – cut of A is convex, so let $\lambda = \min[\mu_{\underset{\sim}{A}}(r), \mu_{\underset{\sim}{A}}(s)]$. Since $\lambda r + (1-\lambda)s \in A\lambda$, we know

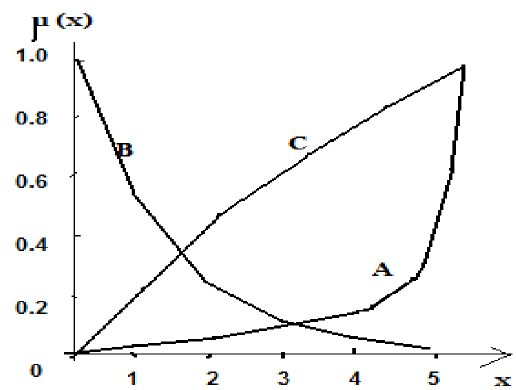
$$\mu_{\underset{\sim}{A}}[\lambda r + (1-\lambda)s] \geq \lambda = \min[\mu_{\underset{\sim}{A}}(r), \mu_{\underset{\sim}{B}}(s)]$$

4.3 For $x = 0,1,2,3,4,5$ on the universe , let us define the three fuzzy sets as follows:

$A = \{\frac{0.008}{0} + \frac{0.01}{1} + \frac{0.02}{2} + \frac{0.01}{3} + \frac{0.17}{4} + \frac{1}{5}\}$

$B = \{\frac{1}{0} + \frac{0.5}{1} + \frac{0.25}{2} + \frac{0.125}{3} + \frac{0.06}{4} + \frac{0.03}{5}\}$

$C = \{\frac{0}{0} + \frac{0.33}{1} + \frac{0.57}{2} + \frac{0.75}{3} + \frac{0.89}{4} + \frac{1}{5}\}$



$A_i = A_{ii} = A_{iii} = A_{iv} = \{5\}$
$B_i = \{0,1,2\}$, $B_{ii} = B_{iii} = B_{iv} = \{0\}$
$C_i = \{1,2,3,4,5\}$, $C_{ii} = \{3,4,5\}$
$C_{iii} = C_{iv} = \{5\}$

4.4

$$R_{0.1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$R_{0.2} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$R_{0.3} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$R_{0.4} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$R_{0.5} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_{0.6} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_{0.7} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_{0.8} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_{0.9} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_{1.0} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

4.5

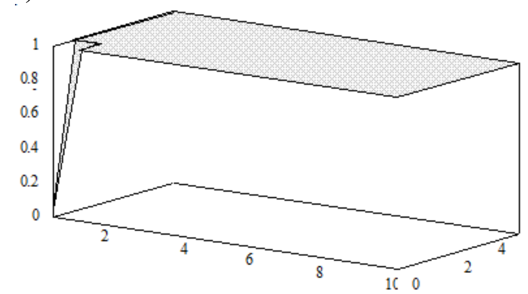
$$R = \begin{bmatrix} 1 & 0.8 & 0.4 & 0.5 & 0.8 \\ 0.8 & 1 & 0.4 & 0.5 & 0.9 \\ 0.4 & 0.4 & 1 & 0.4 & 0.4 \\ 0.5 & 0.5 & 0.4 & 1 & 0.5 \\ 0.8 & 0.9 & 0.4 & 0.5 & 1 \end{bmatrix}$$

$$R_{0+} = R_{0.1} = R_{0.3} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

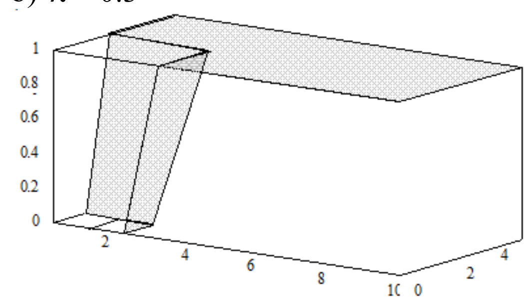
$$R_{0.8} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

4.6

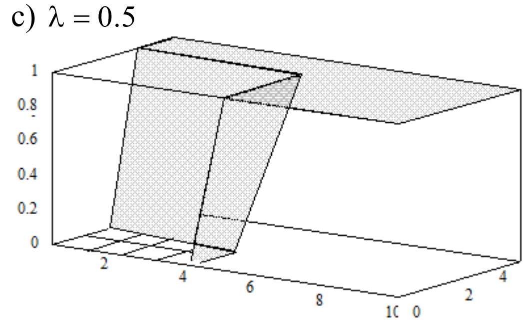
a) $\lambda = 0^{+}$



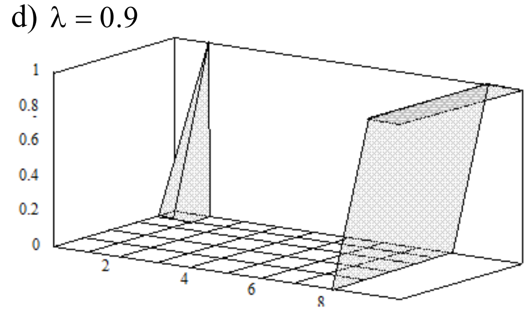
b) $\lambda = 0.3$



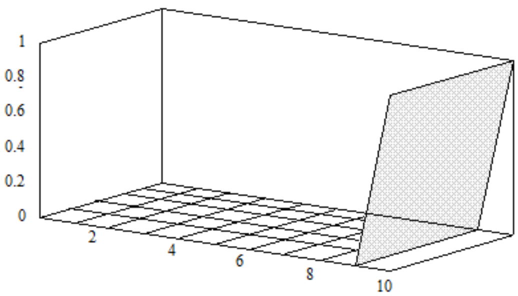
c) $\lambda = 0.5$



d) $\lambda = 0.9$



e) $\lambda = 1.0$



4.7 Let's consider the following relation

$$\underset{\sim}{R} = \begin{bmatrix} 1 & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & 1 & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & 1 & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & 1 & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 \end{bmatrix}$$

The condition of fuzzy tolerance applies only if

(i) $\mu_{\underset{\sim}{R}}(a_i, a_i) = 1$, reflexivity and

(ii) $\mu_{\underset{\sim}{R}}(a_i, a_j) = \mu_{\underset{\sim}{R}}(a_j, a_i)$, symmetry

Now if we consider the same example problem in chapter 3.

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.8 & 0 & 0.1 & 0.2 \\ 0.8 & 1 & 0.4 & 0 & 0.9 \\ 0 & 0.4 & 1 & 0 & 0 \\ 0.1 & 0 & 0 & 1 & 0.9 \\ 0.2 & 0.9 & 0 & 0.5 & 1 \end{bmatrix}$$

This tolerance fuzzy relation applies here because the above two conditions.

When we take $\lambda = 0.5$

$$\underset{\sim}{R}_{\lambda=0.5} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

This is nothing but crisp tolerance relation as $\chi_{R_1}(x_i, x_i) = 1$ and $\chi_{R_1}(x_i, x_j) = \chi_{R_1}(x_j, x_i)$, i.e., we have 1's in all a diagonal elements and $a_{13} = a_{31} = 0$.

4.8 Considering the same example from the previous problem, $\underset{\sim}{R}_1$ is a tolerance fuzzy relation, but not equivalence fuzzy relation. This is because $\mu_{\underset{\sim}{R}_1}(x_4, x_5) \neq \mu_{\underset{\sim}{R}_1}(x_5, x_4)$

So after one composition, we have

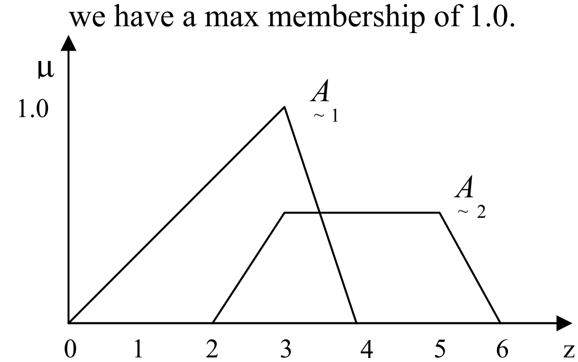
$$\underset{\sim}{R}_1^2 = R_1 \circ R_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix},$$

where transitivity hasn't occurred. Thus,

$$\underset{\sim}{R}_1^3 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

This is a crisp equivalence relation, $\chi_{R_1^3}(x_i, x_i) = 1$, $\chi_{R_1^3}(x_i, x_j) = \chi_{R_1^3}(x_j, x_i)$ and $\chi_{R_1^3}(x_i, x_j)$ and $\chi_{R_1^3}(x_i, x_R) = 1$ thus $\chi_{R_1^3}(x_i, x_k) = 1$

4.9 a) According to max-membership method z* = 3, because, when z = 3, we have a max membership of 1.0.



If we ignore asymmetry of A_1 the weighted average method results in the following:

$$z^* = \frac{3*1.0+4*0.5}{1+0.5} = \frac{3+2}{1.5} = 3.33$$

center of sums:

$$z^* = \frac{2.33*(4*1*0.5)+4*((4+2)*0.5*0.5)}{(4*1*0.5)+((4+2)*0.5*0.5)} = 3.05$$

center of largest area:

$$z^* = \frac{2*(3*1*0.5)+3.25*((1+0.5)*0.5*0.5)}{(3*1*0.5)+((1+0.5)*0.5*0.5)} = 2.25$$

first maxima or last maxima method and max membership method =3

centroid method $z^* =$

$$\frac{\int_0^3 (0.33z)zdz + \int_3^{3.5}(4-z)zdz + \int_{3.5}^5 0.5zdz + \int_5^6 \left(\frac{6-z}{2}\right)zdz}{\int_0^3 (0.33z)dz + \int_3^{3.5}(4-z)dz + \int_{3.5}^5 0.5dz + \int_5^6 \frac{6-z}{2}dz}$$

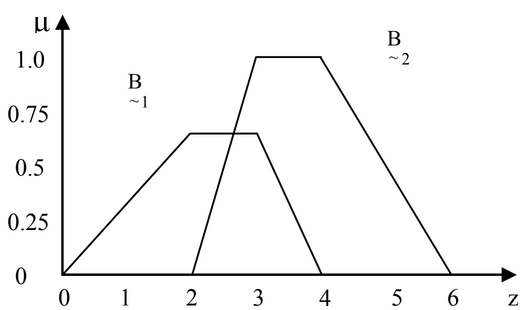
$z^* = 3.04$

mean max method is not applicable.

4.10 Max-membership method does not apply, since there is no peaked output function.

Mean max method:

$$z^* = \frac{3+4}{2} = 3.50$$



weighted average method does not apply due to asymmetry.

center of sums method:

$\bar{z}_1 = 2.2, S_1 = 1.75, \bar{z}_1 = 3.8, S_1 = 2.5$

$$z^* = \frac{2.2*1.75+3.8*2.5}{1.75*2.5} = 3.14$$

center of largest area, z*=

$$\frac{\int_{2.7}^3 (z^2-2z)dx + \int_3^4 zdz + \int_4^6 (-0.5z^2+3z)dz}{\int_{2.7}^3 (z-2)dz + \int_3^4 dz + \int_4^6 (-0.5z+3)dz} = 3.94$$

First of maxima = 3 and last of maxima = 4

Mean max membership=$\frac{3+4}{2}$=3.5

Centroid Method, $z^* =$

$$\frac{\int_0^2 0.35z^2dz + \int_2^{2.7} 0.7zdz + \int_{2.7}^3 (z^2-2z)dx + \int_3^4 zdz + \int_4^6 (-0.5z^2+3z)dz}{\int_0^2 0.35zdz + \int_2^{2.7} 0.7dz + \int_{2.7}^3 (z-2)dz + \int_3^4 dz + \int_4^6 (-0.5z+3)dz}$$
$= 3.19$

4.11

Max membership method is not applicable because there are two maximums.

Centroid method, $z^* =$

$$\frac{\int_0^2 (0.5z)zdz + \int_2^{3.78}(-0.25z+1.5)zdz + \int_{3.78}^6 (0.2z-0.2)zdz + \int_6^{10}(-0.25z+2.5)zdz}{\int_0^2 (0.5z)dz + \int_2^{3.78}(-0.25z+1.5)dz + \int_{3.78}^6 (0.2z-0.2)dz + \int_6^{10}(-0.25z+2.5)dz}$$
$= 4.67$

Mean max membership method does not apply.

Center of sums, $z^* =$

$\bar{z}_1 = 2.67, S_1 = 3.0, \bar{z}_1 = 5.67, S_1 = 4.5$

$$z^* = \frac{2.67*3.0+5.67*4.5}{3.0*4.5} = 4.47$$

Center of largest area method, $z^* =$

$$\frac{\int_{3.78}^6 (0.2z-0.2)zdz + \int_6^{10}(-0.25z+2.5)zdz}{\int_{3.78}^6 (0.2z-0.2)dz + \int_6^{10}(-0.25z+2.5)dz} = 6.25$$

First maxima = 2 and last maxima = 6

4.12 max membership:

$$z^* \rightarrow \mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(z^*) \geq \mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(z), \forall z$$

not applicable, does not yield a unique z^*

centroid:

$$z^* = \frac{\int \mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(z) z dz}{\int \mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(z) dz} = 2.5 \text{MPa}$$

Weighted average

$$z^* = \frac{\sum \mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(\bar{z}) \bar{z}}{\sum \mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(\bar{z})} = 2.5 \text{MPa}$$

Mean-max method is not applicable.

Center of Sums, z^* =2.5MPa

center of largest area is not applicable

First maxima $z^* = 2$

Last maxima $z^* = 3$

4.13 Using the Centroid method

$$z^* = \frac{\int \mu(z) * zdz}{\int \mu(z) dz}$$

$$z^* = \frac{\int_{-2}^{0}\left(\frac{1}{2}x+1\right)xdx + \int_{0}^{1.5}\left(-\frac{1}{2}x+1\right)xdx + \int_{1.5}^{4}\left(\frac{1}{3}x-\frac{1}{3}\right)xdx + \int_{4}^{6}xdx}{\int_{-2}^{0}\left(\frac{1}{2}x+1\right)dx + \int_{0}^{1.5}\left(-\frac{1}{2}x+1\right)dx + \int_{1.5}^{4}\left(\frac{1}{3}x-\frac{1}{3}\right)dx + \int_{4}^{6}dx}$$

$$z^* = \frac{14.34}{5.40} = 2.66$$

Center of Sums Method

$$z^* = \frac{0 * 2 + \frac{3 * 1.5 + 5 * 2}{1.5 + 2} * 3.5}{2 + 3.5} = 2.64$$

Center of Largest Area Method

PFR has the largest area

$$z^* = \frac{\int_{1}^{4}\left(\frac{1}{3}x + \frac{1}{3}\right)xdx + \int_{4}^{6}xdx}{\int_{1}^{4}\left(\frac{1}{3}x - \frac{1}{3}\right)dx + \int_{4}^{6}1dx}$$

$$z^* = \frac{\left.\left(\frac{x^3}{9} - \frac{x^2}{6}\right)\right|_1^4 + \left.\frac{x}{2}\right|_4^6}{\left.\left(\frac{x^2}{6} - \frac{1}{3}x\right)\right|_1^4 + \left.x\right|_4^6} = \frac{14.5}{3.5} = 4.14$$

First and Last Maxima Methods

First Maxima is located at z = 0.

Last Maxima does not exist.

Weighted Average Method

Not applicable since the memberships are not symmetrical

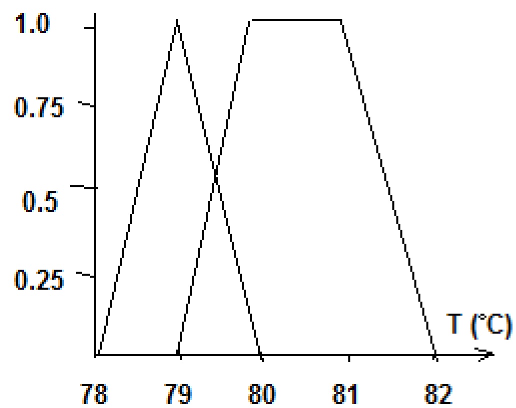
Mean Max Method

Does not apply

Max Membership Method

Does not apply.

4.15



Max member principle not applicable

Centroid method

$$T^* = \frac{\int \mu_{\underset{\sim}{C}}(T) T dT}{\int \mu_{\underset{\sim}{C}}(T) dT} = 80.05^o C$$

Weighted average method

$$T^* = \frac{79(1) + 80.5(1)}{1+1} = 79.75^o C$$

Mean Max membership is not applicable

Center of Sums

$$T^* = \frac{\sum_{k=1}^{n} T \int_T \mu_{\underset{\sim}{C}_k}(T)\,dT}{\sum_{k=1}^{n} \int_T \mu_{\underset{\sim}{C}_k}(T)\,dT} = 80.00^o C$$

$$T^* = \frac{\int_T \mu_{\underset{\sim}{C}_m}(T)\,T\,dT}{\int_T \mu_{\underset{\sim}{C}_m}(T)\,dT} = 80.59^o C$$

First Maxima is located at z = 79.00.
Last Maxima does not exist.

Center of largest area

CHAPTER 5

Logic and Fuzzy Systems

5.1

P	Q	$\overline{P}$	$P \to Q = \overline{P} \vee Q$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	0	1

i.e. if T(P) =T(Q) or P is false and Q is true, then P→Q is a tautology.

5.2

P	Q	$\overline{P} \wedge Q$	$P \wedge \overline{Q}$	$P \oplus Q = (\overline{P} \wedge Q) \vee (P \wedge \overline{Q})$
0	0	0	0	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

$P \vee Q$
0
1
1
1

Therefore, $T(P \oplus Q) \neq T(P \vee Q)$

5.3 a)

P	Q	$\overline{P}$	$\overline{Q}$	$(P \to Q) \leftrightarrow (\overline{Q} \to \overline{P})$, *if* $T(\overline{P} \vee Q) = T(Q \vee \overline{P})$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

$\overline{Q} \to \overline{P}$ $Q \vee \overline{P}$	$P \to Q$ $\overline{P} \vee Q$
1	1
1	1
0	0
1	1

b)

P	Q	$\overline{P}$	$\overline{Q}$	$(Q \to P) \leftrightarrow (\overline{P} \to \overline{Q})$, if $T(\overline{Q} \vee P) = T(P \vee \overline{Q})$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

$Q \to P$ $\overline{Q} \vee P$	$\overline{P} \to \overline{Q}$ $P \vee \overline{Q}$
1	1
0	0
1	1
1	1

5.4 Duality: $(P \wedge Q) \wedge (\overline{P} \vee \overline{Q}) \leftrightarrow 0$

P	Q	$\overline{P}$	$\overline{Q}$	$(P \wedge Q)$	$(\overline{P} \vee \overline{Q})$

0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	1	1

$(P\wedge Q)\wedge(\overline{P}\vee\overline{Q})$	$(P\wedge Q)\wedge(\overline{P}\vee\overline{Q}) \leftrightarrow 0$
0	1
0	1
0	1
0	1

5.5 DeMorgan's Laws

$$\overline{A\vee B} = \overline{A}\wedge\overline{B}, \overline{A\wedge B} = \overline{A}\vee\overline{B}$$

Duality:

$$\overline{A\wedge B} \Leftrightarrow \overline{A}\vee\overline{B}, \overline{A\vee B} = \overline{A}\vee\overline{B}$$

A	B	$\overline{A}$	$\overline{B}$	$(\overline{A\wedge B})$	$(\overline{A}\vee\overline{B})$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

$\overline{A\wedge B} \Leftrightarrow \overline{A}\vee\overline{B}$
1
1
1
1

The same proof for $\overline{A\vee B} \Leftrightarrow \overline{A}\wedge\overline{B}$

5.6

P	Q	$\overline{R}$	$\overline{S}$	$\overline{P}$	$\overline{Q}$	$\overline{R}$	$\overline{S}$
0	0	0	0	1	1	1	1
0	0	0	1	1	1	1	0
0	0	1	0	1	1	0	1
0	0	1	1	1	1	0	0
0	1	0	0	1	0	1	1
0	1	0	1	1	0	1	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	0	0
1	0	0	0	0	1	1	1
1	0	0	1	0	1	1	0
1	0	1	0	0	1	0	1
1	0	1	1	0	1	0	0
1	1	0	0	0	0	1	1
1	1	0	1	0	0	1	0
1	1	1	0	0	0	0	1
1	1	1	1	0	0	0	0

$P\rightarrow Q = \overline{P}\vee Q$ 3	$R\rightarrow\overline{S}$ $\overline{R}\vee\overline{S}$ 4	$Q\rightarrow R = \overline{Q}\vee R$ 5	$P\rightarrow\overline{S}$ $\overline{P}\vee\overline{S}$ 6
1	1	1	1
1	1	1	1
1	1	1	1
1	0	1	1
1	1	1	1
1	1	1	1
1	1	1	1
1	0	1	0
1	1	1	1
0	1	1	0
0	1	1	1
0	0	1	0
1	1	0	1
1	1	0	1
1	1	1	1
1	0	1	0

$3\wedge 4\wedge 5$ 7	$7\rightarrow 6 = \overline{7}\vee 6$ 8
1	$\overline{1}\vee 1 = 1$
1	$\overline{1}\vee 1 = 1$
1	$\overline{1}\vee 1 = 1$
0	$\overline{0}\vee 1 = 1$
1	$\overline{1}\vee 1 = 1$
1	$\overline{1}\vee 1 = 1$
1	$\overline{1}\vee 1 = 1$
0	$\overline{0}\vee 1 = 1$
1	$\overline{1}\vee 1 = 1$
0	$\overline{0}\vee 0 = 1$

0	$\overline{0} \vee 1 = 1$
0	$\overline{0} \vee 0 = 1$
0	$\overline{0} \vee 1 = 1$
0	$\overline{0} \vee 1 = 1$
1	$\overline{1} \vee 1 = 1$
0	$\overline{0} \vee 0 = 1$

From column 8, it can be seen that $((P \to Q) \wedge (R \to \overline{S}) \wedge (Q \to R)) \to (P \to S)$ is a tautology.

5.7 a) P: animals that are called ducks

Q: one who waltzes

R: people who are officers

S: animals that are considered poultry

Statement:

$((P \to \overline{Q}) \wedge (R \to Q) \wedge (S \to P)) \to (S \to \overline{R})$

The same method as 5.6 to prove the statement.

b) P: babies

Q: illogical people

R: despised people

S: one who can manage crocodiles

Statement:

$((P \to Q) \wedge (R \to \overline{S}) \wedge (P \to R)) \to (P \to \overline{S})$

The same method as 5.6 to prove the statement.

c) P: people who do not keep promise

Q: people who are untrustworthy

R: people who drink wine

S: people who are very communicative

T: people who are honest

V: people who are pawnbrokers

Statement :

$$\begin{pmatrix} (P \to Q) \wedge (R \to S) \wedge (\overline{P} \to T) \wedge (V \to R) \\ \wedge (S \to \overline{Q}) \end{pmatrix} \to (U \to T)$$

The same method as 5.6 to prove the statement.

5.8 $((P \to Q) \wedge P) \to Q$ by contradiction:

$((P \to Q) \wedge P) \wedge \overline{Q}$

P	Q	$\overline{P}$	$\overline{Q}$	$(P \to Q) = \overline{P} \vee Q$	$(P \to Q) \wedge P$
0	0	1	1	1	0
0	1	1	0	1	0
1	0	0	1	0	0
1	1	0	0	1	1

$((P \to Q) \wedge P) \wedge \overline{Q}$
0
0
0
0

Therefore, $((P \to Q) \wedge P) \to Q$ is true.

b) $((P \to \overline{Q}) \wedge (Q \vee \overline{R}) \wedge (R \wedge \overline{S})) \to \overline{P}$ and by contradiction:

$((P \to \overline{Q}) \wedge (Q \vee \overline{R}) \wedge (R \wedge S)) \wedge \overline{P}$

P	Q	R	S	$\overline{P}$	$\overline{Q}$	$\overline{R}$	$\overline{S}$
0	0	0	0	1	1	1	1
0	0	0	1	1	1	1	0
0	0	1	0	1	1	0	1
0	0	1	1	1	1	0	0
0	1	0	0	1	0	1	1
0	1	0	1	1	0	1	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	0	0
1	0	0	0	0	1	1	1
1	0	0	1	0	1	1	0
1	0	1	0	0	1	0	1
1	0	1	1	0	1	0	0

1	1	0	0	0	0	1	1
1	1	0	1	0	0	1	0
1	1	1	0	0	0	0	1
1	1	1	1	0	0	0	0

0
0
0
0

$P \to \overline{Q} = \overline{P} \vee Q$ 3	$Q \vee \overline{R}$ 4	$R \wedge \overline{S}$ 5	$3 \wedge 4 \wedge 5$ 6
1	1	0	0
1	1	0	0
1	0	1	0
1	0	0	0
1	1	0	0
1	1	0	0
1	1	1	1
1	1	0	0
1	1	0	0
1	1	0	0
1	0	1	0
1	0	0	0
0	1	0	0
0	1	0	0
0	1	1	0
0	1	0	0

$6 \wedge P$ 7
0
0
0
0
0
0
0
0
0
0
0
0

5.9

$$\left(\left(P \to \overline{Q}\right) \wedge \left(R \to \overline{Q}\right) \wedge \left(P \vee R\right)\right) \to R$$

P	Q	R	$\overline{P}$	$\overline{Q}$	$\overline{R}$	$P \to \overline{Q} = \overline{P} \vee \overline{Q}$ 3
0	0	0	1	1	1	1
0	0	1	1	1	0	1
0	1	0	1	0	1	1
0	1	1	1	0	0	1
1	0	0	0	1	1	1
1	0	1	0	1	0	1
1	1	0	0	0	1	0
1	1	1	0	0	0	0

$R \to \overline{Q} = \overline{R} \vee \overline{Q}$ 4	$P \vee R$ 5	$3 \wedge 4 \wedge 5$ 6	$6 \wedge \overline{R}$ 7
1	0	0	0
1	1	1	0
1	0	0	0
0	1	0	0
1	1	1	1
1	1	1	0
1	1	0	0
0	1	0	0

5.10 a) $\left(\left(P \to Q\right) \wedge P\right) \to Q$ see 5.8 (a)

b) $P \to \left(P \vee Q\right)$

P	Q	$\overline{P}$	$\overline{Q}$	$P \to \left(P \vee Q\right) = \overline{P} V \left(P \vee Q\right)$
0	0	1	1	1

0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

c) $(P \wedge Q) \rightarrow P$

By Contradiction: $(P \wedge Q) \wedge \overline{P}$

P	Q	$\overline{P}$	$\overline{Q}$	$(P \vee Q)$	$(P \wedge Q) \wedge \overline{P}$
0	0	1	1	0	0
0	1	1	0	0	0
1	0	0	1	0	0

d) $((P \vee Q) \wedge \overline{P}) \rightarrow Q$ by contradiction:

$((P \vee Q) \wedge \overline{P}) \wedge \overline{Q}$ is false.

P	Q	$\overline{P}$	$\overline{Q}$	$(P \vee Q)$	$(P \wedge Q) \wedge \overline{P}$
0	0	1	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	1	0	0	0	0

5.11Crisp tautology

A	B	C	$A \rightarrow B = \overline{A} \vee B$	$B \rightarrow C = \overline{B} \vee C$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	1
1	1	0	1	0
1	1	1	1	1

$(A \rightarrow B) \wedge (B \rightarrow C)$ $(\overline{A} \vee B) \wedge (\overline{B} \vee C)$	$A \rightarrow C = \overline{A} \wedge C$	$[(A \rightarrow B) \wedge (B \rightarrow C)] \rightarrow (A \rightarrow C)$
1	1	1
1	1	1
0	1	1
1	1	1
0	1	1
0	0	1
0	0	1
1	1	1

5.12 a) Mamdani Relation, $\underset{\sim}{R} = \underset{\sim}{A} \circ \underset{\sim}{B}$

$$\underset{\sim}{R} = \begin{Bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.5 & 0.5 & 0.0 \\ 0.0 & 0.6 & 1.0 & 0.6 & 0.0 \\ 0.0 & 0.5 & 0.5 & 0.5 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.0 & 0.0 \end{Bmatrix}$$

Product Relation, $\underset{\sim}{R} = \underset{\sim}{A} \circ \underset{\sim}{B}$

$$\underset{\sim}{R} = \begin{Bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.3 & 0.5 & 0.3 & 0.0 \\ 0.0 & 0.6 & 1.0 & 0.6 & 0.0 \\ 0.0 & 0.3 & 0.5 & 0.3 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{Bmatrix}$$

b)

$$\underset{\sim}{B}' = \underset{\sim}{A}' \circ \underset{\sim M}{R} = \frac{0}{0} + \frac{0.5}{1} + \frac{0.5}{2} + \frac{0.5}{3} + \frac{0}{4}$$

$$\underset{\sim}{B}' = \underset{\sim}{A}' \circ \underset{\sim P}{R} = \frac{0}{0} + \frac{0.8}{1} + \frac{0.3}{2} + \frac{0.18}{3} + \frac{0}{4}$$

5.13 a) After discritization:

$$A = \{\frac{1}{0} + \frac{0.8}{1} + \frac{0.6}{2} + \frac{0.4}{3} + \frac{0.2}{4} + \frac{0}{5}\}$$

$$B = \{\frac{0}{0} + \frac{0.25}{1} + \frac{0.5}{2} + \frac{0.75}{3} + \frac{1}{4}\}$$

$$\underset{\sim}{A} \times \underset{\sim}{B} = \begin{Bmatrix} 0 & 0.25 & 0.50 & 0.75 & 1 \\ 0 & 0.25 & 0.50 & 0.75 & 0.80 \\ 0 & 0.25 & 0.50 & 0.60 & 0.60 \\ 0 & 0.25 & 0.40 & 0.40 & 0.40 \\ 0 & 0.20 & 0.20 & 0.20 & 0.20 \\ 0 & 0 & 0 & 0 & 0 \end{Bmatrix}$$

$$\bar{A} = \{\frac{0}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5}\}$$

$$Y = \left\{\frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right\}$$

$$\bar{A} \cup Y = \begin{Bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 1 & 1 & 1 & 1 & 1 \end{Bmatrix}$$

Classical Implication Operation:

R=$(A \times B) \cup (\bar{A} \cup Y) =$

$$\begin{Bmatrix} 0 & 0.25 & 0.50 & 0.75 & 1 \\ 0.2 & 0.25 & 0.5 & 0.75 & 0.8 \\ 0.4 & 0.4 & 0.5 & 0.6 & 0.6 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 1 & 1 & 1 & 1 & 1 \end{Bmatrix}$$

b) $\underset{\sim}{A}' = \frac{1}{3}$

$$\underset{\sim}{B}' = \underset{\sim}{A}' \circ \underset{\sim}{R} = \frac{0}{0} + \frac{0.25}{1} + \frac{0.5}{2} + \frac{0.75}{3} + \frac{1}{4}$$

5.14 a)

$\underset{\sim}{R} =$

$$\begin{Bmatrix} 1 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.5 & 0.7 & 0.7 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.8 & 1.0 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{Bmatrix}$$

b) $\underset{\sim}{B}' =$

$$\frac{1.0}{160} + \frac{1.0}{165} + \frac{1.0}{170} + \frac{1.0}{175} + \frac{1.0}{180} + \frac{1.0}{185} + \frac{1.0}{190}$$

5.15 a) Classical

$$\underset{\sim}{R} = \begin{Bmatrix} 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.2 & 0.2 & 0.2 & 0.5 & 0.8 & 0.8 & 0.2 \\ 0.0 & 0.0 & 0.2 & 0.5 & 1.0 & 0.8 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.5 & 0.8 & 0.8 & 0.2 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{Bmatrix}$$

Mamdani

$$\underset{\sim}{R} = \begin{Bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.0 & 0.0 & 0.2 & 0.5 & 0.5 & 0.5 & 0.2 \\ 0.0 & 0.0 & 0.2 & 0.5 & 0.8 & 0.8 & 0.2 \\ 0.0 & 0.0 & 0.2 & 0.5 & 1.0 & 0.8 & 0.2 \\ 0.0 & 0.0 & 0.2 & 0.5 & 0.8 & 0.8 & 0.2 \\ 0.0 & 0.0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{Bmatrix}$$

Product

$$\underset{\sim}{R} = \begin{Bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.04 & 0.1 & 0.2 & 0.16 & 0.04 \\ 0.0 & 0.0 & 0.1 & 0.25 & 0.5 & 0.4 & 0.1 \\ 0.0 & 0.0 & 0.16 & 0.4 & 0.8 & 0.64 & 0.16 \\ 0.0 & 0.0 & 0.2 & 0.5 & 1.0 & 0.8 & 0.2 \\ 0.0 & 0.0 & 0.16 & 0.4 & 0.8 & 0.64 & 0.16 \\ 0.0 & 0.0 & 0.04 & 0.1 & 0.2 & 0.16 & 0.04 \end{Bmatrix}$$

b)

$$\underset{\sim}{K}' = \frac{0}{1e3} + \frac{0.8}{1e4} + \frac{0.1}{1e5} + \frac{0}{5e5} + \frac{0}{1e6} + \frac{0.8}{5e6} + \frac{0.2}{1e7}$$

max-min composition using classic, $f_1' =$

$$\frac{0.8}{100} + \frac{0.8}{200} + \frac{0.8}{500} + \frac{0.8}{800} + \frac{0.8}{1000} + \frac{0.8}{2000} + \frac{0.8}{5000}$$

max product from a(i): $f' =$

$$\frac{0.64}{100} + \frac{0.64}{200} + \frac{0.64}{500} + \frac{0.64}{800} + \frac{0.64}{1000} + \frac{0.64}{2000} + \frac{0.64}{5000}$$

5.16 a) Mamdani

$$\underset{\sim}{R} = \begin{bmatrix} 0.2 & 0.3 & 0.8 & 1 \\ 0.2 & 0.3 & 0.8 & 1 \\ 0.2 & 0.3 & 0.6 & 0.6 \\ 0.2 & 0.3 & 0.8 & 1 \end{bmatrix}$$

b) $D' = \frac{0.2}{0} + \frac{0.3}{0.5} + \frac{0.8}{1} + \frac{1}{0.7}$

5.17 a) Classic

$$\underset{\sim}{R} = \begin{Bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0 & 0.3 & 0.4 & 0.4 & 0.4 \\ 0 & 0.3 & 0.6 & 0.7 & 0.7 \\ 0 & 0.3 & 0.6 & 1 & 0.8 \end{Bmatrix}$$

b) $F' = \frac{0}{0} + \frac{0.3}{2} + \frac{0.6}{4} + \frac{0.7}{6} + \frac{0.7}{8}$

5.18 a) Classic

$$\underset{\sim}{R} = \begin{Bmatrix} 0 & 0 & 0 & 0 \\ 0.3 & 0.6 & 0.6 & 0.6 \\ 03 & 0.8 & 0.9 & 0.9 \\ 0.3 & 0.8 & 0.9 & 1 \end{Bmatrix}$$

b) $B' = \frac{0.3}{10} + \frac{0.8}{20} + \frac{0.9}{30} + \frac{0.9}{40}$

5.19 a) Classical

$$\underset{\sim}{R_W} = V_{in} \times I_{out} = \begin{Bmatrix} 0.3 & 0.5 & 1 \\ 0.3 & 0.5 & 0.9 \\ 0.3 & 0.5 & 0.7 \\ 0.2 & 0.2 & 0.2 \end{Bmatrix}$$

b) Mamdani

$$\underset{\sim V}{R} = Z \times I_{out} = \begin{Bmatrix} 0.0 & 0.0 & 0.0 \\ 0.3 & 0.3 & 0.3 \\ 0.3 & 0.5 & 0.6 \\ 0.3 & 0.5 & 1 \end{Bmatrix}$$

5.20 a) i)

$$\mu_{R}(x,y) = \max\{\mu_{B}(y), 1 - \mu_{A}(x)\}$$

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0.4 & 0.4 & 0.6 & 0.8 & 0.4 & 0.4 \\ 0 & 0.4 & 0.8 & 0.8 & 0.3 & 0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{bmatrix}$$

ii) $\mu_{\underset{\sim}{R}}(x,y) = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)\}$

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0.6 & 0.3 & 0 \\ 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0 \end{bmatrix}$$

iii)

$$\mu_{\underset{\sim}{R}}(x,y) = \min\{1, 1 - \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(y)\}$$

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0.4 & 0.8 & 1 & 1 & 0.7 & 0.4 \\ 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 0.8 & 1 & 1 & 1 & 1 & 0.8 \end{bmatrix}$$

iv) $\mu_{\underset{\sim}{R}}(x,y) = \{\mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(y)\}$

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.24 & 0.6 & 0.48 & 0.18 & 0 \\ 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 0 & 0.08 & 0.2 & 0.16 & 0.06 & 0 \end{bmatrix}$$

v)

$$\mu_{\underset{\sim}{R}}(x,y) = \begin{cases} 1, \text{ for } \mu_{\underset{\sim}{A}}(x) \le \mu_{\underset{\sim}{B}}(y) \\ \mu_{\underset{\sim}{B}}(y), \text{ otherwise} \end{cases}$$

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0.4 & 1 & 1 & 0.3 & 0 \\ 0 & 0.4 & 1 & 0.8 & 0.3 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

5.21 Classical Implication, Eq 5.19

A	B	$A \to B$
0	1	1
1	0	0
0.5	0.5	0.5
0.2	0.7	0.8
0.8	0.4	0.4

Mamdani Implication, Eq 5.20

A	B	A → B
0	1	0
1	0	0
0.5	0.5	0.5
0.2	0.7	0.2
0.8	0.4	0.4

Lukasiewicz's Implication, Eq 5.21

A	B	A → B
0	1	1
1	0	0
0.5	0.5	1
0.2	0.7	1
0.8	0.4	0.6

Correlation product Implication, Eq 5.22

A	B	A → B
0	1	1
1	0	0
0.5	0.5	0.25
0.2	0.7	0.14
0.8	0.4	0.32

Brouwerian Implication, Eq. 5.23

A	B	A → B
0	1	1
1	0	0
0.5	0.5	1
0.2	0.7	1
0.8	0.4	0.4

The implications give results that are very similar to the classical implication. For binary values, 0 or 1, of A and B, these implications produce the same results. For fuzzy A and B they produce numbers that are almost the same.

5.22 a) (i) temperature not very low

$$=\frac{0.0}{40}+\frac{0.51}{50}+\frac{0.75}{60}+\frac{0.91}{70}+\frac{1.0}{80}$$

(ii) Temperature not very high

$$=\frac{1}{40}+\frac{0.96}{50}+\frac{0.84}{60}+\frac{0.51}{70}+\frac{1.0}{80}$$

(iii) Termperature not very high and not very low

$$=\frac{0.0}{40}+\frac{0.51}{50}+\frac{0.74}{60}+\frac{0.51}{70}+\frac{0}{80}$$

b) (i) Water content slightly high

$$=\frac{0.0}{1}+\frac{0.45}{2}+\frac{0.63}{3}+\frac{0.95}{4}+\frac{1}{5}$$

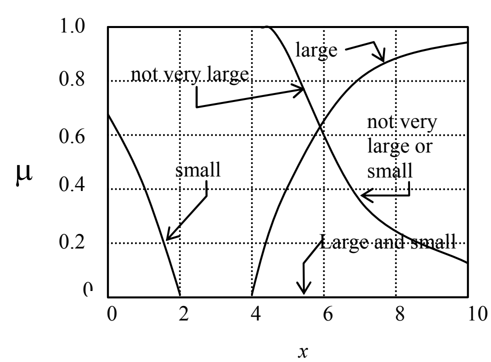
(ii) Water content fairly high

$$=\frac{0.0}{1}+\frac{0.34}{2}+\frac{0.54}{3}+\frac{0.93}{4}+\frac{1}{5}$$

(iii) Water content not very low or fairly low

$$=\frac{1}{1}+\frac{0.89}{2}+\frac{0.77}{3}+\frac{0.84}{4}+\frac{0.96}{5}$$

5.23



5.24 a)

$$=\frac{1}{0}+\frac{0.81}{10}+\frac{0.25}{20}+\frac{0.25}{30}+\frac{0.64}{40}+\frac{1}{50}$$

b)

$$=\frac{0}{0}+\frac{0.1}{10}+\frac{0.5}{20}+\frac{0.5}{30}+\frac{0.2}{40}+\frac{0}{50}$$

c)

$$= \frac{0}{0} + \frac{0.1}{10} + \frac{0.5}{20} + \frac{0.5}{30} + \frac{0.8}{40} + \frac{1}{50}$$

5.25 a) $= \frac{1}{0} + \frac{0.81}{1} + \frac{0.64}{2} + \frac{0.49}{3}$

b) $= \frac{1}{0} + \frac{0.93}{1} + \frac{0.86}{2} + \frac{0.79}{3}$

c) $= \frac{0}{0} + \frac{0.0001}{1} + \frac{0.0016}{2} + \frac{0.0081}{3}$

d) $= \frac{1}{0} + \frac{0.66}{1} + \frac{0.41}{2} + \frac{0.24}{3}$

5.26 a) $= \frac{1}{0} + \frac{0.8}{1} + \frac{0.5}{2} + \frac{0.1}{3} + \frac{0}{4}$

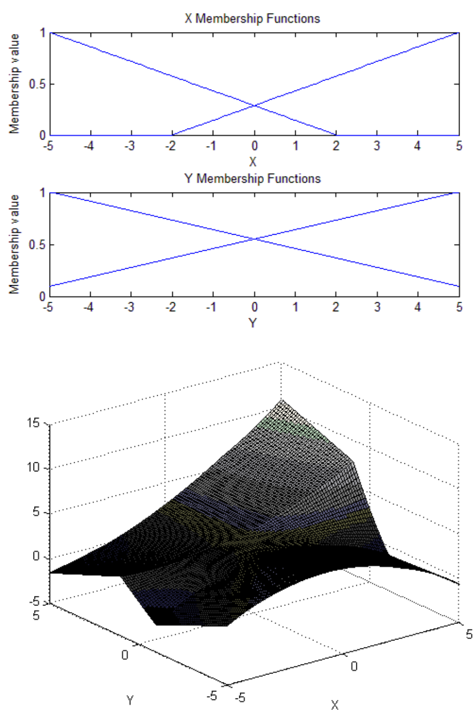
b) $= \frac{1}{0} + \frac{0.04}{1} + \frac{0.16}{2} + \frac{0.81}{3} + \frac{1}{4}$

c) $= \frac{1}{0} + \frac{0.8}{1} + \frac{0.5}{2} + \frac{0.9}{3} + \frac{1}{4}$

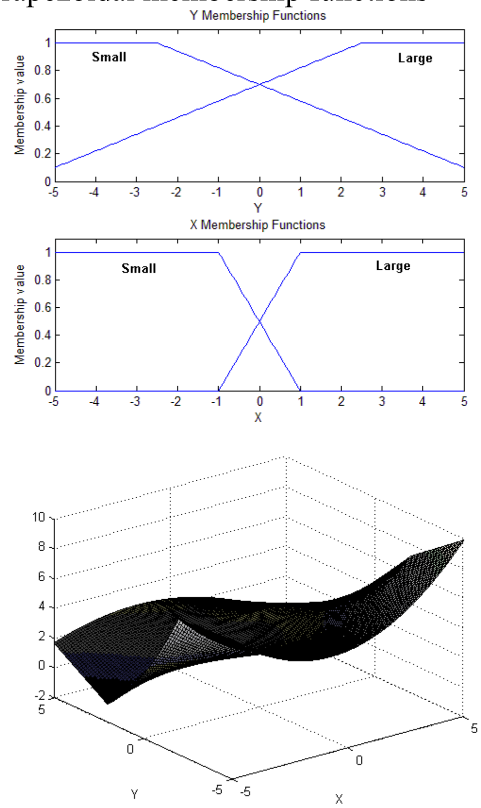
5.27 If $(x_1 \cap x_2)$, Then y

5.28 If $(x_1 \cap x_2)$, Then t

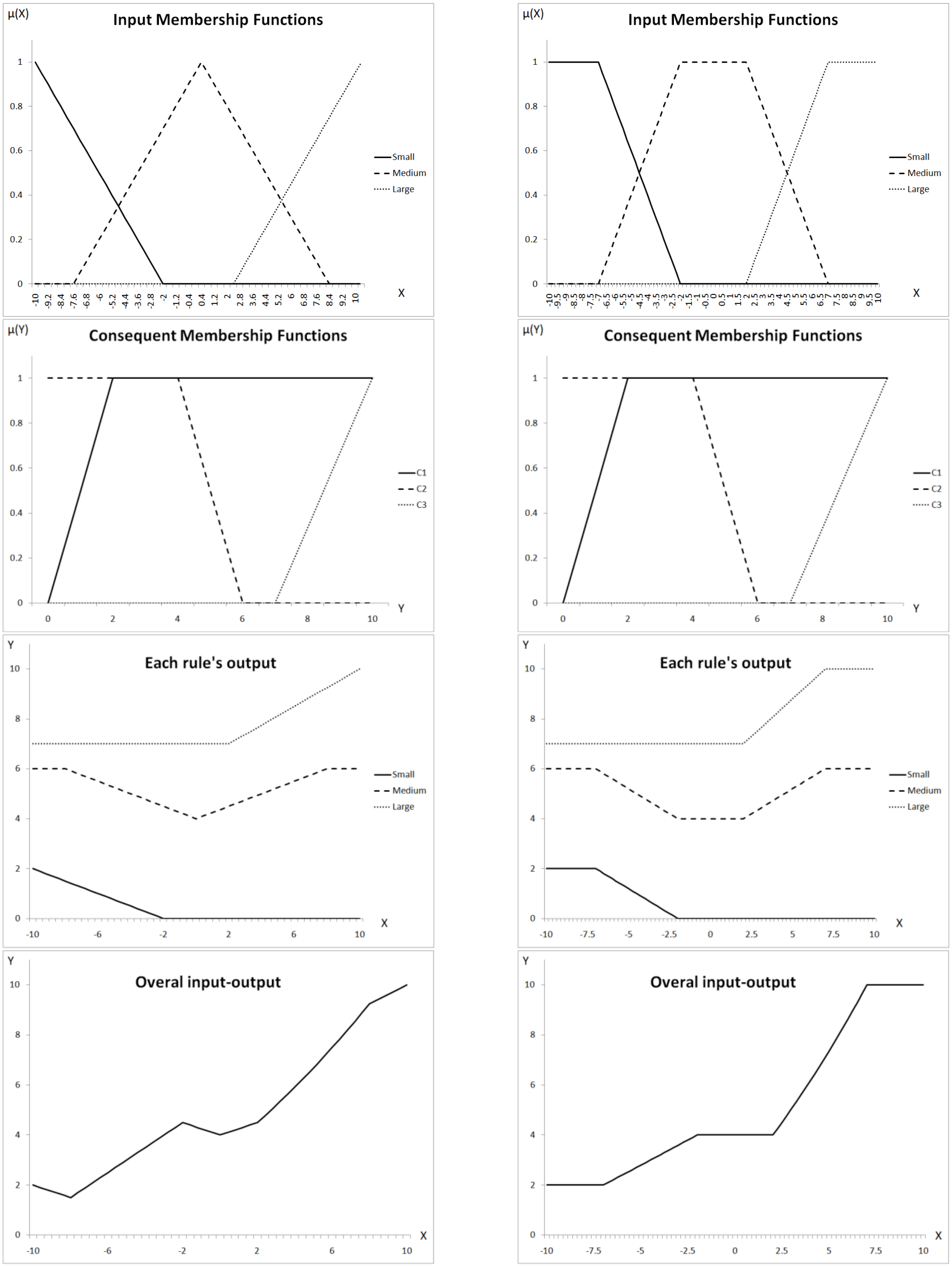
5.29 Triangular membership functions



Trapezoidal membership functions



5.30 Triangular membership functions



Trapezoidal membership functions

5.31 For Sugeno method rules are as follows:

Rule1: T_1=320K Then γ=1.5

Rule2: T_1=300K Then γ=1.4

Rule3: T_1=300K Then γ=1.3

For problem T_1 is fixed to 300K and the fuzzy model will predict P_2 for the given variable P_1 ad T_2. In other words… what is the final pressure of the system if the temperature is changed to T_2 from a pressure equal to T_1?

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$ and in this problem T_1=300K

Input: P_1=1.6bar and T_2=415K

Rule2 and Rule3 are fired since T_1=300K.

From rule2:	γ=1.4	thus	P_2=5.0bar
From rule2:	γ=1.3	thus	P_2=6.5bar

Weighted average: $$P_2{*} = \frac{0.25*5.0+0.5*6.5}{0.25+0.5} = 6.0bar$$

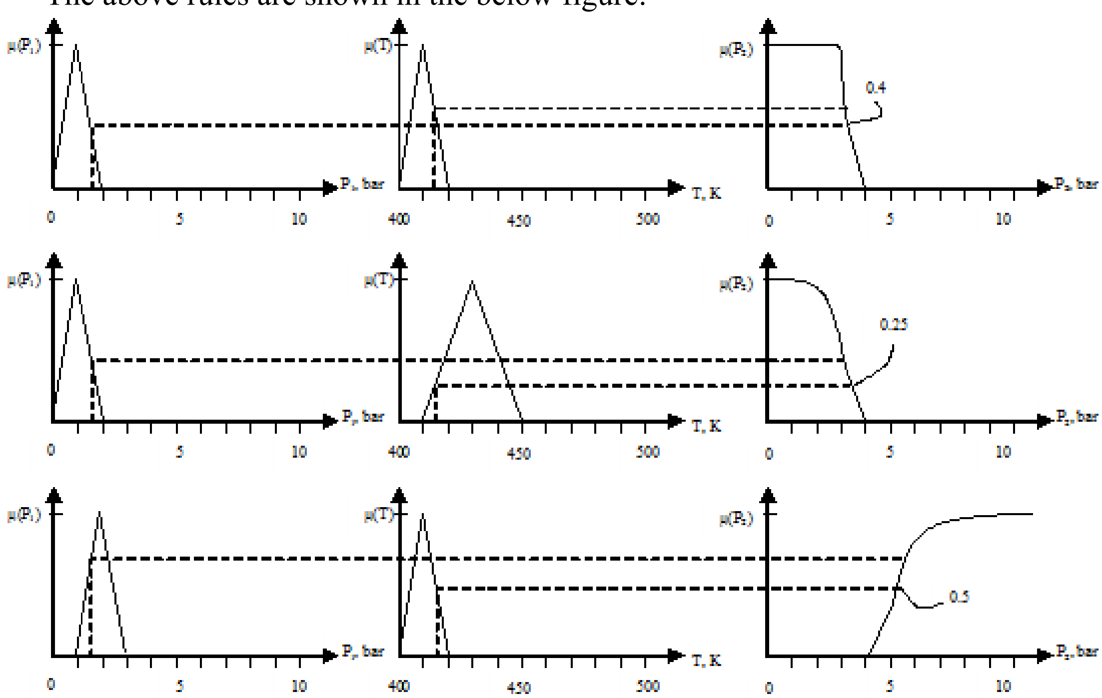
The rules used for Tsukamoto method are:

R1 $\rightarrow$ IF P_1 =atmP AND T_2 = lowT THEN P_2 = lowP

R2 $\rightarrow$ IF P_1 =atmP AND T_2 = midT THEN P_2 = midP

R3 $\rightarrow$ IF P_1 =lowP AND T_2 = lowT THEN P_2 = very highP

The above rules are shown in the below figure:



From the graph of rule2 and rule3 we have (P_2=3.5, μ_2=0.25) and (P_2=5, μ_2=0. 5) respectively.

Thus from the weighted average: $$P_2{*} = \frac{0.25*3.5+0.5*5}{0.25+0.5} = 4.5bar$$

5.32

In finding the Nusselt number (a dimensionless number for determining heat transfer) for a hexagonal cylinder in cross flow, there are two correlations.

$$N_{u1} = 0.16R_e^{0.638} Pr^{1/3} \qquad 5000 < R_e < 19650$$

$$N_{u2} = 0.0385R_e^{0.728} Pr^{1/3} \qquad R_e < 19650$$

R_e is the Reynolds number and P_r is the Prandtl number.

The Nusselt number is a function of convective heat transfer (*h*), diameter of the hexagonal cylinder (*D*) over which cooling fluid travels, and the conductivity of the material (*K*).

$$N_u = \frac{hD}{K},$$

R_e's and P_r's both can be fuzzy due to uncertainty in the variables in velocity. It would be convenient to find N_u (Output) based on R_e and P_r (inputs) without having to do all the calculations.

Rules:

If R_e is high and P_r is low $\rightarrow$ N_u is low
If R_e is low and P_r is low $\rightarrow$ N_u is low
If R_e is high and P_r is high $\rightarrow$ N_u is medium
If R_e is low and P_r is high $\rightarrow$ N_u is medium

In the Mandami method:

INPUT and $P_r \rightarrow$ propagate minimum to μ (Nu) and use weighted average defuzzification.

INPUT Re = 19.65×10^3
μ (Re) = 0.25
Pr = 275 μ (Pr) = 0.25

In the Sugeno method:

Use the correlations N_{u1} and N_{u2} to get Z_1 and Z_2

$$Z = \frac{\mu(N_{u1})z_1 + \mu(N_{u2})z_2}{\mu(N_{u1}) + \mu(N_{u2})}$$

Mandami

Input R_e = 19.65E3
$\mu(R_{eH}) = \mu(R_{eL}) = 0.25$
P_r = 275 $\mu(P_{rL})=0.75$ and $\mu(P_{rH})=0.25$

Rule1: N_{uL}min(0.25,0.75)=0.25
Rule2: N_{uL}=min(0.25,0.75)=0.25
Rule 1 and 2 : max is 0.25

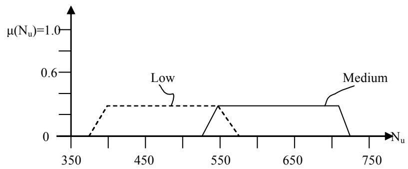
Defuzzification for N_{uL}=0.25 yields to z=487.5

Rule3: N_{uH}=min(0.25,0.25)=0.25
Rule4: N_{uH}=min(0.25,0.25)=0.25
Rule 3 and 4 : max is 0.25

Defuzzification for N_{uH}=0.25 yields to z=612.5

Weighted average :

$$z = \frac{0.25*487.5 + 0.25*612.5}{0.5} = 550$$



For **Sugeno** we have the following input

Input $R_e = 1.965E4$ $\mu(R_e)=0.25$
$P_r = 275$ $\mu(P_{rL})=0.75$ and
$\mu(P_{rH})=0.25$

And from our rule base and the following equations

$$N_{u1} = 0.16 R_e^{0.638} Pr^{1/3} \qquad 5000 < R_e < 19650$$

$$N_{u2} = 0.0385 R_e^{0.728} Pr^{1/3} \qquad R_e < 19650$$

we have the following results:

Nu1 = 560.0993666

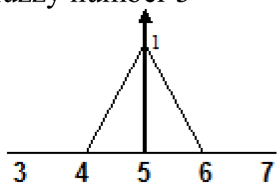
Nu2 = 559.5643482

z = 559.8318574

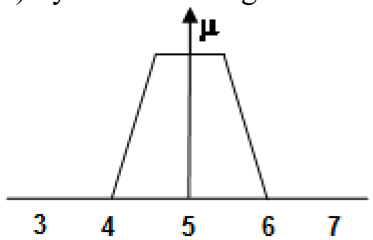
CHAPTER 6

Development of Membership Functions

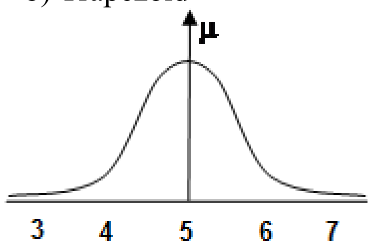
6.1 Fuzzy membership function for the fuzzy number 3



a) Symmetric triangle

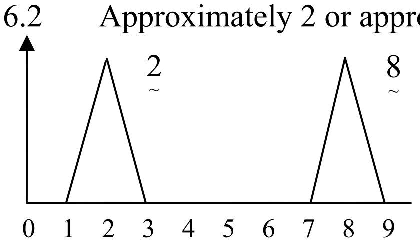


b) Trapezoid

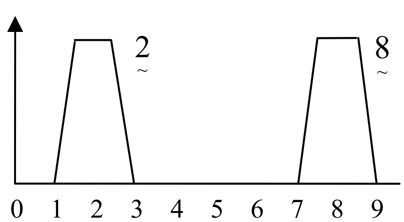


c) Gaussian function

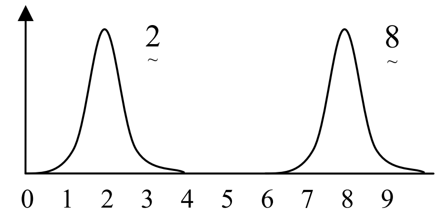
6.2 Approximately 2 or approximately 8



a) Symmetric Triangles



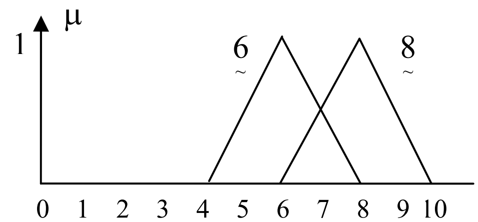
b) Trapezoids



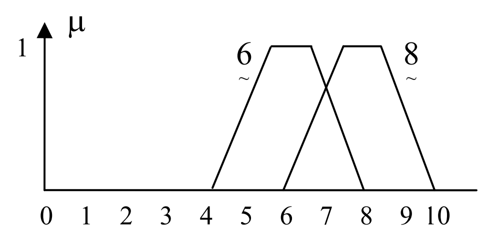
c) Gaussian Function

6.3 Approximately 6 or approximately 8
$\lfloor \underset{\sim}{6} \cup \underset{\sim}{8} \rfloor$

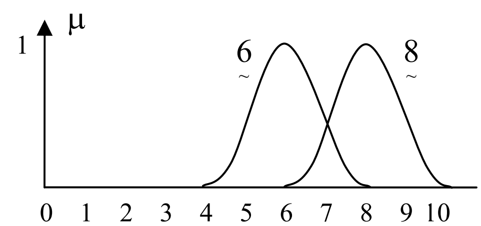
a) Symmetric triangle



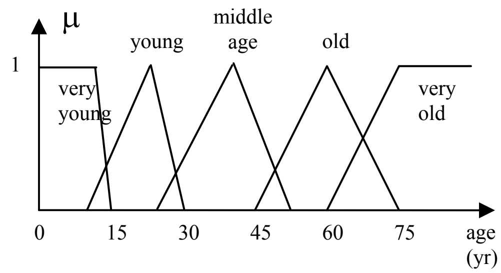
b) Trapezoids



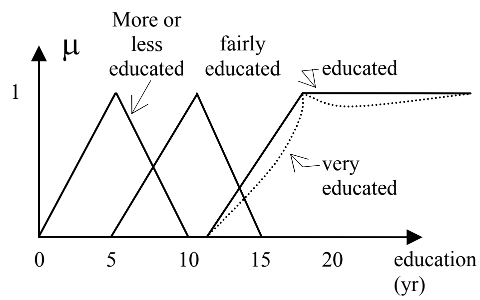
c) Gaussian



6.4 a)



b)



6.5 Given:

A + B + C = 180, $A \geq B \geq C \geq 0$

$$\mu_I = 1 - \frac{\min((A-B),(B-C))}{60}$$

$$\mu_R = 1 - \left|\frac{A-90}{90}\right|$$

$$\mu_E = 1 - \frac{A-C}{180}$$

$$\mu_{IR} = I \cap R = \min(\mu_I, \mu_R)$$

$$\mu_{IR} = \overline{I \cup R \cup E} = \bar{I} \cap \bar{R} \cap \bar{E}$$

$$\mu_{IR} = \min((1-\mu_I),(1-\mu_R),(1-\mu_E))$$

	A, B, C	Isosceles	Right	Right Isosceles
a	80, 75, 25	0.92	0.89	0.89
b	65, 60, 55	0.92	0.72	0.72
c	75, 60, 45	0.75	0.83	0.75
d	120, 50, 10	0.33	0.67	0.33

	A, B, C	Equilateral	other
a	80, 75, 25	0.69	0.083
b	65, 60, 55	0.94	0.056
c	75, 60, 45	0.83	0.167
d	120, 50, 10	0.39	0.333

6.6 Assume that the sides of the rectangular may be given as a and b, such that $a \leq b$ we may develop a membership function for a square as:

$$\mu_{(sqr)} = \frac{a}{b}$$

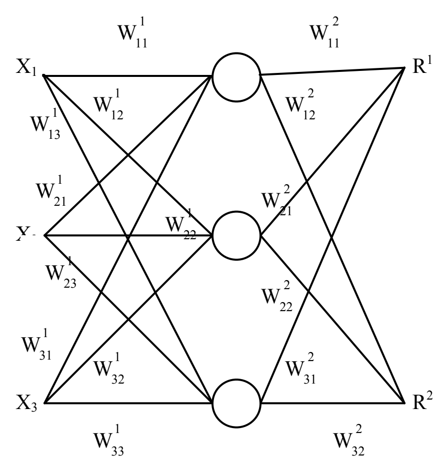
6.7

pref-er-ence	P	B	M	L	I	Σ	%
P	-	21	15	41	33	110	11
B	79	-	77	63	55	274	27.4
M	85	23	-	65	52	225	22.5
L	59	37	35	-	49	180	18.0
I	67	45	48	51	-	211	21.1

Rank order

P	B	M	L	M
5	1	2	4	3

6.8 Using a [3 x 3 x 2] Neural Network



Assigning random values to the weights

$W^1_{11} = 0.8, W^1_{21} = 0.5, W^1_{31} = 0.6$

$W^1_{12} = 0.6, W^1_{22} = 0.4, W^1_{32} = 0.4$

$W^1_{13} = 0.5, W^1_{23} = 0.7, W^1_{33} = 0.6$

$W^2_{11} = 0.25, W^2_{21} = 0.70, W^2_{31} = 0.65$

$W^2_{12} = 0.30, W^2_{22} = 0.75, W^2_{32} = 0.95$

Assuming threshold value t = 0, we have

$$O_1^2 = \frac{1}{1+\exp[-(0.5*0.8+3*0.5+2.5*0.6-0)}$$

$$O_2^2 = \frac{1}{1+\exp[-(0.5*0.6+0.3*0.4+2.5*0.4-0)}$$

$$O_3^2 = \frac{1}{1+\exp[-(0.5*0.5+3*0.7+2.5*0.6-0)}$$

$O_1^2 = 0.9692 \quad O_1^3 = 0.9241 \quad O_2^2 = 0.9792$

$$O_1^3 = \frac{1}{1+\exp[-(0.9692*0.25+0.9241*0.7+0.9792*0.65-0)}$$

$$O_2^3 = \frac{1}{1+\exp[-(0.9692*0.30+0.9241*0.75+0.9792*0.95-0)}$$

$O_1^3 = 0.8214 \quad O_2^3 = 0.8715$

Determine errors:

$R^1 : E_1^3 = O_{1actual}^3 - O_1^3 = 0 - 0.8214 = -0.8214$

$R^1 : E_2^3 = O_{2actual}^3 - O_2^3 = 1 - 0.8715 = 0.1285$

Assigning Errors to the 2nd layer

$E_1^2 = O_{1actual}^2 - O_1^2$
$= 0.9692(1-0.9692)*(0.25*-0.8214+0.3*0.1285)$
$= -0.0050$

$E_2^2 = O_{2actual}^2 - O_2^2$
$= 0.9241(1-0.9241)*(0.70*-0.8214+0.75*0.1285)$
$= -0.0336$

$E_3^2 = O_{3actual}^2 - O_3^2$
$= 0.9792(1-0.9792)*(0.65*-0.8214+0.95*0.1285)$
$= -0.0084$

Updating the weights, assuming $\alpha = 0.3$

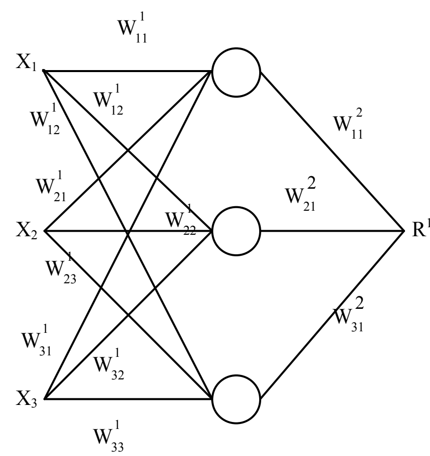
$W_{11}^2 = 0.25 + 0.30(-0.8214)0.9692 = 0.0112$
$W_{21}^2 = 0.7 + 0.30(-0.8214)0.9241 = 0.4723$
$W_{31}^2 = 0.65 + 0.30(-0.8214)0.9792 = 0.4087$
$W_{12}^2 = 0.30 + 0.30(0.1285)0.9692 = 0.3374$
$W_{22}^2 = 0.75 + 0.30(0.1285)0.9241 = 0.7856$
$W_{32}^2 = 0.95 + 0.30(0.1285)0.9672 = 0.9877$

$W_{11}^1 = 0.8 + 0.3(-0.0050)0.5 = 0.7993$
$W_{21}^1 = 0.5 + 0.3(-0.0050)3.0 = 0.4955$
$W_{31}^1 = 0.6 + 0.3(-0.0050)2.5 = 0.5963$
$W_{12}^1 = 0.6 + 0.3(-0.0336)0.5 = 0.5950$
$W_{22}^1 = 0.4 + 0.3(-0.0336)0.5 = 0.3698$
$W_{32}^1 = 0.4 + 0.3(-0.0336)2.5 = 0.3748$
$W_{13}^1 = 0.5 + 0.3(-0.0084)0.5 = 0.3748$
$W_{23}^1 = 0.7 + 0.3(-0.0084)3.0 = 0.6924$
$W_{33}^1 = 0.6 + 0.3(-0.0084)2.5 = 0.5937$

6.9

a) using a [3 x 3 x 1] Neural Network

X_1	X_2	X_3	R^1	R^2
1	0.5	2.3	1	0



Assigning random values be the weights as

$W_{11}^1 = 0.5, W_{21}^1 = 0.2, W_{31}^1 = 0.1, W_{11}^2 = 0.5$
$W_{12}^1 = 0.3, W_{22}^1 = 0.6, W_{32}^1 = 0.3, W_{21}^2 = 0.3$
$W_{13}^1 = 0.1, W_{23}^2 = 0.2, W_{33}^1 = 0.8, W_{31}^2 = 0.8$

threshold value t = 0, we have the output

$$0 = \frac{1}{1+e^{-\Sigma X_i W_i}}$$

Output for the 2nd layer

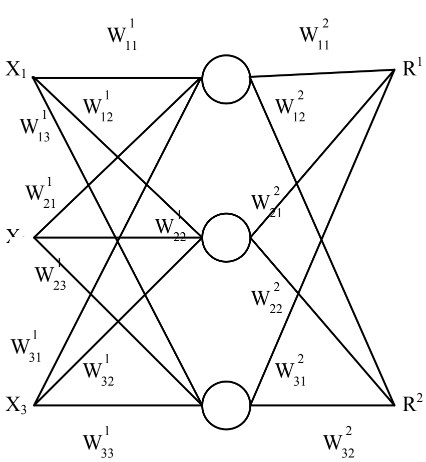
$O_1^2 = 0.696$
$O_2^2 = 0.784$
$O_3^2 = 0.885$

Output for 3rd layer
$O_1^3 = 0.784$

Determining error
$R^1 : E_1^3 = O_{1actual}^3 - O_1^3 = 1 - 0.784 = 0.216$
Assigning errors to the 2nd layer
$E_1^2 = 0.023$
$E_2^2 = 0.011$
$E_3^2 = 0.018$

Updating weights
$W_{11}^2 = 0.5 + 0.3(0.216)0.696 = 0.55$
$W_{21}^2 = 0.3 + 0.3(0.216)0.784 = 0.35$
$W_{31}^2 = 0.8 + 0.3(0.216)0.885 = 0.86$
$W_{11}^1 = 0.5 + 0.3(0.023)1 = 0.51$
$W_{12}^2 = 0.3 + 0.3(0.011)1 = 0.30$
$W_{13}^2 = 0.1 + 0.3(0.018)1 = 0.11$
$W_{21}^1 = 0.2 + 0.3(0.023)0.5 = 0.2$
$W_{22}^1 = 0.6 + 0.3(0.011)0.5 = 0.6$
$W_{23}^1 = 0.2 + 0.3(0.018)0.5 = 0.2$
$W_{31}^1 = 0.1 + 0.3(0.023)2.3 = 0.12$
$W_{32}^1 = 0.3 + 0.3(0.011)2.3 = 0.31$
$W_{33}^1 = 0.8 + 0.3(0.018)2.3 = 0.81$

b) using a [3 x 3 x 2] Neural Network



Assigning random values to the weights
$W_{11}^1 = 0.4,\ W_{21}^1 = 0.5,\ W_{31}^1 = 0.8$
$W_{12}^1 = 0.8,\ W_{22}^1 = 0.4,\ W_{32}^1 = 0.2$
$W_{13}^1 = 0.7,\ W_{23}^1 = 0.1,\ W_{33}^1 = 0.4$
$W_{11}^2 = 0.3,\ W_{21}^2 = 0.9,\ W_{31}^2 = 0.2$
$W_{12}^2 = 0.7,\ W_{22}^2 = 0.4,\ W_{32}^2 = 0.4$
Assuming threshold value t = 0, we have
$O_1^2 = 0.923 \quad O_2^2 = 0.812 \quad O_3^2 = 0.842$
$O_1^3 = 0.761 \quad O_2^3 = 0.787$

Determine errors:
$R^1 : E_1^3 = O_{1actual}^3 - O_1^3 = 1 - 0.761 = 0.239$
$R^1 : E_2^3 = O_{2actual}^3 - O_2^3 = 0 - 0.787 = -0.787$

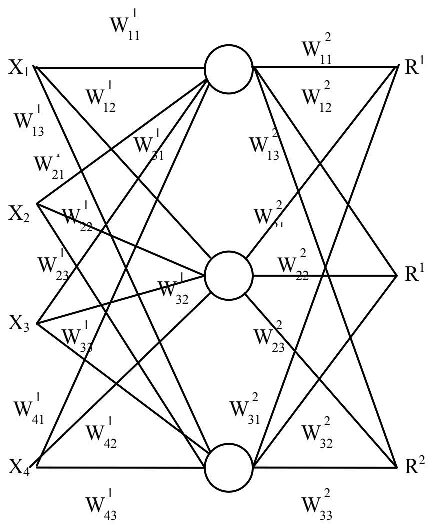
Assigning Errors to the 2nd layer
$E_1^2 = -0.034$, $E_2^2 = -0.015$, $E_3^2 = -0.036$

Updating the weights, assuming $\alpha = 0.3$
$W_{11}^2 = 0.3 + 0.3(0.239)0.923 = 0.366$
$W_{12}^2 = 0.7 + 0.3(-0.787)0.923 = 0.482$
$W_{21}^2 = 0.9 + 0.3(0.239)0.812 = 0.958$
$W_{22}^2 = 0.4 + 0.3(-0.787)0.812 = 0.208$
$W_{31}^2 = 0.2 + 0.3(0.239)0.842 = 0.260$
$W_{32}^2 = 0.4 + 0.3(0.788)0.842 = 0.201$
$W_{11}^1 = 0.4 + 0.3(-0.034)1 = 0.390$
$W_{12}^1 = 0.8 + 0.3(-0.015)1 = 0.796$
$W_{13}^1 = 0.7 + 0.3(-0.036)1 = 0.689$
$W_{21}^1 = 0.5 + 0.3(-0.034)0.5 = 0.495$
$W_{22}^1 = 0.4 + 0.3(-0.015)0.5 = 0.398$
$W_{23}^1 = 0.1 + 0.3(-0.036)0.5 = 0.095$
$W_{31}^1 = 0.8 + 0.3(-0.034)2.3 = 0.777$
$W_{32}^1 = 0.2 + 0.3(-0.015)2.3 = 0.190$
$W_{33}^1 = 0.4 + 0.3(-0.036)2.3 = 0.375$

Theoretically there should be no difference of results from (a) and (b). It may be noticed that when the output variables R^1 and R^2 have binary values (0, 1), then it is sufficient to have just one outlet. If an output value is 1 for the Region R^1 then it has a 0 membership in region R^2 and vice-versa.

6.10

X_1	X_2	X_3	X_4	R^1	R^2	R^3
10	0	-4	2	0	1	0



Assigning random values to the weights

$W^1_{11} = 0.1,\ W^1_{21} = 0.8,\ W^1_{31} = 0.5,\ W^1_{41} = 0.2$

$W^1_{12} = 0.4,\ W^1_{22} = 0.4,\ W^1_{32} = 0.9,\ W^1_{42} = 0.8$

$W^1_{13} = 0.9,\ W^1_{23} = 0.6,\ W^1_{33} = 0.1,\ W^1_{43} = 0.5$

$W^2_{11} = 0.5,\ W^2_{21} = 0.8,\ W^2_{31} = 0.5$

$W^2_{12} = 0.4,\ W^2_{22} = 0.1,\ W^2_{32} = 0.4$

$W^2_{13} = 0.1,\ W^2_{23} = 0.6,\ W^2_{33} = 0.8$

Assuming threshold value t=0, we have

$O^2_1 = 0.354 \quad O^2_2 = 0.881 \quad O^2_3 = 1.000$

$O^3_1 = 0.799 \quad O^3_2 = 0.652 \quad O^3_3 = 0.796$

Determining errors:

$R^1 : E^3_1 = -0.799$

$R^2 : E^3_2 = 1 - 0.652 = 0.348$

$R^1 : E^3_3 = -0.796$

Assigning errors to the 2nd layer

$$E^2_1 = 0.354(1 - 0.354)* \begin{bmatrix} (0.5 * -0.799) + (0.348 * 0.4) + \\ (-0.796 * 0.1) \end{bmatrix} = -0.078$$

$$E^2_2 = 0.881(1 - 0.881)* \begin{bmatrix} (0.8 * -0.799) + (0.348 * 0.1) + \\ (-0.796 * 0.6) \end{bmatrix} = -0.114$$

$$E^2_3 = 1(1-1)[\ldots] = 0$$

Updating the weights, assuming $\alpha = 0.3$

$W^2_{11} = 0.5 + 0.3(-0.799)0.354 = 0.415$

$W^2_{12} = 0.4 + 0.3(0.348)0.354 = 0.437$

$W^2_{13} = 0.1 + 0.3(-0.796)0.354 = 0.015$

$W^2_{21} = 0.8 + 0.3(-0.799)0.881 = 0.589$

$W^2_{22} = 0.1 + 0.3(0.348)0.881 = 0.192$

$W^2_{23} = 0.6 + 0.3(-0.796)0.881 = 0.390$

$W^2_{31} = 0.5 + 0.3(-0.799)1.0 = 0.260$

$W^2_{32} = 0.4 + 0.3(0.348)1.0 = 0.504$

$W^2_{33} = 0.8 + 0.3(-0.796)1.0 = 0.561$

$W^1_{11} = 0.1 + 0.3(-0.078)10 = -0.134$

$W^1_{12} = 0.4 + 0.3(-0.114)10 = 0.058$

$W^1_{13} = 0.9 + 0.3(0)10 = 0.9$

$W^1_{21} = 0.8 + 0.3(-0.078)0 = 0.8$

$W^1_{22} = 0.4 + 0.3(-0.114)0 = 0.4$

$W^1_{23} = 0.6 + 0.3(0)0 = 0.6$

$W^1_{31} = 0.5 + 0.3(-0.078)(-4) = 0.594$

$W^1_{32} = 0.9 + 0.3(-0.114)(-4) = 1.037$

$W^1_{33} = 0.1 + 0.3(0)(-4) = 0.1$

$W^1_{41} = 0.2 + 0.3(-0.078)(2) = 0.153$

$W^1_{42} = 0.8 + 0.3(-0.114)(2) = 0.732$

$W^1_{43} = 0.5 + 0.3(0)(2) = 0.5$

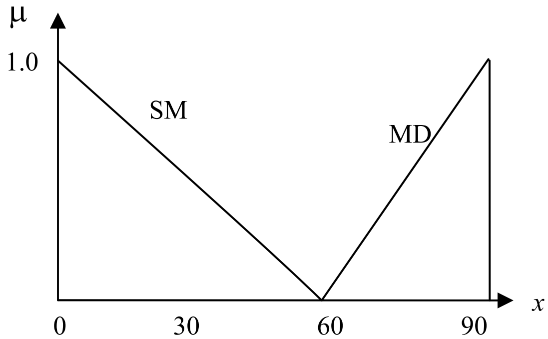
6.11 Use 6 bit strings *x*- C_{min} = 0, C_{max} = 90; *y*-C_{min} = 0, C_{max} =1.0

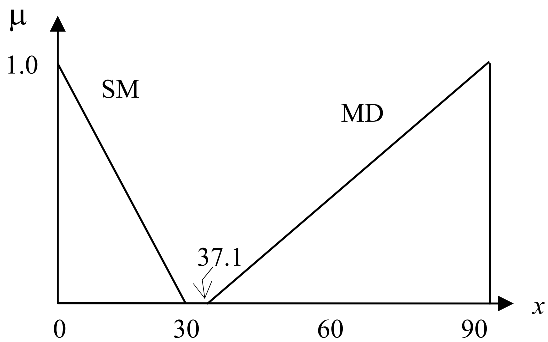
String

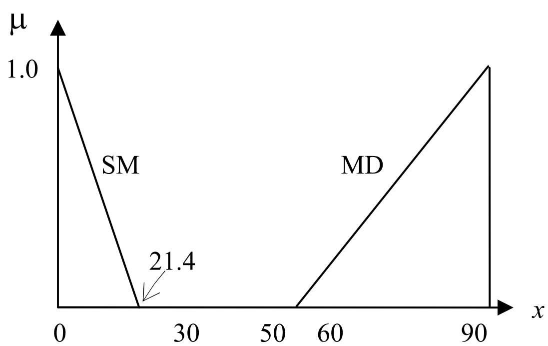
101010	010101	000101	100001
010101	100101	010110	001101
001111	011100	001001	010110
100110	011010	101111	101011

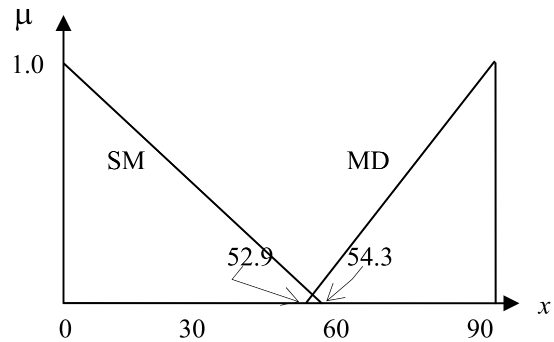
Base1	Base2	Base3	Base4
42	21	5	33
21	37	22	13
15	28	9	22
38	26	47	43

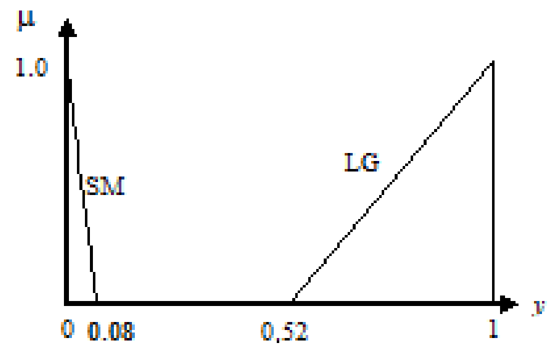
Base1	Base2	Base3	Base4
60	30	0.08	0.52
30	52.9	0.35	0.21
21.4	40	0.14	0.35
54.3	37.1	0.75	0.68

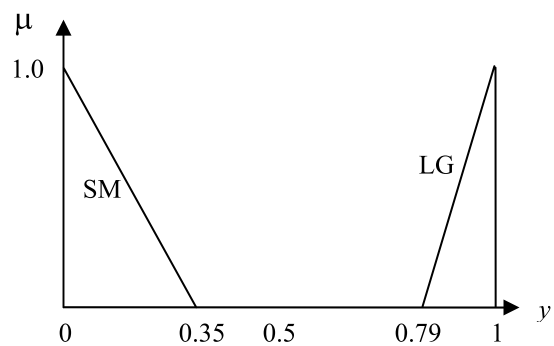


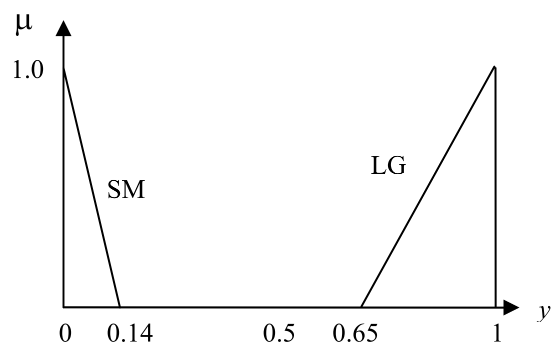


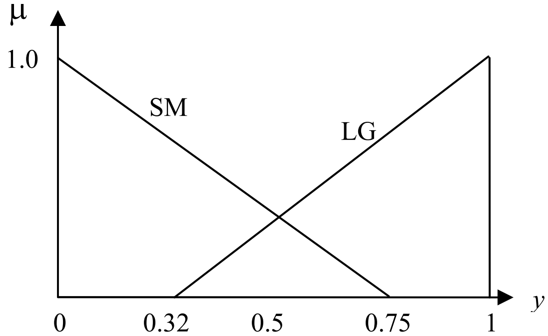












$y'(x=0)$	$y'(x=45)$	$y'(x=90)$
0	0.06	1
0	0.82	1
0	0	1
0	0.62	1

$1-\sum\left(y_i - y_i'\right)^2$	exp. count	actual count
0.578	0.76	0
0.988	1.29	2
0.496	0.65	0
0.99	1.3	2

sum = 3.052, average = 0.763, max=0.99, cut off at 0.8

stg. #1	Selected strings			
1	010101	100101	01|0110	001101
2	100110	011010	10|1111	101011
3	100110	011|010	101111	101011
4	010101	100|101	010110	001101

stg. #2	Selected strings			
1	010101	100101	011111	101011
2	100110	011010	100110	001101
3	100110	011101	010110	001101
4	010101	100010	101111	101011

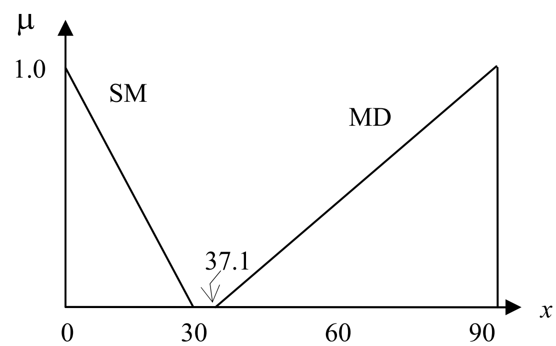
stg. #2	Base 1	Base 2	Base 3	Base 4
1	21	37	31	43
2	38	26	38	13
3	38	29	22	13
4	21	34	47	43

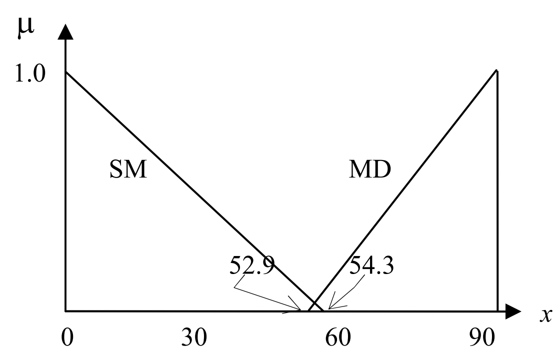
Base1	Base 2	Base 3	Base 4
30	52.9	0.49	0.68
54.3	37.1	0.60	0.21
54.3	41.4	0.35	0.21
30	48.6	0.75	0.68

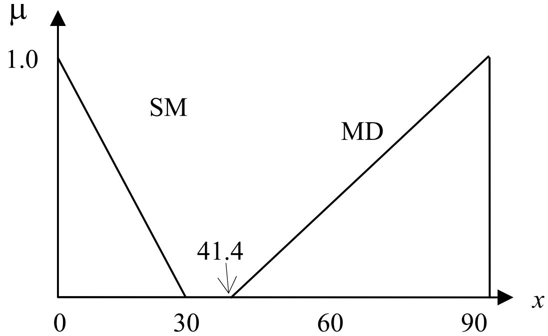
$y'(x=0)$	$y'(x=45)$	$y'(x=90)$
0	0.42	1
0	0.50	1
0	0.29	1
0	0.37	1

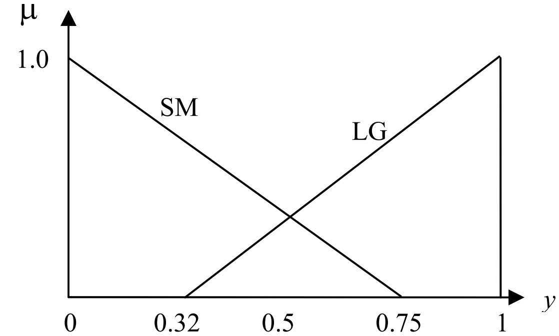
$1-\sum\left(y_i - y_i'\right)^2$	exp. count	actual count
0.916	1.023	1
0.956	1.068	2
0.824	0.92	0
0.884	0.988	1

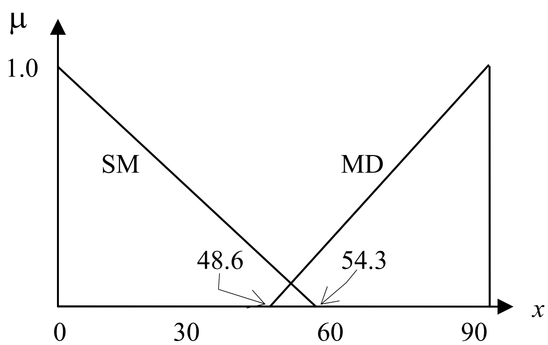
sum = 3.58, average = 0.895, max=0.956, cut off at 0.95

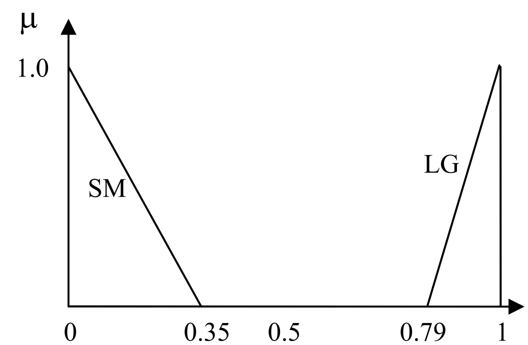


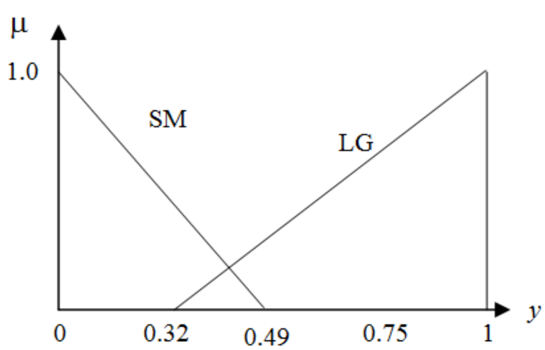


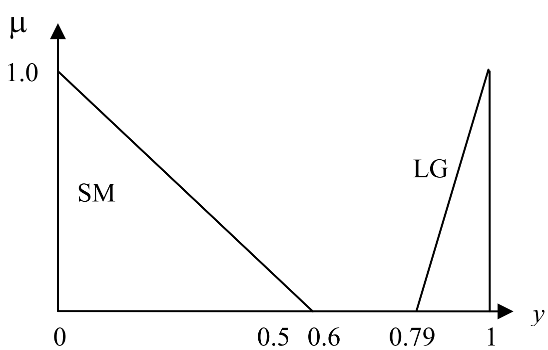












6.12 use 6 bit strings

for x, $C_{min} = 0$, $C_{max} = 100$

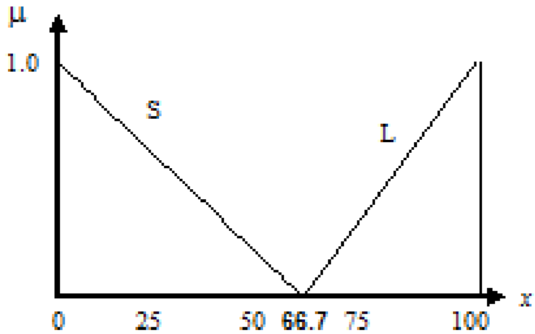
for y, $C_{min} = 0$; $C_{max} = 1.0$

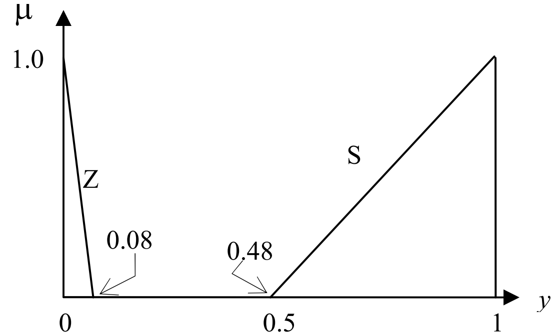
String

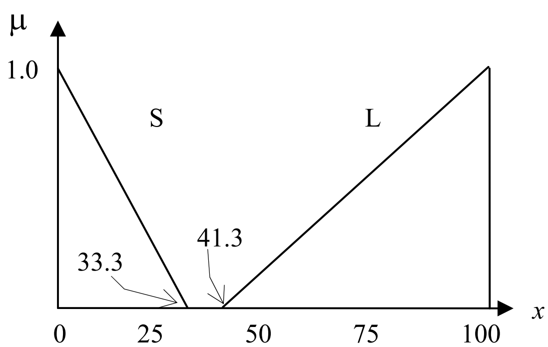
101010	010101	000101	100001
010101	100101	010110	001101
001111	011100	001001	010110
100110	011010	101111	101011

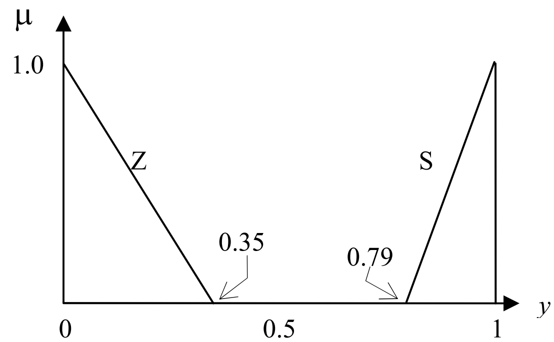
Base1	Base2	Base3	Base4
42	21	5	33
21	37	22	13
15	28	9	22
38	26	47	43

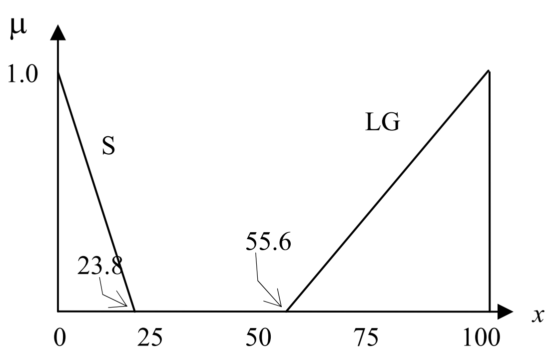
Base1	Base2	Base3	Base4
66.7	33.3	0.08	0.52
33.3	58.7	0.35	0.21
23.8	44.4	0.14	0.35
60.3	41.3	0.75	0.68

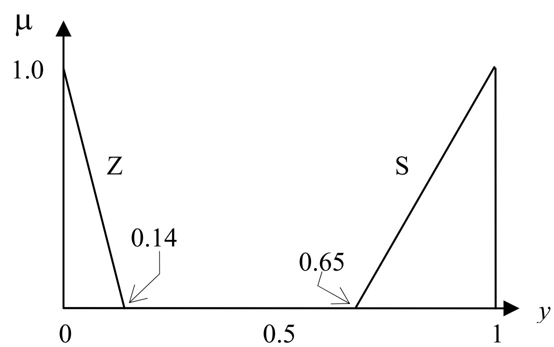


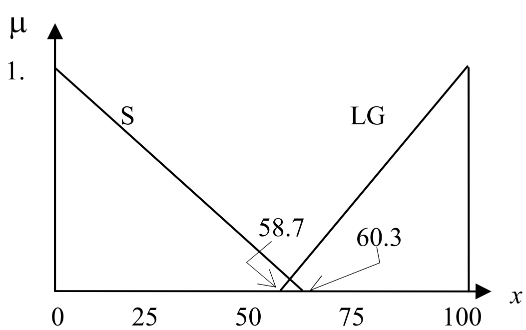


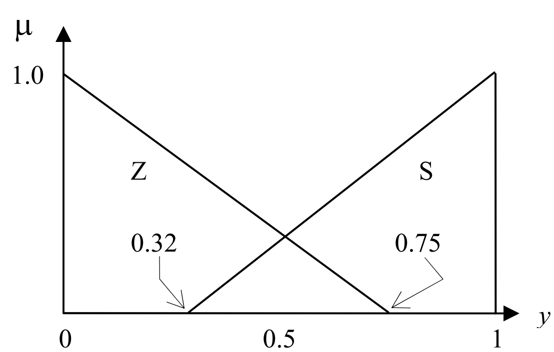












y' $(x=0)$	y' $(x=0.3)$	y' $(x=0.6)$	y' $(x=1.0)$	y' $(x=100)$
1	0.998	0.995	0.992	0
1	0.998	0.996	0.994	0
1	0.996	0.991	0.985	0
1	0.997	0.993		0

$1-\sum\left(y_i - y_i'\right)^2$	exp. count	actual count
0.3485	0.9886	0
0.3451	0.9790	0
0.3618	1.0264	2
0.3545	1.0057	2

sum = 1.4099, average = 0.3525, max=0.3618, cut off at 1.0

$1-\sum\left(y_i - y_i'\right)^2$	exp. count	actual count
0.388	1.069	2
0.344	0.948	0
0.368	1.014	1
0.352	0.970	1

sum = 1.452, average = 0.363, max=0.388, cut off at 0.95

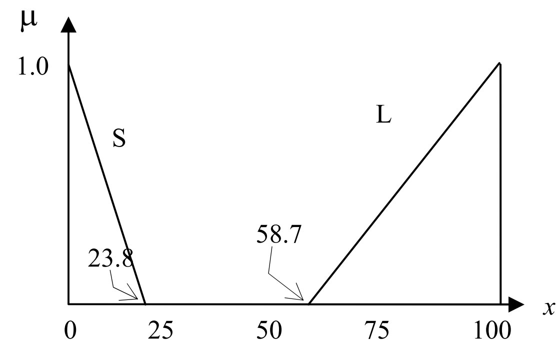
stg. #1	Selected strings			
1	001111	011100	001|001	010110
2	100110	011010	101|111	101011
3	001111	011100	001001	01|0110
4	100110	011010	101111	10|1011

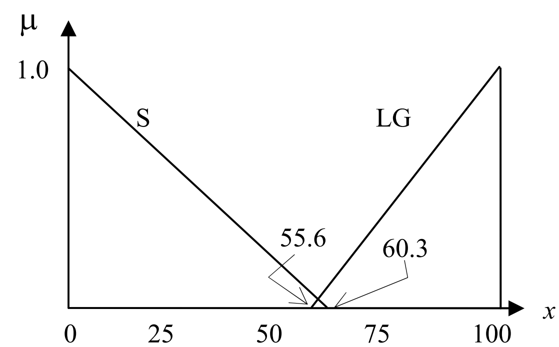
stg. #1	Selected strings			
1	001111	011010	101111	101011
2	100110	011100	001001	010110
3	001111	011100	001001	011011
4	100110	011010	101111	100110

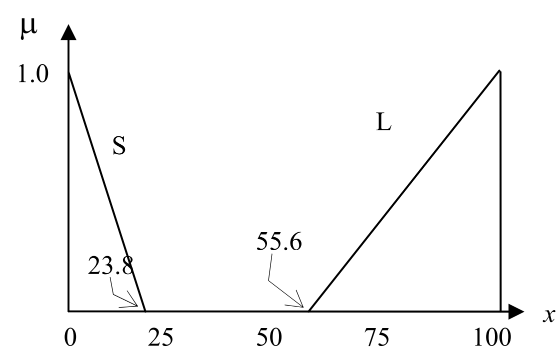
stg. #2	Base 1	Base 2	Base 3	Base 4
1	15	26	47	43
2	38	28	9	22
3	15	28	9	27
4	38	26	47	38

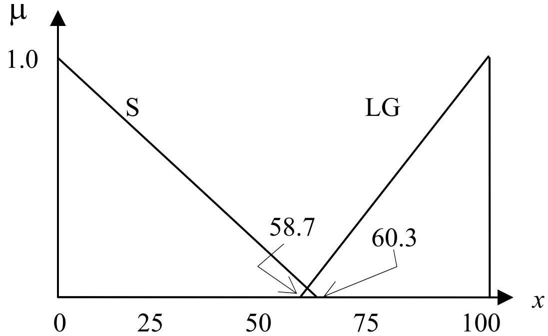
Base1	Base 2	Base 3	Base 4
23.8	41.3	0.75	0.68
60.3	44.4	0.14	0.35
23.8	44.4	0.14	0.43
60.3	41.3	0.75	0.60

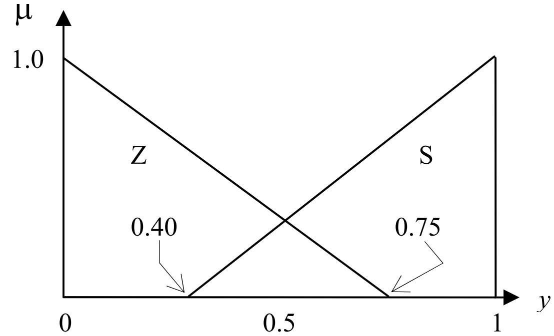
y' $(x=0)$	y' $(x=0.3)$	y' $(x=0.6)$	y' $(x=1.0)$	y' $(x=100)$
1	0.991	0.983	0.971	0
1	0.998	0.997	0.994	0
1	0.995	0.989	0.982	0
1	0.997	0.994	0.990	0

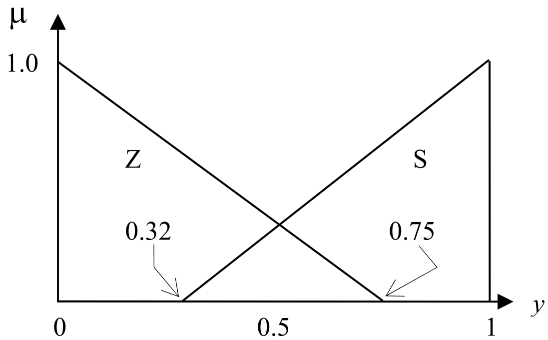


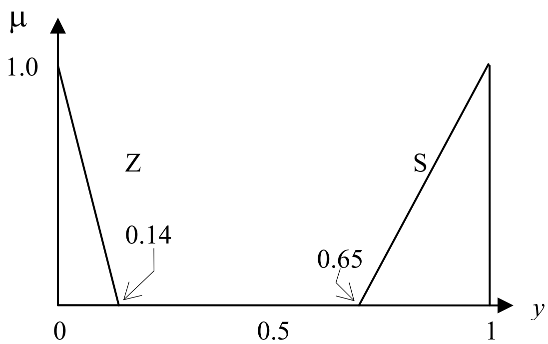


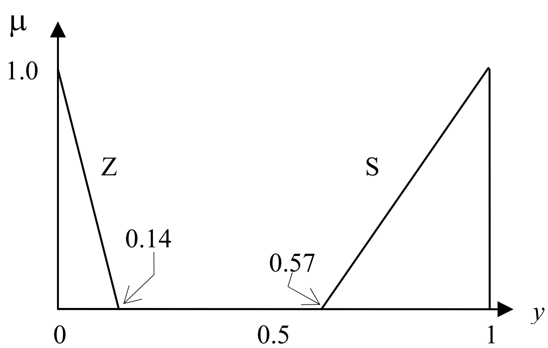












6.13 labeling the class as follows:

Economy -1, Midsize -2, Luxury -3

Similar to Equations 6.19 to 6.21 in the text we get equation

$S(x) = P(x)S_p(x) + q(x)S_q(x)$

where,

$$S_p(x) = -\left[\sum_{i=1}^{3} P_i(x) \ln P_i(x)\right]$$

and $S_q(x) = -\left[\sum_{i=1}^{3} q_i(x) \ln q_i(x)\right]$

Also, Equations 6.22 - 6.27 may be used as such. The calculations are shown in the following 3 tables. The 3 membership functions are shown after the tables.

$	5.5	5.8	7.5	7.9	8.2	8.5	9.2	10.4
Class	E	E	E	E	E	E	E	E

$	11.2	11.9	12.5	13.2	13.5	14.9	15.6	17.8
Class	E	M	M	M	E	M	M	M

$	18.2	19.5	20.5	22.0	23.5	24.0	25.0	26.0
Class	M	M	M	L	L	M	L	L

$	27.5	29.0	32.0	37.0	43.0	47.5
Class	L	L	L	L	L	L

x	13	14	15
p_1	$\frac{9+1}{11+1}=\frac{10}{12}$	$\frac{11}{14}$	$\frac{11}{15}$
p_2	$\frac{2+1}{11+1}=\frac{3}{12}$	$\frac{3+1}{13+1}=\frac{4}{14}$	$\frac{5}{15}$
p_3	$\frac{0+1}{11+1}=\frac{1}{12}$	$\frac{0+1}{13+1}=\frac{1}{14}$	$\frac{1}{15}$
q_1	$\frac{0+1}{19+1}=\frac{1}{20}$	$\frac{0+1}{17+1}=\frac{1}{18}$	$\frac{1}{17}$
q_2	$\frac{8+1}{19+1}=\frac{9}{20}$	$\frac{7+1}{17+1}=\frac{8}{18}$	$\frac{7}{17}$
q_3	$\frac{10+1}{19+1}=\frac{11}{20}$	$\frac{10+1}{17+1}=\frac{11}{18}$	$\frac{11}{17}$
$p(x)$	$\frac{11}{30}$	$\frac{13}{30}$	$\frac{14}{30}$
$q(x)$	$\frac{19}{30}$	$\frac{17}{30}$	$\frac{16}{30}$
$S_p(x)$	0.706	0.736	0.774
$S_q(x)$	0.838	0.822	0.814
S	0.790	**0.785**	0.795

$	5.5	5.8	7.5	7.9	8.2	8.5	9.2	10.4
Class	E	E	E	E	E	E	E	E

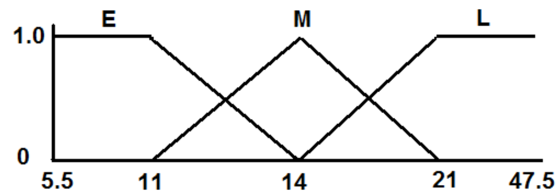
$	11.2	11.9	12.5	13.2	13.5
Class	E	M	M	M	E

x	10	11	12
p_1	1	1	$\frac{10}{11}$
p_2	$\frac{1}{8}$	[illegible]	$\frac{2}{11}$
q_1	$\frac{4}{7}$	$\frac{3}{6}$	$\frac{2}{4}$
q_2	$\frac{4}{7}$	$\frac{4}{6}$	$\frac{3}{4}$
$p(x)$	$\frac{7}{13}$	$\frac{8}{13}$	$\frac{10}{13}$
$q(x)$	$\frac{6}{13}$	$\frac{5}{13}$	$\frac{3}{13}$
$S_p(x)$	0.260	0.244	0.397
$S_q(x)$	0.640	0.617	0.562
S	0.435	0.**387**	0.435

$	14.9	15.6	17.8	18.2	19.5	20.5	22.0	23.5	24.0
Class	M	M	M	M	M	M	L	L	M

$	25.0	26.0	27.5	29.0	32.0	37.0	43.0	47.5
Class	L	L	L	L	L	L	L	L

x	20	21	23
p_1	1	1	$\frac{7}{8}$
p_2	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{2}{8}$
q_1	$\frac{3}{13}$	$\frac{2}{12}$	$\frac{2}{11}$
q_2	$\frac{11}{13}$	$\frac{11}{12}$	$\frac{10}{11}$
$p(x)$	$\frac{5}{17}$	$\frac{6}{17}$	$\frac{7}{17}$
$q(x)$	$\frac{12}{17}$	$\frac{11}{17}$	$\frac{10}{17}$
$S_p(x)$	0.299	0.278	0.463
$S_q(x)$	0.480	0.378	0.397
S	0.427	0.**342**	0.424



6.14

x	0.11	0.10	0.08	0.06	0.04	0.03	0.01	0.009	0.007	0.005	0.003	0
state	1	1	1	1	1	2	1	1	2	2	2	2

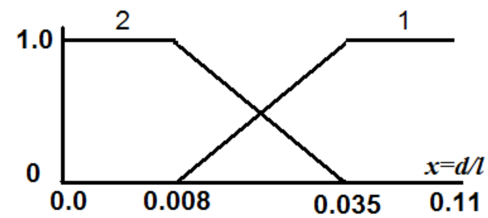
x	0.035	0.02	0.0095	0.008
p_1	$\frac{5+1}{5+1}=1$	$\frac{5+1}{6+1}=\frac{6}{7}$	$\frac{6+1}{7+1}=\frac{7}{8}$	$\frac{7+1}{8+1}=\frac{8}{9}$
p_2	$\frac{0+1}{5+1}=\frac{1}{6}$	$\frac{1+1}{6+1}=\frac{2}{7}$	$\frac{1+1}{7+1}=\frac{1}{4}$	$\frac{1+1}{8+1}=\frac{2}{9}$
q_1	$\frac{2+1}{7+1}=\frac{3}{8}$	$\frac{2+1}{6+1}=\frac{3}{7}$	$\frac{1+1}{5+1}=\frac{1}{3}$	$\frac{0+1}{4+1}=\frac{1}{5}$
q_2	$\frac{5+1}{7+1}=\frac{3}{4}$	$\frac{4+1}{6+1}=\frac{5}{7}$	$\frac{4+1}{5+1}=\frac{5}{6}$	$\frac{4+1}{4+1}=1$
p(x)	$\frac{5}{12}$	$\frac{6}{12}=\frac{1}{2}$	$\frac{7}{12}$	$\frac{8}{12}=\frac{2}{3}$
q(x)	$\frac{7}{12}$	$\frac{6}{12}=\frac{1}{2}$	$\frac{5}{12}$	$\frac{4}{12}=\frac{1}{3}$
$S_p(x)$	0.299	0.490	0.463	0.439
$S_q(x)$	0.584	0.603	0.518	0.322
S	0.465	0.547	0.486	**0.400**

x	0.11	0.10	0.08	0.06	0.04	0.03	0.01	0.009
state	1	1	1	1	1	2	1	1

x	0.007	0.005	0.003	0
state	2	2	2	2

x	0.035	0.02	0.0095	0.008
p_1	$\frac{5+1}{5+1}=1$	$\frac{5+1}{6+1}=\frac{6}{7}$	$\frac{6+1}{7+1}=\frac{7}{8}$	$\frac{7+1}{8+1}=\frac{8}{9}$
p_2	$\frac{0+1}{5+1}=\frac{1}{6}$	$\frac{1+1}{6+1}=\frac{2}{7}$	$\frac{1+1}{7+1}=\frac{1}{4}$	$\frac{1+1}{8+1}=\frac{2}{9}$
q_1	$\frac{2+1}{3+1}=\frac{3}{4}$	$\frac{2+1}{2+1}=1$	$\frac{1+1}{1+1}=1$	
q_2	$\frac{1+1}{3+1}=\frac{2}{4}$	$\frac{1}{2+1}=\frac{1}{3}$	$\frac{1}{1+1}=\frac{1}{2}$	
p(x)	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	1
q(x)	$\frac{3}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0
$S_p(x)$	0.299	0.490	0.463	0.439
$S_q(x)$	0.562	0.366	0.347	0
S	**0.398**	0.459	0.449	0.439

x	0.008
p_1	
p_2	
q_1	$\frac{0+1}{4+1}=\frac{1}{5}$
q_2	$\frac{4+1}{4+1}=1$
p(x)	0
q(x)	1
$S_p(x)$	0
$S_q(x)$	0.322
S	**0.322**



CHAPTER 7

Automated Methods for Fuzzy Systems

7.1 After two cycles,

$$\hat{\theta} = \{0.3646, 8.1779\},$$

$$Y = 1.4320, 4.0883, 6.4798$$

There is no change in the predicted outputs.

7.2 a) After two cycles,

$$\hat{\theta} = \{1.2976, 7.5519\},$$

x_1	x_2	y predicted	y actual
0	2	1.6319	1
2	4	3.8453	5
3	6	6.5228	6

b) After two cycles

$$\hat{\theta} = \{1.2701, 6.5711\},$$

x_1	x_2	y predicted	y actual
0	2	1.6472	1
2	4	3.8286	5
3	6	6.5243	6

7.3 To improve the results of the recursive least squares algorithm we try modifying the rule base parameters of of Table 7.4. This is done using the gradient method which should modify the rule base parameters. Using the gradient method software, the input parameters of Table 7.4 in text, the output found in Table 7.3, the training set and testing set of Table7.2, along with 20 cycles we attempt to improve the rule base. We set a value equal to 0.01, for which no input membership function spread can decrease below (call this value sigmabar) and designate the step size for our lambda all equal to 1. From this we obtain the following results.

B(*10^{-2})	c_1	c_2	σ_1	σ_2
2.9715	12600	9000	250	250
3.8824	12250	3750	166.7	250
3.1504	11500	6500	357.1	250
2.1933	11250	9000	119	250
1.6516	11000	11000	119	250
2.5575	11960	9000	138.1	250
4.2514	13000	3750	190.5	250
5.0554	13800	3750	380.9	250
3.5990	12950	6500	23.80	250
2.0233	12930	13000	9.524	250
2.3660	13000	12500	23.81	250
1.9561	12000	12000	19.00	250
2.4149	14490	14450	328.6	250
3.8163	13220	6500	104.8	250

Testing set			Predicted output
Major (x2 psi)	Minor (x1 psi)	δ_L (inch)	δ_L (inch)
12911.1	12927	2.0273E-02	2.02E-02
11092.4	10966.4	1.5974E-02	1.65E-02
14545.8	14487.1	2.4083E-02	2.41E-02
13012.1	12963.8	2.0216E-02	2.37E-02
12150	3750	3.8150E-02	3.88E-02
12904	3744.8	4.2895E-02	4.30E-02
14000	3770.4	5.0554E-02	5.06E-02
11406	6520.3	2.8397E-02	3.15E-02
12100	6535.5	3.2120E-02	3.15E-02
13109	6525.3	3.6977E-02	3.82E-02
11017.6	9000.7	2.1197E-02	2.19E-02
12105.9	8975.1	2.5744E-02	2.64E-02
12700	900	2.8504E-02	4.26E-02

The results indicate no change in the rule base input parameters and only a slight change in the output parameters.

For the test points mentioned in Table 7.2 GM shows more output error than RLS. In this exercise, increasing the number of cylces to 200 does not significantly change the parameters nor does increasing lambda.

7.4 After the second time steps, the input parameters for the rule base and the input and output tuples are as follows:

$B = \{0.9307, 5.0719\}$

$$\mu = \begin{bmatrix} 1.0000 & 0.0183 & 0.0000 \\ 0.0148 & 0.9999 & 0.0763 \end{bmatrix}$$

$\text{f} = \{0.9889, 4.9255, 4.9985\}$

$$C = \begin{bmatrix} -0.0107 & 2.0101 \\ 1.9893 & 4.0101 \end{bmatrix}$$

$$\sigma = \begin{bmatrix} 0.9786 & 0.9797 \\ 0.9786 & 0.9797 \end{bmatrix}$$

After the third time step, the input parameters for the rule base and the input and output tuples are as follows:

$B = \{0.9307, 6.0001\}$

$$\mu = \begin{bmatrix} 0.9999 & 0.0147 & 0.0000 \\ 0.0148 & 0.9999 & 0.0763 \end{bmatrix}$$

$\text{f} = \{0.9889, 4.9255, 4.9985\}$

$$C = \begin{bmatrix} -0.0110 & 2.0102 \\ 1.9889 & 4.0103 \end{bmatrix}$$

$$\sigma = \begin{bmatrix} 0.9776 & 0.9798 \\ 0.9769 & 0.9801 \end{bmatrix}$$

7.5 When using the clustering method to develop a fuzzy model for the input-output data presented in Table 7.2, we must speicify certain parameters. The most important of these parameters is the maximum distance between cluster centers. In this case we have two inputs thus we should specify two values for the maximum distance between cluster centers. We specify these values as 1200 for the first input and 4800 for the second input. Only one spread is designated and is equal to 250. We initiate the algorithm by designating the first input tuple as the initial cluster centers, v_{x1}=12250 and v_{x2}=3750. Also, A = 3.92176 which is the output for the first data tuple. Using all the training data set we obtain the following 15 rules base parameters.

v_1	v_2
12250	3750
12250	9000
11000	11000
12250	9000
13800	3750
12250	9000
12250	11110
12250	13000
12250	12500
12250	9000
12250	9000
12250	12000
14490	14450
12250	9000
14000	3750

Testing set			Predicted output
Major (x2 psi)	Minor (x1 psi)	δ_L (inch)	δ_L (inch)
12911.1	12927	2.0273E-02	2.13E-02
11092.4	10966.4	1.5974E-02	1.63E-02
14545.8	14487.1	2.4083E-02	2.41E-02
13012.1	12963.8	2.0216E-02	2.12E-02
12150	3750	3.8150E-02	4.24E-01
12904	3744.8	4.2895E-02	4.14E-01
14000	3770.4	5.0554E-02	4.99E-02
11406	6520.3	2.8397E-02	2.74E-02
12100	6535.5	3.2120E-02	2.67E-02
13109	6525.3	3.6977E-02	2.72E-02
11017.6	9000.7	2.1197E-02	2.60E-02
12105.9	8975.1	2.5744E-02	2.60E-02
12700	900	2.8504E-02	4.23E-01

7.6 By changing the test factors for x_1 and x_2 to 2.1 and 5, respectively, the number of rules produced in the rules base decreases to 9; although, the predictability decreases slightly for the testing set when this is done. If the initiating rule base parameter inputs are also changed to 12140 and 6510 and output to 3.22360, the number of rules produced by the algorithm is increased to 12. The predictability also decreases slightly from that of Example 7.1.

Changing the test factor affects the size of the spread in the membership function thus increasing or decreasing the test factor will change the predictability of the model. Whether it improves the predictability of the model depends on how much it is changed; however, in this case the change in the test factor did not improve the predictability of the model.

7.7 The resulting rule base parameters produced in exercise 7.4 is used as the rules base inputs in the batch least squares algorithm to improve the predictability of the model as follows:

$$\hat{\theta} = \{0.9256, 5.5364\}$$
$$f = \{0.9932, 5.4705, 5.5363\}$$

7.8 Number of developed rules is 3.

$$B = \{2,5,1\}$$
$$C = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 2 \end{bmatrix}$$
$$\sigma = \begin{bmatrix} 0.5 & 1.0 & 0.5 \\ 0.5 & 1.0 & 1.0 \end{bmatrix}$$
$$f = \{2.0540, 4.9281, 1.3034, 1.0600\}$$

7.9 Number of developed rules is 4.

$$B = \{1,3,6,7\}$$
$$C = \begin{bmatrix} 1 & 3 & 5 & 6 \\ 0 & 2 & 4 & 5 \end{bmatrix}$$
$$\sigma = \begin{bmatrix} 0.5 & 1.0 & 1.0 & 0.5 \\ 0.5 & 1.0 & 1.0 & 0.5 \end{bmatrix}$$
$$f = \{2.0540, 4.9281, 1.3034, 1.0600\}$$

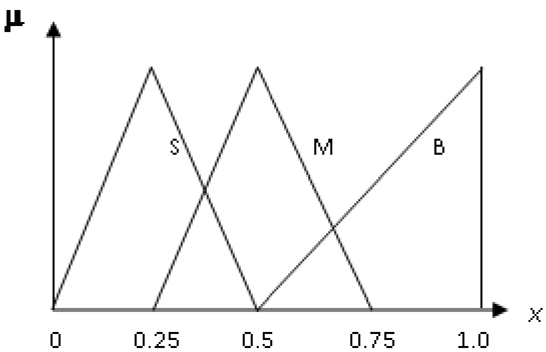
7.10 Number of developed rules is 4.

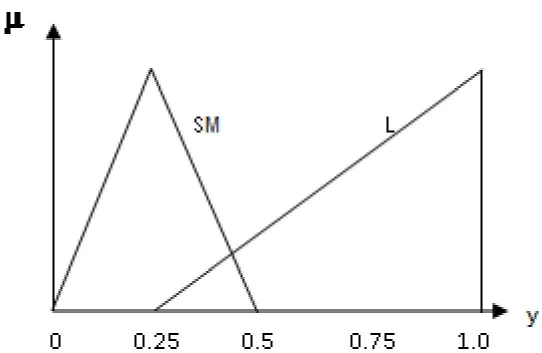
$$B = \{1,3,8,9\}$$
$$C = \begin{bmatrix} 0 & 2 & 6 & 7 \\ 1 & 5 & 10 & 11 \end{bmatrix}$$
$$\sigma = \begin{bmatrix} 0.5 & 1.0 & 2.0 & 0.5 \\ 0.5 & 2.0 & 2.5 & 0.5 \end{bmatrix}$$
$$f = \{1.0361, 3.0899, 8.0179, 8.5511\}$$

CHAPTER 8

Fuzzy Systems Simulation

8.1





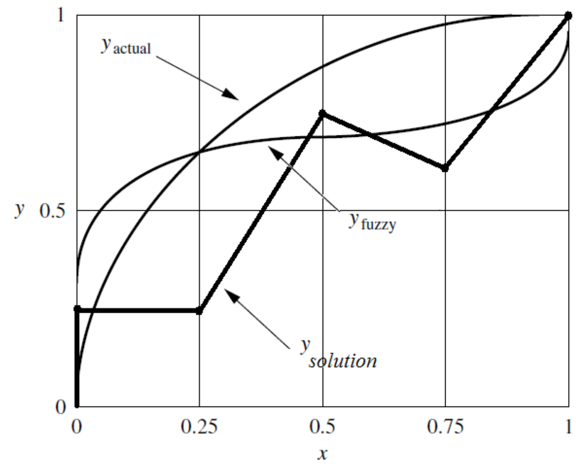
Formulating rules
1) IF x is S THEN y is SM
2) IF x is M THEN y is L
3) IF x is L THEN y is L

crisp inputs
a) $x = 0$, then rule 1 is fired, $y = 0.25$ (using center area method)
b) $x = 0.25$, then rule 1 is fired $y = 0.25$
c) $x = 0.5$, then rule 2 is fired, $y = 0.75$
d) $x = 0.75$, then rule 3 is fired,

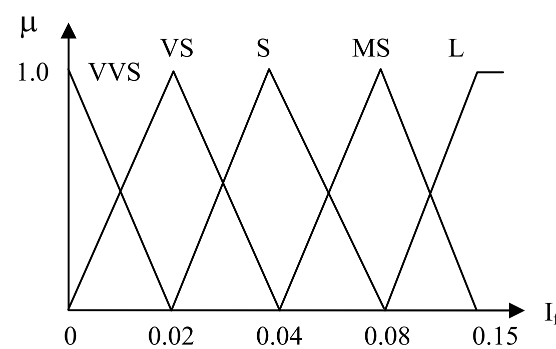
$$y = 0.5(1 + 0.25) = 0.625$$

e) x = 1.0, then rule 3 is fired and y = 1

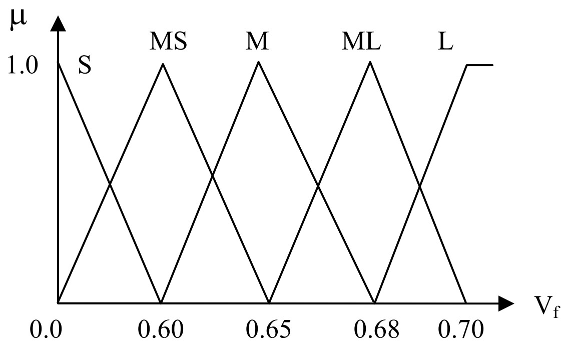
The solution can be improved by using a greater number of values and fuzzy classes.



8.2 a) Partitioning of the input space (I_f) and the output space (V_f).
A good partitioning scheme would have a large number of partition for $I_{f(a)}$ = 0.0-0.15, and a single partition covering $I_{f(a)} \ge 0.15$, similarly for V_f we require a large number of partition for V_f = 0.6-0.68 and a single partition can cover the region $V_f \le 0.6$. A simple division would be as shown below:



8.2 Cont.



b) Some proposed rules:

IF I_f is VVS THEN V_f is S
IF I_f is VS THEN V_f is MS
IF I_f is S THEN V_f is M
IF I_f is MS THEN V_f is ML
IF I_f is L THEN V_f is L

8.3 a) i) IF x_1 THEN G_1: $R_1 = x_1 \circ G_1$

$$R_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.5 \\ 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 = \left\{ \begin{matrix} 1 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0.25 & 0 & 0 & 0 \\ 1 & 0.5 & 0 & 0 & 0 \end{matrix} \right\}$$

ii)IF x_2 THEN G_2: $R_2 = x_2 \circ G_2$

$$R_2 = \left\{ \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 1 \\ 0 & 0 & 0 & 0.5 & 1 \\ 0 & 0 & 0 & 0.25 & 0.5 \\ 0 & 0 & 0 & 0.5 & 1 \\ 0 & 0 & 0 & 0.25 & 0.5 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \right\}$$

iii) IF x_3 THEN G_2: $R_3 = x_3 \circ G_3$

$$R_3 = \left\{ \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0.25 & 0.5 \\ 0 & 0.5 & 1 & 0.5 & 1 \\ 0 & 0.25 & 0.5 & 0.25 & 0.5 \\ 0 & 0.5 & 1 & 0.5 & 1 \\ 0 & 0.25 & 0.5 & 0.25 & 0.5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \right\}$$

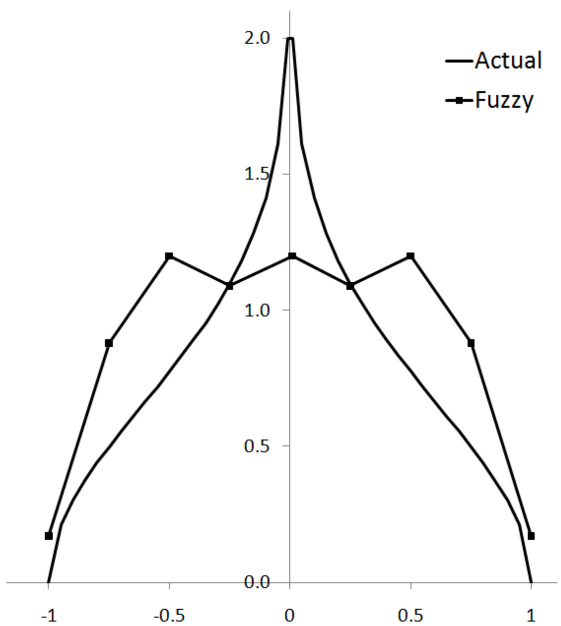
b) R = max (R1, R2, R3)

$$R = \left\{ \begin{matrix} 1.0 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0.25 & 0.5 & 0.25 & 0.5 \\ 0 & 0.5 & 1.0 & 0.5 & 1.0 \\ 0 & 0.25 & 0.5 & 0.25 & 0.5 \\ 0 & 0.5 & 1.0 & 0.5 & 1.0 \\ 0 & 0.25 & 0.5 & 0.25 & 0.5 \\ 0 & 0 & 0 & 0.5 & 1 \\ 0.5 & 0.25 & 0 & 0.25 & 0.5 \\ 1.0 & 0.5 & 0 & 0 & 0 \end{matrix} \right\}$$

8.3 Cont.

c)

x	G_{fuzzy}	G_{actual}
-1.00	0.17	0
-0.75	0.88	0.500
-0.5	1.20	0.776
-0.25	1.09	1.097
0.01	1.2	2
0.25	1.09	1.097
0.5	1.20	0.776
0.75	0.88	0.500
1.00	0.17	0



The above results will correlate better with the actual results as the number of rules is increased. The increase in the number of values will create a better relation matrix.

8.4 We will conduct simulation for

i) $\theta = \pm 180^{\circ}$, rule 3 is fired thus EP = -1.

ii) $\theta = \pm 90^{\circ}$, rule 2 is fired thus EP = 0.

iii) $\theta = \pm 0^{\circ}$, rule 1 is fired thus EP = 1.

iv) $\theta = \pm 45^{\circ}$, rule 1 and 2 is fired. For rule 1 EP =

$$\frac{\frac{2}{3}\times 0.25\times 0.25\times 0.5\times 0.5+\frac{1.25}{2}\times 0.75\times 0.5}{0.25\times 0.5\times 0.5+0.75\times 0.5} = 0.56$$

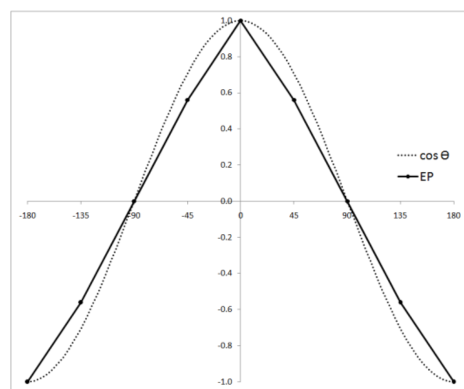
and for rule 2 EP = 0

max(|0|,|0.56|)=|0.56| thus EP=0.56

v) $\theta = \pm 135^{\circ}$, rule 2 and 3 is fired. For rule 2 EP = 0.5,

and for rule 3 similarly EP = -0.56

max(|0|,|-0.56|)=|-0.56| thus EP= -0.56



8.5

$$Z = \frac{0}{-5}+\frac{0.2}{-4}+\frac{0.6}{-2}+\frac{1}{0}+\frac{0.6}{2}+\frac{0.2}{4}+\frac{0}{5}$$

$NS =$

$$\frac{0}{-10}+\frac{0.4}{-8}+\frac{0.8}{-6}+\frac{1}{-5}+\frac{0.8}{-4}+\frac{0.4}{-2}+\frac{0}{0}$$

$$PS = \frac{0}{0}+\frac{0.4}{2}+\frac{0.8}{4}+\frac{1}{5}+\frac{0.8}{6}+\frac{0.4}{8}+\frac{0}{10}$$

$$NB = \frac{1}{-10}+\frac{0.6}{-8}+\frac{0.2}{-6}+\frac{0}{-5}$$

$$PB = \frac{0}{5}+\frac{0.2}{6}+\frac{0.6}{8}+\frac{1}{10}$$

$$N = \frac{1}{0}+\frac{0.6}{4}+\frac{0.2}{16}+\frac{0}{25}$$

$$S = \frac{0}{0}+\frac{0.4}{4}+\frac{0.8}{16}+\frac{1}{25}+\frac{0.8}{36}+\frac{0.4}{64}+\frac{0}{100}$$

$$\mathrm{V} = \frac{0}{25} + \frac{0.2}{36} + \frac{0.6}{64} + \frac{1}{100}$$

$$\underset{\sim}{\mathrm{R}}_1 = \begin{array}{c|ccccccc} & -5 & -4 & -2 & 0 & 2 & 4 & 5 \\ \hline 0 & 0 & 0.2 & 0.6 & 1 & 0.6 & 0.2 & 0 \\ 4 & 0 & 0.2 & 0.6 & 0.6 & 0.6 & 0.2 & 0 \\ 16 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 \\ 25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\underset{\sim}{\mathrm{R}}_2 = \begin{array}{c|ccccccccccccc} & -10 & -8 & -6 & -5 & -4 & -2 & 0 & 2 & 4 & 5 & 6 & 8 & 10 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0 \\ 16 & 0 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0 \\ 25 & 0 & 0.4 & 0.8 & 1 & 0.8 & 0.4 & 0 & 0.4 & 0.8 & 1 & 0.8 & 0.4 & 0 \\ 36 & 0 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0 \\ 64 & 0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0 \\ 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\underset{\sim}{\mathrm{R}}_3 = \begin{array}{c|cccccccc} & -10 & -8 & -6 & -5 & 5 & 6 & 8 & 10 \\ \hline 25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 36 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0.2 & 0.2 & 0.2 \\ 64 & 0.6 & 0.6 & 0.2 & 0 & 0 & 0.2 & 0.6 & 0.6 \\ 100 & 1 & 0.6 & 0.2 & 0 & 0 & 0.2 & 0.6 & 1 \end{array}$$

$$\underset{\sim 1}{R} \cup \underset{\sim 2}{R} \cup \underset{\sim 3}{R} = \begin{array}{c|ccccccccccccc} & -10 & -8 & -6 & -5 & -4 & -2 & 0 & 2 & 4 & 5 & 6 & 8 & 10 \\ \hline 0 & 0 & 0 & 0 & 0 & 0.2 & 0.6 & 0 & 0.6 & 0.2 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.6 & 1 & 0.6 & 0.4 & 0.4 & 0.4 & 0.4 & 0 \\ 16 & 0 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0.6 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0 \\ 25 & 0 & 0.4 & 0.8 & 1 & 0.8 & 0.4 & 0.25 & 0.4 & 0.8 & 1 & 0.8 & 0.4 & 0 \\ 36 & 0.2 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0.2 \\ 64 & 0.6 & 0.6 & 0.4 & 0.4 & 0.4 & 0.4 & 0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.6 & 0.6 \\ 100 & 1 & 0.6 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.6 & 1 \end{array}$$

Simulation

x = -10

$$y = \frac{0.2}{36} + \frac{0.6}{64} + \frac{1}{100} \Rightarrow y = 100 \text{ max}$$

memb.

x = -8

$$y = \frac{0}{0} + \frac{0.4}{4} + \frac{0.4}{16} + \frac{0.4}{25} + \frac{0.4}{36} + \frac{0.6}{64} + \frac{0.6}{100}$$

$$y = \frac{64+100}{2} = 82$$

x = -6

$$y = \frac{0}{0} + \frac{0.4}{4} + \frac{0.8}{16} + \frac{0.8}{25} + \frac{0.8}{36} + \frac{0.4}{64} + \frac{0.2}{100}$$

$$y = \frac{1}{3}(16+25+36) \approx 26$$

x = -5

$$y = \frac{0}{0} + \frac{0.4}{4} + \frac{0.8}{16} + \frac{1}{25} + \frac{0.8}{36} + \frac{0.4}{64} + \frac{0.2}{100}$$

$$y = \frac{16+32}{2} = 26$$

x = -4

$$y = \frac{0.2}{0} + \frac{0.4}{4} + \frac{0.8}{16} + \frac{0.8}{25} + \frac{0.8}{36} + \frac{0.4}{64} + \frac{0}{100}$$

$$y = \frac{1}{3}(16+25+36) \approx 26$$

x = -2

$$y = \frac{0.6}{0} + \frac{0.6}{4} + \frac{0.4}{16} + \frac{0.4}{25} + \frac{0.4}{36} + \frac{0.4}{64} + \frac{0}{100}$$

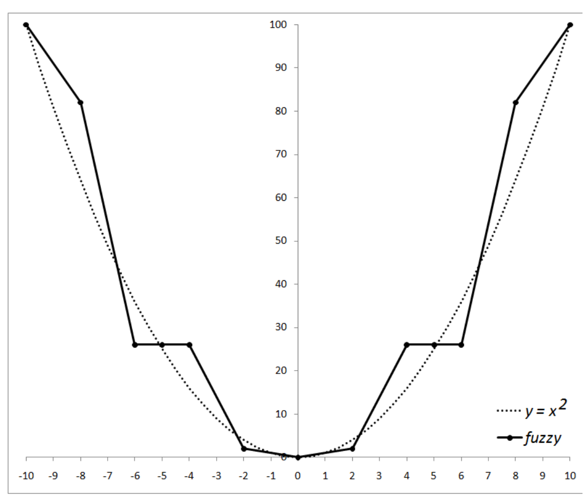
$$y = \frac{0+4}{2} = 2$$

x = 0

$$y = \frac{1}{0} + \frac{0.6}{4} + \frac{0.2}{16} + \frac{0}{25} + \frac{0}{36} + \frac{0}{64} + \frac{0}{100}$$

$$y = 0$$

y	100	82	26	26	26	2	0	2	26	26	26	82	100
x	-10	-8	-6	-5	-4	-2	0	2	4	5	6	8	10



8.6

In this problem we extend the MFs for HSAC and HSC in a linear fashion until they reach the zero-membership line. In addition the curved portions of the MFs are approximated as lines, so that the calculation of the centroid is easier. Also, in order to apply the centroid method, we will integrate approximately the area under the curves up to the required level of the membership functions in figure P8.6c.

I = -8 leads to NS = 0.6 thus firing rule 3. One can achieve N = -2000 applying centroid method for the area under HSAC membership function curve up to μ = 0.6.

I = -8 also leads to NB = 0.4 thus firing rule 5. One can achieve N = -700 applying centroid method for the area under MSAC membership function curve up to μ = 0.4.

So, for I = -8 the greater absolute value is N = -2000.

I = -2 leads to Z = 0.6 thus firing rule 1. One can achieve N = 2000 applying centroid method for the area under HSC membership function curve up to μ = 0.6.

I = -2 also leads to NS = 0.4 thus firing rule 3. One can achieve N = 700 applying centroid method for the area under MSAC membership function curve up to μ = 0.4.

So, for I = -2 the greater absolute value is N = 2000.

I = 3 leads to Z = 0.4 thus firing rule 1. One can achieve N = 2050 applying centroid method for the area under HSC membership function curve up to μ = 0.4.

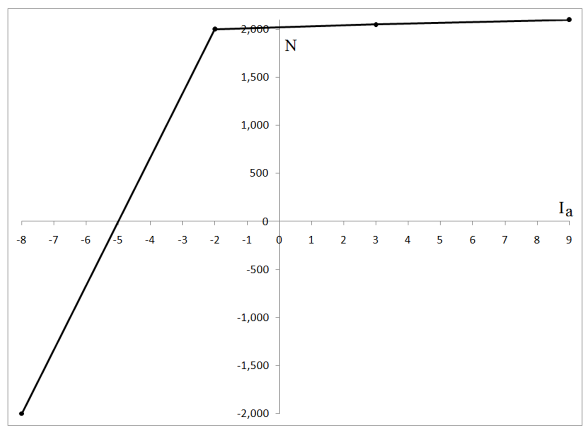
I = 3 also leads to PS = 0.6 thus firing rule 2. One can achieve N = 2000 applying centroid method for the area under HSC membership function curve up to μ = 0.6.

So, for I = 3 the greater absolute value is N = 2050.

I = 9 leads to PS = 0.2 thus firing rule 2. One can achieve N = 2100 applying centroid method for the area under HSC membership function curve up to μ = 0.2.

I = 9 also leads to PB = 0.8 thus firing rule 4. One can achieve N = 600 applying centroid method for the area under MSC membership function curve up to μ = 0.8.

So, for I = 9 the greater absolute value is N = 2100.



8.7 This problem is posed inappropriately. We need to change the rules as follows:

Rule 1: If x is Z, then y is S
Rule 2: If x is NS, then y is NM
Rule 3: If x is NB, then y is NL
Rule 4: If x is PS, then y is PM
Rule 5: If x is PB, then y is PL

We will use max-min composition method. As the function is symmetric, we need to analyze only one half of it.

a) $Z = \{\frac{0}{-0.5} + \frac{0.2}{-0.4} + \frac{0.6}{-0.2} + \frac{1}{0}\}$

$S = \{\frac{0}{-0.5} + \frac{0.2}{-0.4} + \frac{0.6}{-0.2} + \frac{1}{0}\}$

$$
R_1 = \begin{array}{c|cccc} & x=-0.5 & -0.4 & -0.2 & 0 \\ \hline y=-0.5 & 0 & 0 & 0 & 0 \\ -0.4 & 0 & 0.2 & 0.2 & 0.2 \\ -0.2 & 0 & 0.2 & 0.6 & 0.6 \\ 0 & 0 & 0.2 & 0.6 & 0 \end{array}
$$

$NS = \{\frac{0}{-1} + \frac{0.4}{-0.8} + \frac{0.8}{-0.6} + \frac{1}{-0.5} + \frac{0.8}{-0.4} + \frac{0.4}{-0.2} + \frac{0}{0}\}$

$NM = \{\frac{0}{-0.81} + \frac{0.6}{-0.7} + \frac{1}{-0.5} + \frac{0.8}{-0.4} + \frac{0.4}{-0.2} + \frac{0}{0}\}$

$$
R_2 = \begin{array}{c|ccccccc} & x=-1 & -0.8 & -0.6 & -0.5 & -0.4 & -0.2 & -0 \\ \hline y=-0.81 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.7 & 0 & 0.4 & 0.6 & 0.6 & 0.6 & 0.4 & 0 \\ -0.5 & 0 & 0.4 & 0.8 & 1 & 0.8 & 0.4 & 0 \\ -0.4 & 0 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0 \\ -0.2 & 0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}
$$

$NB = \{\frac{1}{-1} + \frac{0.6}{-0.8} + \frac{0.2}{-0.6} + \frac{0}{-0.5}\}$

$NL = \{\frac{1}{-0.81} + \frac{0.4}{-0.7} + \frac{0}{-0.5}\}$

$$
R_3 = \begin{array}{c|cccc} & x=-1 & -0.8 & -0.6 & 0.5 \\ \hline y=-0.81 & 1 & 0.6 & 0.2 & 0 \\ -0.4 & 0.4 & 0.4 & 0.2 & 0 \\ -0.2 & 0 & 0 & 0 & 0 \end{array}
$$

$R = R_1 \cup R_2 \cup R_3 =$

$$
\begin{array}{c|ccccccc} & x=-1 & -0.8 & -0.6 & -0.5 & -0.4 & -0.2 & -0 \\ \hline y=-0.81 & 1 & 0.6 & 0.2 & 0 & 0 & 0 & 0 \\ -0.7 & 0.4 & 0.4 & 0.6 & 0.6 & 0.6 & 0.4 & 0 \\ -0.5 & 0 & 0.4 & 0.8 & 1 & 0.8 & 0.4 & 0 \\ -0.4 & 0 & 0.4 & 0.8 & 0.8 & 0.8 & 0.4 & 0.2 \\ -0.2 & 0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.6 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0.6 & 1 \end{array}
$$

b)

$x = -1 \Rightarrow \underset{\sim}{y} = \frac{1}{-0.81} + \frac{0.4}{-0.7} \Rightarrow y = -0.81$

$x = -0.8 \Rightarrow$

$\underset{\sim}{y} = \frac{0.6}{-0.81} + \frac{0.4}{-0.7} + \frac{0.4}{-0.5} + \frac{0.4}{-0.4} + \frac{0.4}{-0.2}$

$\Rightarrow y = -0.81$

$x = -0.6 \Rightarrow \underset{\sim}{y} = \frac{0.2}{-0.81} + \frac{0.6}{-0.7} + \frac{0.8}{-0.5} + \frac{0.8}{-0.4} + \frac{0.4}{-0.2}$

$\Rightarrow y = \frac{(-0.5)+(-0.4)}{2} = -0.45$

$x = -0.5 \Rightarrow$

$\underset{\sim}{y} = \frac{0.6}{-0.7} + \frac{1}{-0.5} + \frac{0.8}{-0.4} + \frac{0.4}{-0.2} + \frac{0.2}{0}$

$\Rightarrow y = -0.5$

$x = -0.4 \Rightarrow \underset{\sim}{y} = \frac{0.6}{-0.7} + \frac{0.8}{-0.5} + \frac{0.8}{-0.4} + \frac{0.4}{-0.2} + \frac{0.2}{0}$

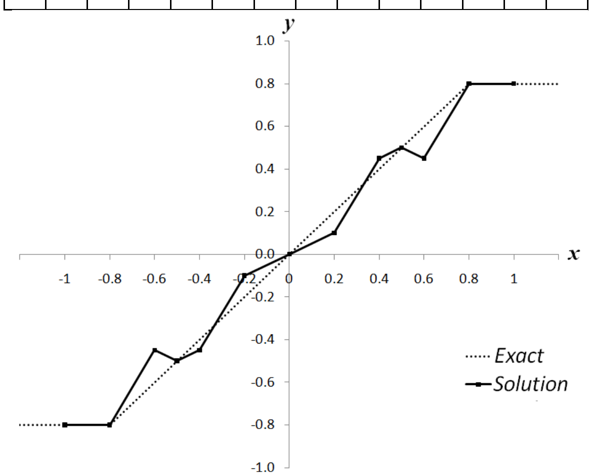
$\Rightarrow y = \frac{(-0.5)+(-0.4)}{2} = -0.45$

$x = -0.2 \Rightarrow \underset{\sim}{y} = \frac{0.4}{-0.7} + \frac{0.4}{-0.5} + \frac{0.4}{-0.4} + \frac{0.6}{-0.2} + \frac{0.6}{0}$

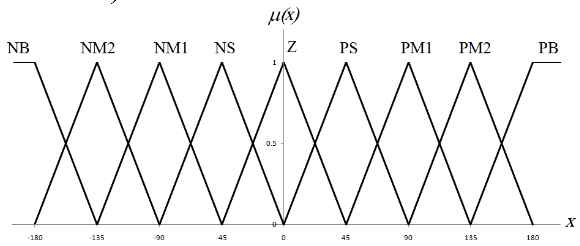
$\Rightarrow y = \frac{0+(-0.2)}{2} = -0.1$

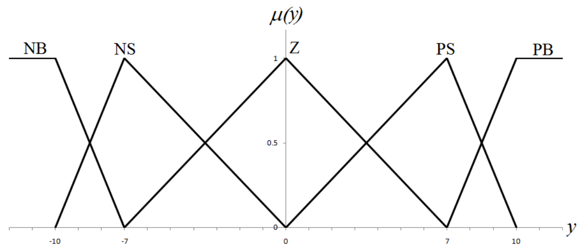
$x = 0 \Rightarrow \underset{\sim}{y} = \frac{0.2}{-0.4} + \frac{0.6}{-0.2} + \frac{1}{0} \Rightarrow y = 0$

x	-1	-0.8	-0.6	-0.5	-0.4	-0.2	0	0.2	0.4	0.5	0.6	0.8	1
y	-0.81	-0.81	-0.45	-0.5	-0.45	-0.1	0	0.1	0.45	0.5	0.45	0.81	0.81



8.8 a)





Rules:

1. If x is Z or PB then y is Z
2. If x is PS or PM2 then y is PM
3. If x is PM1 then y is PB
4. If x is Z or NB then y is Z
5. If x is NS or NM2 then y is NM
6. If x is NM1 then y is NB

$x = \{-180, -157.5, -112.5, -62.5, -22.5, 0, 22.5, 67.5, 112.5, 157.5, 180\}$

x = -180 fires rule 4 from wich y = 0

x = -157.5 fires rules 4 & 5
μ(x)=0.5 thus μ(y)=0.5
from R4: y=0
or from R5:

$$y = -\frac{{}^{2}/_{3}*3.5*3.5*0.5*0.5+6*0.5*5+9*1.5*0.5*0.5}{3.5*0.5*0.5+5*0.5+1.5*0.5*0.5} = -5.4$$

Therefore y = -5.4

x = -112.5 fires rules 5 & 6
μ(x)=0.5 thus μ(y)=0.5
from R5: y = -5.4
or from R6:

$$y = -10 + \frac{0.75*1.5^2*0.5+2*1.5^2*0.5*0.5}{1.5^2*0.5+1.5^2*0.5*0.5} = -8.8$$

Therefore y = -8.8

x = -67.5 fires rules 5 & 6 from which
y = -5.4 or y = -8.8 respectively.
Therefore y = -8.8

x = -22.5 fires rules 1 & 4 & 5 from which
y = 0 or y= 0 or y = -5.4 respectively.
Therefore y = -5.4

x = 0 fires rule1 from which y = 0

x = 22.5 fires rules 1 & 2 & 4 from which
y = 0 or y = 5.4 or y= 0 respectively.
Therefore y = 5.4

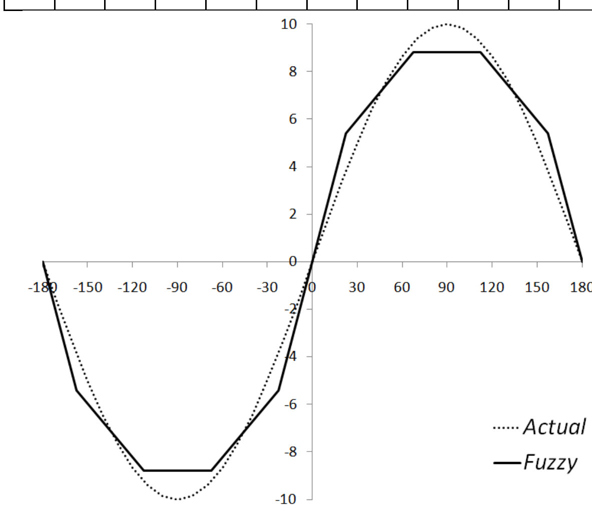
x = 67.5 fires rules 2 & 3 from which
y = 5.4 or y = 8.8 respectively.
Therefore y = 8.8

x = 112.5 fires rules 2 & 3 from which
y = 5.4 or y = 8.8 respectively.
Therefore y = 8.8

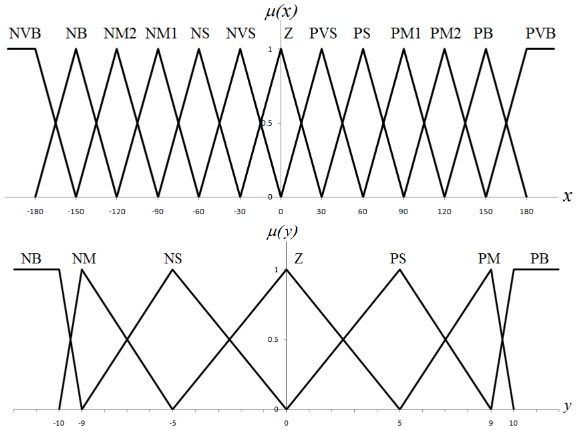
x = 157.5 fires rules 1 & 2 from which
y = 0 or y = -5.4 respectively.
Therefore y = 5.4

x = -180 fires rule 4 from which y = 0

x	y
-180	0
-157.5	-5.4
-112.5	-8.8
-67.5	-8.8
-22.5	-5.4
0	0
22.5	5.4
67.5	8.8
112.5	8.8
157.5	5.4
180	0



b)



Rules:

1. If x is Z or PVB then y is Z
2. If x is PVS or PB then y is PS
3. If x is PS or PM2 then y is PM
4. If x is PM1 NB then y is PB
5. If x is Z or NVB then y is Z
6. If x is NVS or NB then y is NS
7. If x is NS or NM2 then y is NM
8. If x is NM1 then y is NB

$$x = \{-180, -165, -135, -105, -75, -45, -15, 0, 15, 45, 75, 105, 135, 165, 180\}$$

x = -180 then y is Z thus y = 0

x = -165 then y is NS or Z thus

$$y = -\frac{2/3*2.5*0.5*0.5+4.75*4.5*0.5+8.33*2*0.5*0.5}{2.5*0.5*0.5+4.5*0.5+2*0.5*0.5} = -4.5$$

x = -135 then y is NS or NM thus

$$y = -5 - \frac{1.33*2*0.5*0.5+3.25*2.5*0.5+4.83*0.5*0.5*0.5}{2*0.5*0.5+2.5*0.5+0.5*0.5*0.5} = -7.8$$

x = -105 then y is NM or NB thus

$$y = -9 - \frac{0.33*0.5*0.5*0.5+0.75*0.5*0.5}{0.5*0.5*0.5+0.5*0.5} = -9.6$$

x = -75 then y is NM or NB thus y = -9.6

x = -45 then y is NS or NM thus y = -7.8

x = -15 then y is NS or Z thus y = -4.5

x = 0 then y is Z thus y = 0

x = 15 then y is Z or PS thus y = 4.5

x = 45 then y is PS or PM thus y = 7.8

x = 75 then y is PM or PB thus y = 9.6

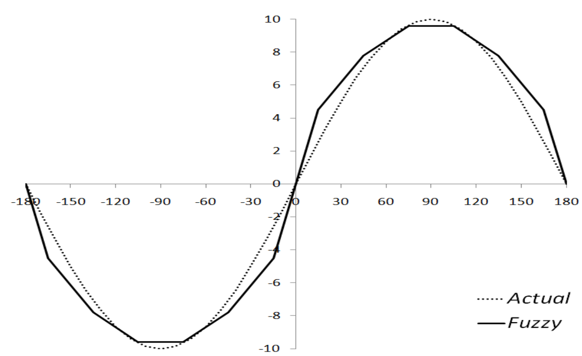
x = 105 then y is PM or PB thus y = 9.6

x = 135 then y is PS or PM thus y = 7.8

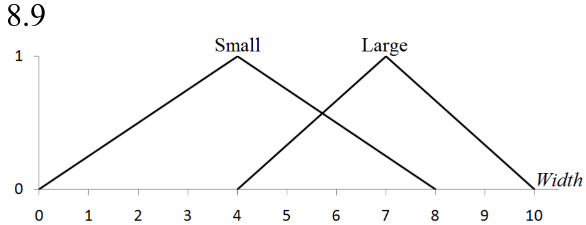
x = 165 then y is PS or Z thus y = 4.5

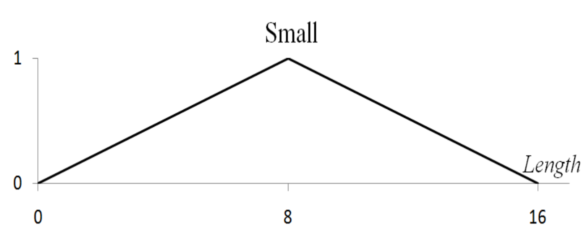
x = 180 then y is Z thus y = 0

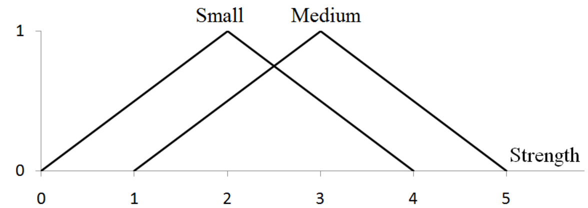
y	0	-4.5	-7.8	-9.6	-9.6	-7.8	-4.5	0	4.5	7.8	9.6	9.6	7.8	4.5	0
x	-180	-165	-135	-105	-75	-45	15	0	15	45	75	105	135	165	180



8.9







We use max-product composition in this problem.

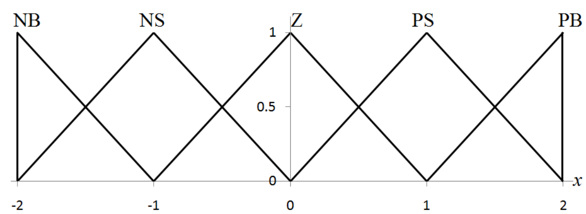
W = 6 & L = 10 fires rules 1 & 2

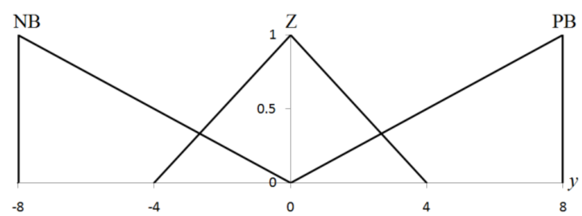
Rule 1: W=Small=0.5 & L=Small=0.75 then Strength=Small=0.5*0.75*0.375 thus Stength= 2

Rule 2: W=Large=0.67 & L=Small=0.75 then Strength=Medium=0.67*0.75= 0.5025 thus Stength=3

Therefore Strength=max(2, 3) = 3

8.10





Rule	x	y
1	NS or Z or PS	Z
2	PB	PB
3	NB	NB

x = -2 then NB = 1 fires R3 thus

$y = {}^{2}/_{3} * -8 = -5.3$

x =-1.5 then NB=0.5 fires R3 thus

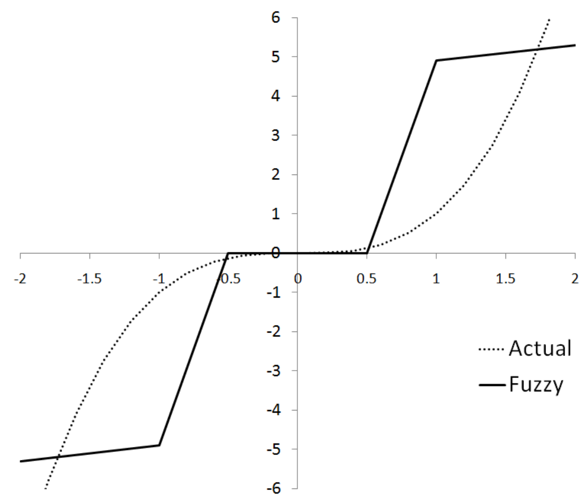
$$y = \frac{-6*2-2.7*1}{2+1} = -4.9$$

or NS=0.5 fires R1 thus y=0
so, y = -4.9

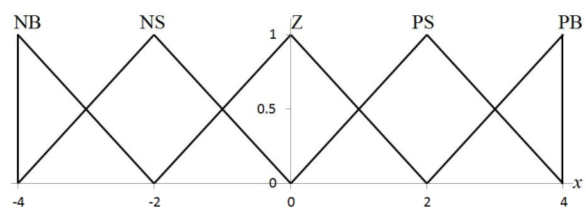
x =-0.5 then NS=0.5 fires R1 thus y=0
or Z=0.5 fires R1 thus y=0
so, y=0

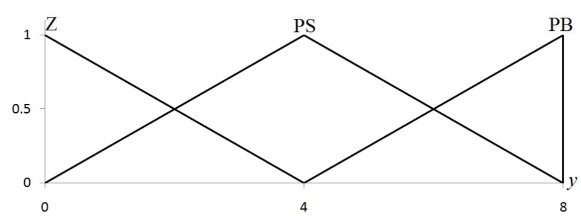
x=0 then Z=0 fires R1 thus y=0

Similarly for x= 0.5, 1.5, and 2 one achieves y=0, 4.9, and 5.3 respectively.



8.11





Rule	x	y
1	Z	Z
2	NB or PB	PB
3	NS or PS	PS

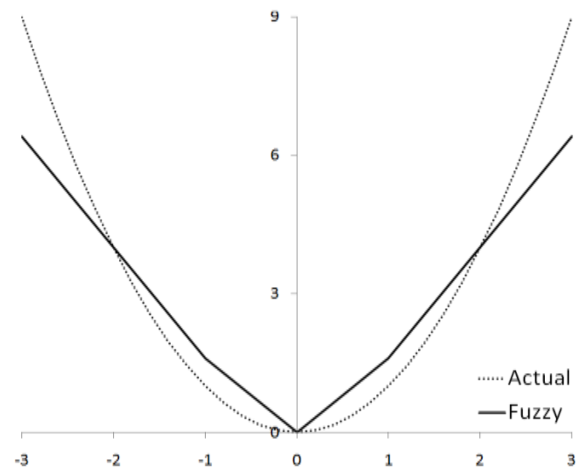
x = -3 then NB=0.5 fires R2, from NB:
$y = \frac{5.3*0.5+7*1}{0.5+1} = 6.4$
or NS=0.5 fires R3, from PS: y=4
thus, y=6.4

x= -1 then NS=0.5 fires R3, fro PS: y=4
or Z=0.5 fires R1, from Z:
$y = \frac{1*1+0.5*2.7}{1+0.5} = 1.6$
thus, y=4

x=0 then Z=1 fires R1, from Z: y=1.6

x=1 then PS=0.5 fires R3, from PS: y=4
or Z=0.5 fires R1, from Z: y=1.6
thus, y=4

x=3 then PB=0.5 fires R2, from PB: y=6.4
or PS=0.5 fires R3, from PS: y=4
thus, y=6.4



CHAPTER 9

Decision Making with Fuzzy Information

9.1 $T(I_1 \geq I_2) = 0.7$, $T(I_1 \geq I_3) = 0.8$

$T(I_2 \geq I_1) = 1$, $T(I_2 \geq I_3) = 1$,

$T(I_3 \geq I_1) = 1$, $T(I_3 \geq I_2) = 0.7$

$T(I_1 \geq I_2, I_3) = 0.7$, $T(I_2 \geq I_1, I_3) = 1$

$T(I_3 \geq I_1, I_2) = 0.7$

9.2 The evaluation vector

$$\underset{\sim}{e} = \{0.1, 0.35, 0.4, 0.15\} \circ \begin{bmatrix} 0.3 & 0.4 & 0.3 \\ 0.1 & 0.5 & 0.4 \\ 0.5 & 0.4 & 0.1 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$$

using max-min composition,

$$\underset{\sim}{e} = \{0.4, 0.4, 0.35\}$$

9.3 The evaluation vector,

$$\underset{\sim}{e} = \{0.2, 0.25, 0.3, 0.25\} \circ \begin{bmatrix} 0 & 0.7 & 0.3 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.5 & 0.4 \\ 0.7 & 0.2 & 0.1 \end{bmatrix}$$

using max-min composition,

$$\underset{\sim}{e} = \{0.25, 0.3, 0.3\}$$

9.4 The evaluation vector

$$\underset{\sim}{e} = \{0.6, 0.2, 0.1, 0.1\} \circ$$

$$\begin{bmatrix} 0.3 & 0.6 & 0.1 & 0 & 0 \\ 0.1 & 0.2 & 0.5 & 0.1 & 0.1 \\ 0.3 & 0.4 & 0.2 & 0.1 & 0 \\ 0 & 0.2 & 0.6 & 0.1 & 0.1 \end{bmatrix}$$

using max-min composition,

$$\underset{\sim}{e} = \{0.3, 0.6, 0.2, 0.1, 0.1\}$$

9.5 The comparison matrix is:

	P_1	P_2	P_3	P_4	min	*Ranking*
P_1	1	0.6	1	0	0.6	2
P_2	1	1	1	1	1	1
P_3	1	1	1	1	1	1
P_4	1	1	1	1	1	1

9.6 Using equation 9.6, the comparison matrix,

	x_1	x_2	x_3	x_4	min	*Ranking*
x_1	1	0.83	1	0.33	0.33	3
x_2	1	1	1	0.25	0.25	4
x_3	0.56	0.88	1	0.6	0.56	2
x_4	1	1	1	1	1	1

The overall ranking as given by the max value in the minimum column is x_4, x_3, x_1, x_2

9.7

	x_1	x_2	x_3	min	*Ranking*
x_1	1.0	0.89	0.29	0.29	3
x_2	1.0	1.0	1.0	1.0	1
x_3	1.0	0.6	1.0	0.6	2
x_4					

Particle x_2 (cedar) is the closest.

9.8

$$C = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{bmatrix} 1 & 0.33 & 0.43 & 1 \\ 1 & 1 & 0.25 & 0.83 \\ 1 & 1 & 1 & 1 \\ 0.56 & 1 & 0.5 & 1 \end{bmatrix} \end{array} \begin{array}{cc} \text{min} & \text{Ranking} \\ 0.33 & 3 \\ 0.25 & 4 \\ 1 & 1 \\ 0.5 & 2 \end{array}$$

x_3 tastes more like the expensive wine.

9.9

$R \times R =$

$$\begin{bmatrix} 0 & 0.5 & 0.8 & 0.4 \\ 0.5 & 0 & 0.9 & 0.2 \\ 0.2 & 0.1 & 0 & 0.1 \\ 0.6 & 0.8 & 0.9 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0.5 & 0.8 & 0.4 \\ 0.5 & 0 & 0.9 & 0.2 \\ 0.2 & 0.1 & 0 & 0.1 \\ 0.6 & 0.8 & 0.9 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0.25+0.16+0.24 & & & \\ & 0.25+0.09+0.16 & & \\ & & 0.16+0.09+0.09 & \\ & & & 0.24+0.16+0.09 \end{bmatrix} =$$

$$\begin{bmatrix} 0.65 & & & \\ & 0.50 & & \\ & & 0.34 & \\ & & & 0.49 \end{bmatrix}$$

$$F(R) = \frac{tr(R^2)}{n(n-1)/2} = \frac{0.65+0.50+0.34+0.49}{4(4-1)/2} = 0.33$$

$$C(R) = 1 - F(R) = 1 - 0.33 = 0.67$$

$$m(R) = 1 - (2C(R) - 1)^{\frac{1}{2}}$$
$$= 1 - (2 * 0.67 - 1)^{\frac{1}{2}} = 0.417$$

$$For\ M_1^*:\ m = 1 - (2/n)^{\frac{1}{2}} = 1 - (2/4)^{\frac{1}{2}}$$
$$= 0.293$$

Thus, Distance from M_1^* consensus is 0.417 - 0.293 = 0.124

9.10 Degree of preference measures from equation9.13, average certainty,

$$\underset{\sim}{R} \cdot \underset{\sim}{R}^T = \begin{bmatrix} 0 & 0.1 & 0.7 & 0.2 \\ 0.9 & 0 & 0.6 & 1 \\ 0.3 & 0.4 & 0 & 0.5 \\ 0.8 & 0 & 0.5 & 0 \end{bmatrix} \times$$

$$\begin{bmatrix} 0 & 0.9 & 0.3 & 0.8 \\ 0.1 & 0 & 0.4 & 0 \\ 0.7 & 0.6 & 0 & 0.5 \\ 0.2 & 1 & 0.5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.54 & & & \\ & 2.17 & & \\ & & 0.50 & \\ & & & 0.89 \end{bmatrix}$$

Note: off-diagonal elements are not required

$$tr(RR^T) = 4.10$$

$$C(\underset{\sim}{R}) = 0.683$$

$$F(\underset{\sim}{R}) = 0.317$$

$$m(\underset{\sim}{R}) = 0.394$$

$$m\left(\underset{\sim}{M_1}\right) = 1 - (2/n)^{1/2} = 0.293$$

$$m\left(\underset{\sim}{M_2}\right) = 0$$

$$\text{Distance to type-I} = \frac{1-0.394}{1-0.293} = 86\%$$

$$\text{Distance to type-II} = 1 - 0.394 = 61\%$$

Therefore, the group is closer in the way toward a type-II consensus.

9.11

$$F(\underset{\sim}{R}) = 0.413$$

$$C(\underset{\sim}{R}) = 0.587$$

$$m(\underset{\sim}{R}) = 1 - (2 * 0.587 - 1)^{1/2} = 0.673$$

$m(M_1^*) = 1 - (2/n)^{1/2} = 0.293$

$\frac{1-0.584}{(1-0.293)} = 59\%$ of the way to Type I consensus

$1 - 0.584 = 42\%$ of the way to Type II consensus

9.12

$F(\underset{\sim}{R}) = 0.287$ Average fuzziness

$C(\underset{\sim}{R}) = 0.713$ Average certainty

$m(\underset{\sim}{R}) = 1 - (2 * 0.713 - 1)^{1/2} = 0.347$

$m(M_1^*) = 1 - (2/n)^{1/2} = 0.293$

$m(\underset{\sim}{R}) = 0$ For a Type II (M_2^*) consensus relation.

$\frac{1-0.347}{(1-0.293)} = 92\%$ of the way to Type I consensus

$1 - 0.347 = 65\%$ of the way to Type II consensus

9.13

$F(\underset{\sim}{R}) = 0.19$ Average fuzziness

$C(\underset{\sim}{R}) = 0.81$ Average certainty

$m(\underset{\sim}{R}) = 1 - (2 * 0.81 - 1)^{1/2} = 0.213$

$m(M_1^*) = 1 - (2/n)^{1/2} = 0.293$ For a Type (M_1^*) consensus relation.

$m(\underset{\sim}{R}) = 0$ For a Type II (M_2^*) consensus relation.

Distance from a Type II consensus 1- 0.213 = 0.787= 79% of the way from a Type II consensus.

or at $0.787/(1-0.293) = 1.11 = 111\%$ of the way toward a Type I consensus.

9.14

$P = \{0.6, 0.8, 0.7, 0.5\}$

$\bar{b}_1 = 0.4, \bar{b}_2 = 0.2, \bar{b}_3 = 0.3, \bar{b}_4 = 0.5$

$$D(a_1) = (0.4 \vee 0.7) \wedge (0.2 \vee 0.3) \wedge (0.3 \vee 0.7) \wedge (0.5 \vee 0.9)$$
$$D(a_1) = (0.7) \wedge (0.3) \wedge (0.7) \wedge (0.9) = 0.3$$
$$D(a_2) = (0.4 \vee 1.0) \wedge (0.2 \vee 0.6) \wedge (0.3 \vee 0.3) \wedge (0.5 \vee 0.5)$$
$$D(a_2) = (1.0) \wedge (0.6) \wedge (0.3) \wedge (0.5) = 0.3$$
$$D(a_3) = (0.4 \vee 0.3) \wedge (0.2 \vee 0.8) \wedge (0.3 \vee 0.5) \wedge (0.5 \vee 0.5)$$
$$D(a_3) = (0.4) \wedge (0.8) \wedge (0.5) \wedge (0.5) = 0.4$$
$$D^* = \vee\{D(a_1), D(a_2), D(a_3)\} = 0.4$$

Therefore, the best choice is a_3, BTPA.

9.15

$P = \{0.7, 0.8, 0.9, 0.5\}$

$\bar{b}_1 = 0.3, \bar{b}_2 = 0.2, \bar{b}_3 = 0.1, \bar{b}_4 = 0.5$

$$D(a_1) = (0.3 \vee 0.8) \wedge (0.2 \vee 0.5) \wedge (0.1 \vee 0.6) \wedge (0.5 \vee 0.6)$$
$$D(a_1) = (0.8) \wedge (0.5) \wedge (0.6) \wedge (0.6) = 0.5$$
$$D(a_2) = (0.3 \vee 0.4) \wedge (0.2 \vee 0.8) \wedge (0.1 \vee 0.3) \wedge (0.5 \vee 0.7)$$
$$D(a_2) = (0.4) \wedge (0.8) \wedge (0.3) \wedge (0.7) = 0.3$$
$$D(a_3) = (0.3 \vee 0.3) \wedge (0.2 \vee 0.7) \wedge (0.1 \vee 0.5) \wedge (0.5 \vee 0.6)$$
$$D(a_3) = (0.3) \wedge (0.7) \wedge (0.5) \wedge (0.6) = 0.3$$
$$D^* = \vee\{D(a_1), D(a_2), D(a_3)\} = 0.5$$

Therefore, the best choice is a_1, Masonry.

9.16

$P = \{0.8, 0.8, 0.6, 0.5\}$

$\bar{b}_1 = 0.2, \bar{b}_2 = 0.2, \bar{b}_3 = 0.4, \bar{b}_4 = 0.5$

$$D(a_1) = (0.2 \vee 0.8) \wedge (0.2 \vee 0.9) \wedge (0.4 \vee 0.8) \wedge (0.5 \vee 0.4)$$
$$D(a_1) = (0.8) \wedge (0.9) \wedge (0.4) \wedge (0.5) = 0.4$$
$$D(a_2) = (0.2 \vee 1.0) \wedge (0.2 \vee 0.5) \wedge (0.4 \vee 0.7) \wedge (0.5 \vee 0.8)$$
$$D(a_2) = (1.0) \wedge (0.5) \wedge (0.7) \wedge (0.8) = 0.5$$
$$D(a_3) = (0.2 \vee 0.7) \wedge (0.2 \vee 0.5) \wedge (0.4 \vee 0.4) \wedge (0.5 \vee 0.9)$$
$$D(a_3) = (0.7) \wedge (0.5) \wedge (0.4) \wedge (0.9) = 0.4$$
$$D^* = \vee\{D(a_1), D(a_2), D(a_3)\} = 0.5$$

Therefore, the best choice is a_2, Ozone.

9.17

First scenario: $P = \{0.8, 0.4, 0.8, 0.6\}$

$\overline{b}_1 = 0.2, \overline{b}_2 = 0.6, \overline{b}_3 = 0.2, \overline{b}_4 = 0.4$

$$D(a_1) = (0.2 \vee 0.2) \wedge (0.6 \vee 0.6) \wedge (0.2 \vee 1.0) \wedge (0.4 \vee 0.7)$$
$$D(a_1) = (0.2) \wedge (0.6) \wedge (1.0) \wedge (0.7) = 0.2$$
$$D(a_2) = (0.2 \vee 0.9) \wedge (0.6 \vee 1.0) \wedge 2(0.4 \vee 0.6) \wedge (0.4 \vee 0.7)$$
$$D(a_2) = (0.9) \wedge (1.0) \wedge (0.6) \wedge (0.7) = 0.6$$
$$D(a_3) = (0.2 \vee 0.4) \wedge (0.6 \vee 0.2) \wedge (0.2 \vee 0.8) \wedge (0.4 \vee 0.2)$$
$$D(a_3) = (0.4) \wedge (0.6) \wedge (0.8) \wedge (0.4) = 0.4$$
$$D^* = \vee\{D(a_1), D(a_2), D(a_3)\} = 0.6$$

Therefore, the best choice for the first senario is a_2, GOES.

Second scenario: $P = \{0.4, 0.6, 0.4, 0.7\}$

$\overline{b}_1 = 0.6, \overline{b}_2 = 0.4, \overline{b}_3 = 0.6, \overline{b}_4 = 0.3$

$$D(a_1) = (0.6 \vee 0.2) \wedge (0.4 \vee 0.6) \wedge (0.6 \vee 1.0) \wedge (0.3 \vee 0.7)$$
$$D(a_1) = (0.6) \wedge (0.6) \wedge (1.0) \wedge (0.7) = 0.6$$
$$D(a_2) = (0.6 \vee 0.9) \wedge (0.4 \vee 1.0) \wedge 2(0.6 \vee 0.6) \wedge (0.3 \vee 0.7)$$
$$D(a_2) = (0.9) \wedge (1.0) \wedge (0.6) \wedge (0.7) = 0.6$$
$$D(a_3) = (0.6 \vee 0.4) \wedge (0.4 \vee 0.2) \wedge (0.6 \vee 0.8) \wedge (0.3 \vee 0.2)$$
$$D(a_3) = (0.6) \wedge (0.4) \wedge (0.8) \wedge (0.3) = 0.4$$
$$D^* = \vee\{D(a_1), D(a_2), D(a_3)\} = 0.6$$

There is a tie between alternatives 1 and 2.

$$\hat{D}(a_1) = (1.0) \wedge (0.7) = 0.7$$
$$\hat{D}(a_2) = (0.9) \wedge (1.0) \wedge (0.7) = 0.7$$

There is still a tie between alternatives 1 and 2.

$$\hat{\hat{D}}(a_1) = (1.0) = 1.0$$
$$\hat{\hat{D}}(a_2) = (0.9) \wedge (1.0) = 0.9$$
$$D^* = \vee\{D(a_1), D(a_2)\} = 1.0$$

The tie is now broken; thus, the best choice for the second scenario is a_1, LANTSANT7.

9.18

$P = \{0.6, 0.5, 0.6, 0.8, 0.6\}$

$\overline{b}_1 = 0.4, \overline{b}_2 = 0.5, \overline{b}_3 = 0.4, \overline{b}_4 = 0.2, \overline{b}_5 = 0.4$

$$D(Pipe) = (\overline{b}_1 \vee O_1) \wedge (\overline{b}_2 \vee O_2) \wedge (\overline{b}_3 \vee O_3) \wedge (\overline{b}_4 \vee O_4) \wedge (\overline{b}_5 \vee O_5)$$
$$D(Pipe) = (0.4 \vee 0.8) \wedge (0.5 \vee 0.9) \wedge (0.4 \vee 0.6) \wedge (0.2 \vee 0.4) \wedge (0.4 \vee 0.7)$$
$$= (0.8) \wedge (0.9) \wedge (0.6) \wedge (0.4) \wedge (0.7) = 0.4$$
$$D(Pond) = (0.4 \vee 0.5) \wedge (0.5 \vee 0.4) \wedge (0.4 \vee 0.8) \wedge (0.2 \vee 0.9) \wedge (0.4 \vee 0.4)$$
$$= (0.5) \wedge (0.5) \wedge (0.8) \wedge (0.9) \wedge (0.4) = 0.4$$

There is a tie between alternatives 1 and 2.

$$\hat{D}(a_1) = (0.8) \wedge (0.9) \wedge (0.6) \wedge (0.7) = 0.6$$
$$\hat{D}(a_2) = (0.5) \wedge (0.5) \wedge (0.8) \wedge (0.9) = 0.5$$
$$D^* = \vee\{D(a_1), D(a_2)\} = 0.6$$

The tie is broken now; thus the first choice, pipe, is preferred.

9.19 To find: $P(\underset{\sim}{D}_2 \mid \underset{\sim}{M}_3)$ use Equation 9.31 to determine $p(x_k)$.

$$p(x_k) = \sum_{i=1}^{n} p(x_k \mid s_i) \cdot p(s_i)$$

$p(x_1) = 0.12, p(x_2) = 0.18,$
$p(x_3) = 0.29, p(x_4) = 0.15$
$p(x_5) = 0.15,\ p(x_6) = 0.11,$

Now, using Equation 9.52b for imperfect information.

$$p(\underset{\sim}{D}_2 \mid M_3) = 0.477$$

For perfect information

$p(x_1) = 0.1, p(x_2) = 0.1,$
$p(x_3) = 0.2, p(x_4) = 0.1$
$p(x_5) = 0.4,\ p(x_6) = 0.1,$

$$p(\underset{\sim}{D}_2 \mid \underset{\sim}{M}_3) = 0.691$$

b) To find: $E(U_1|M_2)$, for imperfect info

$$p(\underset{\sim}{D}_1 \mid \underset{\sim}{M}_2) = 0.361$$
$$p(\underset{\sim}{D}_2 \mid \underset{\sim}{M}_2) = 0.512$$
$$p(\underset{\sim}{D}_3 \mid \underset{\sim}{M}_2) = 0.129$$

Expected Utility using eq. 9.53b

$E(u_1 \mid \underset{\sim}{M}_2) = 2.340$,

For perfect information,

$p(\underset{\sim}{D}_1 \mid \underset{\sim}{M}_2) = 0.559$

$p(\underset{\sim}{D}_2 \mid \underset{\sim}{M}_2) = 0.332$

$p(\underset{\sim}{D}_3 \mid \underset{\sim}{M}_2) = 0.108$

$E(u_1 \mid \underset{\sim}{M}_2) = 2.795$

9.20

Using Equation 9.31:

$$p(x_k) = \sum_{i=1}^{n} p(x_k \mid s_i) \cdot p(s_i)$$

$p(x_1) = 0.11,\ p(x_2) = 0.235,$
$p(x_3) = 0.305,\ p(x_4) = 0.35$

a) $P(F_2|M_1) = 0.211$
$P(F_3|M_3) = 0.751$

b) $P(F_1|M_3) = 0.010$
$P(F_2|M_3) = 0.238$
$P(F_3|M_3) = 0.751$

$E(U_1|M_3) = 9.950$

9.21 a)

$p(x_1) = 0.22,\ p(x_2) = 0.18,$
$p(x_3) = 0.165,\ p(x_4) = 0.155,$
$p(x_5) = 0.28$

$P(F_1|x_1)$ =0.164, $P(F_1|x_2)$ =0.306
$P(F_1|x_3)$ =0.221, $P(F_1|x_4)$ =0.506
$P(F_1|x_5)$ =0.193, $P(F_2|x_1)$ =0.623
$P(F_2|x_2)$ =0.325, $P(F_2|x_3)$ =0.409
$P(F_2|x_4)$ =0.461, $P(F_2|x_5)$ =0.698
$P(F_3|x_1)$ =0.214, $P(F_3|x_2)$ =0.369
$P(F_3|x_3)$ =0.370, $P(F_3|x_4)$ =0.032
$P(F_3|x_5)$ =0.109

$E(U_1|x_1) = 0.250, E(U_1|x_2) = 0.319$
$E(U_1|x_3) = 0.742, E(U_1|x_4) = -2.371$
$E(U_1|x_5) = -0.420, E(U_2|x_1) = 0.745$
$E(U_2|x_2) = 0.011, E(U_2|x_3) = -0.667$
$E(U_2|x_4) = 5.665, E(U_2|x_5) = 2.236$

$E(U^* | x_1) = 0.745, E(U^*|x_2) = 0.319$
$E(U^*|x_3) = 0.742, E(U^*|x_4) = 5.665$
$E(U^*|x_5) = 2.236$

$E(U^*_{\Phi}) = 1.848$

$P(F_1) = 0.26, P(F_2) = 0.53, P(F_3) = 0.21$

$E(U^*) = 0.53$

V(x) = 1.848 – 0.53 = 1.318

b) $p(x_1) = 0.15,\ p(x_2) = 0.12,$
$p(x_3) = 0.08,\ p(x_4) = 0.3,$
$p(x_5) = 0.35$

$P(F_1|x_1)$ =0.1, $P(F_1|x_2)$ =0
$P(F_1|x_3)$ =0, $P(F_1|x_4)$ =0.7
$P(F_1|x_5)$ =0.1, $P(F_2|x_1)$ =0.8
$P(F_2|x_2)$ =0.2, $P(F_2|x_3)$ =0.2
$P(F_2|x_4)$ =0.3, $P(F_2|x_5)$ =0.8
$P(F_3|x_1)$ =0.1, $P(F_3|x_2)$ =0.8
$P(F_3|x_3)$ =0.8, $P(F_3|x_4)$ =0
$P(F_3|x_5)$ =0.1

$E(U_1|x_1) = 0, E(U_1|x_2) = 4$
$E(U_1|x_3) = 4, E(U_1|x_4) = -3.5$
$E(U_1|x_5) = 0, E(U_2|x_1) = 1.6$
$E(U_2|x_2) = -7.6, E(U_2|x_3) = -7.6$
$E(U_2|x_4) = 7.6, E(U_2|x_5) = 1.6$

$E(U^*_{x_1}) = 1.6, E(U^*_{x_2}) = 4$
$E(U^*_{x_3}) = 4, E(U^*_{x_4}) = 7.6$
$E(U^*_{x_5}) = 1.6$

$E(U^*_{xp}) = 3.88$

$E(U^*) = 0.53$

V(x_p) = 3.88 – 0.53 = 3.35

Note: $V(x_p) = 3.35 > V(x) = 1.318$
indicates that computations are correct.

CHAPTER 10

Fuzzy Classification and Pattern Recognition

10.1

$$\underset{\sim}{R}_1 = \begin{bmatrix} 1 & 0.7 & 0 & 0.2 & 0.1 \\ 0.7 & 1 & 0.9 & 0 & 0.4 \\ 0 & 0.9 & 1 & 0 & 0.3 \\ 0.2 & 0 & 0 & 1 & 0.5 \\ 0.1 & 0.4 & 0.3 & 0.5 & 1 \end{bmatrix}$$

$$\underset{\sim}{R}_e = \underset{\sim}{R}_3 = \begin{bmatrix} 1 & 0.7 & 0.7 & 0.4 & 0.4 \\ 0.7 & 1 & 0.9 & 0.4 & 0.4 \\ 0.7 & 0.9 & 1 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 1 & 0.5 \\ 0.4 & 0.4 & 0.4 & 0.5 & 1 \end{bmatrix}$$

$$\lambda = 0.9 \quad \begin{array}{c} x_1 \; x_2 \; x_3 \; x_4 \; x_5 \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

4 classes $x_1, \{x_2, x_3\}, x_4, x_5$

$$\lambda = 0.7 \quad \begin{array}{c} x_1 \; x_2 \; x_3 \; x_4 \; x_5 \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$3\ classes \{x_1, x_2, x_3\}, x_4, x_5$

$$\lambda = 0.5 \quad \begin{array}{c} x_1 \; x_2 \; x_3 \; x_4 \; x_5 \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$2\ classes \{x_1, x_2, x_3\}, \{x_4, x_5\}$

$$\lambda = 0.4 \quad \begin{array}{c} x_1 \; x_2 \; x_3 \; x_4 \; x_5 \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$1\ class \{x_1, x_2, x_3, x_4, x_5\}$

10.2 $$\mathrm{r_{ij}} = \frac{\sum_{R=1}^{3} \min(x_{ik}, x_{jk})}{\sum_{R=1}^{m} \max(x_{ik}, x_{jk})}$$

$$R(1,2) = \frac{0.2+0.3+0.1}{0.6+0.3+0.5} = 0.429$$

$$R(1,3) = \frac{0+0.3+0.1}{0.6+0.8+0.2} = 0.250$$

$$R(1,4) = \frac{0.6+0.1+0.1}{0.8+0.3+0.1} = 0.667$$

$$R(2,3) = \frac{0+0.3+0.2}{0.2+0.8+0.5} = 0.333$$

$$R(2,4) = \frac{0.2+0.1+0.1}{0.8+0.3+0.5} = 0.250$$

$$R(3,4) = \frac{0+0.1+0.1}{0.8+0.8+0.2} = 0.111$$

$$\underset{\sim}{R}_1 = \begin{bmatrix} 1 & 0.429 & 0.250 & 0.667 \\ 0.429 & 1 & 0.333 & 0.250 \\ 0.250 & 0.333 & 1 & 0.11 \\ 0.667 & 0.250 & 0.111 & 1 \end{bmatrix}$$

$$\underset{\sim e}{R} = \underset{\sim 2}{R} = \begin{bmatrix} 1 & 0.429 & 0.333 & 0.667 \\ 0.429 & 1 & 0.333 & 0.429 \\ 0.333 & 0.333 & 1 & 0.333 \\ 0.667 & 0.429 & 0.333 & 1 \end{bmatrix}$$

$x_1 \quad x_2 \quad x_3 \quad x_4$

$$\lambda = 0.5 \quad \begin{array}{c} x_1 \; x_2 \; x_3 \; x_4 \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

3 classes $\{x_1, x_4\}, x_2, x_3$

10.3 a) It is a tolerance relation.
b) Equivalence relation

$$\underset{\sim e}{R} = \underset{\sim 2}{R} = \begin{bmatrix} 1 & 0.7 & 0.5 & 0.7 \\ 0.7 & 1 & 0.5 & 0.8 \\ 0.5 & 0.5 & 1 & 0.5 \\ 0.7 & 0.8 & 0.5 & 1 \end{bmatrix}$$

$$\lambda = 0.4 \quad \underset{\sim 0.4}{R} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$1\,class\;\{x_1, x_2, x_3, x_4\}$

$$\lambda = 0.6 \quad \underset{\sim 0.6}{R} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$2\;classes\,\{x_1, x_2, x_4\}, x_3$

$$\lambda = 0.7 \quad \underset{\sim 0.7}{R} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$2\;classes\,\{x_1, x_2, x_4\}, x_3$

10.4 By using m=2 and $\varepsilon_L \le 0.01$ and assuming $U^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

$$U^{(25)} = \begin{bmatrix} 0.911 & 0.824 & 0.002 & 0.906 \\ 0.089 & 0.176 & 0.998 & 0.094 \end{bmatrix}$$

$V_1 = \{8.458 \quad 1.634\}; V_2 = \{7.346 \quad 1.792\}$

10.5 After 4 cycles :

$$U^{(4)} = \begin{bmatrix} 0.963 & 0.820 & 0.054 & 0.029 \\ 0.037 & 0.180 & 0.946 & 0.971 \end{bmatrix}$$

$V_1 = \{5.431 \quad 6.282\}; V_2 = \{8.470 \quad 10.974\}$

$\varepsilon_L = 0.005$

10.6 After 7 cycles :

$$U^{(7)} = \begin{bmatrix} 0.145 & 0.979 & 0.950 & 0.199 & 0.300 & 0.105 & 0.862 \\ 0.855 & 0.021 & 0.050 & 0.801 & 0.700 & 0.895 & 0.138 \end{bmatrix}$$

$V_1 = \{8.029 \quad 3.047\}; V_2 = \{4.757 \quad 8.094\}$

$\varepsilon_L = 0.006$

10.7 After 4 cycles :

$$U^{(4)} = \begin{bmatrix} 0.999 & 0.175 & 0.055 \\ 0.001 & 0.825 & 0.945 \end{bmatrix}$$

$V_1 = \{2.034 \quad 1.530\}; V_2 = \{3.563 \quad 2.218\}$

$\varepsilon_L = 0.005$

10.8 After 3 cycles: (not converged)

$$U^{(3)} = \begin{bmatrix} 0.941 & 0.969 & 0.731 & 0.003 \\ 0.059 & 0.031 & 0.289 & 0.997 \end{bmatrix}$$

$V_1 = \{-2.619 \quad 1.337\}; V_2 = \{-0.106 \quad 1.767\}$

$\varepsilon_L = 0.125 > 0.01$

After 11 cycles: (converged)

$$U^{(11)} = \begin{bmatrix} 0.941 & 0.969 & 0.731 & 0.003 \\ 0.059 & 0.031 & 0.289 & 0.997 \end{bmatrix}$$

$V_1 = \{-2.153 \quad 1.511\}; V_2 = \{0.845 \quad 1.962\}$

$\varepsilon_L = 0.005$

10.9 After 2 cycles: (not converged)

$$U^{(2)} = \begin{bmatrix} 0.469 & 0.999 & 0.959 & 0.007 & 0.0002 \\ 0.531 & 0.001 & 0.041 & 0.993 & 0.9998 \end{bmatrix}$$

$V_1 = \{233.62 \quad 57.22\}; V_2 = \{753.98 \quad 15.41\}$

$\varepsilon_L = 0.141 > 0.01$

After 6 cycles: (converged)

$$U^{(6)} = \begin{bmatrix} 0.351 & 0.987 & 0.976 & 0.014 & 0.002 \\ 0.649 & 0.013 & 0.024 & 0.986 & 0.998 \end{bmatrix}$$

$V_1 = \{196.15 \quad 63.84\}; V_2 = \{762.92 \quad 14.97\}$

$\varepsilon_L = 0.006$

10.10 Iteration 1

$$v_{1j} = \frac{\sum_{k=1}^{5} \mu_{1k}^{2} x_{kj}}{\mu_{1k}^{2}}, v_{2j} = \frac{\sum_{k=1}^{5} \mu_{2k}^{2} x_{kj}}{\mu_{2k}^{2}}$$

$$v_{11} = \frac{1\times 3.5 + 1\times 4}{1+1} = 3.75$$

$$v_{12} = \frac{1\times 35 + 1\times 25}{1+1} = 30$$

$$v_1 = \{3.75 \quad 30\}$$

$$v_{21} = \frac{1\times 5 + 1\times 7 + 1\times 8}{1+1+1} = 6.667$$

$$v_{22} = \frac{1\times 20 + 1\times 10 + 1\times 22}{1+1+1} = 17.333$$

$$v_2 = \{6.667 \quad 17.333\}$$

$$d_{11} = \sqrt{(5-3.75)^2 + (20-30)^2} = 10.078$$

$$d_{21} = \sqrt{(5-6.667)^2 + (20-17.333)^2} = 3.145$$

$$d_{12} = \sqrt{(3.5-3.75)^2 + (35-30)^2} = 5.006$$

$$d_{22} = \sqrt{(3.5-6.667)^2 + (35-17.333)^2} = 17.949$$

$$d_{13} = \sqrt{(4-3.75)^2 + (25-30)^2} = 5.006$$

$$d_{23} = \sqrt{(4-6.667)^2 + (25-17.333)^2} = 8.118$$

$$d_{14} = \sqrt{(7-3.75)^2 + (10-30)^2} = 20.262$$

$$d_{24} = \sqrt{(7-6.667)^2 + (10-17.333)^2} = 7.341$$

$$d_{15} = \sqrt{(8-3.75)^2 + (22-30)^2} = 9.059$$

$$d_{25} = \sqrt{(8-6.667)^2 + (22-17.333)^2} = 4.854$$

$$\mu_{11} = \left[\left(\frac{d_{11}}{d_{11}}\right)^2 + \left(\frac{d_{11}}{d_{21}}\right)^2\right]^{-1} = \left[1 + \left(\frac{10.078}{3.145}\right)^2\right]^{-1} = 0.089$$

$$\mu_{21} = 1 - \mu_{11} = 1 - 0.089 = 0.911$$

$$\mu_{12} = \left[\left(\frac{d_{12}}{d_{12}}\right)^2 + \left(\frac{d_{12}}{d_{22}}\right)^2\right]^{-1} = \left[1 + \left(\frac{5.005}{17.949}\right)^2\right]^{-1} = 0.928$$

$$\mu_{22} = 1 - \mu_{12} = 1 - 0.928 = 0.072$$

$$\mu_{13} = \left[\left(\frac{d_{13}}{d_{13}}\right)^2 + \left(\frac{d_{13}}{d_{23}}\right)^2\right]^{-1} = \left[1 + \left(\frac{5.006}{8.118}\right)^2\right]^{-1} = 0.724$$

$$\mu_{23} = 1 - \mu_{13} = 1 - 0.724 = 0.276$$

$$\mu_{14} = \left[\left(\frac{d_{14}}{d_{14}}\right)^2 + \left(\frac{d_{14}}{d_{24}}\right)^2\right]^{-1} = \left[1 + \left(\frac{20.262}{7.341}\right)^2\right]^{-1} = 0.116$$

$$\mu_{24} = 1 - \mu_{14} = 1 - 0.116 = 0.884$$

$$\mu_{15} = \left[\left(\frac{d_{15}}{d_{15}}\right)^2 + \left(\frac{d_{15}}{d_{25}}\right)^2\right]^{-1} = \left[1 + \left(\frac{9.059}{4.854}\right)^2\right]^{-1} = 0.223$$

$$\mu_{25} = 1 - \mu_{15} = 1 - 0.223 = 0.777$$

$$\underset{\sim}{U}^{(1)} = \begin{bmatrix} 0.089 & 0.928 & 0.724 & 0.116 & 0.223 \\ 0.911 & 0.072 & 0.276 & 0.884 & 0.777 \end{bmatrix}$$

$$v_{11} = \frac{0.089^2 \times 5 + 0.928^2 \times 3.5 + 0.724^2 \times 4 + 0.116^2 \times 7 + 0.223^2 \times 8}{0.089^2 + 0.928^2 + 0.724^2 + 0.116^2 + 0.223^2} = 3.874$$

$$v_{12} = \frac{0.089^2 \times 20 + 0.928^2 \times 35 + 0.724^2 \times 25 + 0.116^2 \times 10 + 0.223^2 \times 22}{0.089^2 + 0.928^2 + 0.724^2 + 0.116^2 + 0.223^2} = 30.641$$

v_{21}
$= \frac{0.911^2 \times 5 + 0.072^2 \times 3.5 + 0.276^2 \times 4 + 0.884^2 \times 7 + 0.777^2 \times 8}{0.911^2 + 0.072^2 + 0.276^2 + 0.884^2 + 0.777^2}$
$= 6.433$
v_{22}
$= \frac{0.911^2 \times 20 + 0.072^2 \times 35 + 0.276^2 \times 25 + 0.884^2 \times 10 + 0.777^2 \times 22}{0.911^2 + 0.072^2 + 0.276^2 + 0.884^2 + 0.777^2}$
$= 17.323$

$$v_1 = \{3.874 \quad 30.641\};\ v_2 = \{6.433 \quad 17.323\}$$

Iteration 2

$$d_{11} = \sqrt{(5-3.874)^2 + (20-30.641)^2} = 10.701$$
$$d_{21} = \sqrt{(5-6.433)^2 + (20-17.323)^2} = 3.037$$
$$d_{12} = \sqrt{(3.5-3.874)^2 + (35-30.641)^2} = 5.643$$
$$d_{22} = \sqrt{(3.5-6.433)^2 + (35-17323)^2} = 17.919$$
$$d_{13} = \sqrt{(4-3.874)^2 + (25-30.641)^2} = 5.643$$
$$d_{23} = \sqrt{(4-6.433)^2 + (25-17.323)^2} = 8.054$$
$$d_{14} = \sqrt{(7-3.874)^2 + (10-30.641)^2} = 20.877$$
$$d_{24} = \sqrt{(7-6.433)^2 + (10-17.323)^2} = 7.345$$
$$d_{15} = \sqrt{(8-3.874)^2 + (22-30.641)^2} = 9.576$$
$$d_{25} = \sqrt{(8-6.433)^2 + (22-17.323)^2} = 4.933$$

$$\mu_{11} = \left[\left(\frac{d_{11}}{d_{11}}\right)^2 + \left(\frac{d_{11}}{d_{21}}\right)^2\right]^{-1} = \left[1 + \left(\frac{10.701}{3.037}\right)^2\right]^{-1} = 0.076$$
$$\mu_{21} = 1 - \mu_{11} = 1 - 0.076 = 0.925$$
$$\mu_{12} = \left[\left(\frac{d_{12}}{d_{12}}\right)^2 + \left(\frac{d_{12}}{d_{22}}\right)^2\right]^{-1} = \left[1 + \left(\frac{4.375}{17.919}\right)^2\right]^{-1} = 0.944$$
$$\mu_{22} = 1 - \mu_{12} = 1 - 0.944 = 0.056$$
$$\mu_{13} = \left[\left(\frac{d_{13}}{d_{13}}\right)^2 + \left(\frac{d_{13}}{d_{23}}\right)^2\right]^{-1} = \left[1 + \left(\frac{5.643}{8.054}\right)^2\right]^{-1} = 0.671$$
$$\mu_{23} = 1 - \mu_{13} = 1 - 0.671 = 0.329$$
$$\mu_{14} = \left[\left(\frac{d_{14}}{d_{14}}\right)^2 + \left(\frac{d_{14}}{d_{24}}\right)^2\right]^{-1} = \left[1 + \left(\frac{20.877}{8.054}\right)^2\right]^{-1} = 0.110$$
$$\mu_{24} = 1 - \mu_{14} = 1 - 0.110 = 0.890$$
$$\mu_{15} = \left[\left(\frac{d_{15}}{d_{15}}\right)^2 + \left(\frac{d_{15}}{d_{25}}\right)^2\right]^{-1} = \left[1 + \left(\frac{9.576}{4.933}\right)^2\right]^{-1} = 0.210$$
$$\mu_{25} = 1 - \mu_{15} = 1 - 0.210 = 0.790$$

$$\underset{\sim}{U}^{(1)} = \begin{bmatrix} 0.076 & 0.944 & 0.671 & 0.110 & 0.210 \\ 0.924 & 0.056 & 0.329 & 0.890 & 0.790 \end{bmatrix}$$

$$\varepsilon_L = \left\| U^{(2)} - U^{(1)} \right\| = 0.054 \geq 0.01$$

not converged

After 11 cycles converges:

$$\underset{\sim}{U}^{(2)} = \begin{bmatrix} 0.026 & 0.984 & 0.447 & 0.115 & 0.108 \\ 0.974 & 0.016 & 0.553 & 0.885 & 0.892 \end{bmatrix}$$

$$v_1 = \{3.672 \quad 32.856\};\ v_2 = \{6.293 \quad 18.309\}$$
$$\varepsilon_L = 0.008$$

10.10

$$\underset{\sim}{U}^{(0)} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$v_{ij} = \frac{\sum_{k=1}^{n} (\mu_{ik})^{m'} \cdot x_{kj}}{\sum_{k=1}^{n} (\mu_{ik})^{m'}}$$

where for c =1,

$$v_{1j} = \frac{\mu_1^2 x_{1j} + \mu_2^2 x_{2j} + \mu_3^2 x_{3j}}{\mu_1^2 + \mu_2^2 + \mu_3^2}$$
$$v_{1j} = \frac{1x_{1j} + 1x_{2j} + 0x_{3j}}{\mu_1^2 + \mu_2^2 + \mu_3^2}$$
$$v_{11} = \frac{50+40}{2} = 45$$
$$v_{11} = \frac{10+12}{2} = 11$$
$$v_1 = \{45, 11\}$$

and for c =2,

$$v_{21} = \frac{20}{1} = 20$$
$$v_{22} = \frac{5}{1} = 5$$
$$v_2 = \{20, 5\}$$

Now calculate the distance measures using Eq (11.29).

$$d_{11} = \sqrt{(50-45)^2 + (10-11)^2} = 5.0990$$
$$d_{12} = \sqrt{(40-45)^2 + (12-11)^2} = 5.0990$$
$$d_{13} = \sqrt{(20-45)^2 + (5-11)^2} = 25.7099$$
$$d_{21} = \sqrt{(50-20)^2 + (10-5)^2} = 30.4138$$
$$d_{22} = \sqrt{(40-20)^2 + (12-5)^2} = 21.1896$$

$$d_{23} = \sqrt{(20-20)^2 + (5-5)^2} = 0$$

$$\mu_{11} = \left[\left(\frac{d_{11}}{d_{11}}\right)^2 + \left(\frac{d_{11}}{d_{21}}\right)^2\right]^{-1} = \left[\left(\frac{5.099}{5.099}\right)^2 + \left(\frac{5.099}{30.4138}\right)^2\right]^{-1} = 0.9727$$

$$\mu_{12} = \left[\left(\frac{d_{12}}{d_{12}}\right)^2 + \left(\frac{d_{12}}{d_{22}}\right)^2\right]^{-1} = \left[1^2 + \left(\frac{5.099}{21.1896}\right)^2\right]^{-1} = 0.9453$$

$$\mu_{13} = \left[\left(\frac{d_{13}}{d_{13}}\right)^2 + \left(\frac{d_{13}}{d_{23}}\right)^2\right]^{-1} = \left[1^2 + \left(\frac{25.7099}{0}\right)^2\right]^{-1} = 0$$

$$\mu_{21} = \left[\left(\frac{d_{21}}{d_{11}}\right)^2 + \left(\frac{d_{21}}{d_{21}}\right)^2\right]^{-1} = \left[\left(\frac{30.4138}{5.099}\right)^2 + (1)^2\right]^{-1} = 0.0273$$

$$\mu_{22} = \left[\left(\frac{d_{22}}{d_{12}}\right)^2 + \left(\frac{d_{22}}{d_{22}}\right)^2\right]^{-1} = \left[\left(\frac{21.1896}{5.099}\right)^2 + (1)^2\right]^{-1} = 0.05474$$

$$\mu_{23} = \left[\left(\frac{d_{23}}{d_{13}}\right)^2 + \left(\frac{d_{23}}{d_{23}}\right)^2\right]^{-1} = \left[\left(\frac{0}{25.7099}\right)^2 + 1^2\right]^{-1} = 1$$

$$\underset{\sim}{U}^{(1)} = \begin{bmatrix} 0.9727 & 0.9453 & 0 \\ 0.0273 & 0.0547 & 1 \end{bmatrix}$$

where for c =1,

$$v_{1j} = \frac{\mu_1^2 x_{1j} + \mu_2^2 x_{2j} + \mu_3^2 x_{3j}}{\mu_1^2 + \mu_2^2 + \mu_3^2}$$

$$v_{1j} = \frac{0.9727 x_{1j} + 0.9453 x_{2j} + 0.x_{3j}}{\mu_1^2 + \mu_2^2 + \mu_3^2}$$

$$v_{11} = 46.9888$$

$$v_{12} = 11.4530$$

$$v_1 = \{46.9888, 11.4530\}$$

and for c =2,

$$v_{21} = 23.4653$$

$$v_{22} = 5.9073$$

$$v_2 = \{23.4653, 5.9073\}$$

Now calculate the distance measures using Eq (11.29).

$$d_{11} = \sqrt{(50-46.9888)^2 + (10-11.4530)^2} = 3.3434$$

$$d_{12} = \sqrt{(40-46.9888)^2 + (12-11.4530)^2} = 7.010$$

$$d_{13} = \sqrt{(20-46.9888)^2 + (5-11.4530)^2} = 27.7495$$

$$d_{21} = \sqrt{(50-23.4653)^2 + (10-5.9073)^2} = 26.8485$$

$$d_{22} = \sqrt{(40-23.4653)^2 + (12-5.9073)^2} = 17.6215$$

$$d_{23} = \sqrt{(20-23.4653)^2 + (5-5.9073)^2} = 3.5821$$

$$\mu_{11} = \left[\left(\frac{d_{11}}{d_{11}}\right)^2 + \left(\frac{d_{11}}{d_{21}}\right)^2\right]^{-1} = \left[(1)^2 + \left(\frac{3.1219}{26.8485}\right)^2\right]^{-1} = 0.984$$

$$\mu_{12} = \left[\left(\frac{d_{12}}{d_{12}}\right)^2 + \left(\frac{d_{12}}{d_{22}}\right)^2\right]^{-1} = \left[1^2 + \left(\frac{7.010}{17.6215}\right)^2\right]^{-1} = 0.8634$$

$$\mu_{13} = \left[\left(\frac{d_{13}}{d_{13}}\right)^2 + \left(\frac{d_{13}}{d_{23}}\right)^2\right]^{-1} = \left[1^2 + \left(\frac{28.0455}{3.5821}\right)^2\right]^{-1} = 0.01605$$

$$\mu_{21} = \left[\left(\frac{d_{21}}{d_{11}}\right)^2 + \left(\frac{d_{21}}{d_{21}}\right)^2\right]^{-1} = \left[\left(\frac{26.8485}{3.3434}\right)^2 + (1)^2\right]^{-1} = 0.01527$$

$$\mu_{22} = \left[\left(\frac{d_{22}}{d_{12}}\right)^2 + \left(\frac{d_{22}}{d_{22}}\right)^2\right]^{-1} = \left[\left(\frac{17.6215}{7.010}\right)^2 + (1)^2\right]^{-1} = 0.1366$$

$$\mu_{23} = \left[\left(\frac{d_{23}}{d_{13}}\right)^2 + \left(\frac{d_{23}}{d_{23}}\right)^2\right]^{-1} = \left[\left(\frac{3.5821}{27.7495}\right)^2 + 1^2\right]^{-1} = 0.9836$$

$$\underset{\sim}{U}^{(2)} = \begin{bmatrix} 0.984 & 0.8643 & 0.01605 \\ 0.0152 & 0.1366 & 0.9836 \end{bmatrix}$$

10.11 a)

$$U^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$V_1(0.679, 85.5, 2.25)$

$V_2\left(\frac{0.7056+0.701+0.718}{3}, \frac{93+91+98.3}{3}, \frac{2.177+2.253+2.177}{3}\right)$

$= V_2(0.7082, 94.1, 2.202)$

$d_{11} = 0$

$d_{12} = \sqrt{(0.679-0.7082)^2 + (85.3-94.1)^2 + (2.25-2.202)^2} = 8.6$

$d_{21} = \sqrt{(0.7056-0.679)^2 + (93-85.5)^2 + (2.177-2.25)^2} = 7.5$

$d_{22} = \sqrt{(0.7056-0.7082)^2 + (93-94.1)^2 + (2.177-2.202)^2} = 1.1$

$d_{31} = \sqrt{(0.701-0.679)^2 + (91-85.5)^2 + (2.253-2.25)^2} = 5.5$

$d_{32} = \sqrt{(0.701-0.7082)^2 + (91-94.1)^2 + (2.253-2.202)^2} = 3.1$

$d_{41} = \sqrt{(0.718-0.679)^2 + (98.3-85.5)^2 + (2.177-2.25)^2} = 12.8$

$d_{42} = \sqrt{(0.718-0.7082)^2 + (98.3-94.1)^2 + (2.177-2.202)^2} = 4.2$

Therefore $U^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

$U^{(1)} = U^{(0)}$ converged

b) Assuming $U^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

$$U^{(7)} = \begin{bmatrix} 0.992 & 0.065 & 0.392 & 0.080 \\ 0.008 & 0.935 & 0.608 & 0.920 \end{bmatrix}$$

$V_1(0.682, 86.309, 2.250), V_2(0.710, 94.769, 2.191)$

$\varepsilon_L = 0.008 < 0.01$ converged

c) Using U in part b

$$F_c(U) = \frac{tr(U * U^T)}{4} = 0.810$$

d) Using equation 10.41 and U in part b

$r_{11} = 1$

$r_{12} = \min(0.992, 0.065) + \min(0.008, 0.935)$
$= 0.065 + 0.008 = 0.073$

$r_{13} = \min(0.992, 0.392) + \min(0.008, 0.608)$
$= 0.392 + 0.008 = 0.400$

$r_{14} = \min(0.992, 0.008) + \min(0.008, 0.920)$
$= 0.080 + 0.008 = 0.088$

$r_{22} = 1$

$r_{23} = \min(0.065, 0.392) + \min(0.935, 0.608)$
$= 0.065 + 0.608 = 0.673$

$r_{24} = \min(0.065, 0.080) + \min(0.935, 0.920)$
$= 0.065 + 0.920 = 0.985$

$r_{33} = 1$

$r_{34} = \min(0.392, 0.080) + \min(0.608, 0.920)$
$= 0.080 + 0.608 = 0.688$

$r_{44} = 1$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.073 & 0.400 & 0.088 \\ & 1 & 0.673 & 0.985 \\ & & 1 & 0.688 \\ sym. & & & 1 \end{bmatrix}$$

10.12 a)

$$\underset{\sim}{U} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$V_1(1.35, 395), V_2(6.5, 278.33)$

b) Assuming $\underset{\sim}{U}^{(0)} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

$$U^{(3)} = \begin{bmatrix} 0.971 & 0.934 & 0.015 & 0.138 & 0.008 \\ 0.029 & 0.066 & 0.985 & 0.862 & 0.992 \end{bmatrix}$$

$V_1(1.412, 395.14), V_2(6.303, 275.87)$

$\varepsilon_L = 0.003 < 0.01$ converged

c) Using U in part b

$F_c(U) = \frac{tr(U * U^T)}{5} = 0.907$

d) Using equation 10.41 and U in part b

$r_{11} = 1$

$r_{12} = 0.934 + 0.029 = 0.963$

$r_{13} = 0.015 + 0.029 = 0.044$

$r_{14} = 0.138 + 0.029 = 0.167$

$r_{15} = 0.008 + 0.029 = 0.037$

$r_{22} = 1$

$r_{23} = 0.015 + 0.066 = 0.081$

$r_{24} = 0.138 + 0.066 = 0.204$

$r_{25} = 0.008 + 0.066 = 0.074$

$r_{33} = 1$

$r_{34} = 0.015 + 0.862 = 0.877$

$r_{35} = 0.008 + 0.985 = 0.993$

$r_{44} = 1$

$r_{45} = 0.008 + 0.862 = 0.870$

$r_{55} = 1$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.963 & 0.044 & 0.167 & 0.037 \\ & 1 & 0.081 & 0.204 & 0.074 \\ & & 1 & 0.877 & 0.993 \\ & sym. & & 1 & 0.870 \\ & & & & 1 \end{bmatrix}$$

10.13 a)

$$\underset{\sim}{U} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$V_1(-55, 3.125), V_2(1.667, 3.083)$

b) Assuming $\underset{\sim}{U}^{(0)} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

$$U^{(3)} = \begin{bmatrix} 0.929 & 0.986 & 0.790 & 0.088 & 0.039 \\ 0.071 & 0.014 & 0.230 & 0.912 & 0.961 \end{bmatrix}$$

$V_1(-46.61, 3.222), V_2(14.50, 2.899)$

$\varepsilon_L = 0.0096 < 0.01$ converged

c) Using U in part b

$F_c(U) = \frac{tr(U * U^T)}{5} = 0.850$

d) Using equation 10.40 and U in part b

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.943 & 0.841 & 0.160 & 0.111 \\ & 1 & 0.784 & 0.103 & 0.054 \\ & & 1 & 0.319 & 0.270 \\ & sym. & & 1 & 0.951 \\ & & & & 1 \end{bmatrix}$$

CHAPTER 11

Fuzzy Classification and Pattern Recognition

11.1

$\underset{\sim}{a} = \underset{\sim}{b}$

$\underset{\sim}{a} \cdot \underset{\sim}{b}^T = \underset{\sim}{a} \cdot \underset{\sim}{a}^T$

$\overset{n}{\underset{i=1}{\vee}} (a_i \wedge a_i) = \overset{n}{\underset{i=1}{\max}} a_i$

$\underset{\sim}{a} \oplus \underset{\sim}{b}^T = \underset{\sim}{a} \oplus \underset{\sim}{a}^T$

$\overset{n}{\underset{i=1}{\wedge}} (a_i \vee a_i) = \overset{n}{\underset{i=1}{\min}} a_i$

11.2

$\hat{a} = \hat{b} = 1; \quad \underset{\wedge}{a} = \underset{\wedge}{b} = 0; \quad \underset{\sim}{a} = \underset{\sim}{b}$

i) $\underset{\sim}{a} \cdot \underset{\sim}{b}^T = \underset{\sim}{a} \cdot \underset{\sim}{a}^T$

$\overset{n}{\underset{i=1}{\vee}} (a_i \wedge a_i) = \overset{n}{\underset{i=1}{\max}} a_i = \hat{a} = 1$

ii) $\underset{\sim}{a} \oplus \underset{\sim}{b}^T = \underset{\sim}{a} \oplus \underset{\sim}{a}^T$

$= \overset{n}{\underset{i=1}{\wedge}} (a_i \vee a_i) = \overset{n}{\underset{i=1}{\min}} a_i = \underset{\wedge}{a} = 0$

11.3

a) $a = \{a_1, a_2, \ldots, a_n\}$ and

$b = \{b_1, b_2, \ldots, b_n\}$

$\overline{a \cdot b} = 1 - a \cdot b = 1 - \{a_1, a_2, \ldots, a_n\} \cdot \{b_1, b_2, \ldots, b_n\}$

$= 1 - \vee \{a_1 \wedge b_1, a_2 \wedge b_2, \ldots, a_n \wedge b_n\}$

$= \wedge \left\{ \begin{matrix} 1 - (a_1 \wedge b_1), 1 - (a_2 \wedge b_2), \ldots, \\ 1 - (a_n \wedge b_n) \end{matrix} \right\}$

but $1 - (a_i \wedge b_i) = (1 - a_i) \vee (1 - b_i)$

thus $\overline{a \cdot b} =$

$= \wedge \left\{ \begin{matrix} (1 - a_1) \vee (1 - b_1), (1 - a_2) \vee \\ (1 - b_2), \ldots, (1 - a_n) \vee (1 - b_n) \end{matrix} \right\}$

$= \{1 - a_1, 1 - a_2, \ldots 1 - a_n\} \oplus \{1 - b_1, 1 - b_2, \ldots 1 - b_n\}$

$= \{\overline{a_1}, \overline{a_2}, \ldots, \overline{a_n}\} \oplus \{\overline{b_1}, \overline{b_2}, \ldots, \overline{b_n}\}$

$= \overline{a} \oplus \overline{b}$

b)

$a \cdot \overline{a} = \{a_1, a_2, \ldots, a_n\} \cdot \{1 - a_1, 1 - a_2, \ldots, 1 - a_n\}$

$= \vee \left\{ \begin{matrix} a_1 \wedge (1 - a_1), a_2 \wedge (1 - a_2), \ldots, \\ a_n \wedge (1 - a_n) \end{matrix} \right\}$

therefore $0 \le a_i \le 1$.

If $a_i < 0.5, \rightarrow (1 - a_i) > 0.5$

therefore,

$a_i \wedge 1 - a_i = \min[a_i, (1 - a_i)] < 0.5$.

If $a_i > 0.5, \rightarrow (1 - a_i) < 0.5$

therefore, $a_i \wedge 1 - a_i < 0.5$.

If $a_i = 0.5, \rightarrow (1 - a_i) = 0.5$

therefore, $a_i \wedge 1 - a_i = 0.5$.

We showed that whatever a_i is, the expression $a \cdot \overline{a} \le 0.5$ is correct.

11.4 a) The expression posed in this problem is wrong. If, for example, A is a normal fuzzy set, then we get the following:

$A = [a_1, a_2, \ldots, a_n]$

$(A, A)_1 = [(A.A) \wedge \overline{(A \oplus A)}]$

$(A, A)_2 = \frac{1}{2}[(A.A) + \overline{(A \oplus A)}]$

$A.A = \max\left[\min(a_1,a_1), \min(a_2,a_2), \ldots, \min(a_n,a_n)\right] = \max[a_1,a_2,\ldots,a_n] = \hat{a} = 1$

$A \oplus A = \min\left[\min(a_1,a_1), \max(a_2,a_2), \ldots, \max(a_n,a_n)\right] = \min[a_1,a_2,\ldots,a_n] = \underset{\wedge}{a}$

$\overline{A \oplus A} = 1 - \underset{\wedge}{a}$

$(A,A)_1 = 1 \wedge \left(1 - \underset{\wedge}{a}\right) = 1 - \underset{\wedge}{a}$

$(A,A)_2 = \frac{1}{2}[1 + 1 - \underset{\wedge}{a}] = 1 - \underset{\wedge}{a}\ /\ 2$

(

b)

$\mu_1 = \underset{\sim}{A} \cdot \overline{\underset{\sim}{A}} \qquad \mu_2 = \underset{\sim}{A} \oplus \overline{\underset{\sim}{A}} \qquad \overline{\mu_2} = \overline{\underset{\sim}{A} \oplus \overline{\underset{\sim}{A}}}$

From equation 11.11 we know that $\mu_1 \le 0.5$ and $\mu_2 \le 0.5$ thus $\overline{\mu_2} \ge 0.5$.

$$\left(\underset{\sim}{A}, \overline{\underset{\sim}{A}}\right)_1 = \underset{\sim}{A} \cdot \overline{\underset{\sim}{A}} \wedge \overline{\underset{\sim}{A} \oplus \overline{\underset{\sim}{A}}} = \mu_1 \wedge \overline{\mu}_2$$

$$= \mu_1 \le 0.5$$

The second part of part b is wrong. The following expression must be replaced:

$$\left(\underset{\sim}{A}, \overline{\underset{\sim}{A}}\right)_2 \ge \frac{1}{4}$$

The proof:

$$\left(\underset{\sim}{A}, \overline{\underset{\sim}{A}}\right)_2 = \frac{1}{2}\left[\underset{\sim}{A} \cdot \overline{\underset{\sim}{A}} + \overline{\underset{\sim}{A} \oplus \overline{\underset{\sim}{A}}}\right] = \frac{1}{2}\left[\mu_1 + \overline{\mu}_2\right]$$

$$\ge \frac{1}{2}[0 + 0.5] \ge \frac{1}{4}$$

11.5

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1 = \left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) \wedge \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 = \frac{1}{2}\left[\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) \wedge \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)\right]$$

case I: Let $\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) = \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$

then $\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1 = \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 = \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$$

case II: Let $\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) < \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1 = \left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right)$$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 > \frac{1}{2}\left[\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) + \left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right)\right] =$$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 \ge \left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right)$$

case II: Let $\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) > \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$

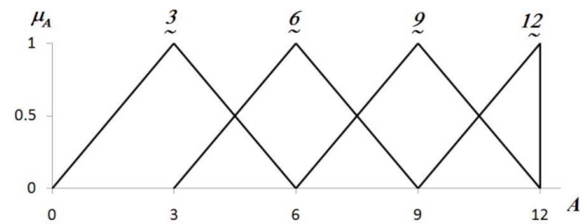
$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1 = \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$$

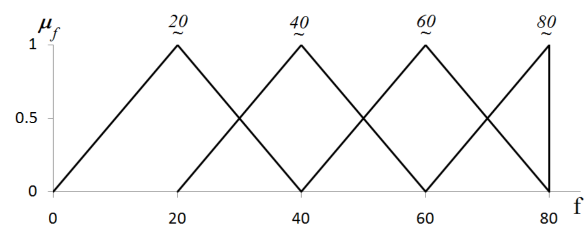
$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 > \frac{1}{2}\left[\left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right) + \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)\right] = \overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}$$

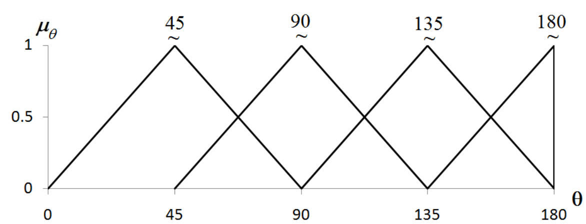
In all three cases,

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 \ge \left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1$$

11.6







C_1=[3V, 20Hz, 45°]
C_2=[6V, 40Hz, 90°]
C_3=[9V, 60Hz, 135°]
C_4=[12V, 80Hz, 180°]

$A_{11}=0 \qquad A_{21}=0 \qquad A_{31}=0.78$
$A_1 = 0.5\times0 + 0.25\times0 + 0.25\times0.78 = 0.19$

$A_{12}=0.67 \qquad A_{22}=0.5 \qquad A_{32}=0.22$
$A_2 = 0.5\times0.67 + 0.25\times05 + 0.25\times0.22 = 0.52$

A_{13}=0.33 A_{23}=0.5 A_{33}=0
A_3= 0.5×0.33 + 0.25×0.5 + 0.25×0 = 0.29

A_{14}=0 A_{24}=0 A_{34}=0
A_4= 0.5×0 + 0.25×0 + 0.25×0 = 0

Therefore, the best match is C_2.

11.7
Voltage

$$\underset{\sim}{3}=\left\{\frac{0}{0}+\frac{0.33}{1}+\frac{0.67}{2}+\frac{1}{3}+\frac{0.67}{4}+\frac{0.33}{5}+\frac{0}{6}\right\}$$
$$\underset{\sim}{6}=\left\{\frac{0}{3}+\frac{0.33}{4}+\frac{0.67}{5}+\frac{1}{6}+\frac{0.67}{7}+\frac{0.33}{8}+\frac{0}{9}\right\}$$
$$\underset{\sim}{9}=\left\{\frac{0}{6}+\frac{0.33}{7}+\frac{0.67}{8}+\frac{1}{9}+\frac{0.67}{10}+\frac{0.33}{11}+\frac{0}{12}\right\}$$
$$\underset{\sim}{12}=\left\{\frac{0}{9}+\frac{0.33}{10}+\frac{0.67}{11}+\frac{1}{12}\right\}$$

Frequency

$$\underset{\sim}{20}=\left\{\frac{0}{0}+\frac{0.5}{10}+\frac{1}{20}+\frac{0.5}{30}+\frac{0}{40}\right\}$$
$$\underset{\sim}{40}=\left\{\frac{0}{20}+\frac{0.5}{30}+\frac{1}{40}+\frac{0.5}{50}+\frac{0}{60}\right\}$$
$$\underset{\sim}{60}=\left\{\frac{0}{40}+\frac{0.5}{50}+\frac{1}{60}+\frac{0.5}{70}+\frac{0}{80}\right\}$$
$$\underset{\sim}{80}=\left\{\frac{0}{60}+\frac{0.5}{70}+\frac{1}{80}\right\}$$

Phase

$$\underset{\sim}{45}=\left\{\frac{0}{0}+\frac{0.50}{22.5}+\frac{1}{45}+\frac{0.89}{50}+\frac{0.44}{70}+\frac{0}{90}\right\}$$
$$\underset{\sim}{90}=\left\{\frac{0}{45}+\frac{0.11}{50}+\frac{0.56}{70}+\frac{1}{90}+\frac{0.56}{110}+\frac{0.33}{120}+\frac{0.11}{130}\right\}$$
$$\underset{\sim}{135}=\left\{\frac{0}{90}+\frac{0.44}{110}+\frac{0.67}{120}+\frac{0.89}{130}+\frac{1}{135}+\frac{0}{180}\right\}$$
$$\underset{\sim}{180}=\left\{\frac{0}{135}+\frac{0.5}{158.5}+\frac{1}{180}\right\}$$

$$A_{11}=(B_{voltage},\underset{\sim}{3})_{1\ or\ 2}$$
$$(A_{11})_1=(B_{voltage}\cdot\underset{\sim}{3})\wedge\overline{(B_{voltage}\oplus\underset{\sim}{3})}$$
$$B_{voltage}\cdot\underset{\sim}{3}=\vee\left\{\frac{0}{1}+\frac{0.3}{2}+\frac{0.7}{3}+\frac{0.67}{4}+\frac{0.33}{5}+\frac{0}{6}\right\}=0.7$$
$$B_{voltage}\oplus\underset{\sim}{3}=\wedge\left\{\frac{0.33}{1}+\frac{0.67}{2}+\frac{1}{3}+\frac{0.85}{4}+\frac{0.1}{5}+\frac{0}{6}\right\}=0$$
$$\overline{(B_{voltage}\oplus\underset{\sim}{3})}=1 \qquad (A_{11})_1=0.7$$
$$(A_{11})_2=\frac{1}{2}\left[(B_{voltage}\cdot\underset{\sim}{3})+\overline{(B_{voltage}\oplus\underset{\sim}{3})}\right]=0.85$$

$$A_{21}=(B_{frequency},\underset{\sim}{20})_{1\ or\ 2}$$
$$(A_{21})_1=(B_{frequency}\cdot\underset{\sim}{20})\wedge\overline{(B_{frequency}\oplus\underset{\sim}{20})}$$
$$B_{frequency}\cdot\underset{\sim}{20}=\vee(0,0.5)=0.5$$
$$B_{frequency}\oplus 20=0 \qquad \overline{(B_{frequency}\oplus 20)}=1$$
$$(A_{21})_1=0.5$$
$$(A_{21})_2=\frac{1}{2}\left[(B_{frequency}\cdot\underset{\sim}{20})+\overline{(B_{frequency}\oplus\underset{\sim}{20})}\right]=0.75$$

$$A_{31}=(B_{phase},\underset{\sim}{45})_{1\ or\ 2}$$
$$(A_{31})_1=(B_{phase}\cdot\underset{\sim}{45})\wedge\overline{(B_{phase}\oplus\underset{\sim}{45})}$$
$$B_{phase}\cdot\underset{\sim}{45}=\vee(0,0.3,0)=0.3$$
$$B_{phase}\oplus\underset{\sim}{45}=0 \qquad \overline{(B_{phase}\oplus\underset{\sim}{45})}=1$$
$$(A_{31})_1=0.3$$
$$(A_{31})_2=\frac{1}{2}\left[(B_{phase}\cdot\underset{\sim}{45})+\overline{(B_{phase}\oplus\underset{\sim}{45})}\right]=0.65$$

$$(A_1)_1=0.5\times0.7+0.25\times0.5+0.25\times0.3=0.55$$
$$(A_1)_2=0.5\times0.85+0.25\times0.75+0.25\times0.65=0.78$$

$$A_{12}=(B_{voltage},\underset{\sim}{6})_{1\ or\ 2}$$
$$(A_{12})_1=(B_{voltage}\cdot\underset{\sim}{6})\wedge\overline{(B_{voltage}\oplus\underset{\sim}{6})}$$
$$B_{voltage}\cdot\underset{\sim}{6}=0.67$$
$$B_{voltage}\oplus 6=0 \qquad \overline{(B_{voltage}\oplus 6)}=1$$
$$(A_{12})_1=0.67$$
$$(A_{12})_2=\frac{1}{2}\left[(B_{voltage}\cdot\underset{\sim}{6})+\overline{(B_{voltage}\oplus\underset{\sim}{6})}\right]=0.83$$

$$A_{22}=(B_{frequency},\underset{\sim}{40})_{1\ or\ 2}$$
$$(A_{22})_1=(B_{frequency}\cdot\underset{\sim}{40})\wedge\overline{(B_{frequency}\oplus\underset{\sim}{40})}$$
$$B_{frequency}\cdot\underset{\sim}{40}=1$$
$$B_{frequency}\oplus 40=0 \qquad \overline{(B_{frequency}\oplus 40)}=1$$
$$(A_{22})_1=1$$
$$(A_{22})_2=\frac{1}{2}\left[(B_{frequency}\cdot\underset{\sim}{40})+\overline{(B_{frequency}\oplus\underset{\sim}{40})}\right]=1$$

$$A_{32}=(B_{phase},\underset{\sim}{90})_{1\ or\ 2}$$
$$(A_{32})_1=(B_{phase}\cdot\underset{\sim}{90})\wedge\overline{(B_{phase}\oplus\underset{\sim}{90})}$$
$$B_{phase}\cdot\underset{\sim}{90}=0.6$$
$$B_{phase}\oplus\underset{\sim}{90}=0 \qquad \overline{(B_{phase}\oplus\underset{\sim}{90})}=1$$
$$(A_{32})_1=0.6$$
$$(A_{32})_2=\frac{1}{2}\left[(B_{phase}\cdot\underset{\sim}{90})+\overline{(B_{phase}\oplus\underset{\sim}{90})}\right]=0.8$$

$$(A_2)_1=0.5\times0.67+0.25\times1+0.25\times0.6=0.74$$
$$(A_2)_2=0.5\times0.83+0.25\times1+0.25\times0.8=0.87$$

$$A_{13}=(B_{voltage},\underset{\sim}{9})_{1\ or\ 2}$$
$$(A_{13})_1=(B_{voltage}\cdot\underset{\sim}{9})\wedge\overline{(B_{voltage}\oplus\underset{\sim}{9})}$$
$$B_{voltage}\cdot\underset{\sim}{9}=0$$

$B_{voltage} \oplus 9 = 0 \qquad \overline{(B_{voltage} \oplus 9)} = 1$

$(A_{13})_1 = 0$

$(A_{13})_2 = \frac{1}{2}\left[(B_{voltage} \cdot \underset{\sim}{9}) + \overline{(B_{voltage} \oplus \underset{\sim}{9})}\right] = 0.5$

$A_{23} = (B_{frequency}, \underset{\sim}{60})_{1\ or\ 2}$

$(A_{23})_1 = (B_{frequency} \cdot \underset{\sim}{60}) \wedge \overline{(B_{frequency} \oplus \underset{\sim}{60})}$

$B_{frequency} \cdot \underset{\sim}{60} = 0.4$

$B_{frequency} \oplus 60 = 0 \qquad \overline{(B_{frequency} \oplus 60)} = 1$

$(A_{23})_1 = 0.4$

$(A_{23})_2 = \frac{1}{2}\left[(B_{frequency} \cdot \underset{\sim}{60}) + \overline{(B_{frequency} \oplus \underset{\sim}{60})}\right] = 0.7$

$A_{33} = (B_{phase}, \underset{\sim}{135})_{1\ or\ 2}$

$(A_{33})_1 = (B_{phase} \cdot \underset{\sim}{135}) \wedge \overline{(B_{phase} \oplus \underset{\sim}{135})}$

$B_{phase} \cdot \underset{\sim}{135} = 0.67$

$B_{phase} \oplus \underset{\sim}{135} = 0 \qquad \overline{(B_{phase} \oplus \underset{\sim}{135})} = 1$

$(A_{33})_1 = 0.67$

$(A_{33})_2 = \frac{1}{2}\left[(B_{phase} \cdot \underset{\sim}{135}) + \overline{(B_{phase} \oplus \underset{\sim}{135})}\right] = 0.83$

$(A_3)_1 = 0.5 \times 0 + 0.25 \times 0.4 + 0.25 \times 0.67 = 0.27$
$(A_3)_2 = 0.5 \times 0.5 + 0.25 \times 0.7 + 0.25 \times 0.83 = 0.63$

$A_{14} = (B_{voltage}, \underset{\sim}{12})_{1\ or\ 2}$

$(A_{14})_1 = (B_{voltage} \cdot \underset{\sim}{12}) \wedge \overline{(B_{voltage} \oplus \underset{\sim}{12})}$

$B_{voltage} \cdot \underset{\sim}{12} = 0$

$B_{voltage} \oplus 12 = 0 \qquad \overline{(B_{voltage} \oplus 12)} = 1$

$(A_{14})_1 = 0$

$(A_{14})_2 = \frac{1}{2}\left[(B_{voltage} \cdot \underset{\sim}{12}) + \overline{(B_{voltage} \oplus \underset{\sim}{12})}\right] = 0.5$

$A_{24} = (B_{frequency}, \underset{\sim}{80})_{1\ or\ 2}$

$(A_{24})_1 = (B_{frequency} \cdot \underset{\sim}{80}) \wedge \overline{(B_{frequency} \oplus \underset{\sim}{80})}$

$B_{frequency} \cdot \underset{\sim}{80} = 0$

$B_{frequency} \oplus 80 = 0 \qquad \overline{(B_{frequency} \oplus 80)} = 1$

$(A_{24})_1 = 0$

$(A_{24})_2 = \frac{1}{2}\left[(B_{frequency} \cdot \underset{\sim}{80}) + \overline{(B_{frequency} \oplus \underset{\sim}{80})}\right] = 0.5$

$A_{34} = (B_{phase}, \underset{\sim}{180})_{1\ or\ 2}$

$(A_{34})_1 = (B_{phase} \cdot \underset{\sim}{180}) \wedge \overline{(B_{phase} \oplus \underset{\sim}{180})}$

$B_{phase} \cdot \underset{\sim}{180} = 0$

$B_{phase} \oplus \underset{\sim}{180} = 0 \qquad \overline{(B_{phase} \oplus \underset{\sim}{180})} = 1$

$(A_{34})_1 = 0$

$(A_{34})_2 = \frac{1}{2}\left[(B_{phase} \cdot \underset{\sim}{180}) + \overline{(B_{phase} \oplus \underset{\sim}{180})}\right] = 0.5$

$(A_4)_1 = 0.5 \times 0 + 0.25 \times 0 + 0.25 \times 0 = 0$
$(A_4)_2 = 0.5 \times 0.5 + 0.25 \times 0.5 + 0.25 \times 0.5 = 0.5$

Therefore, the best match is C_2.

11.8

C_1=[Depth=*very low*, Clay=*very low*, moisture=*very low*]

C_2=[Depth=*low*, Clay= *low*, moisture=*low*]

C_3=[Depth=*medium*, Clay=*medium*, moisture=*medium*]

C_4=[Depth=*high*, Clay=*high*, moisture=*high*]

$A_{11}=0 \quad A_{21}=0 \quad A_{31}=0$
$A_1=0.4\times0 + 0.4\times0 + 0.2\times0 = 0$

$A_{12}=0 \quad A_{22}=0.704 \quad A_{32}=0.3$
$A_2=0.4\times0 + 0.4\times0.704 + 0.2\times0.3 = 0.34$

$A_{13}=0.5 \quad A_{23}=0.296 \quad A_{33}=0.7$
$A_3=0.4\times0.5 + 0.4\times0.296 + 0.2\times0.7 = 0.46$

$A_{14}=0.5 \quad A_{24}=0 \quad A_{34}=0$
$A_4=0.4\times0.5 + 0.4\times0 + 0.2\times0 = 0.20$

Therefore, based on the metric 1 and 2 the best match is C_3.

11.9

$A_1=0.4(A_{11}) + 0.4(A_{21}) + 0.2(A_{31})$

$(A_{11})_1= (B_{depth}, Depth_{very\ low})_1$
$=(B_{depth} \bullet Depth_{very\ low}) \wedge \overline{(B_{depth} \oplus Depth_{very\ low})}$

$(A_{11})_2= (B_{depth}, Depth_{very\ low})_2$
$= \frac{1}{2}[(B_{depth} \bullet Depth_{very\ low}) + \overline{(B_{depth} \oplus Depth_{very\ low})}]$

$B_{depth} \bullet Depth_{very\ low}$
$= \vee \{[0, 0.4, 1, 0.6, 0] \wedge [0, 0, 0, 0, 0]\} = 0$

$B_{depth} \oplus Depth_{very\ low}$
$= \wedge\{[0, 0.4, 1, 0.6, 0] \vee [0, 0, 0, 0, 0]\} = 0$

$\overline{B_{depth} \oplus Depth_{very\ low}}=1$

$(A_{11})_1=0 \qquad (A_{11})_2=0.5$

$(A_{21})_1= (B_{clay}, Clay_{very\ low})_1$
$=(B_{Clay} \bullet Clay_{very\ low}) \wedge \overline{(B_{Clay} \oplus Clay_{very\ low})}$

$(A_{21})_2 = (B_{clay}, Clay_{very\ low})_2$
$= \frac{1}{2}[(B_{clay} \bullet Clay_{very\ low}) + (\overline{B_{Clay} \oplus Clay_{very\ low}})]$
$B_{Clay} \bullet Clay_{very\ low}$
$= \vee \{[0, 0.7, 1, 0.5, 0] \wedge [0, 0, 0, 0, 0]\} = 0$
$B_{Clay} \oplus Clay_{very\ low}$
$= \wedge\{[0, 0.7, 1, 0.5, 0] \vee [0, 0, 0, 0, 0]\} = 0$
$\overline{B_{Clay} \oplus Clay_{very\ low}} = 1$
$(A_{21})_1 = 0 \qquad (A_{21})_2 = 0.5$

$(A_{31})_1 = (B_{moisture}, Moisture_{very\ low})_1$
$= (B_{Moisture} \bullet Moisture_{very\ low}) \wedge (\overline{B_{Moisture} \oplus Moisture_{very\ low}})$
$(A_{31})_2 = (B_{moisture}, Moisture_{very\ low})_2$
$= \frac{1}{2}[(B_{clay} \bullet Moisture_{very\ low}) + (\overline{B_{Moisture} \oplus Moisture_{very\ low}})]$
$B_{Moisture} \bullet Moisture_{very\ low}$
$= \vee \{[0, 0.5, 1, 0.8, 0] \wedge [0, 0, 0, 0, 0]\} = 0$
$B_{Moisture} \oplus Moisture_{very\ low}$
$= \wedge\{[0, 0.5, 1, 0.8, 0] \vee [0, 0, 0, 0, 0]\} = 0$
$\overline{B_{Clay} \oplus Moisture_{very\ low}} = 1$
$(A_{31})_1 = 0 \qquad (A_{31})_2 = 0.5$

$(A_1)_1 = 0.4\times0 + 0.4\times0 + 0.2\times0 = 0$
$(A_1)_2 = 0.4\times0.5 + 0.4\times0.5 + 0.2\times0.5 = 0.5$

$A_2 = 0.4(A_{12}) + 0.4(A_{22}) + 0.2(A_{32})$
$(A_{12})_1 = (B_{depth}, Depth_{low})_1$
$= (B_{depth} \bullet Depth_{low}) \wedge (\overline{B_{depth} \oplus Depth_{low}})$
$(A_{12})_2 = (B_{depth}, Depth_{low})_2$
$= \frac{1}{2}[(B_{depth} \bullet Depth_{low}) + (\overline{B_{depth} \oplus Depth_{low}})]$
$B_{depth} \bullet Depth_{low}$
$= \vee \{[0, 0.4, 1, 0.6, 0] \wedge [0, 0, 0, 0, 0]\} = 0$
$B_{depth} \oplus Depth_{low}$
$= \wedge\{[0, 0.4, 1, 0.6, 0] \vee [0, 0, 0, 0, 0]\} = 0$
$\overline{B_{depth} \oplus Depth_{low}} = 1$
$(A_{12})_1 = 0 \qquad (A_{12})_2 = 0.5$

$(A_{22})_1 = (B_{clay}, Clay_{low})_1$
$= (B_{Clay} \bullet Clay_{low}) \wedge (\overline{B_{Clay} \oplus Clay_{low}})$
$(A_{22})_2 = (B_{clay}, Clay_{low})_2$
$= \frac{1}{2}[(B_{clay} \bullet Clay_{low}) + (\overline{B_{Clay} \oplus Clay_{low}})]$

$B_{Clay} \bullet Clay_{low}$
$= \vee \{[0, 0.7, 1, 0.5, 0] \wedge [0.8, 0.68, 0.47, 0.42, 0.3]\}$
$= \vee [0, 0.68, 0.47, 0.42, 0] = 0.68$
$B_{Clay} \oplus Clay_{low}$
$= \wedge\{[0, 0.7, 1, 0.5, 0] \vee [0.8, 0.68, 0.47, 0.42, 0]\}$
$= \wedge[0.8, 0.7, 1, 0.5, 0.3] = 0.3$
$\overline{B_{Clay} \oplus Clay_{low}} = 0.7$
$(A_{22})_1 = 0.3 \qquad (A_{22})_2 = 0.69$

$(A_{32})_1 = (B_{moisture}, Moisture_{low})_1$
$= (B_{Moisture} \bullet Moisture_{low}) \wedge (\overline{B_{Moisture} \oplus Moisture_{low}})$
$(A_{32})_2 = (B_{moisture}, Moisture_{low})_2$
$= \frac{1}{2}[(B_{clay} \bullet Moisture_{low}) + (\overline{B_{Moisture} \oplus Moisture_{low}})]$
$B_{Moisture} \bullet Moisture_{low} =$
$\vee \{[0, 0.5, 1, 0.8, 0] \wedge [0.8, 0.65, 0.5, 0.35, 0.2]\}$
$= 0.5$
$B_{Moisture} \oplus Moisture_{low}$
$= \wedge\{[0, 0.5, 1, 0.8, 0] \vee [0.8, 0.65, 0.5, 0.35, 0.2]\}$
$= 0.2$
$\overline{B_{Clay} \oplus Moisture_{low}} = 0.8$
$(A_{32})_1 = 0.5 \qquad (A_{32})_2 = 0.65$

$(A_2)_1 = 0.4\times0 + 0.4\times0.3 + 0.2\times0.5 = 0.22$
$(A_2)_2 = 0.4\times0.5 + 0.4\times0.69 + 0.2\times0.65 = 0.61$

$A_3 = 0.4(A_{13}) + 0.4(A_{23}) + 0.2(A_{33})$
$(A_{13})_1 = (B_{depth}, Depth_{medium})_1$
$= (B_{depth} \bullet Depth_{medium}) \wedge (\overline{B_{depth} \oplus Depth_{medium}})$
$(A_{13})_2 = (B_{depth}, Depth_{medium})_2$
$= \frac{1}{2}[(B_{depth} \bullet Depth_{medium}) + (\overline{B_{depth} \oplus Depth_{medium}})]$
$B_{depth} \bullet Depth_{medium}$
$= \vee \{[0, 0.4, 1, 0.6, 0] \wedge [0.6, 0.55, 0.5, 0.45, 0.4]\}$
$= 0.5$
$B_{depth} \oplus Depth_{medium}$
$= \wedge\{[0, 0.4, 1, 0.6, 0] \vee [0.6, 0.55, 0.5, 0.45, 0.4]\}$
$= 0.4$
$\overline{B_{depth} \oplus Depth_{medium}} = 0.6$
$(A_{13})_1 = 0.5 \qquad (A_{13})_2 = 0.55$

$(A_{23})_1 = (B_{clay}, Clay_{medium})_1$
$= (B_{Clay} \bullet Clay_{medium}) \wedge (\overline{B_{Clay} \oplus Clay_{medium}})$
$(A_{23})_2 = (B_{clay}, Clay_{medium})_2$
$= \frac{1}{2}[(B_{clay} \bullet Clay_{medium}) + (\overline{B_{Clay} \oplus Clay_{medium}})]$

$B_{Clay} \bullet Clay_{medium} = \vee\{[0, 0.7, 1, 0.5, 0] \wedge [0.2, 0.32, 0.53, 0.58, 0.7]\} = 0.53$

$B_{Clay} \oplus Clay_{medium} = \wedge\{[0, 0.7, 1, 0.5, 0] \vee [0.2, 0.32, 0.53, 0.58, 0.7]\} = 0.2$

$\overline{B_{Clay} \oplus Clay_{medium}} = 0.8$

$(A_{23})_1 = 0.53 \quad (A_{23})_2 = 0.67$

$(A_{33})_1 = (B_{moisture}, Moisture_{medium})_1$
$= (B_{Moisture} \bullet Moisture_{medium}) \wedge (\overline{B_{Moisture} \oplus Moisture_{medium}})$
$(A_{33})_2 = (B_{moisture}, Moisture_{medium})_2$
$= \frac{1}{2}[(B_{clay} \bullet Moisture_{medium}) + (\overline{B_{Moisture} \oplus Moisture_{medium}})]$

$B_{Moisture} \bullet Moisture_{medium} = \vee\{[0, 0.5, 1, 0.8, 0] \wedge [0.2, 0.35, 0.5, 0.65, 0.8]\} = 0.65$

$B_{Moisture} \oplus Moisture_{medium} = \wedge\{[0, 0.5, 1, 0.8, 0] \vee [0.2, 0.35, 0.5, 0.65, 0.8]\} = 0.2$

$\overline{B_{Clay} \oplus Moisture_{medium}} = 0.8$

$(A_{33})_1 = 0.65 \quad (A_{33})_2 = 0.73$

$(A_3)_1 = 0.4 \times 0.5 + 0.4 \times 0.53 + 0.2 \times 0.65 = 0.22$
$(A_3)_2 = 0.4 \times 0.55 + 0.4 \times 0.67 + 0.2 \times 0.73 = 0.63$

$A_4 = 0.4(A_{14}) + 0.4(A_{24}) + 0.2(A_{34})$
$(A_{14})_1 = (B_{depth}, Depth_{high})_1$
$= (B_{depth} \bullet Depth_{high}) \wedge (\overline{B_{depth} \oplus Depth_{high}})$
$(A_{14})_2 = (B_{depth}, Depth_{high})_2$
$= \frac{1}{2}[(B_{depth} \bullet Depth_{high}) + (\overline{B_{depth} \oplus Depth_{high}})]$

$B_{depth} \bullet Depth_{high} = \vee\{[0, 0.4, 1, 0.6, 0] \wedge [0.4, 0.45, 0.5, 0.55, 0.6]\} = 0.55$

$B_{depth} \oplus Depth_{high} = \wedge\{[0, 0.4, 1, 0.6, 0] \vee [0.4, 0.45, 0.5, 0.55, 0.6]\} = 0.4$

$\overline{B_{depth} \oplus Depth_{high}} = 0.6$

$(A_{14})_1 = 0.55 \quad (A_{14})_2 = 0.58$

$(A_{24})_1 = (B_{clay}, Clay_{high})_1$
$= (B_{Clay} \bullet Clay_{high}) \wedge (\overline{B_{Clay} \oplus Clay_{high}})$
$(A_{24})_2 = (B_{clay}, Clay_{high})_2$
$= \frac{1}{2}[(B_{clay} \bullet Clay_{high}) + (\overline{B_{Clay} \oplus Clay_{high}})]$

$B_{Clay} \bullet Clay_{high} = \vee\{[0, 0.7, 1, 0.5, 0] \wedge [0, 0, 0, 0, 0]\} = 0$

$B_{Clay} \oplus Clay_{high} = \wedge\{[0, 0.7, 1, 0.5, 0] \vee [0, 0, 0, 0, 0]\} = 0$

$\overline{B_{Clay} \oplus Clay_{high}} = 1$

$(A_{24})_1 = 0 \quad (A_{24})_2 = 0.5$

$(A_{34})_1 = (B_{moisture}, Moisture_{high})_1$
$= (B_{Moisture} \bullet Moisture_{high}) \wedge (\overline{B_{Moisture} \oplus Moisture_{high}})$
$(A_{34})_2 = (B_{moisture}, Moisture_{high})_2$
$= \frac{1}{2}[(B_{clay} \bullet Moisture_{high}) + (\overline{B_{Moisture} \oplus Moisture_{high}})]$

$B_{Moisture} \bullet Moisture_{high} = \vee\{[0, 0.5, 1, 0.8, 0] \wedge [0, 0, 0, 0, 0]\} = 0$

$B_{Moisture} \oplus Moisture_{high} = \wedge\{[0, 0.5, 1, 0.8, 0] \vee [0, 0, 0, 0, 0]\} = 0$

$\overline{B_{Clay} \oplus Moisture_{high}} = 1$

$(A_{34})_1 = 0 \quad (A_{34})_2 = 0.5$

$(A_4)_1 = 0.4 \times 0.55 + 0.4 \times 0 + 0.2 \times 0 = 0.22$
$(A_4)_2 = 0.4 \times 0.58 + 0.4 \times 0.5 + 0.2 \times 0.5 = 0.53$

Therefore, based on metric 1 and 2 the best match is C_3.

11.10 B=[8, 120] W=[0.4, 0.6]

(B, Low flow rate) = 0.4 (B_V, L-vel) + 0.6 (B_A, S-area)
(B, Moderate flow rate) = 0.4 (B_V, M-vel) + 0.6 (B_A, M-area)
(B, High flow rate) = 0.4 (B_V, H-vel) + 0.6 (B_A, B-area)

$(B_V, \text{L-vel}) = \exp\left(-\frac{(8-5)^2}{3^2}\right) = 0.368$
$(B_V, \text{M-vel}) = \exp\left(-\frac{(8-10)^2}{5^2}\right) = 0.852$
$(B_V, \text{H-vel}) = \exp\left(-\frac{(8-15)^2}{3^2}\right) = 0.004$

$(B_A, \text{S-area}) = \exp\left(-\frac{(120-50)^2}{26^2}\right) = 0.001$
$(B_A, \text{M- area}) = \exp\left(-\frac{(120-100)^2}{18^2}\right) = 0.291$
$(B_A, \text{B- area}) = \exp\left(-\frac{(120-150)^2}{4^2}\right) = 0$

(B, Low flow rate) = 0.4×0.368 + 0.6×0.001 = 0.148
(B, Moderate flow rate) = 0.4×0.852 + 0.6×0.291 = 0.515
(B, High flow rate) = 0.4×0.004 + 0.6×0 = 0.002

Therefore, the best match is Moderate flow rate.

11.11

This problem is the same as problem 11.10 but B = [11, 130]

(B, Low flow rate) = 0.4 (B_V, L-vel) + 0.6 (B_A, S-area)
(B, Moderate flow rate) = 0.4 (B_V, M-vel) + 0.6 (B_A, M-area)
(B, High flow rate) = 0.4 (B_V, H-vel) + 0.6 (B_A, B-area)
(B_V, L-vel) = 0.018
(B_V, M-vel) = 0.961
(B_V, H-vel) = 0.169

(B_A, S-area) = 0
(B_A, M- area) = 0.062
(B_A, B- area) = 0

(B, Low flow rate) = 0.4×0.018 + 0.6×0 = 0.007
(B, Moderate flow rate) = 0.4×0.961 + 0.6×0.062 = 0.422
(B, High flow rate) = 0.4×0.169 + 0.6×0 = 0.068
Therefore, the best match is Moderate flow rate.

11.12

$(B, A_{i1})_1 = (B \bullet A_{i1}) \wedge (\overline{B \oplus A_{11}})$
$(B, A_{i1})_2 = ½[(B \bullet A_{i1}) + (\overline{B \oplus A_{11}})]$

$B \bullet A_{11} = \vee\{[1] \wedge [0.1]\} = \vee[0.1] = 0.1$
$B \oplus A_{11} = \wedge\{[1] \vee [0.1]\} = \wedge[1] = 1$
$\overline{B \oplus A_{11}} = 0$
$(B, A_{11})_1 = 0$ $(B, A_{11})_2 = 0.05$
$B \bullet A_{21} = \vee\{[1] \wedge [0.2]\} = \vee[0.2] = 0.2$
$B \oplus A_{21} = \wedge\{[1] \vee [0.2]\} = \wedge[1] = 1$
$\overline{B \oplus A_{21}} = 0$
$(B, A_{21})_1 = 0$ $(B, A_{21})_2 = 0.1$
$B \bullet A_{31} = \vee\{[1] \wedge [0.7]\} = \vee[0.7] = 0.7$
$B \oplus A_{31} = \wedge\{[1] \vee [0.7]\} = \wedge[1] = 1$
$\overline{B \oplus A_{31}} = 0$
$(B, A_{31})_1 = 0$ $(B, A_{31})_2 = 0.35$

$(A_1)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_1)_2 = 0.3\times0.05 + 0.3\times0.1 + 0.4\times0.35 = 0.185$

$(B, A_{i2})_1 = (B \bullet A_{i2}) \wedge (\overline{B \oplus A_{12}})$
$(B, A_{i2})_2 = ½[(B \bullet A_{i2}) + (\overline{B \oplus A_{12}})]$

$B \bullet A_{12} = \vee\{[1] \wedge [0.3]\} = \vee[0.3] = 0.3$
$B \oplus A_{12} = \wedge\{[1] \vee [0.3]\} = \wedge[1] = 1$
$\overline{B \oplus A_{12}} = 0$
$(B, A_{12})_1 = 0$ $(B, A_{12})_2 = 0.15$

$B \bullet A_{22} = \vee\{[1] \wedge [0.4]\} = \vee[0.4] = 0.4$
$B \oplus A_{22} = \wedge\{[1] \vee [0.4]\} = \wedge[1] = 1$
$\overline{B \oplus A_{22}} = 0$
$(B, A_{22})_1 = 0$ $(B, A_{22})_2 = 0.2$

$B \bullet A_{32} = \vee\{[1] \wedge [0.8]\} = \vee[0.8] = 0.8$
$B \oplus A_{32} = \wedge\{[1] \vee [0.8]\} = \wedge[1] = 1$
$\overline{B \oplus A_{32}} = 0$
$(B, A_{32})_1 = 0$ $(B, A_{32})_2 = 0.4$

$(A_2)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_2)_2 = 0.3\times0.15 + 0.3\times0.2 + 0.4\times0.4 = 0.265$

$(B, A_{i3})_1 = (B \bullet A_{i3}) \wedge (\overline{B \oplus A_{13}})$
$(B, A_{i3})_2 = ½[(B \bullet A_{i3}) + (\overline{B \oplus A_{13}})]$

$B \bullet A_{13} = \vee\{[1] \wedge [0.3]\} = \vee[0.3] = 0.3$
$B \oplus A_{13} = \wedge\{[1] \vee [0.3]\} = \wedge[1] = 1$
$\overline{B \oplus A_{13}} = 0$
$(B, A_{13})_1 = 0$ $(B, A_{13})_2 = 0.15$

$B \bullet A_{23} = \vee\{[1] \wedge [0.6]\} = \vee[0.6] = 0.6$
$B \oplus A_{23} = \wedge\{[1] \vee [0.6]\} = \wedge[1] = 1$
$\overline{B \oplus A_{23}} = 0$
$(B, A_{23})_1 = 0$ $(B, A_{23})_2 = 0.3$

$B \bullet A_{33} = \vee\{[1] \wedge [0.8]\} = \vee[0.8] = 0.8$
$B \oplus A_{33} = \wedge\{[1] \vee [0.8]\} = \wedge[1] = 1$
$\overline{B \oplus A_{33}} = 0$
$(B, A_{33})_1 = 0$ $(B, A_{33})_2 = 0.4$

$(A_3)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_3)_2 = 0.3\times0.15 + 0.3\times0.3 + 0.4\times0.4$
$= 0.295$

$(B, A_{i4})_1 = (B \bullet A_{i4}) \wedge (\overline{B \oplus A_{14}})$
$(B, A_{i4})_2 = ½[(B \bullet A_{i4}) + (\overline{B \oplus A_{14}})]$

$B \bullet A_{14} = \vee\{[1] \wedge [0.4]\} = \vee[0.4] = 0.4$
$B \oplus A_{14} = \wedge\{[1] \vee [0.4]\} = \wedge[1] = 1$
$\overline{B \oplus A_{14}} = 0$
$(B, A_{14})_1 = 0$ $(B, A_{14})_2 = 0.2$

$B \bullet A_{24} = \vee\{[1] \wedge [0.8]\} = \vee[0.8] = 0.8$
$B \oplus A_{24} = \wedge\{[1] \vee [0.8]\} = \wedge[1] = 1$
$\overline{B \oplus A_{24}} = 0$
$(B, A_{24})_1 = 0$ $(B, A_{24})_2 = 0.4$

$B \bullet A_{34} = \vee\{[1] \wedge [0.9]\} = \vee[0.9] = 0.9$
$B \oplus A_{34} = \wedge\{[1] \vee [0.9]\} = \wedge[1] = 1$
$\overline{B \oplus A_{34}} = 0$
$(B, A_{34})_1 = 0$ $(B, A_{34})_2 = 0.45$

$(A_4)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_4)_2 = 0.3\times0.2 + 0.3\times0.4 + 0.4\times0.45$
$= 0.36$

$(B, A_{i5})_1 = (B \bullet A_{i5}) \wedge (\overline{B \oplus A_{15}})$
$(B, A_{i5})_2 = ½[(B \bullet A_{i5}) + (\overline{B \oplus A_{15}})]$

$B \bullet A_{15} = \vee\{[1] \wedge [0.6]\} = \vee[0.6] = 0.6$
$B \oplus A_{15} = \wedge\{[1] \vee [0.6]\} = \wedge[1] = 1$
$\overline{B \oplus A_{15}} = 0$
$(B, A_{15})_1 = 0$ $(B, A_{15})_2 = 0.5$

$B \bullet A_{25} = \vee\{[1] \wedge [0.9]\} = \vee[0.9] = 0.9$
$B \oplus A_{25} = \wedge\{[1] \vee [0.9]\} = \wedge[1] = 1$
$\overline{B \oplus A_{25}} = 0$
$(B, A_{25})_1 = 0$ $(B, A_{25})_2 = 0.45$

$B \bullet A_{35} = \vee\{[1] \wedge [1]\} = \vee[1] = 1$
$B \oplus A_{35} = \wedge\{[1] \vee [1]\} = \wedge[1] = 1$
$\overline{B \oplus A_{35}} = 0$
$(B, A_{35})_1 = 0$ $(B, A_{35})_2 = 0.5$

$(A_5)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_5)_2 = 0.3\times0.5 + 0.3\times0.45 + 0.4\times0.5$
$= 0.485$

Based on metric 1 it is not possible to choose any match. But based on the metric 2 the best match is A_5.

11.13
B = [6.5, 825, 750] W = [0.5, 0.3, 0.2]

(B, PGO) = 0.5 (B_{color}, $Color_{PGO}$) + 0.3 ($B_{viscosity}$, $Viscosity_{PGO}$) + 0.2 ($B_{flash\ point}$, Flash $point_{PGO}$) = 0.5×0 + 0.3×0 + 0.2×0 = 0

(B, 100N) = 0.5 (B_{color}, $Color_{100N}$) + 0.3 ($B_{viscosity}$, $Viscosity_{100N}$) + 0.2 ($B_{flash\ point}$, Flash $point_{100N}$) = 0.5×0 + 0.3×0 + 0.2×0 = 0

(B, 150N) = 0.5 (B_{color}, $Color_{150N}$) + 0.3 ($B_{viscosity}$, $Viscosity_{150N}$) + 0.2 ($B_{flash\ point}$, Flash $point_{150N}$) = 0.5×0 + 0.3×0.375 + 0.2×0.25 = 0.16

(B, HSN) = 0.5 (B_{color}, $Color_{HSN}$) + 0.3 ($B_{viscosity}$, $Viscosity_{HSN}$) + 0.2 ($B_{flash\ point}$, Flash $point_{HSN}$) = 0.5×0.75 + 0.3×0.625 + 0.2×0.75 = 0.71

(B, 500N) = 0.5 (B_{color}, $Color_{500N}$) + 0.3 ($B_{viscosity}$, $Viscosity_{500N}$) + 0.2 ($B_{flash\ point}$, Flash $point_{500N}$) = 0.5×0.25 + 0.3×0 + 0.2×0 = 0.13
Therefore, the best match is HSN.

11.14

$B = [0.3, 0.3, 0.3]$ $W = [0.4, 0.4, 0.2]$

$(B, A_{i1})_1 = (B \bullet A_{i1}) \wedge (\overline{B \oplus A_{11}})$
$(B, A_{i1})_2 = ½[(B \bullet A_{i1}) + (\overline{B \oplus A_{11}})]$

$B \bullet A_{11} = \vee\{[0.3] \wedge [0.1]\} = \vee[0.1] = 0.1$
$B \oplus A_{11} = \wedge\{[0.3] \vee [0.1]\} = \wedge[0.3] = 0.3$
$\overline{B \oplus A_{11}} = 0.7$
$(B, A_{11})_1 = 0.1$ $(B, A_{11})_2 = 0.4$

$B \bullet A_{21} = \vee\{[0.3] \wedge [0.15]\} = \vee[0.15] = 0.15$
$B \oplus A_{21} = \wedge\{[0.3] \vee [0.15]\} = \wedge[0.3] = 0.3$
$\overline{B \oplus A_{21}} = 0.7$
$(B, A_{21})_1 = 0.15$ $(B, A_{21})_2 = 0.425$

$B \bullet A_{31} = \vee\{[0.3] \wedge [0.2]\} = \vee[0.2] = 0.2$
$B \oplus A_{31} = \wedge\{[0.3] \vee [0.2]\} = \wedge[0.3] = 0.3$
$\overline{B \oplus A_{31}} = 0.7$
$(B, A_{31})_1 = 0.2$ $(B, A_{31})_2 = 0.45$

$(A_1)_1 = 0.3\times0.1 + 0.3\times0.15 + 0.4\times0.2 = 0.155$
$(A_1)_2 = 0.3\times0.4 + 0.3\times0.425 + 0.4\times0.45$
$= 0.4275$

$(B, A_{i2})_1 = (B \bullet A_{i2}) \wedge (\overline{B \oplus A_{12}})$
$(B, A_{i2})_2 = ½[(B \bullet A_{i2}) + (\overline{B \oplus A_{12}})]$

$B \bullet A_{12} = \vee\{[0.3] \wedge [0.2]\} = \vee[0.2] = 0.2$
$B \oplus A_{12} = \wedge\{[0.3] \vee [0.2]\} = \wedge[0.3] = 0.3$
$\overline{B \oplus A_{12}} = 0.7$
$(B, A_{12})_1 = 0.2$ $(B, A_{12})_2 = 0.45$

$B \bullet A_{22} = \vee\{[0.3] \wedge [0.2]\} = \vee[0.2] = 0.2$
$B \oplus A_{22} = \wedge\{[0.3] \vee [0.2]\} = \wedge[0.3] = 0.3$
$\overline{B \oplus A_{22}} = 0.7$
$(B, A_{22})_1 = 0.2$ $(B, A_{22})_2 = 0.45$

$B \bullet A_{32} = \vee\{[0.3] \wedge [0.3]\} = \vee[0.3] = 0.3$
$B \oplus A_{32} = \wedge\{[0.3] \vee [0.3]\} = \wedge[0.3] = 0.3$
$\overline{B \oplus A_{32}} = 0.7$
$(B, A_{32})_1 = 0.3$ $(B, A_{32})_2 = 0.5$

$(A_2)_1 = 0.3\times0.2 + 0.3\times0.2 + 0.4\times0.3 = 0.24$
$(A_2)_2 = 0.3\times0.45 + 0.3\times0.45 + 0.4\times0.5 = 0.47$

$(B, A_{i3})_1 = (B \bullet A_{i3}) \wedge (\overline{B \oplus A_{13}})$
$(B, A_{i3})_2 = ½[(B \bullet A_{i3}) + (\overline{B \oplus A_{13}})]$

$B \bullet A_{13} = \vee\{[0.3] \wedge [0.5]\} = \vee[0.3] = 0.3$
$B \oplus A_{13} = \wedge\{[0.3] \vee [0.5]\} = \wedge[0.5] = 0.5$
$\overline{B \oplus A_{13}} = 0.5$
$(B, A_{13})_1 = 0.3$ $(B, A_{13})_2 = 0.4$

$B \bullet A_{23} = \vee\{[0.3] \wedge [0.7]\} = \vee[0.3] = 0.3$
$B \oplus A_{23} = \wedge\{[0.3] \vee [0.7]\} = \wedge[0.7] = 0.7$
$\overline{B \oplus A_{23}} = 0.3$
$(B, A_{23})_1 = 0.3$ $(B, A_{23})_2 = 0.3$

$B \bullet A_{33} = \vee\{[0.3] \wedge [0.5]\} = \vee[0.3] = 0.3$
$B \oplus A_{33} = \wedge\{[0.3] \vee [0.5]\} = \wedge[0.5] = 0.5$
$\overline{B \oplus A_{33}} = 0.5$
$(B, A_{33})_1 = 0.3$ $(B, A_{33})_2 = 0.4$

$(A_3)_1 = 0.3\times0.3 + 0.3\times0.3 + 0.4\times0.3 = 0.3$
$(A_3)_2 = 0.3\times0.4 + 0.3\times0.3 + 0.4\times0.4 = 0.37$

$(B, A_{i4})_1 = (B \bullet A_{i4}) \wedge (\overline{B \oplus A_{14}})$
$(B, A_{i4})_2 = ½[(B \bullet A_{i4}) + (\overline{B \oplus A_{14}})]$

$B \bullet A_{14} = \vee\{[0.3] \wedge [0.9]\} = \vee[0.3] = 0.3$
$B \oplus A_{14} = \wedge\{[0.3] \vee [0.9]\} = \wedge[0.9] = 0.9$
$\overline{B \oplus A_{14}} = 0.1$
$(B, A_{14})_1 = 0.1$ $(B, A_{14})_2 = 0.2$

$B \bullet A_{24} = \vee\{[0.3] \wedge [0.9]\} = \vee[0.3] = 0.3$
$B \oplus A_{24} = \wedge\{[0.3] \vee [0.9]\} = \wedge[0.9] = 0.9$
$\overline{B \oplus A_{24}} = 0.1$
$(B, A_{24})_1 = 0.1$ $(B, A_{24})_2 = 0.2$

$B \bullet A_{34} = \vee\{[0.3] \wedge [0.9]\} = \vee[0.3] = 0.3$
$B \oplus A_{34} = \wedge\{[0.3] \vee [0.9]\} = \wedge[0.9] = 0.9$
$\overline{B \oplus A_{34}} = 0.1$
$(B, A_{34})_1 = 0.1$ $(B, A_{34})_2 = 0.2$

$(A_4)_1 = 0.3\times0.1 + 0.3\times0.1 + 0.4\times0.1 = 0.1$
$(A_4)_2 = 0.3\times0.2 + 0.3\times0.2 + 0.4\times0.2 = 0.2$

Based on metric 1, the best match is A_3 (SEASAT) whereas, based on metric 2, the best match is A_2 (DMSP).

11.15

The Gaussian distribution given in this problem is incorrect. It should be in the following form:

$$\mu_{A_{ij}}(x) = \exp\left[-\left(\frac{x_j - a_{ij}}{\sigma_{a_{ij}}}\right)^2\right]$$

(B, Light) = 0.5 ($B_{Quality}$, $Quality_{Light}$) + 0.3 ($B_{Visibility}$, $Visibility_{Light}$) + 0.2 ($B_{Geometery}$, $Geometery_{Light}$)

($B_{Quality}$, $Quality_{Light}$) =

$$\exp\left(-\left(\frac{45-30}{15}\right)^2\right) = 0.368$$

($B_{Visibility}$, $Visibility_{Light}$) =

$$\exp\left(-\left(\frac{55-40}{12}\right)^2\right) = 0.210$$

($B_{Geometery}$, $Geometery_{Light}$) =

$$\exp\left(-\left(\frac{35-20}{7}\right)^2\right) = 0.010$$

(B, Light) = 0.5×0.368 + 0.3×0.210 + 0.2×0.010 = 0.249

(B, Moderate) = 0.5 ($B_{Quality}$, $Quality_{Moderate}$) + 0.3 ($B_{Visibility}$, $Visibility_{Moderate}$) + 0.2 ($B_{Geometery}$, $Geometery_{Moderate}$)

($B_{Quality}$, $Quality_{Moderate}$) =

$$\exp\left(-\left(\frac{45-40}{20}\right)^2\right) = 0.939$$

($B_{Visibility}$, $Visibility_{Moderate}$) =

$$\exp\left(-\left(\frac{55-40}{5}\right)^2\right) = 0.000$$

($B_{Geometery}$, $Geometery_{Moderate}$) =

$$\exp\left(-\left(\frac{35-35}{10}\right)^2\right) = 1$$

(B, Moderate) = 0.5×0.939 + 0.3×0.000 + 0.2×1 = 0.670

(B, Good) = 0.5 ($B_{Quality}$, $Quality_{Good}$) + 0.3 ($B_{Visibility}$, $Visibility_{Good}$) + 0.2 ($B_{Geometery}$, $Geometery_{Good}$)

($B_{Quality}$, $Quality_{Good}$) =

$$\exp\left(-\left(\frac{45-50}{15}\right)^2\right) = 0.895$$

($B_{Visibility}$, $Visibility_{Good}$) =

$$\exp\left(-\left(\frac{55-50}{10}\right)^2\right) = 0.779$$

($B_{Geometery}$, $Geometery_{Good}$) =

$$\exp\left(-\left(\frac{35-60}{10}\right)^2\right) = 0.002$$

(B, Good) = 0.5×0.895 + 0.3×0.779 + 0.2×0.002 = 0.682

(B, Rush-hour)=0.5($B_{Quality}$, $Quality_{Rush\text{-}hour}$) + 0.3 ($B_{Visibility}$, $Visibility_{Rush\text{-}hour}$) + 0.2 ($B_{Geometery}$, $Geometery_{Rush\text{-}hour}$)

($B_{Quality}$, $Quality_{Rush\text{-}hour}$) =

$$\exp\left(-\left(\frac{45-60}{10}\right)^2\right) = 0.105$$

($B_{Visibility}$, $Visibility_{Rush\text{-}hour}$) =

$$\exp\left(-\left(\frac{55-60}{6}\right)^2\right) = 0.499$$

($B_{Geometery}$, $Geometery_{Rush\text{-}hour}$) =

$$\exp\left(-\left(\frac{35-70}{15}\right)^2\right) = 0.004$$

(B, Rush-hour) = 0.5×0.105 + 0.3×0.499 + 0.2×0.004 = 0.203

The best match is Good capacity.

11.16

(B, Light) = 0.5 ($B_{Quality}$, $Quality_{Light}$) + 0.3 ($B_{Visibility}$, $Visibility_{Light}$) + 0.2 ($B_{Geometery}$, $Geometery_{Light}$)

($B_{Quality}$, $Quality_{Light}$)$_1$ =

$$\exp\left(-\left(\frac{45-30}{10+15}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Quality}, Quality_{Light})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{45-30}{10+15}\right)^2\right)+1\right] = 0.500$$

$(B_{Visibility}, Visibility_{Light})_1 =$

$$\exp\left(-\left(\frac{55-40}{12-12}\right)^2\right) \wedge 1 = 0$$

$(B_{Visibility}, Visibility_{Light})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{55-40}{12-12}\right)^2\right)+1\right] = 0.500$$

$(B_{Geometery}, Geometery_{Light})_1 =$

$$\exp\left(-\left(\frac{35-20}{8-7}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Geometery}, Geometery_{Light})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{35-20}{8-7}\right)^2\right)+1\right] = 0.500$$

$(B, Light)_1 = 0.5\times0 + 0.3\times0 + 0.2\times0 = 0$
$(B, Light)_2 = 0.5\times0.5+0.3\times0.5+0.2\times0.5 = 0.5$

$(B, Moderate) = 0.5\ (B_{Quality}, Quality_{Moderate}) + 0.3\ (B_{Visibility}, Visibility_{Moderate}) + 0.2\ (B_{Geometery}, Geometery_{Moderate})$

$(B_{Quality}, Quality_{Moderate})_1 =$

$$\exp\left(-\left(\frac{45-40}{10-20}\right)^2\right) \wedge 1 = 0.779$$

$(B_{Quality}, Quality_{Moderate})_2 =$

$$\frac{1}{2}\left[exp\left(-\left(\frac{45-40}{10-20}\right)^2\right)+1\right] = 0.889$$

$(B_{Visibility}, Visibility_{Moderate})_1 =$

$$\exp\left(-\left(\frac{55-40}{12-5}\right)^2\right) \wedge 1 = 0.010$$

$(B_{Visibility}, Visibility_{Moderate})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{55-40}{12-5}\right)^2\right)+1\right] = 0.505$$

$(B_{Geometery}, Geometery_{Moderate})_1 =$

$$\exp\left(-\left(\frac{35-35}{8-10}\right)^2\right) \wedge 1 = 1$$

$(B_{Geometery}, Geometery_{Moderate})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{35-35}{8-10}\right)^2\right)+1\right] = 1$$

$(B, Moderate)_1 = 0.5\times0.779 + 0.3\times0.010 + 0.2\times1 = 0.593$
$(B, Moderate)_2 = 0.5\times0.889 + 0.3\times0.505 + 0.2\times1 = 0.593$

$(B, Good) = 0.5\ (B_{Quality}, Quality_{Good}) + 0.3\ (B_{Visibility}, Visibility_{Good}) + 0.2\ (B_{Geometery}, Geometery_{Good})$

$(B_{Quality}, Quality_{Good})_1 =$

$$\exp\left(-\left(\frac{45-50}{10-15}\right)^2\right) \wedge 1 = 0.368$$

$(B_{Quality}, Quality_{Good})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{45-50}{10-15}\right)^2\right)+1\right] = 0.684$$

$(B_{Visibility}, Visibility_{Good})_1 =$

$$\exp\left(-\left(\frac{55-50}{12-10}\right)^2\right) \wedge 1 = 0.002$$

$(B_{Visibility}, Visibility_{Good})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{55-50}{12-10}\right)^2\right)+1\right] = 0.501$$

$(B_{Geometery}, Geometery_{Good})_1 =$

$$\exp\left(-\left(\frac{35-60}{8-10}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Geometery}, Geometery_{Good})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{35-60}{8-10}\right)^2\right)+1\right] = 0.500$$

$(B, Good)_1 = 0.5\times0.368 + 0.3\times0.002 + 0.2\times0 = 0.185$
$(B, Good)_2 = 0.5\times0.684 + 0.3\times0.501 + 0.2\times0 = 0.492$

$(B, Rush\text{-}hour)=0.5(B_{Quality}, Quality_{Rush\text{-}hour}) + 0.3\ (B_{Visibility}, Visibility_{Rush\text{-}hour}) + 0.2\ (B_{Geometery}, Geometery_{Rush\text{-}hour})$

$(B_{Quality}, Quality_{Rush\text{-}hour})_1 =$

$$\exp\left(-\left(\frac{45-60}{10-10}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Quality}, Quality_{Rush\text{-}hour})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{45-60}{10-10}\right)^2\right)+1\right] = 0.500$$

$(B_{Visibility}, \text{Visibility}_{Rush\text{-}hour})_1 =$

$$\exp\left(-\left(\frac{55-60}{12-6}\right)^2\right) \wedge 1 = 0.499$$

$(B_{Visibility}, \text{Visibility}_{Rush\text{-}hour})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{55-60}{12-6}\right)^2\right) + 1\right] = 0.750$$

$(B_{Geometery}, \text{Geometery}_{Rush\text{-}hour})_1 =$

$$\exp\left(-\left(\frac{35-70}{8-15}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Geometery}, \text{Geometery}_{Rush\text{-}hour})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{35-70}{8-15}\right)^2\right) + 1\right] = 0.500$$

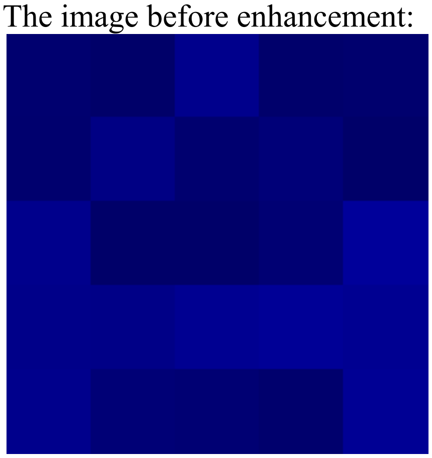
$(B, \text{Rush-hour})_1 = 0.5\times 0 + 0.3\times 0.499 + 0.2\times 0 = 0.150$

$(B, \text{Rush-hour})_2 = 0.5\times 0.500 + 0.3\times 0.750 + 0.2\times 0.500 = 0.575$

Therefore, based on both metric 1 and 2, the best match is Good capacity.

11.17

The image before enhancement:



Scaling the pixel values between 0 and 1 by dividing by 255 we get:

0.44	0.41	0.55	0.42	0.43
0.43	0.52	0.44	0.47	0.41
0.55	0.41	0.41	0.45	0.60
0.54	0.53	0.57	0.59	0.57
0.55	0.46	0.45	0.43	0.58

using the algorithm explained in the book.

$$\mu = \begin{cases} 2(x)^2 & \forall x \le 0.5 \\ 1-2(1-x)^2 & \forall x > 0.5 \end{cases}$$

where x = scaled values

(1)

0.38	0.34	0.59	0.35	0.37
0.37	0.53	0.38	0.44	0.34
0.59	0.34	0.34	0.41	0.69
0.57	0.56	0.63	0.66	0.63
0.59	0.43	0.41	0.37	0.65

(2)

0.29	0.23	0.67	0.25	0.28
0.28	0.57	0.29	0.39	0.23
0.67	0.23	0.23	0.33	0.80
0.63	0.61	0.72	0.77	0.72
0.67	0.37	0.33	0.27	0.75

(3)

0.16	0.11	0.78	0.12	0.15
0.15	0.62	0.16	0.31	0.11
0.78	0.11	0.11	0.22	0.92
0.73	0.69	0.85	0.89	0.85
0.78	0.27	0.22	0.14	0.88

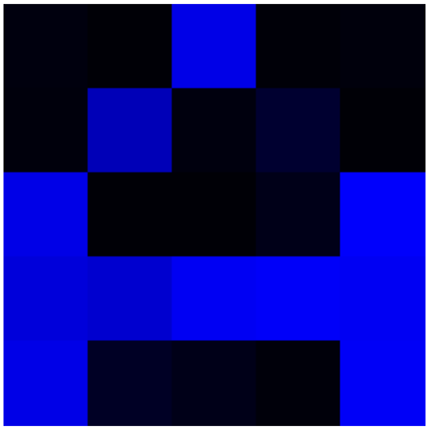
(4)

0.05	0.02	0.90	0.03	0.05
0.05	0.72	0.05	0.19	0.02
0.90	0.02	0.02	0.10	0.99
0.86	0.81	0.95	0.98	0.95
0.90	0.14	0.10	0.04	0.97

Restoring the original scale of the pixel values by multiplying by 255 we get:

14	6	231	8	12
12	183	14	48	6
231	6	6	24	252
218	207	243	249	243
231	37	24	10	247

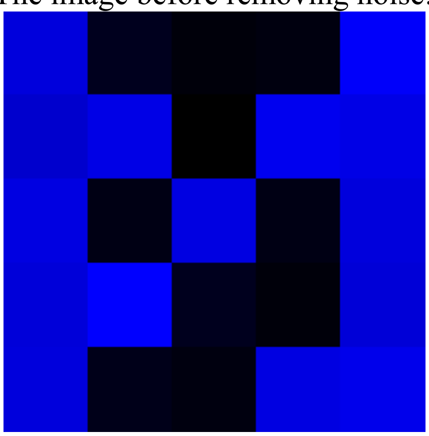
The achieved image after enhancement:



This is the letter A.

11.18

The image before removing noise:



Scaling the pixel values between 0 and 1 by dividing by 255 we get:

0.86	0.12	0.04	0.06	0.98
0.8	0.9	0	0.94	0.9
0.88	0.08	0.88	0.08	0.86
0.85	1	0.12	0.04	0.84
0.86	0.1	0.06	1	0.92

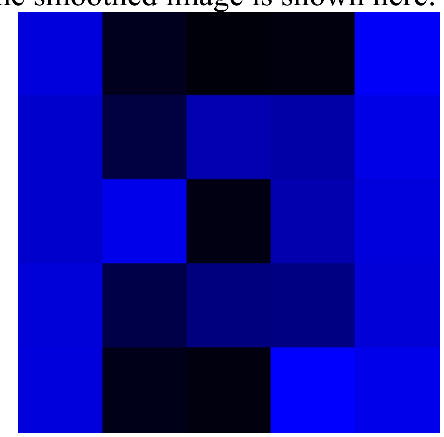
Using the algorithm explained in the text, i.e. $\mu_{00} = ¼(\mu_{-1,0}+\mu_{1,0}+\mu_{0,1}+\mu_{0,-1})$, we get:

0.86	0.12	0.04	0.06	0.98
0.8	0.25	0.69	0.26	0.9
0.88	0.92	0.07	0.68	0.86
0.85	0.29	0.5	0.51	0.84
0.86	0.1	0.06	1	0.92

Note: the pixels on the edges of the pixel graph (screen) would not be affected by the algorithm. Restoring the original scale of the pixel values by multiplying by 255 we get:

220	30	10	15	250
205	63.8	176	66.3	230
225	234	17.5	174	220
217	73	126	130	215
220	25	15	255	235

The smoothed image is shown here:



CHAPTER 12

Fuzzy Arithmetic and Extension Principle

12.1

a) $[2,3]+[3,4]=[5,7]$

b) $[1,2]*[2,3]=$

$[\min(1,2,3,6),\max(1,2,3,6)]=[1,6]$

c) $[3,6]\div[1,3]=[3,6]\div[\frac{1}{3},1]$

$=[\min(3,6,1,2),\max(3,6,1,2)]$

$=[1,6]$

d)

$[2,5]-[4,6]=[2-6,5-4]=[-4,1]$

12.2

$$\underset{\sim}{I}=\underset{\sim}{3}=\frac{0.2}{2}+\frac{1}{3}+\frac{0.2}{4}$$

$$\underset{\sim}{J}=\underset{\sim}{2}=\frac{0.1}{1}+\frac{1}{2}+\frac{0.1}{3}$$

$$\underset{\sim}{K}=\underset{\sim}{6}=\underset{\sim}{I}\times\underset{\sim}{J}=$$

$$\frac{\min(0.1,0.2)}{2}+\frac{\min(1,0.1)}{3}+$$

$$\frac{\max(\min(0.2,0.1),\min(0.2,1))}{4}+$$

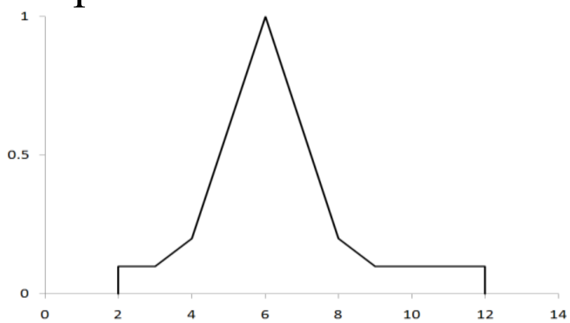
$$\frac{\max(\min(0.2,0.1),\min(1,1))}{6}+$$

$$\frac{\min(0.2,1)}{8}+\frac{\min(0.1,1)}{9}+$$

$$\frac{\min(0.2,0.1)}{12}$$

$$=\left\{\frac{0.1}{2}+\frac{0.1}{3}+\frac{0.2}{4}+\frac{1}{6}+\frac{0.2}{8}+\frac{0.1}{9}+\frac{0.1}{12}\right\}$$

By plotting the above points it can be shown that it is non-convex as show hereafter.



12.3

a) $z=3x-2$

$$=\frac{0.0}{3(0)-2}+\frac{0.3}{3(1)-2}+\frac{0.1}{3(2)-2}+$$

$$\frac{0.8}{3(3)-2}+\frac{1.0}{3(4)-2}+\frac{0.7}{3(5)-2}+$$

$$\frac{0.2}{3(6)-2}+\frac{0.0}{3(7)-2}$$

$$=\frac{0}{-2}+\frac{0.3}{1}+\frac{0.6}{4}+\frac{0.8}{7}+\frac{1.0}{10}+\frac{0.7}{13}+$$

$$\frac{0.2}{16}+\frac{0}{19}$$

b) $$z=\frac{0}{3}+\frac{0.3}{7}+\frac{0.6}{19}+\frac{0.8}{39}+\frac{1.0}{67}+\frac{0.7}{103}+$$

$$\frac{0.2}{147}+\frac{0}{199}$$

c)

$$z=\frac{0}{0}+\frac{0}{1}+\frac{0.3}{2}+\frac{0}{4}+\frac{0.6}{5}+\frac{0.6}{8}+\frac{0}{9}+$$

$$\frac{0.8}{10}+\frac{0.8}{13}+\frac{0}{16}+\frac{1.0}{17}+\frac{0.5}{18}+\frac{0.9}{20}+$$

$$\frac{0.5}{25}+\frac{0.7}{26}+\frac{0.7}{29}+\frac{0.2}{32}+\frac{0.5}{34}+\frac{0}{36}+$$

$$\frac{0.2}{37}+\frac{0.2}{40}+\frac{0.2}{41}+\frac{0.2}{45}+\frac{0}{49}+\frac{0.7}{50}+$$

$$\frac{0.2}{52}+\frac{0}{53}+\frac{0}{58}+\frac{0.1}{61}+\frac{0}{65}+\frac{0}{72}+$$

$$\frac{0}{74}+\frac{0}{85}+\frac{0}{98}$$

d) $z = \frac{0.0}{0-0} + \frac{0.3}{1-1} + \frac{0.6}{2-2} + \frac{0.8}{3-3} + \frac{1.0}{4-4} + \frac{0.7}{5-5} + \frac{0.1}{6-6} + \frac{0.0}{7-7} = \frac{1}{0}$

e)

$$\frac{0}{0} + \frac{0.3}{1} + \frac{0.6}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.1}{5} + \frac{0}{6} + \frac{0}{7}$$

12.4

a) $\left\{\frac{0}{-11} + \frac{0}{-10} + \frac{0}{-8} + \frac{0}{-7} + \frac{0}{-6} + \frac{0.5}{-4} + \frac{0}{-3} + \frac{0}{-2} + \frac{0.5}{0} + \frac{1}{1} + \frac{0}{2} + \frac{0}{4} + \frac{0.5}{5} + \frac{0}{6} + \frac{0}{8} + \frac{0}{9} + \frac{0}{10} + \frac{0}{12} + \frac{0}{13} + \frac{0}{16} + \frac{0}{17}\right\}$

b) $\frac{\partial f}{\partial x_1} = 2x_1 - 4 = 0 \qquad x_1 = 2$

$\frac{\partial f}{\partial x_2} = 2x_2 = 0 \qquad x_2 = 0$

$E(2,0) \qquad f(E) = 0$

$I_{0^+} \quad x_1[1,4] \qquad x_2[-1,1]$

$(a) \quad x_1 = 1 \quad x_2 = -1 \quad f(a) = 2$

$(b) \quad x_1 = 1 \quad x_2 = 1 \quad f(b) = 2$

$(c) \quad x_1 = 4 \quad x_2 = -1 \quad f(c) = 5$

$(d) \quad x_1 = 4 \quad x_2 = 1 \quad f(d) = 5$

$B_{0^+} = [\min(2,2,5,5,0), \max(2,2,5,5,0)] = [0,5]$

$I_{0.5} \quad x_1[2,3.5] \qquad x_2[-0.5,0.5]$

$(a) \quad x_1 = 2 \quad x_2 = -0.5 \quad f(a) = 0.25$

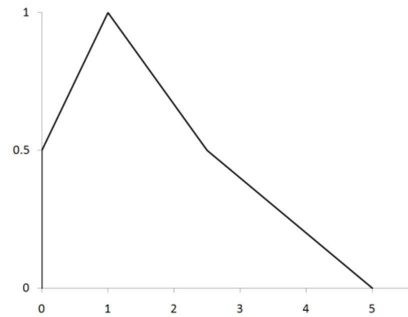
$(b) \quad x_1 = 2 \quad x_2 = 0.5 \quad f(b) = 0.25$

$(c) \quad x_1 = 3.5 \quad x_2 = -0.5 \quad f(c) = 2.5$

$(d) \quad x_1 = 3.5 \quad x_2 = 0.5 \quad f(d) = 2.5$

$B_{0.5} = [\min(0.25,0.25,2.5,2.5,0), \max(0.25,0.25,2.5,2.5,0)] = [0,5]$

$I_1 \quad (a)\ x_1 = 3 \quad x_2 = 0 \quad f(a) = 1 \quad B_1 = 1$



c) $I_{0^+} \quad x_1[1,4] \qquad x_2[-1,1]$

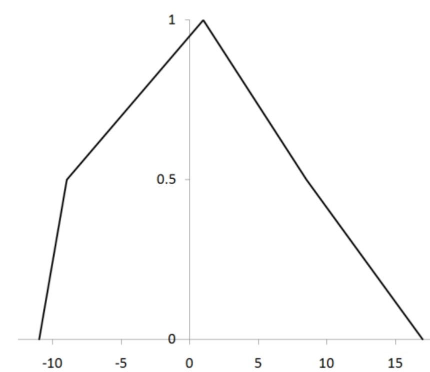
$B_{0^+} = [1^2, 4^2] + [0, 1^2] - 4[1,4] + 4$
$= [1,16] + [0,1] + [-16,-4] + 4$
$= [1 + 0 - 16 + 4, 16 + 1 - 4 + 4] = [-11, 17]$

$I_{0.5} \quad x_1[2,3.5] \qquad x_2[-0.5,0.5]$

$B_{0^+} = [2^2, 3.5^2] + [0, 0.5^2] - 4[2,3.5] + 4$
$= [4 + 0 - 17 + 4, 12.25 + 0.25 - 8 + 4] = [-9, 8.5]$

$I_1 \quad x_1 = 3 \quad x_2 = 0$

$B_1 = 3^2 + 0 - 4 \times 3 + 4 = 1$



12.5

$$\left\{\frac{0}{0} + \frac{0.7}{0.5} + \frac{1}{1} + \frac{0.7}{1.5} + \frac{0}{2}\right\} \times \left\{\frac{0.5}{500} + \frac{0.8}{750} + \frac{1}{1000} + \frac{0.8}{1250} + \frac{0.5}{1500}\right\}$$
$$= \{\frac{0.8}{0} + \frac{0.5}{250} + \frac{0.7}{375} + \frac{0.7}{500} + \frac{0.7}{625} + \frac{0.8}{750} + \frac{1}{1000} + \frac{0.7}{1125} + \frac{0.8}{1250} + \frac{0.7}{1500} + \frac{0.7}{1875} + \frac{0}{2000} + \frac{0.5}{2250} + \frac{0}{2500} + \frac{0}{3000}\}$$

12.6

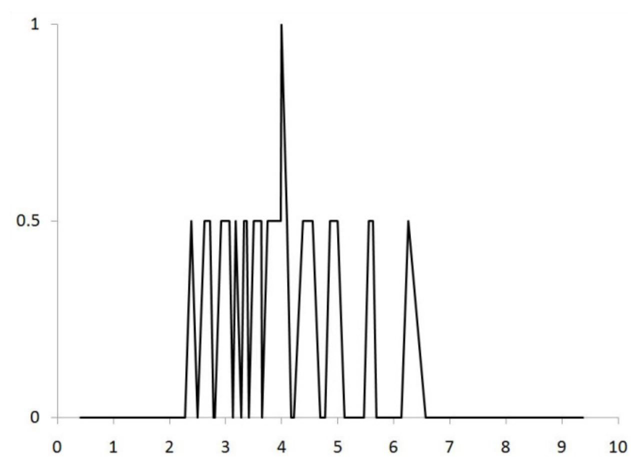
$$\left\{\frac{0.5}{3}+\frac{1}{4}+\frac{0.6}{5}\right\}+\left\{\frac{0.4}{8}+\frac{1}{9}+\frac{0.3}{10}\right\}$$
$$=\frac{0.4}{11}+\frac{0.5}{12}+\frac{0.3}{13}+\frac{0.4}{12}+\frac{1}{13}+\frac{0.3}{14}+\frac{0.4}{13}+\frac{0.6}{14}+\frac{0.3}{15}$$
$$=\left\{\frac{0.4}{11}+\frac{0.5}{12}+\frac{1}{13}+\frac{0.6}{14}+\frac{0.3}{15}\right\}$$

12.7

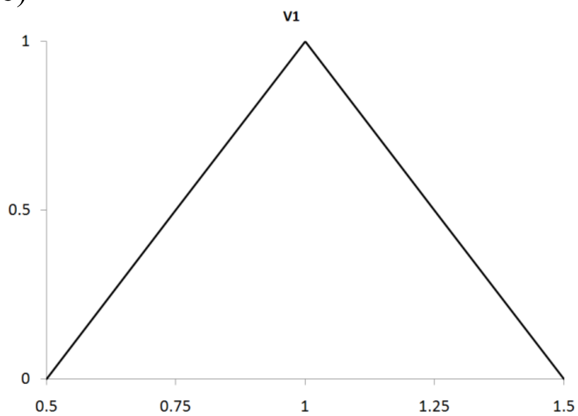
a) $$P_1V_1=\left\{\frac{0}{0.5}+\frac{0.5}{0.75}+\frac{1}{1}+\frac{0.5}{1.25}+\frac{0}{1.5}\right\}\times\left\{\frac{0}{0.5}+\frac{0.5}{1.75}+\frac{1}{2}+\frac{0.5}{2.25}+\frac{0}{2.5}\right\}$$
$$=\left\{\frac{0}{0.25}+\frac{0}{0.375}+\frac{0}{0.5}+\frac{0}{0.625}+\frac{0}{0.75}+\frac{0}{0.875}+\frac{0}{1}+\frac{0}{1.125}+\frac{0}{1.25}+\frac{0.5}{1.3125}+\frac{0.5}{1.5}+\frac{0.5}{1.6875}+\frac{0.5}{1.75}+\frac{0}{1.875}+\frac{1}{2}+\frac{0.5}{2.1875}+\frac{0.5}{2.25}+\frac{0.5}{2.5}+\frac{0}{2.625}+\frac{0.5}{2.8125}+\frac{0}{3}+\frac{0}{3.125}+\frac{0}{3.375}+\frac{0}{3.75}\right\}$$

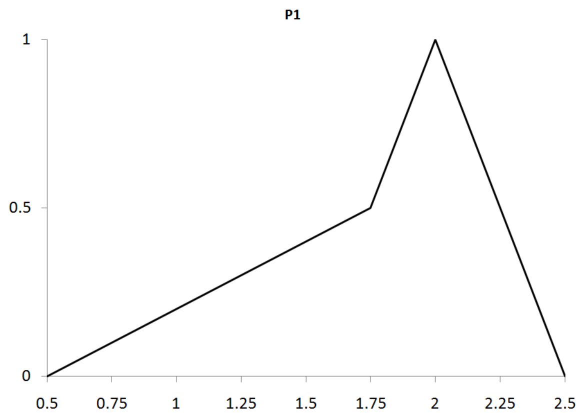
$$P_2=P_1V_1\div V_2=$$
$$P_1V_1\div\left\{\frac{0}{0.4}+\frac{0.5}{0.45}+\frac{1}{0.5}+\frac{0.5}{0.55}+\frac{0}{0.6}\right\}$$

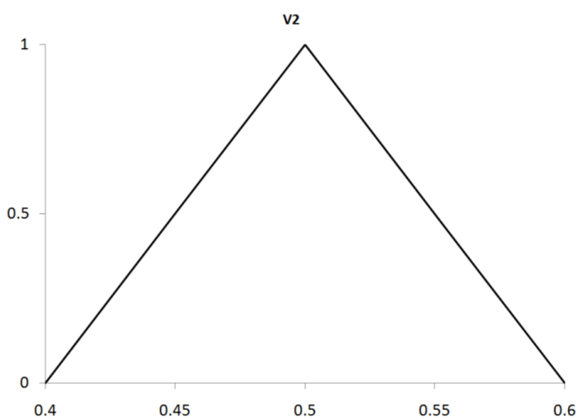
$$=\left\{\frac{0}{0.417}+\frac{0}{0.455}+\frac{0}{0.5}+\frac{0}{0.555}+\frac{0}{0.625}+\frac{0}{0.682}+\frac{0}{0.75}\right.$$
$$+\frac{0}{0.833}+\frac{0}{0.909}+\frac{0}{0.938}+\frac{0}{1}+\frac{0}{1.042}+\frac{0}{1.111}+\frac{0}{1.136}$$
$$+\frac{0}{1.25}+\frac{0}{1.364}+\frac{0}{1.389}+\frac{0}{1.458}+\frac{0}{1.5}+\frac{0}{1.563}+\frac{0}{1.591}$$
$$+\frac{0}{1.667}+\frac{0}{1.75}+\frac{0}{1.818}+\frac{0}{1.875}+\frac{0}{1.944}+\frac{0}{2}+\frac{0}{2.045}$$
$$+\frac{0}{2.083}+\frac{0}{2.188}+\frac{0}{2.222}+\frac{0}{2.25}+\frac{0}{2.273}+\frac{0.5}{2.386}+\frac{0}{2.5}$$
$$+\frac{0.5}{2.625}+\frac{0.5}{2.727}+\frac{0}{2.778}+\frac{0}{2.813}+\frac{0.5}{2.917}+\frac{0.5}{3}+\frac{0.5}{3.068}$$
$$+\frac{0}{3.125}+\frac{0.5}{3.182}+\frac{0}{3.281}+\frac{0.5}{3.333}+\frac{0.5}{3.375}+\frac{0}{3.409}+\frac{0.5}{3.5}$$
$$+\frac{0.5}{3.636}+\frac{0}{3.646}+\frac{0.5}{3.75}+\frac{0.5}{3.889}+\frac{0.5}{3.977}+\frac{1}{4}+\frac{0.5}{4.091}$$
$$+\frac{0}{4.167}+\frac{0}{4.219}+\frac{0.5}{4.375}+\frac{0.5}{4.444}+\frac{0.5}{4.5}+\frac{0.5}{4.545}+\frac{0}{4.688}$$
$$+\frac{0}{4.773}+\frac{0.5}{4.861}+\frac{0.5}{5}+\frac{0}{5.114}+\frac{0}{5.208}+\frac{0}{5.25}+\frac{0}{5.455}$$
$$+\frac{0}{5.469}+\frac{0.5}{5.556}+\frac{0.5}{5.625}+\frac{0}{5.682}+\frac{0}{5.833}+\frac{0}{6}+\frac{0}{6.136}$$
$$+\frac{0.5}{6.25}+\frac{0}{6.563}+\frac{0}{6.667}+\frac{0}{6.75}+\frac{0}{6.818}+\frac{0}{6.944}+\frac{0}{7.031}$$
$$\left.+\frac{0}{7.5}+\frac{0}{7.813}+\frac{0}{8.333}+\frac{0}{8.438}+\frac{0}{9.375}\right\}$$



b)







b-i)

I_{0^+} $V_1[0.5, 1.5]$ $P_1[0.5, 2.5]$ $V_2[0.4, 0.6]$

$V_1 = 0.5$ $P_1 = 0.5$ $V_2 = 0.4$ $f(a) = 0.625$

$V_1 = 0.5$ $P_1 = 0.5$ $V_2 = 0.6$ $f(b) = 0.417$

$V_1 = 0.5$ $P_1 = 2.5$ $V_2 = 0.4$ $f(c) = 3.125$

$V_1 = 0.5$ $P_1 = 2.5$ $V_2 = 0.6$ $f(d) = 2.083$

$V_1 = 1.5$ $P_1 = 0.5$ $V_2 = 0.4$ $f(e) = 1.875$

$V_1 = 1.5$ $P_1 = 0.5$ $V_2 = 0.6$ $f(f) = 1.25$

$V_1 = 1.5$ $P_1 = 2.5$ $V_2 = 0.4$ $f(g) = 9.375$

$V_1 = 1.5$ $P_1 = 2.5$ $V_2 = 0.6$ $f(h) = 6.25$

$$f(V_1, P_1, V_2) = \frac{V_1 P_1}{P_2}$$

$$\frac{\partial f}{\partial V_1} = \frac{P_1}{P_2} = 0 \quad P_1 = 0$$

$$\frac{\partial f}{\partial P_1} = \frac{V_1}{P_2} = 0 \quad V_1 = 0$$

$$\frac{\partial f}{\partial V_1} = -\frac{V_1 P_1}{{P_2}^2} = 0 \quad P_2 \neq 0$$

$E(0, 0, P_2) \notin \{[0.5, 1.5], [0.5, 2.5], [0.4, 0.6]\}$

$B_{0^+} = [\min(0.625, 0.417, 2.083, 1.875, 1.25, 9.375, 6.25), \max(0.625, 0.417, 2.083, 1.875, 1.25, 9.375, 6.25)]$

$B_{0^+} = [0.417, 9.375]$

$I_{0.5}$

$V_1[0.75, 1.25]$ $P_1[1.75, 2.25]$ $V_2[0.45, 0.55]$

$V_1 = 0.75$ $P_1 = 1.75$ $V_2 = 0.45$ $f(a) = 2.917$

$V_1 = 0.75$ $P_1 = 1.75$ $V_2 = 0.55$ $f(b) = 2.386$

$V_1 = 0.75$ $P_1 = 2.25$ $V_2 = 0.45$ $f(c) = 3.75$

$V_1 = 0.75$ $P_1 = 2.25$ $V_2 = 0.55$ $f(d) = 3.068$

$V_1 = 1.25$ $P_1 = 1.75$ $V_2 = 0.45$ $f(e) = 4.861$

$V_1 = 1.25$ $P_1 = 1.75$ $V_2 = 0.55$ $f(f) = 3.977$

$V_1 = 1.25$ $P_1 = 2.25$ $V_2 = 0.45$ $f(g) = 6.25$

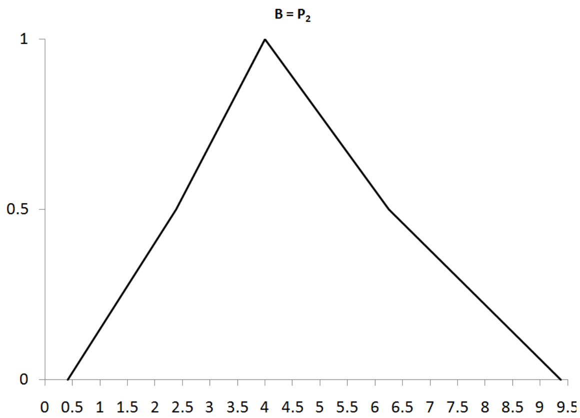
$V_1 = 1.25$ $P_1 = 2.25$ $V_2 = 0.55$ $f(h) = 5.114$

$B_{0.5} = [\min(2.917, 2.386, 3.75, 3.068, 4.861, 3.977, 6.25, 5.114), \max(2.917, 2.386, 3.75, 3.068, 4.861, 3.977, 6.25, 5.114)]$

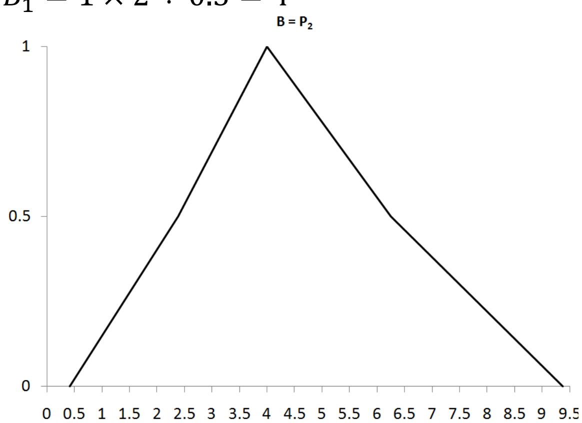
$B_{0.5} = [2.386, 6.25]$

I_1 $V_1 = 1$ $P_1 = 2$ $V_2 = 0.5$

$f(a) = 4$ $B_1 = 4$



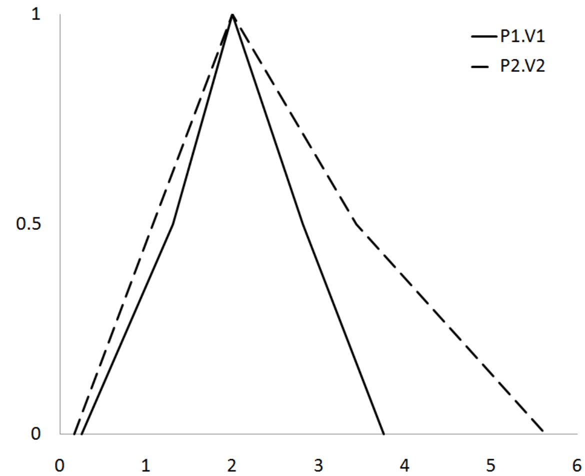
b-ii)
$B_{0^+} = [0.5,1.5].[0.5,2.5] \div [0.4,0.6]$
$= [0.25,3.75] \div [0.4,0.6] = [0.417,9.375]$
$B_{0.5} = [0.75,1.25].[1.75,2.25] \div [0.45,0.55]$
$= [1.313,2.813] \div \{0.45,0.55]$
$= [2.386,6.25]$
$B_1 = 1 \times 2 \div 0.5 = 4$



c) We will use DSW method in this part of the exercise to achieve more comparable results:

P_1V_1: $B_{0^+} = [0.5,1.5].[0.5,2.5]$
$= [0.25,0.375]$
$B_{0.5} = [0.75,1.25].[1.75,2.25]$
$= [1.313,2.813]$
$B_1 = 1 \times 2 = 2$

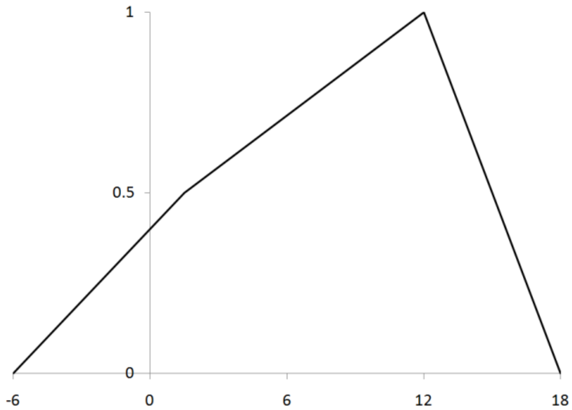
P_2V_2: (Using P_2 calculated in part b-ii)
$B_{0^+} = [0.4,0.6].[0.417,9.937]$
$= [0.167,5.625]$
$B_{0.5} = [0.45,0.55].[2.386,6.25]$
$= [1.074,3.438]$
$B_1 = 0.5 \times 4 = 2$



We can use restricted DSW algorithm in this case: $P_1[a,b]$ $V_1[c,d]$ $V_2[e,f]$
For example, when we calculated I_λ for P_1V_1 we had:
$B_\lambda = [a,b].[c,d] = [ac,bd]$
Also, we calculated P_2 using the equation $P_2 = P_1V_1 \div V_2$, thus for P_2 we obtained the following:
$B_\lambda = [ac,bd] \div [e,f] = \left[\frac{ac}{f},\frac{bd}{e}\right]$
And then we calculated P_2V_2 as follows:
$B_\lambda = \left[\frac{ac}{f},\frac{bd}{e}\right] \times [e,f] = \left[\frac{ace}{f},\frac{bdf}{e}\right]$;
while P_1V_1 we have $B_\lambda = [ac,bd]$.
Therefore, that is evident that we would not have the same P_1V_1 and P_2V_2 due to the property of the interval dividing operation.

12.8 a-i)
I_{0^+} $x_1[1,3]$ $x_2[1,3]$
$x_1 = 1$ $x_2 = 1$ $f(a) = -2$
$x_1 = 1$ $x_2 = 3$ $f(b) = -6$
$x_1 = 3$ $x_2 = 1$ $f(c) = 6$
$x_1 = 3$ $x_2 = 3$ $f(d) = 18$
$B_{0^+} = [\min(-2,-6,6,18), \max(-2,-6,6,18)]$
$= [-6,18]$
$I_{0.5}$ $x_1[2,3]$ $x_2[1.5,2.5]$
$x_1 = 2$ $x_2 = 1.5$ $f(a) = 1.5$
$x_1 = 2$ $x_2 = 2.5$ $f(b) = 2.5$
$x_1 = 3$ $x_2 = 1.5$ $f(c) = 9$
$x_1 = 3$ $x_2 = 2.5$ $f(d) = 15$
$B_{0.5} = [\min(1.5,2.5,9,15), \max(1.5,2.5,9,15)]$
$= [1.5,15]$

$I_1 \quad x_1 = 3 \quad x_2 = 2 \quad f(a) = 12 \quad B_1 = 12$



a-ii) $\frac{\partial y}{\partial x_1} = 2x_1x_2 = 0 \qquad x_1x_2 = 0$

$\frac{\partial y}{\partial x_2} = {x_1}^2 - 3 = 0 \qquad x_1 = \pm\sqrt{3}$

$\rightarrow x_2 = 0$

$E_1(-\sqrt{3}, 0) \rightarrow f(E_1) = 0$

$E_1(+\sqrt{3}, 0) \rightarrow f(E_2) = 0$

$I_{0^+} \quad E_1 \notin \{x_1[1,3], x_2[1,3]\}$

$\quad E_2 \notin \{x_1[1,3], x_2[1,3]\}$

$I_{0.5} \quad E_1 \notin \{x_1[2,3], x_2[1.5,2.5]\}$

$\quad E_2 \notin \{x_1[2,3], x_2[1.5,2.5]\}$

$I_1 \quad E_1 \neq (3,2) \quad , \quad E_2 \neq (3,2)$

Thus, consideration of extreme points does not change the previous result in which we ignored the extreme points.

b) $B_{0^+} = ([1,3])^2 . [1,3] - 3[1,3]$

$= [1,9] . [1,3] - [3,9]$

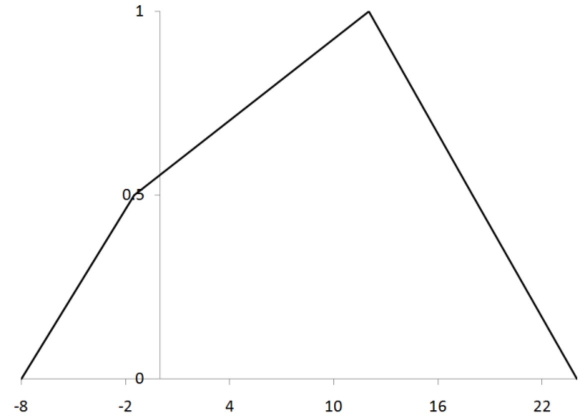
$= [1,27] - [3,9] = [-8,24]$

$B_{0.5} = ([2,3])^2 . [1.5,2.5] - 3[1.5,2.5]$

$= [4,9] . [1.5,2.5] - [4.5,7.5]$

$= [6,22.5] - [4.5,7.5] = [-1.5,18]$

$B_1 = (3)^2 \times 2 - 3 \times 2 = 12$



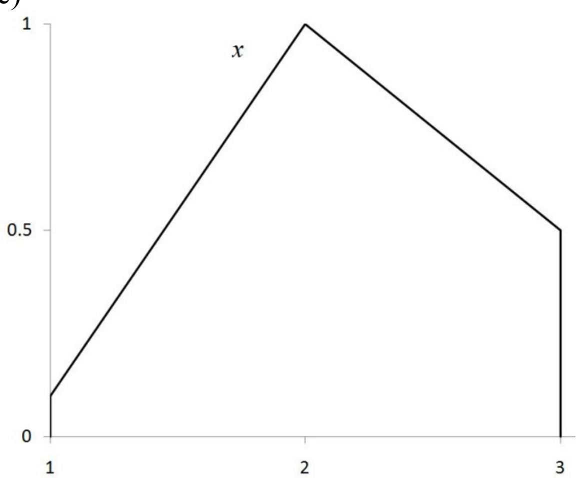
12.9 a) $z = \left\{\frac{0.1}{1} + \frac{1}{4} + \frac{0.5}{9}\right\}$

b) $z = \left\{\frac{0.1}{1} + \frac{1}{2} + \frac{0.5}{3}\right\} \times \left\{\frac{0.1}{1} + \frac{1}{2} + \frac{0.5}{3}\right\}$

$$= \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.1}{3} + \frac{0.1}{2} + \frac{1}{4} + \frac{0.5}{6} + \frac{0.1}{3} + \frac{0.5}{6} + \frac{0.5}{9}$$

$$= \left\{\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.1}{3} + \frac{1}{4} + \frac{0.5}{6} + \frac{0.5}{9}\right\}$$

c)



$z = x^2$

$I_{0^+} \quad x[1,3]$

x = 1 z = 1

x = 3 z = 9

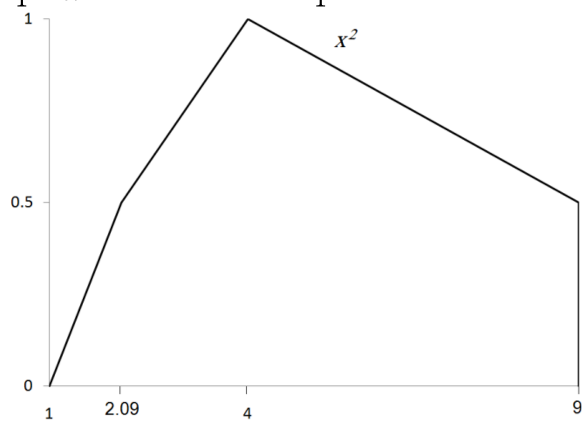
$B_{0^+} = [\min(1,9), \max(1,9)] = [1,9]$

$I_{0.5} \quad x[1.44,3]$

x = 1.44 z = 2.09

x = 3 z = 9

$B_{0.5} = [\min(2.09,9), \max(2.09,9)]$
$= [2.09,9]$
$I_1 \quad x = 2 \quad z = 4 \quad B_1 = 4$



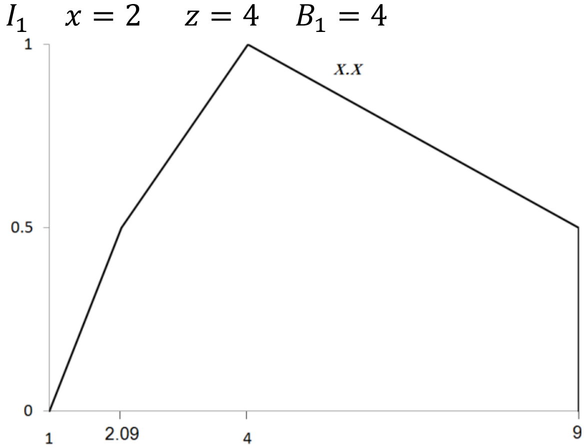
$z = x.x$
$I_{0^+} \quad x[1,3] \quad x[1,3]$

x = 1	x = 1	z = 1
x = 1	x = 3	z = 3
x = 3	x = 1	z = 3
x = 3	x = 3	z = 9

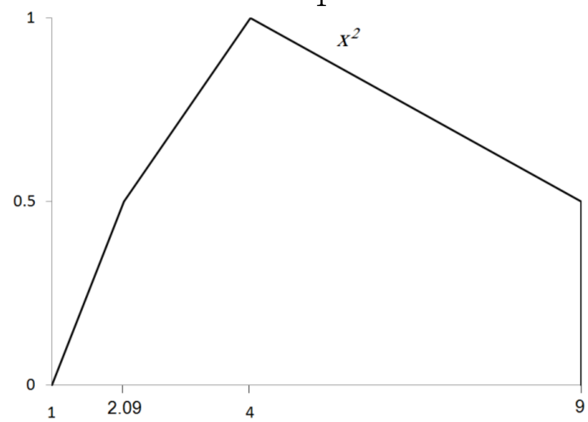
$B_{0^+} = [\min(1,3,9), \max(1,3,9)]$
$= [1,9]$
$I_{0.5} \quad x[1.44,3] \quad x[1.44,3]$

x = 1.44	x = 1.44	z = 2.09
x = 1.44	x = 3	z = 4.32
x = 3	x = 1.44	z = 4.32
x = 3	x = 3	z = 9

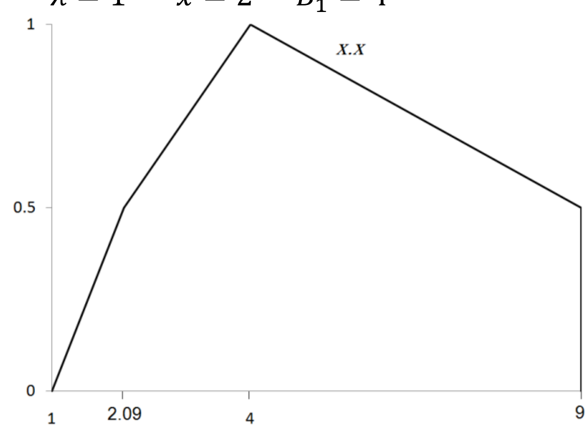
$B_{0.5}$
$= [\min(2.09,4.32,9), \max(2.09,4.32,9)]$
$= [2.0,9]$
$I_1 \quad x = 2 \quad z = 4 \quad B_1 = 4$



d)
$z = x^2$
$\lambda = 0^+ \quad x[1,3]$
$B_{0^+} = ([1,3])^2 = [1,9]$
$\lambda = 0.5 \quad x[1.44,3]$
$B_{0.5} = ([1.44,3])^2 = [2.09,9]$
$\lambda = 1 \quad x = 2 \quad B_1 = 4$



$z = x.x$
$\lambda = 0^+ \quad x[1,3]$
$B_{0^+} = [1,3].[1,3] = [1,9]$
$\lambda = 0.5 \quad x[1.44,3]$
$B_{0.5} = [1.44,3].[1.44,3] = [2.09,9]$
$\lambda = 1 \quad x = 2 \quad B_1 = 4$



e) The answers in the first part (a) and (b) are different because in a) x is a single variable, and when it is squared its value (the crisp value) is squared too. In (b) 'x' and 'x' are treated as two separate variables, i.e. for all practical purpose, they could be 'x' and 'y' with the same membership function.
For both vertex and DSW methods in this exercise there is no difference between $z = x^2$ and $z = x.x$. Also,

outcomes from vertex and DSW are the same.

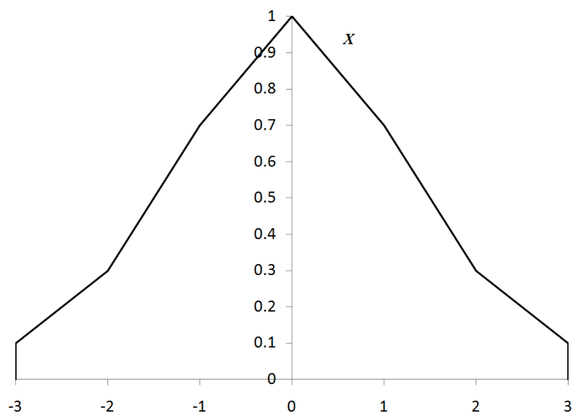
12.10

a) $z = \left\{\frac{1}{0} + \frac{0.7}{1} + \frac{0.3}{4} + \frac{0.1}{9}\right\}$

b) $z = \left\{\frac{0.1}{-3} + \frac{0.3}{-2} + \frac{0.7}{-1} + \frac{1}{0} + \frac{0.7}{1} + \frac{0.3}{2} + \frac{0.1}{3}\right\} \times \left\{\frac{0.1}{-3} + \frac{0.3}{-2} + \frac{0.7}{-1} + \frac{1}{0} + \frac{0.7}{1} + \frac{0.3}{2} + \frac{0.1}{3}\right\}$

$= \{\frac{0.1}{-9} + \frac{0.1}{-6} + \frac{0.3}{-4} + \frac{0.1}{-3} + \frac{0.3}{-2} + \frac{0.7}{-1} + \frac{1}{0} + \frac{0.7}{1} + \frac{0.3}{2} + \frac{0.1}{3} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.1}{9}\}$

c)



$\frac{\partial z}{\partial x} = 2x = 0 \rightarrow x = 0 \rightarrow z = 0$

$z = x^2$

I_{0^+} x[-3, 3]

x = -3 z = 9

x = 3 z = 9

x = 0 z = 0

$B_{0^+} = [\min(0,9), \max(0,9)] = [0,9]$

$I_{0.5}$ x[-1.5, 1.5]

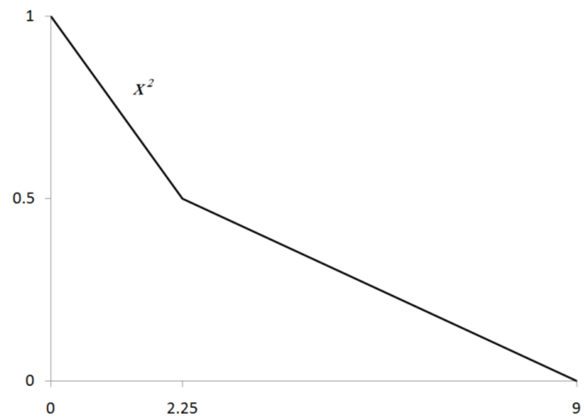
x = -1.5 z = 2.25

x = 1.5 z = 2.25

x = 0 z = 0

$B_{0.5} = [\min(0,2.25), \max(0,2.25)] = [0,2.25]$

I_1 x = 0 z = 0 $B_1 = 0$



z = x.x

I_{0^+} x[-3, 3] x[-3, 3]

x = -3 x = -3 z = 9

x = -3 x = 3 z = -9

x = 3 x = -3 z = -9

x = 3 x = 3 z = 9

$B_{0^+} = [\min(-9,9), \max(-9,9)] = [-9,9]$

$I_{0.5}$ x[-1.5, 1.5] x[-1.5, 1.5]

x = -1.5 x = -1.5 z = 2.25

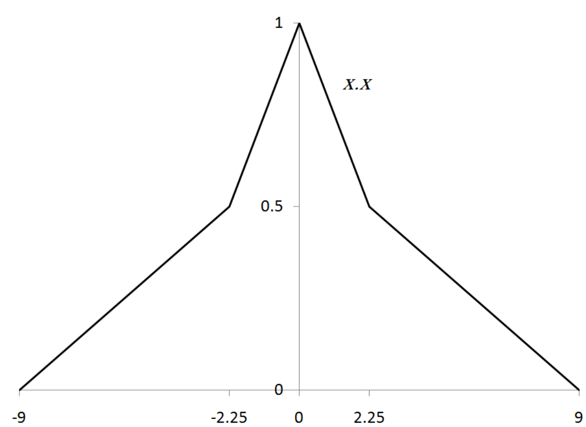
x = -1.5 x = 1.5 z = -2.25

x = 1.5 x = -1.5 z = -2.25

x = 1.5 x = 1.5 z = 2.25

$B_{0.5} = [\min(-2.25,2.25), \max(-2.25,2.25)] = [0,2.25]$

I_1 x = 0 z = 0 $B_1 = 0$



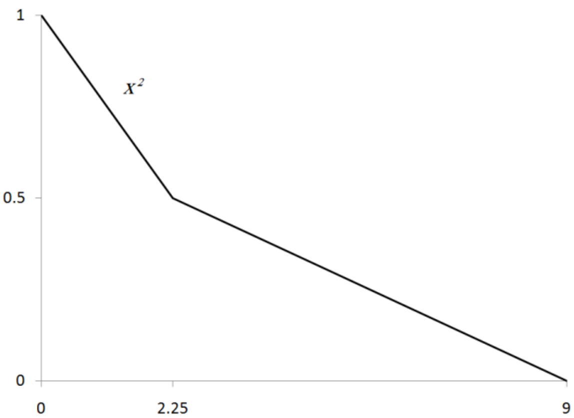
d) $z = x^2$

$\lambda=0^+$ x[-3, 3] $B_{0^+} = ([-3,3])^2 = [0,9]$

$\lambda=0.5$ x[-1.5, 1.5]

$B_{0.5} = ([-1.5,1.5])^2 = [0,2.25]$

$\lambda=1$ x=0 $B_1=0$



z = x.x

$\lambda=0^+$ x[-3, 3] x[-3, 3]

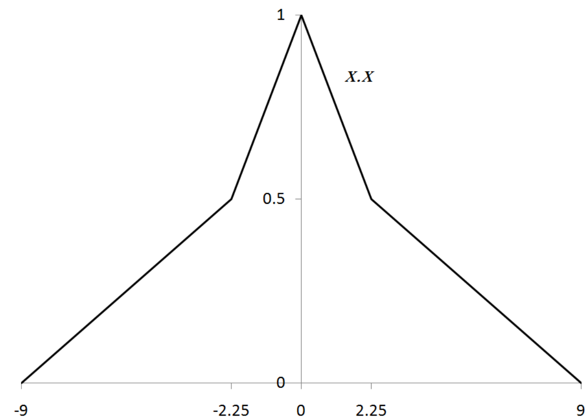
$B_{0^+} = [-3,3].[-3,3] = [-9,9]$

$\lambda=0.5$ x[-1.5, 1.5] x[-1.5, 1.5]

$B_{0.5} = [-1.5,1.5].[-1.5,1.5]$

$= [-2.25,2.25]$

$\lambda=1$ x=0 $B_1=0$



e) Despite the problem 12.9 there is a difference between $z = x^2$ and $z = x \, . \, x$ because the extreme point (x = 0 , z = 0) is located in the internal. The outcomes from both vertex and DSW methods are still the same.

12.11

$$\underset{\sim}{t} = \{\frac{\min(1.0,1.0)}{30} + \frac{\max(\min(1.0,0.6),\min(0.8,1.0))}{40} + \frac{\max(\min(1.0,0.3)\min(0.8,0.6),\min(0.5,1.0))}{50} + \frac{\max(\min(0.8,0.3),\min(0.5,0.6))}{60} + \frac{\min(0.5,0.3)}{70}\}$$

$$\underset{\sim}{t} = \left\{\frac{1.0}{30} + \frac{0.8}{40} + \frac{0.6}{50} + \frac{0.5}{60} + \frac{0.3}{70}\right\}$$

CHAPTER 13

Fuzzy Control Systems

13.1

The membership functions can be expressed as:

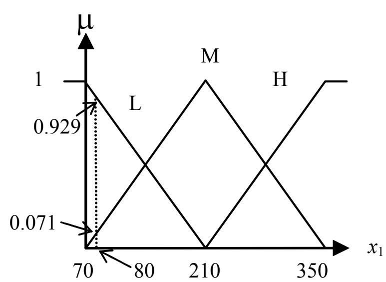
$$\mu_{L(x)} = \begin{cases} 1 & if\ \ x \le 70 \\ 1 - \dfrac{1}{140}(x-70), & if\ \ x \in (70,210) \\ 0, & otherwise \end{cases}$$

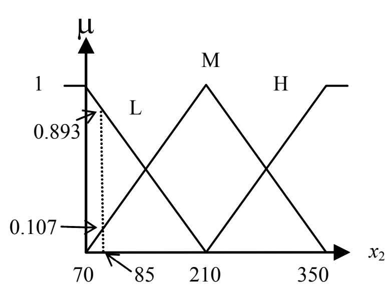
$$\mu_{m(x)} = \begin{cases} \dfrac{1}{140}(x-70), & if\ \ x \in (70,\ 210) \\ 1 - \dfrac{1}{140}(x-210), & if\ \ x \in (210,\ 350) \\ 0, & otherwise \end{cases}$$

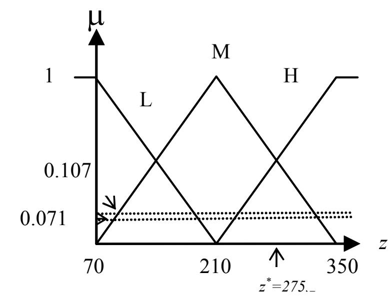
$$\mu_{H(x)} = \begin{cases} 1 & \text{if } x \ge 350 \\ \dfrac{1}{140}(x-210), & \text{if } x \in (210,\ 350) \\ 0, & \text{otherwise} \end{cases}$$

cycle 1: k =0

$x_1(0) = 80^o$, $x_2(0) = 85^o$







From FAM Table, the following rules fire:

$R_1:\quad \min(0.929,\ 0.893) = 0.893(\mathrm{H})$

$R_2:\quad \min(0.071,\ 0.893) = 0.071(\mathrm{H})$

$R_3:\quad \min(0.929,\ 0.107) = 0.107(\mathrm{M})$

$$z^* = \frac{\int_{70}^{85} \frac{1}{40}(x-70)x\mathrm{d}x + \int_{85}^{225} 0.107x\mathrm{d}x + \int_{225}^{335} \frac{1}{140}(x-210)x\mathrm{d}x + \int_{70}^{85} 0.893x\mathrm{d}x}{\int_{70}^{85} \frac{1}{140}(x-70)\mathrm{d}x + \int_{85}^{225} 0.107\mathrm{d}x + \int_{225}^{335} \frac{1}{140}(x-210)\mathrm{d}x + \int_{70}^{85} 0.893\mathrm{d}x}$$

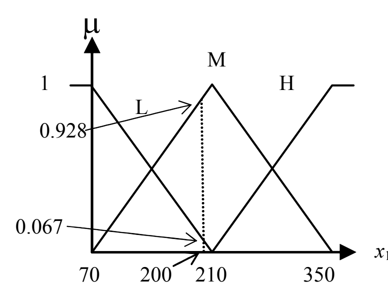
$z^* = 275.20 = \mathrm{U}(0)$

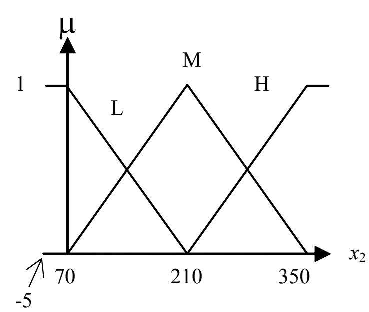
To begin the next cycle (k=1) we find $x_1(1)$ and $x_2(1)$:

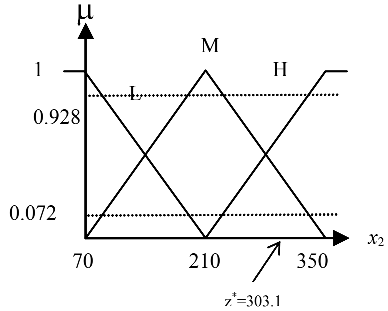
$x_1(1) = -2x_1(0) + x_2(0) + \mathrm{U}(0) = 200.2$

$x_2(1) = x_1(0) - \mathrm{x}_2(0) = -5$

cycle 2







From FAM Table, the following rules fire:

$R_1: \quad \min(0.928, 1) = 0.928(H)$

$R_2: \quad \min(0.072, 1.0) = 0.072(H)$

$z^* = 303.10 = U(1)$

Performing a third cycle (k = 2) the initial points would be.

$x_1(2) = -2x_1(1) + x_2(1) + U(1) = -102.3$

$x_2(2) = x_1(1) - x_2(1) = 205.2$

From FAM Table, the following rules fire:

$R_1: \quad \min(1, 0.964) = 0.964(H)$

$R_2: \quad \min(1, 0.0357) = 0.0357(M)$

$z^* = 292.55 = U(2)$

Performing a fourth cycle (k = 3) the initial points would be.

$x_1(3) = -102.3$

$x_2(3) = -307.5$

From FAM Table:

$R_3: \min(1, 1) = 1(H),\ z^* = 303.33 = U3$

$x_1(3) = -1409.1$

$x_2 4 = -1009.8$

13.2

Cycle 1:

$s(0) = 52.5 \rightarrow \text{High} \,\&\, \text{OK}$

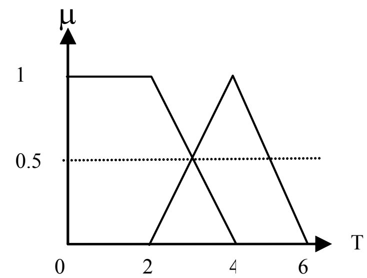
$\theta(0) = -5 \rightarrow \text{Down}$

IF $(s = H \wedge \theta = \text{Down})$ THEN

$(T = \text{Low}) \Rightarrow \min(0.5, 1) = 0.5\ \text{Low}$

IF $(s = OK \wedge \theta = \text{Down})$ THEN

$(T = LM) \Rightarrow \min(0.5, 1) = 0.5\ LM$



T^* =3, Therefore:

$s_1 = 0.9(52.5) + 3 - 0.1(-5) = 50.75$

Cycle 2

$s_1 = 50.75 \rightarrow OK \,\&\, \text{High}$

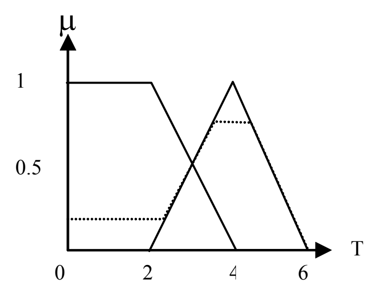
$\theta_1 = -5 \rightarrow \text{Down}$

IF $(s = H \wedge \theta = \text{Down})$ THEN

$(T = \text{Low}) \Rightarrow \min(0.2, 1) = 0.2\ \text{Low}$

IF $(s = OK \wedge \theta = \text{Down})$ THEN

$(T = LM) \Rightarrow \min(0.8, 1) = 0.8\ LM$



T^* =3.8, Therefore: $s_2 = 49.975$

Cycle 3

$s_2 = 49.975 \rightarrow OK \,\&\, \text{Low}$

$\theta_2 = -2.5 \rightarrow \text{Level} \,\&\, \text{Down}$

IF $(s = \text{Low} \wedge \theta = \text{Down})$ THEN

$(T = HM) \Rightarrow \min(.005, 0.5) = 0.05\ HM$

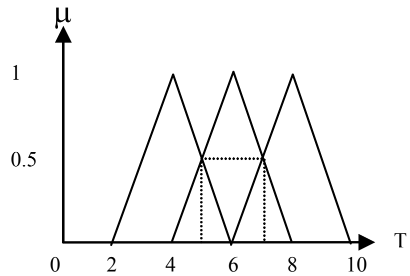
IF $(s = \text{Low} \wedge \theta = \text{Level})$ THEN

$(T = HM) \Rightarrow \min(0.005, 0.5) = 0.005\ HM$

IF $(s = OK \wedge \theta = \text{Down})$ THEN

$(T = LM) \Rightarrow \min(.995, 0.5) = 0.5\ LM$

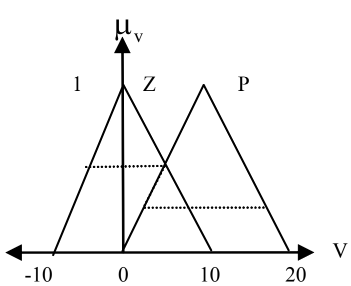
IF $(s = OK \wedge \theta = Level)$ THEN
$(T = M) \Rightarrow \min(0.995, 0.5) = 0.5\ M$



T^* =5, Therefore: $s_3 = 50.228$

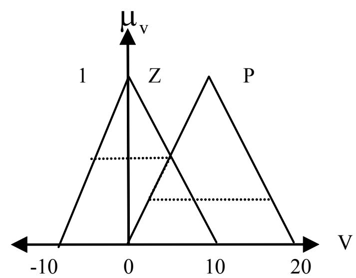
Cycle 43
$s_3 = 50.228 \rightarrow$ OK & HIGH
$\theta_3 = -2.5 \rightarrow$ Level & Down
IF HIGH $\wedge$ DOWN $\Rightarrow \min(0.46, 0.5)$Low
IF HIGH $\wedge$ Level $\Rightarrow \min(0.46, 0.5)$LM
IF OK $\wedge$ DOWN $\Rightarrow \min(0.954, 0.5)$ LM
IF OK $\wedge$ Level $\Rightarrow \min(0.954, 0.5)$ M
T^* =5, Therefore: $s_4 = 50.455$

13.3
Cycle 1:
IF $(x_1 = Z \wedge x_2 = N)$ THEN
$(v = P) \Rightarrow \min(.25, 0.5) = 0.25\ P$
IF $(x_1 = P \wedge x_2 = N)$ THEN
$(v = Z) \Rightarrow \min(0.75, 0.5) = 0.5\ Z$
IF $(x_1 = Z \wedge x_2 = Z)$ THEN
$(v = Z) \Rightarrow \min(0.25, 0.5) = 0.25\ Z$
IF $(x_1 = P \wedge x_2 = Z)$ THEN
$(v = Z) \Rightarrow \min(0.75, 0.5) = 0.5\ Z$



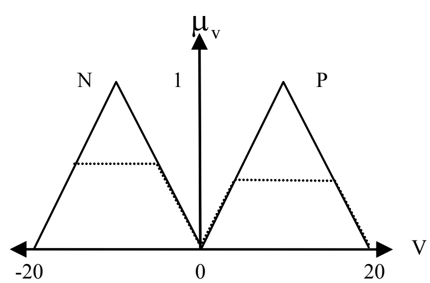
$v^* = 3.44$, $\because x_1(k+1) = x_2(k) + x_1(k)$
$\because x_2(k+1) = 17.5v + 0.36x_2(k)$
$\therefore x_1(1) = -16.4, \quad x_2 = 6.2$

Cycle 2:
IF $(x_1 = N \wedge x_2 = Z)$ THEN
$(v = P) \Rightarrow \min(1, 0.98) = 0.98\ P$
IF $(x_1 = N \wedge x_2 = P)$ THEN
$(v = N) \Rightarrow \min(1, 0.02) = 0.02\ N$



$v^* = 9.6$,
$\therefore x_1(1) = -15.4, \quad x_2 = 170.2$

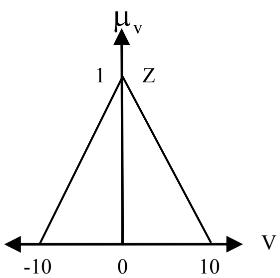
Cycle 3:
IF $(x_1 = N \wedge x_2 = Z)$ THEN
$(v = P) \Rightarrow \min(1, 0.43) = 0.43\ P$
IF $(x_1 = N \wedge x_2 = P)$ THEN
$(v = N) \Rightarrow \min(1, 0.57) = 0.57\ N$



$v^* = -0.09$, $\therefore x_1(1) = 11.7, \quad x_2 = 0.59$

Cycle 4:
IF $(x_1 = P \wedge x_2 = Z)$ THEN
$(v = Z) \Rightarrow \min(1, 1) = 1\ Z$

$$\text{IF } (x_1 = P \wedge x_2 = P) \text{ THEN}$$
$$(v = NB) \Rightarrow \min(1, 0) = 0 \text{ NB}$$



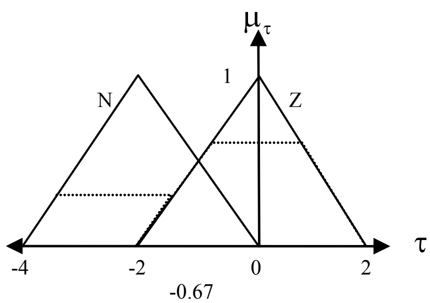
$$v^* = 0.0,$$

13.4

$$\theta(0) = 0.7^\circ, \quad \dot{\theta}(0) = -0.2 \ {}^\circ\!/_{\sec}$$

Cycle 1

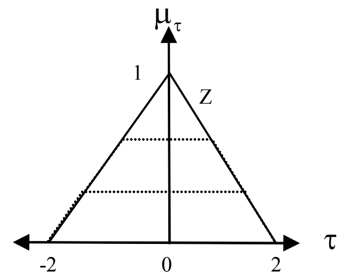
$$\text{IF } (\theta = P \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.8, 0.7) = 0.7 \text{ Z}$$
$$\text{IF } (\theta = P \wedge \dot{\theta} = Z) \text{ THEN}$$
$$(\tau = N) \Rightarrow \min(0.8, 0.3) = 0.3 \text{ Z}$$



$$\tau^* = -0.67 \text{ N} \cdot \text{m},$$
$$\therefore \theta(1) = 0.5, \quad \dot{\theta}(1) = -3.97$$

Cycle 2

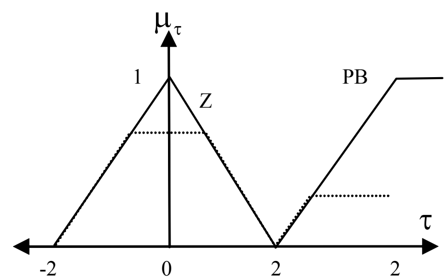
$$\text{IF } (\theta = P \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.62, 1) = 0.62 \text{ Z}$$
$$\text{IF } (\theta = Z \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.37, 1) = 0.37 \text{ Z}$$



$$\tau^* = 0.0 \text{ N} \cdot \text{m},$$
$$\therefore \theta(2) = -0.19, \quad \dot{\theta}(2) = -12.4$$

Cycle 3

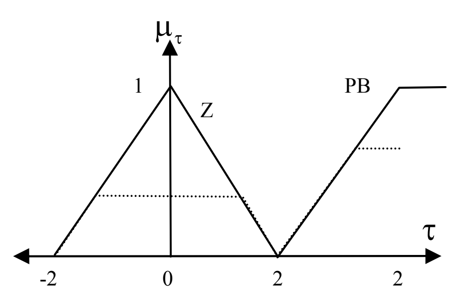
$$\text{IF } (\theta = Z \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.7, 1) = 0.7 \text{ Z}$$
$$\text{IF } (\theta = N \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = PB) \Rightarrow \min(0.25, 1) = 0.25 \text{ PB}$$



$$\tau^* = 1.2 \text{ N} \cdot \text{m},$$
$$\therefore \theta(3) = -0.5, \quad \dot{\theta}(3) = 2.46$$

Cycle 4

$$\text{IF } (\theta = N \wedge \dot{\theta} = P) \text{ THEN}$$
$$(\tau = PB) \Rightarrow \min(0.65, 1) = 0.65 \text{ PB}$$
$$\text{IF } (\theta = Z \wedge \dot{\theta} = P) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.35, 1) = 0.35 \text{ Z}$$



$\tau^* = 1.4056\ \text{N}\cdot\text{m},$

$\therefore \theta(4) = -0.19, \quad \dot{\theta}(4) = 3.63$

13.5

Cycle 1:

$IF\ (I_1 = P \wedge I_2 = N)\ THEN$

$(v = Z) \Rightarrow \min(0.5, 0.5) = 0.25\ Z$

$IF\ (I_1 = P \wedge I_2 = Z)\ THEN$

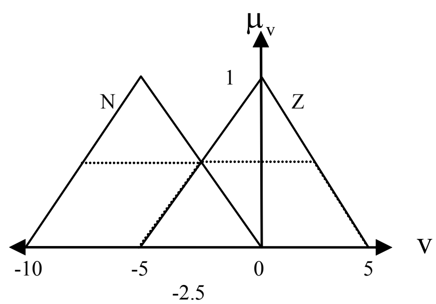
$(v = N) \Rightarrow \min(0.5, 0.5) = 0.5\ N$

$IF\ (I_1 = Z \wedge I_2 = Z)\ THEN$

$(v = Z) \Rightarrow \min(0.5, 0.5) = 0.5\ Z$

$IF\ (I_1 = Z \wedge I_2 = Z)\ THEN$

$(v = Z) \Rightarrow \min(0.5, 0.5) = 0.5\ Z$



$v^* = 2.5,$

$\therefore I_1(1) = -8, \quad I_2(1) = 1.5$

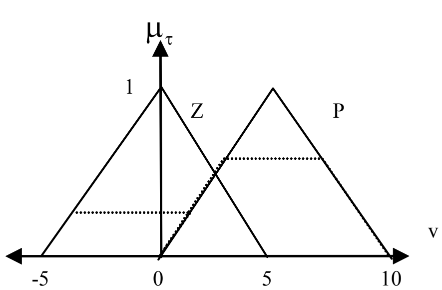
Cycle 2

$IF\ (I_1 = N \wedge I_2 = Z)\ THEN$

$(v = P) \Rightarrow \min(1.0, 0.75) = 0.75\ P$

$IF\ (I_1 = N \wedge I_2 = P)\ THEN$

$(v = Z) \Rightarrow \min(1, 0.25) = 0.25\ Z$



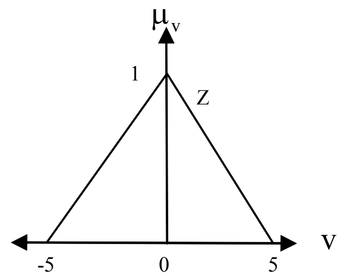
$v^* = 3.08,$

$\therefore I_1(2) = 2.0, \quad I_2(2) = -2.0$

Cycle 3

$IF\ (I_1 = P \wedge I_2 = N)\ THEN$

$(v = Z) \Rightarrow \min(1, 1) = 1.0\ Z$



$v^* = 0,$

$\therefore I_1(2) = -2.0, \quad I_2(2) = 2.0$

Cycle 4

$IF\ (I_1 = N \wedge I_2 = P)\ THEN$

$(v = Z) \Rightarrow \min(1, 1) = 1\ Z$

$v^* = 0,$

$\therefore I_1(2) = 2.0, \quad I_2(2) = 2.0$

13.6

We have a cylindrical tank with cross sectional area, A_c. Liquid flows in at a rate F_i and liquid flows out at a constant rate F_o. We want to control the tank liquid level h using a level controller to change the liquid level set height h_s. The available tank liquid height is H_T. The flow rate in the tank (F_i) is proportional to the percentage that the value is opened. We call this set Flow into the tank F_{is}.

$$F_i - F_o = A_c \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{F_i - F_o}{A_c}$$

The difference between the liquid level set point and the actual tank liquid level is "e"

$e = h - h_s$

and the percent difference is:

$$e = \frac{h - h_s}{h}$$

The percent difference is used to govern the flow into the system with the following rules:

If $e = 0\%$ then $\Delta F_i = 0$
If $e > 10\%$ then $\Delta F_i = 4\%$
If $e < -10\%$ then $\Delta F_i = -4\%$

The percent change in flow into the system (ΔF_i) is:

$$\Delta F_i \% = \frac{F_{is} - F_i}{F_{is}}$$

The initial values are:

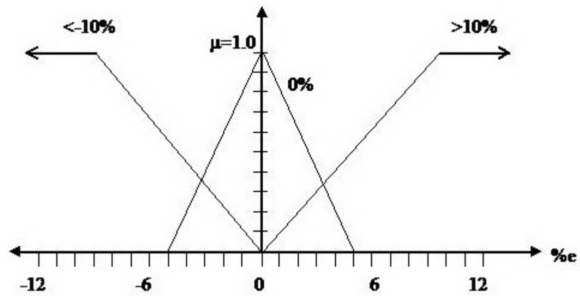
$F_{is} = 0.3\ m^3/s$
$F_{is} = 0.3\ m^3/s$
$H_T = 2\ m$
$A_c = 3\ m^2$
$h_s = 1\ m$

At t = 0 the disturbance in the inlet flow is:

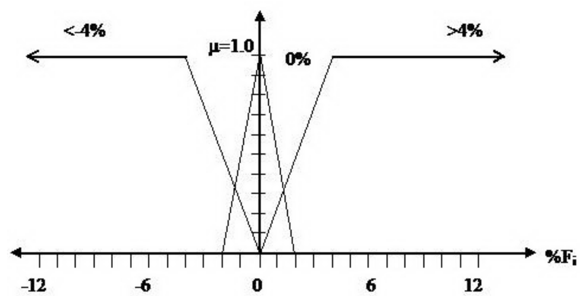
$F_i = 0.4\ m^3/s$
$F_o = 0.3\ m^3/s$
$e = 0$

At t = 0.5s $\Delta h = \frac{0.4 - 0.3}{3} = 0.03\ m$, thus h = 1.03 m and $\%e = \frac{1.03 - 1}{1} = 3\%$.

We now make use of the fuzzy controller. The input membership function is as follows:



While the output membership function is as follows:



The value for percent error is a member of the fuzzy set 0% and >10% and triggers the first and second rule.

$$\mu_{e_{0\%}}(3) = -\frac{1}{5}(3) + 1 = 0.4$$
$$\mu_{e_{10\%}}(3) = -\frac{1}{10}(3) + 1 = 0.3$$

Using the weighted average method we determine the percent change in flow.

$$\Delta F_i \% = \frac{(0.4)(0) + 0.3(4)}{0.7} = 1.7$$

Now let's move on to the system after another 0.5 seconds.

At t = 1 s

$$0.017 = \frac{0.3 - F_i}{0.3} = 0.29$$
$$\Delta h = \frac{0.29 - 0.3}{3} = -0.0033$$
$$h = 1.03 - 0.0033 = 1.027\ m$$
$$\%e = \frac{1.027 - 1}{1} = 2.7$$

Now we find that the percent error is a member of the fuzzy set 0 and >10%.

$$\mu_{e_{0\%}}(3) = -\frac{1}{5}(2.7) + 1 = 0.46$$
$$\mu_{e_{10\%}}(3) = -\frac{1}{10}(2.7) + 1 = 0.27$$

Using the weighted average method we determine the percent change in flow.

$$\Delta F_i\,\% = \frac{(0.46)(0) + 0.27(4)}{0.73} = 1.5$$

Now we move on to the system after another 0.5 seconds.

At t = 1.5 s

$$0.015 = \frac{0.3 - F_i}{0.3} = 0.296$$
$$\Delta h = \frac{0.296 - 0.3}{3} = -0.0013$$
$$h = 1.0257\,\text{m}$$
$$\%e = \frac{1.0257 - 1}{1} = 2.6$$

13.7

The transport of toxic chemicals in water principally depends on two phenomena: advection and dispersion. In advection, the mathematical expression for time-variable diffusion is a partial differential equation accounting for concentration difference in space and time, which is derived from Ficks First Law.

$$J = -DA \cdot \frac{\Delta C}{\Delta x}$$
$$V \cdot \frac{\Delta C}{\Delta t} = -DA \cdot \frac{\Delta C}{\Delta x}, \text{ and } V = A \cdot \Delta\alpha$$

$$\frac{\Delta C}{\Delta t} = -D \cdot \frac{\Delta C}{\Delta x \Delta x} \quad \Rightarrow \quad \frac{\partial C}{\partial t} = -D\frac{\partial^2 C}{\partial x^2}$$

Where J = the mass flux rate due to molecular diffusion, mg/s
D = the molecular diffusion coefficient cm^2/s
A = the area of the cross section, cm2
$\frac{\Delta C}{\Delta t}$ = the concentration gradient $mg/cm^3/s$
Δx = movement distance, cm

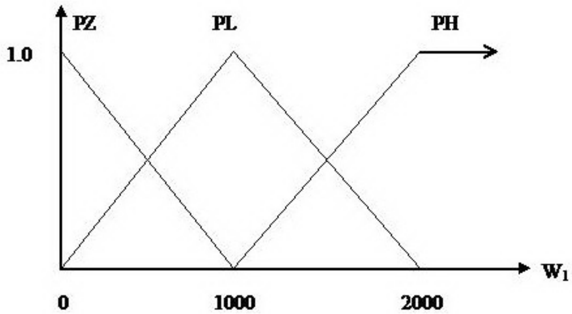
So if we want to control the $\frac{\Delta C}{\Delta t}$, we can set a control as follows:
W_1 = C (concentration, mg/cm^3)
$W_2 = \frac{\Delta C}{\Delta t}$ (concentration gradient, $mg/cm^3/s$)

So $\frac{dW_1}{dx} = W_2$ and
$\frac{dW_1}{dx} = -\alpha\left(if\;\; D = 1.0 cm^2/s\right)$.

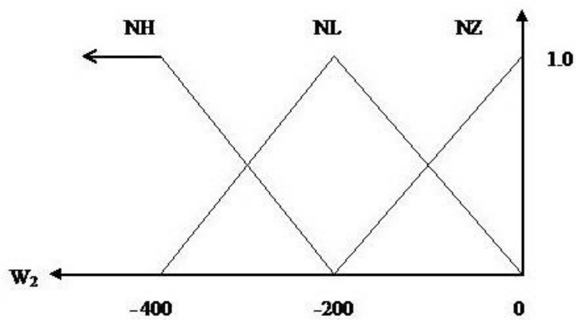
Therefore $W_1(k+1) = W_1(k) + W_2(k)$
and $W_2(k+1) = W_2(k) - \mu(k)$

For this problem, we assume
$0 \le W_1 \le 2000\,\text{mg/cm}^3$
$-400 \le W_2 \le 0\;\; \text{mg/cm}^3$
$0 \le \alpha \le 80\,\text{mg/cm}^3$
(W_2 is negative because flow direction is from high concentration to concentration)

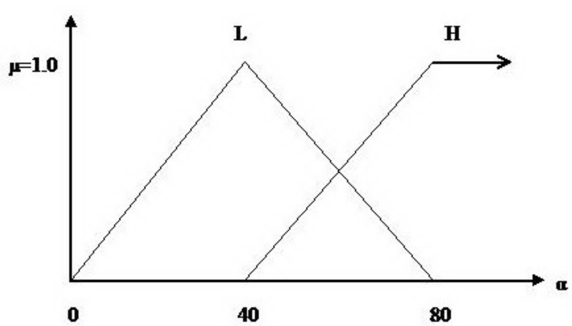
Step 1: Partition W_1 to Zero (PZ), Low (PL), High (PH).



Partition W2 to Zero (Z), Low (NL), High (NH)



Step 2: Partition α to low (L) and High (H).



Step 3: Construct Rules based on experience

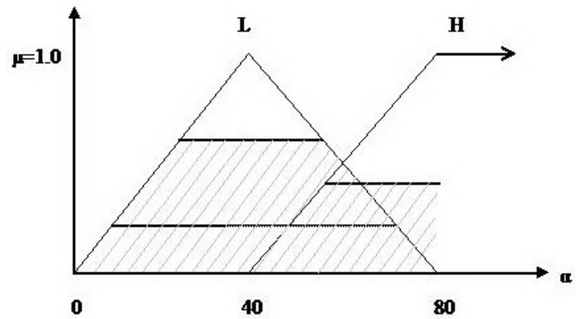
FAM Table

x_1 \ x_2	**NZ**	**NL**	**NH**
PZ	L	L	L
PL	L	L	H
PH	L	H	H

Step 4: Initial Conditions:

$W_1(0) = 800\ \text{mg/cm}^3$,
$W_2(0) = -280\ \text{mg/cm}^3 \cdot \text{cm}$

If $W_1 = PZ$, $W_2 = NL \rightarrow \alpha = L$
and $\wedge(0.2, 0.6) = 0.2(L)$
If $W_1 = PZ$, $W_2 = NH \rightarrow \alpha = L$
and $\wedge(0.2, 0.4) = 0.2(L)$
If $W_1 = PL$, $W_2 = NL \rightarrow \alpha = L$
and $\wedge(0.8, 0.6) = 0.6(L)$
If $W_1 = PL$, $W_2 = NL \rightarrow \alpha = H$
and $\wedge(0.8, 0.4) = 0.4(H)$



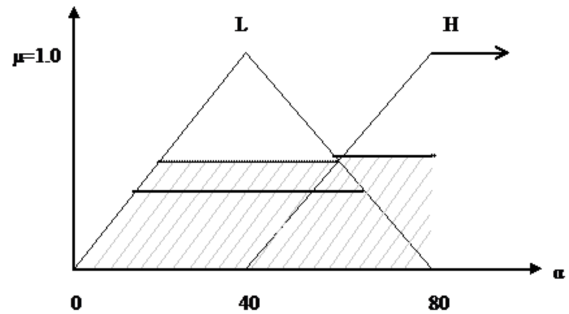
$\alpha^*(0) = 47\ \text{mg/cm}^3 \cdot \text{s}$
$W_1(1) = W_1(0) + W_2(0) =$
$800 - 280 = 520\ \text{mg/cm}^3$

$W_2(1) = W_2(0) + \alpha(0) =$
$-280 - 47 = 327\ \text{mg/cm}^3 \cdot \text{cm}$

Step 5: Second Cycle

If $W_1 = PZ$, $W_2 = NH \rightarrow \alpha = L$
and $\wedge(0.48, 0.64) = 0.48(L)$

If $W_1 = PZ$, $W_2 = NL \rightarrow \alpha = L$
and $\wedge(0.48, 0.36) = 0.36(L)$
If $W_1 = PL$, $W_2 = NH \rightarrow \alpha = H$
and $\wedge(0.52, 0.64) = 0.52(H)$
If $W_1 = PL$, $W_2 = NL \rightarrow \alpha = L$
and $\wedge(0.52, 0.36) = 0.36(L)$



$\alpha^*(1) = 48\ \text{mg}/\text{cm}^3 \cdot \text{s}$

$W_1(2) = W_1(1) + W_2(1) = 193\ \text{mg}/\text{cm}^3$

$W_2(1) = W_2(0) + \alpha(0) =$

$-375\ \text{mg}/\text{cm}^3 \cdot \text{cm}$

13.8

GIS (Global Information System) is a powerful tool in Environmental Modeling. It integrates geographical information with data stored in databases. The main issue in using GIS is selecting the appropriate spatial resolution. If the spatial resolution selected is low, then the mapping tool cannot fully represent the true topograph. On the other hand, if the spatial resolution selected is too high, then the data base size will be larger than necessary thus increasing storage requirement and processing speed.

Two parameters can be used in a fuzzy control system, to govern the GIS. The first one is the digital elevation value (DE). This value is the difference between the highest and lowest elevation in that certain area. The second parameter is the area of coverage (AC). The update equation is defined as follows:

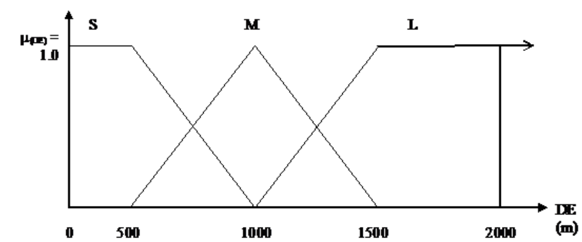
$$DE_{new} = \frac{SR^2}{AC} DE_{old} + DE_{old}$$

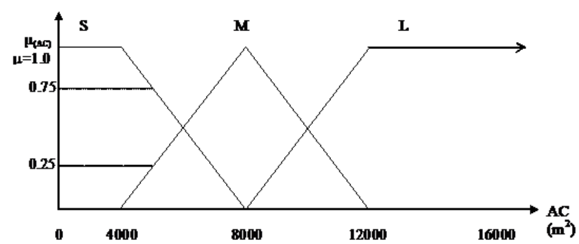
In the above equation SR represents spatial resolution (meters).

The first input is DE and can be either {small, medium, large} (in meters), the second input is AC and can be either {small, medium, large} (in meters^2) and SR is the output, which can either be {increase (I), decrease (D)}.

FAM Table

x_1 / x_2	L	M	S
L	D	I	I
M	I	D	I
S	D	D	D





Initial condition for DE is DE(0) = 3000 m
Initial condition for AC is AC(0) = 5000 m^2

The specified rules used to govern the system are:

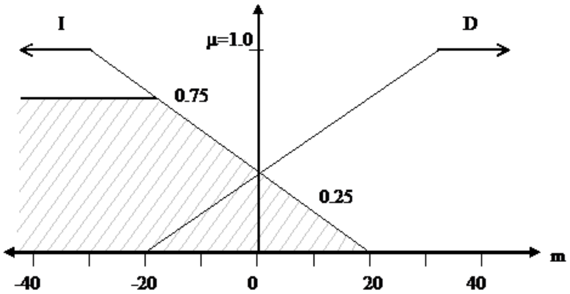
1) If DE is large and AC is small then increase SR.

2) If DE is large and AC is medium then increase SR.

In these rules the initial conditions have the following values:

For rule one, $\wedge(1, 0.75) = 0.75(\mathrm{I})$

For rule two, $\wedge(1, 0.25) = 0.25(\mathrm{I})$



Using the weighted average C = -14.06 meters.

$$DE(1) = \frac{(-14.06)^2}{5000} \cdot 2000 + 2000 = 2079m$$

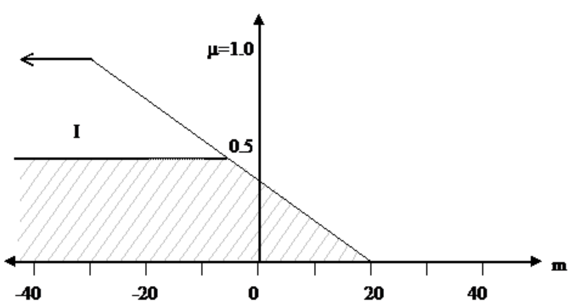
With a measured value of AC(1) = 6000m

Now back to the rule base

1) If DE is large and AC is small then increase SR.
2) If DE is large and AC is medium then increase SR.

In these rules the initial conditions have the following values:

For rule one, $\wedge(1, 0.5) = 0.5(\mathrm{I})$
For rule two, $\wedge(1, 0.5) = 0.5(\mathrm{I})$



The weighted average is C = -17.1 m

13.9

Input membership functions – with input from worker D on day 20.

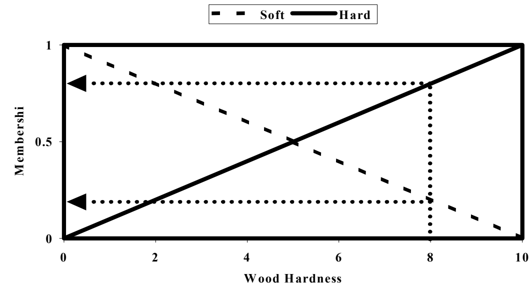


Figure 1. Membership functions for Wood Hardness- example- Wood Hardness = 8, membership in Hard = 0.8 and membership in Soft = 0.2

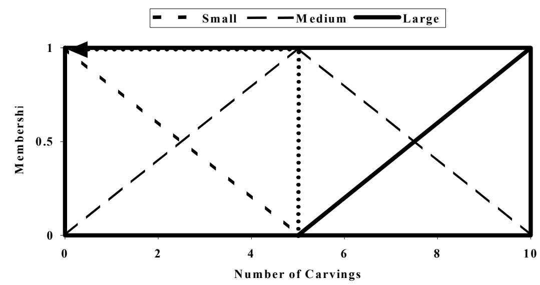


Figure 2. Membership functions for Number of Carvings- example- Number of Carvings = 5, membership in Small =0.0, membership in Medium = 1.0, and membership in Large = 0.0.

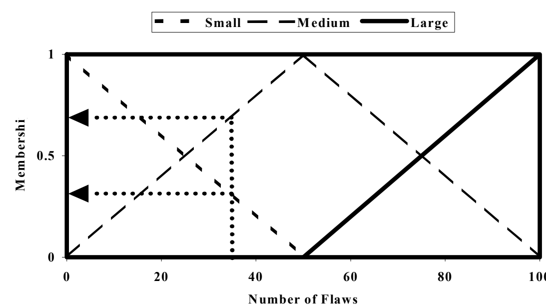


Figure 3. Membership functions for Number of Flaws- example- Number of Flaws = 35, membership in Small =0.3, membership in Medium = 0.7, and membership in Large = 0.0.

Table I. Woodcutters rules fired – with strengths and resolution

Rule Number	Wood Hardness	Number of Carvings	Number of Flaws	Type of Day
4	Soft (0.2)	Medium (1.0)	Small (0.3)	OK (0.2)
5	Soft (0.2)	Medium (1.0)	Medium (0.7)	OK (0.2)
13	Hard (0.8)	Medium (1.0)	Small (0.3)	Fair (0.3)
14	Hard (0.8)	Medium (1.0)	Medium (0.7)	OK (0.7)

On day 20 Worker D made 5 carvings from wood with an average hardness (for all 5 carvings) of 8, containing a total of 35 flaws. The membership values for the rules to be fired are obtained from figures 1-3, as shown above. These values are applied to the rules and four rules are fired as shown in Table I. The Max-Min rule is applied to the rules to find the strength at which each rule is fired (Min-value) and resolve the final output membership, as shown in Table I. The final values for the output membership functions are 0.3 for Fair and 0.7 for OK (Max-value). The output membership functions are "clipped" at these values as shown in figure 4. We used the centroid technique here to defuzzify the "clipped" output membership function. The centroid of this figure is 0.42. This is the type of day for worker D on Day 20. This number is added to the Type of Day for each of the other workers (A= 0.45, B= 0.45, C = 0.43). The average or X bar for Day 20 is then 0.4585and the Range, R, is 0.03. These numbers are added to Table II in the homework problem. The Grand Average (all 20 X bars) is $\overline{\overline{X}} = 0.4585$ and the average Range is $\overline{R} = 0.0615$. If we use the parameters A_2 and D_4 from our abbreviated parameter Table, Table II, we find $A_2 = 0.729$ and $D_4 = 2.282$ for n = 4 (The number of workers).

Table II. Abbreviated Parameter Table

N	A_2	D_3	D_4
2	1.880	-	3.268
3	1.023	-	2.574
4	0.729	-	2.282

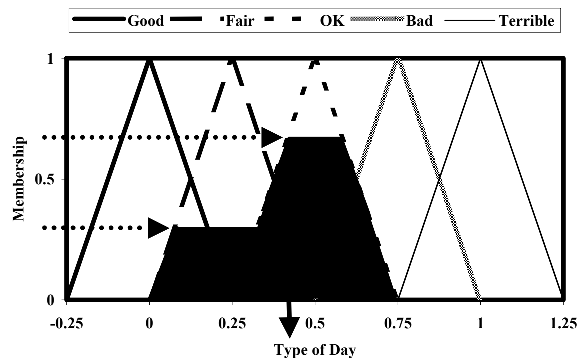


Figure 4. Output membership functions for Type of Day with a centroid at approximately 0.42 for Worker D on Day 20.

We can compute our upper control limit for X bar from equation (1), as = 0.5033 and the lower control limit from equation (2) as = 0.4137. The upper control limit for the range is found from equation (3) to be = 0.140. Recall that there is no lower control limit for the Range for n less than or equal to 6.

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} \quad \textbf{(1)}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} \quad \textbf{(2)}$$

$$UCL_R = D_4\overline{R} \quad \textbf{(3)}$$

The X bar chart for this operation is shown in figure 5. The R chart is shown in figure 6. The X bar chart indicates that the wood carving process was out of control on days 5, 6, and 7. These were most likely the rainy days. Note that there are several points that fall below the lower control limit. This is because there are only 20 data sets and the three out of control sets raised the Grand Average significantly. If data were collected continuously over a long period of time, this would not happen.

Also the out of control situation would be detected immediately. The R chart indicates that there is no significant difference between the workers on a daily basis. The R chart would show, for example if a worker came to work with a hangover and produced more poorly than normal on that day.

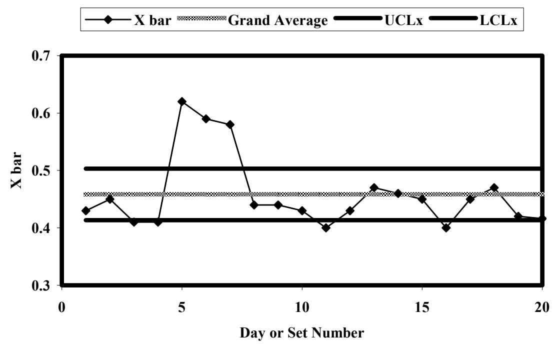


Figure 5. X bar chart for the wood carver problem.

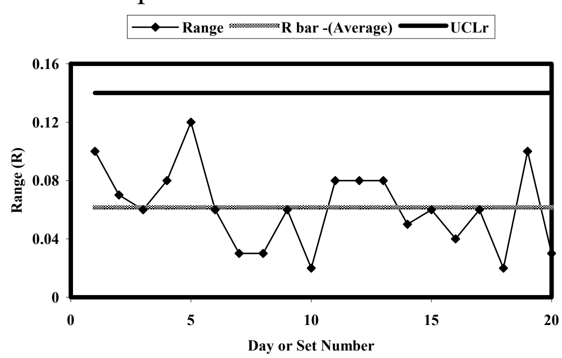


Figure 6. R chart for the wood carver problem.

13.10

Input membership functions – with input for part a.

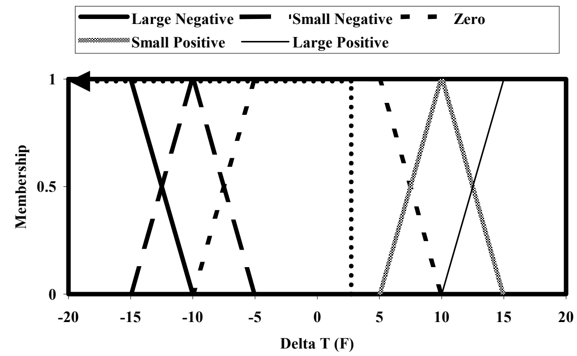


Figure 7. Membership functions for Delta T- For part a) the membership in Zero is 1.0, the membership in all other functions is 0.0.

For part a) with T = 113 F and T_s = 110 F, ΔT = +3. Figure 7. shows that ΔT = +3 means that ΔT has a

membership of 1.0 in the function Zero and a membership of 0.0 in all other functions. Only rule 3 is fired.

If ΔT = Zero (1.0) Then f = Zero (1.0).

The output for part a) is shown in the output membership functions, in figure 8. The centroid is zero and therefore in this case the new valve position is the same as the old one, 0.6. This is an example of using a "dead band" in a control problem.

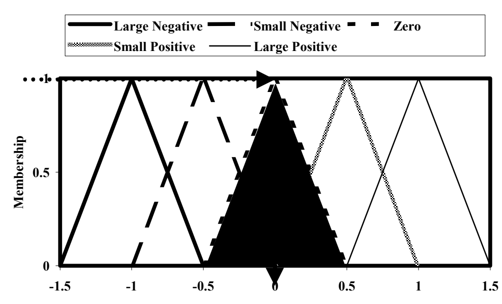


Figure 8. Output membership functions for Valve Fractional Change, f, with a centroid at 0.0 for part a).

Part b) T = 122 F, ΔT = +12. This produces a membership of 0.6 in the set Small

Positive and a membership of 0.4 in the set Large Positive for ΔT, as shown in figure 9.

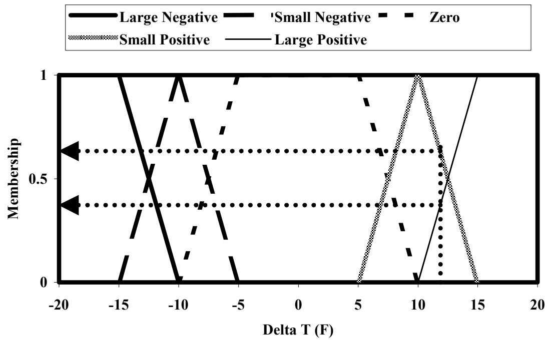


Figure 9. Membership functions for Delta T- For part b) the membership in Small Positive is 0.6, the membership in Large Positive is 0.4, and the membership in all other functions is 0.0.

This causes rules 1 and 2 to be fired with their relative strengths as shown below.

1- If ΔT Large Positive (0.4) Then f is Large Positive (0.4)

2- If ΔT Small Positive (0.6) Then f is Small Positive (0.6)

The output results are shown in figure 10. The centroid of the "clipped" figure is 0.71, the Valve Fractional change, f. Since f is greater than 0.0, the Range is defined as 1.0 – the current valve position (0.6). The Range is 0.4.

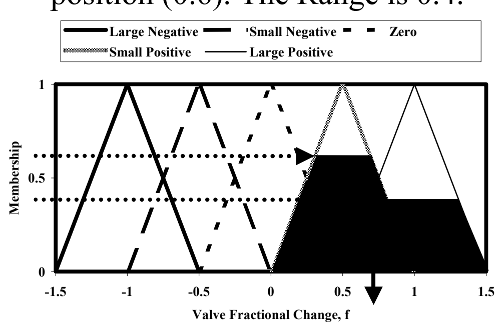


Figure 10. Output membership functions for Valve Fractional Change, f, with a centroid at 0.71 for part b).

The new valve position is equal to the old valve position plus f times the Range (0.6 + 0.71*0.4).
For part b) the new valve position is adjusted to approximately 0.8.

Part c) T = 98 F, ΔT = -12. This produces a membership of 0.6 in the set Small Negative and a membership of 0.4 in the set Large Negative for ΔT, as shown in figure 11.

This causes rules 4 and 5 to be fired with their relative strengths as shown below.

4- If ΔT Small Negative (0.6) Then f is Small Negative (0.6)

5- If ΔT Large Negative (0.4) Then f is Large Negative (0.4)

The output results are shown in figure 12. The centroid of the "clipped" figure is -0.71, the Valve Fractional change, f. Since f is less than 0.0, the Range is defined as the current valve position (0.6). The Range is 0.6. (if we are working from the original valve position)

The new valve position is equal to the old valve position plus f times the Range (0.6 - 0.71*0.6).
For part c) the new valve position is adjusted to approximately 0.174.

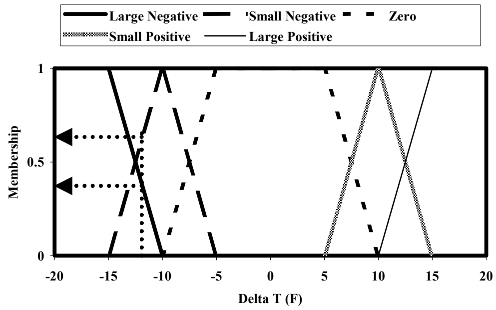


Figure 11. Membership functions for Delta T- For part c) the membership in Small Negative is 0.6, the membership in Large Negative is 0.4, and the membership in all other functions is 0.0.

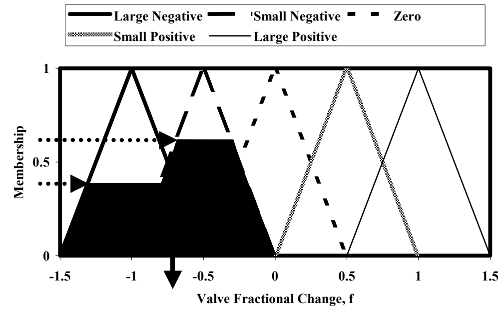


Figure 12. Output membership functions for Valve Fractional Change, f, with a centroid at -0.71 for part c).

CHAPTER 14

Miscellaneous Topics

14.1

The error function for the closed loop system is:

$$\frac{E(s)}{R(s)} = \frac{1}{1+H(s)} = \frac{1}{1+\frac{1}{s+1}} = \frac{s+1}{s+2}$$

Where; E(s) is the error function
R(s) is the input function
H(s) is the system forward transfer function

Now, the input signal is a step function.
So, R(s)= 1/s

$$E(s) = \left(\frac{s+1}{s+2}\right)\frac{1}{s} = \frac{0.5}{s+2} + \frac{0.5}{s}$$

Using Laplace inverse transform, we get

$$e(t) = 0.5e^{-2t} + 0.5$$

Therefore,

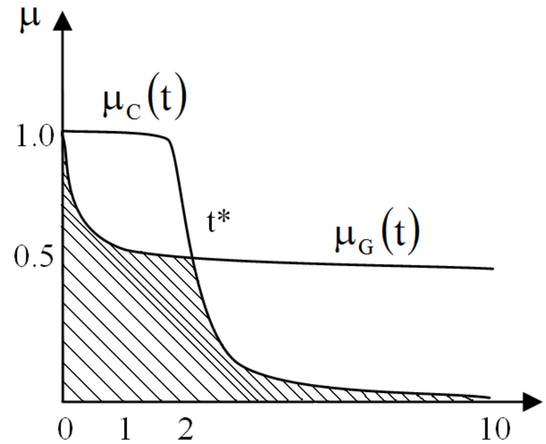
$$\mu_{\underset{\sim}{G}}(t) = \frac{e(t)-0}{1-0} = e(t) = 0.5e^{-2t} + 0.5$$

$$\mu_{\underset{\sim}{D}}(t) = \mu_{\underset{\sim}{C}}(t) \cap \mu_{\underset{\sim}{G}}(t)$$

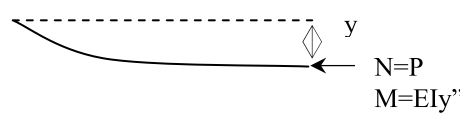
$$\begin{cases} 0.5e^{-2t} + 0.5, & 0 \le t \le t^* \\ e^{1-t}, & t > t^* \end{cases}$$

$$0.5e^{-2t} + 0.5 = e^{1-t}$$

The optimal solution for the minimum error is shown in the figure below.



14.2



$$-Py = M = EIy''$$

$$y'' + \frac{P}{EI}y = 0$$

(1)

So, $y = C_1\sin(kx) + C_2\cos(kx)$ (2)

Consider the boundary conditions for C_1 and C_2:

$x = 0$, $y = 0$, so $C_2 = 0$
$x = l$, $y = 0$, so $C_1\sin(kl) = 0$

Because equation (2) exists, C_1 will not be zero, so $kl = n\pi$, $n = 1,2,\ldots,m$

That is: $k^2 = \frac{P}{EI} = \frac{(\pi n)^2}{l^2}$

$$P = \frac{n^2\pi^2 EI}{l^2}$$

Find $\underset{\sim}{\mu_G}(n)$:

$0 \le n \le 2$, $\quad P(s) = \dfrac{4\pi^2 EI}{l^2}$

$P(0)$=0

$$\underset{\sim}{\mu_G} = \frac{P(n) - P(0)}{P(2) - P(0)} = \frac{n^2}{4}$$

Find the optimal solution n^*

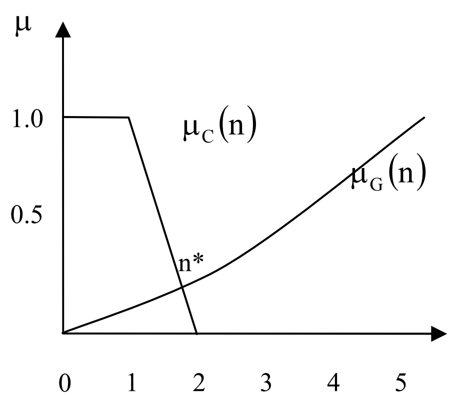
$$\underset{\sim}{\mu_C}(n) = \begin{cases} 1-n, & 1 \le n \le 2 \\ 0, & n > 1 \end{cases}$$

$$\underset{\sim}{\mu_D}(n) = \begin{cases} \dfrac{n^2}{4}, & 0 \le n \le n^* \\ 1-n, & n > n^* \end{cases}$$

$$\frac{n^2}{4} = 1 - n$$

$n^* = 1.86$

Therefore: $P(n^*) = \dfrac{(0.8284)^2 \pi^2 EI}{l^2}$



14.3

Find $\underset{\sim}{\mu_G}(\sigma_b)$

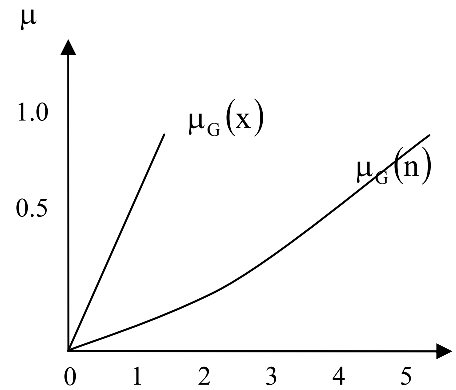
P(0) = 0, $\quad$ P(60M)=$\dfrac{4Wz60M}{l}$

$$\underset{\sim}{\mu_G} = \frac{\sigma_b}{60M}$$

$\because \underset{\sim}{\mu_G}$ is unit less, $\therefore$ let $\dfrac{\sigma_b}{60M} = x$

$\underset{\sim}{\mu_G}(x) = x$

Find μ_G for $\bigcap_{j=1,2} \underset{\sim}{\sigma}_j$

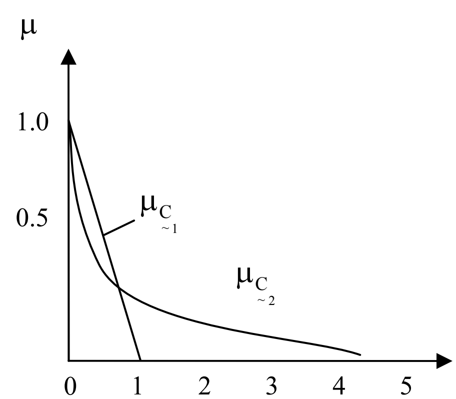


Therfore, $\underset{\sim}{\mu_{G_{\min}}} = \underset{\sim}{\mu_G}(n) = \dfrac{n^2}{25}$

Find, $\mu_{C\min}$:

$$\mu_{C_1}(n) = \begin{cases} 1-n, & 0 \le n \le 1 \\ 0, & n > 1 \end{cases}$$

$$\mu_{C_1}(n) = \begin{cases} \dfrac{1-n}{(1+x)^2}, & 0 \le n \le 1 \\ x, & x > 1 \end{cases}$$



$$\underset{\sim}{\mu_C} = \bigcap_{j=1,2} \mu_{C_j}$$

$$\frac{1}{(1+x)^2} = 1 - x \Rightarrow x = 0.62$$

$$\mu_{\underset{\sim}{c}}(x)=\begin{cases}\dfrac{1}{(1+x)^2}, 0\le x\le 0.62\\ 1-x, 0.62\le x\le 1\end{cases}$$

Find optimal x^* for P:

$$\mu_{\underset{\sim}{D}}=\mu_{\underset{\sim}{C}}\cap\mu_{\underset{\sim}{G}}=\begin{cases}\dfrac{x^2}{25}, & 0\le x\le x^*\\ 1\text{-}x, & x>x^*\end{cases}$$

$$\therefore x^*=0.96$$

$$\sigma_b=0.96*60\text{MPa}=57.6\text{MPa}$$

The minimum P is:

$$P_{\min}=\min\left(\frac{(0.96)^2\pi^2 EI}{l^2},\frac{4Wz(57.6M)}{l^2}\right)$$

14.4

Find out $\mu_{\underset{\sim}{G}}(R)$:

$$\sigma_H(R)=0.564\sqrt{\frac{PE}{LR}},\ 10\le R\le 30$$

So,

$$\sigma_H(10)=0.564\sqrt{\frac{PE}{L}}\sqrt{\frac{1}{10}}$$

$$\sigma_H(30)=0.564\sqrt{\frac{PE}{L}}\sqrt{\frac{1}{30}}$$

$$\mu_{\underset{\sim}{G}}(R)=\frac{\sigma_H(R)-\sigma_H(30)}{\sigma_H(10)-\sigma_H(30)}=\frac{\sqrt{\frac{1}{R}}-0.183}{0.134}$$

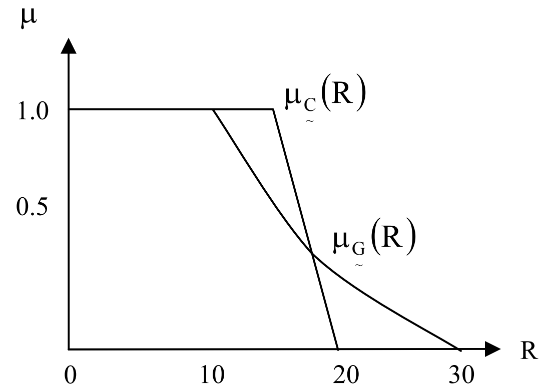
Determine optimal R^*:

$$\mu_{\underset{\sim}{C}}(R)=\begin{cases}1, & 10\le R\le 15\\ 4-\dfrac{R}{5}, & 15<R\le 20\end{cases}$$

$$\mu_{\underset{\sim}{D}}(R)=\begin{cases}\left(\sqrt{\frac{1}{R}}-0.183\right)0.75, & 10\le R\le R^*\\ 4-\dfrac{R}{5}, & R>R^*\end{cases}$$

$$7.5\left(\sqrt{\frac{1}{R}}-0.183\right)=4-\frac{R}{5}$$

$$R^*\approx 18\text{cm}$$



14.5

$[0,1,1,1,-1]$ stabilized in 3 iterations

14.6

$I_1(C_1,C_5)=\min(\text{much},\text{a lot})=\text{much};$

$I_2(C_1,C_5)=\min(\text{much},\text{a lot},\text{much},\text{some})$

$I_2(C_1,C_5)=\text{some}$

$T(C_1,C_5)=\max(I_1,I_2)=\max(\text{much},\text{some})$

$=\text{much}$

Therefore, an increase in "required natural gas" for this facility results in "much economic gain."

14.7 $[0,1,1,1,-1]$ stabilized in 3 iterations

$I_1=\min(\text{a lot},\text{a lot})=\text{a lot};$

$I_2=\min(\text{a lot},\text{some},\text{much},\text{some})$

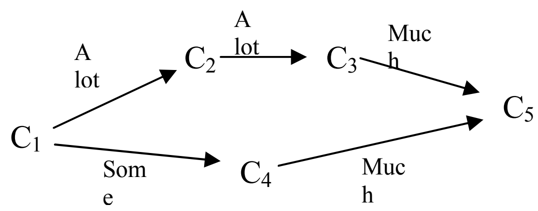
$I_2=\text{some}$

$T=\max(I_1,I_2)=\max(\text{a lot},\text{some})=\text{a lot}$

Therefore, an increase in "CO_2 emissions" for this facility results in "a lot of economic gain"

14.8

$$\begin{bmatrix}0 & 1 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & -1\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

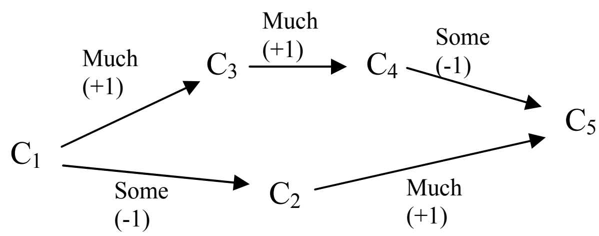


The resulting state vector stabilized after 3 iterations:

[0 0 0 0 -1]

14.9

$$\begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

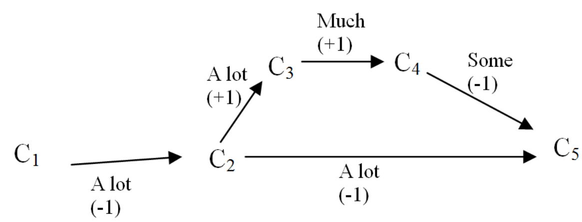


The resulting state vector stabilized after 4 iterations:

[0 0 0 0 0]

14.10

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

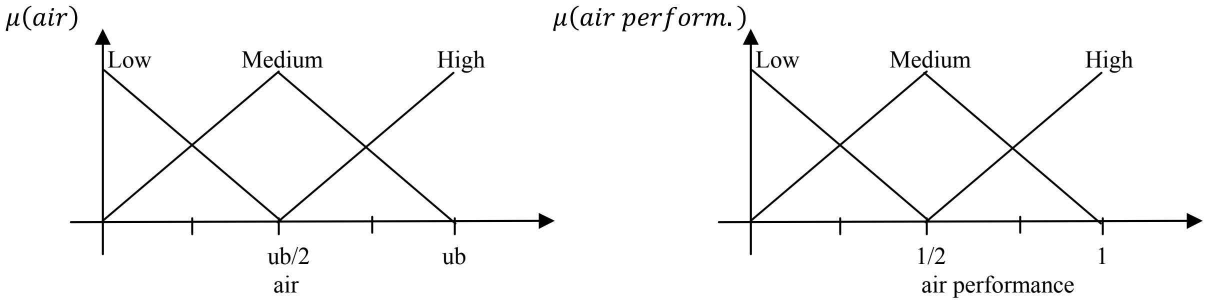


The resulting state vector stabilized after 5 iterations:

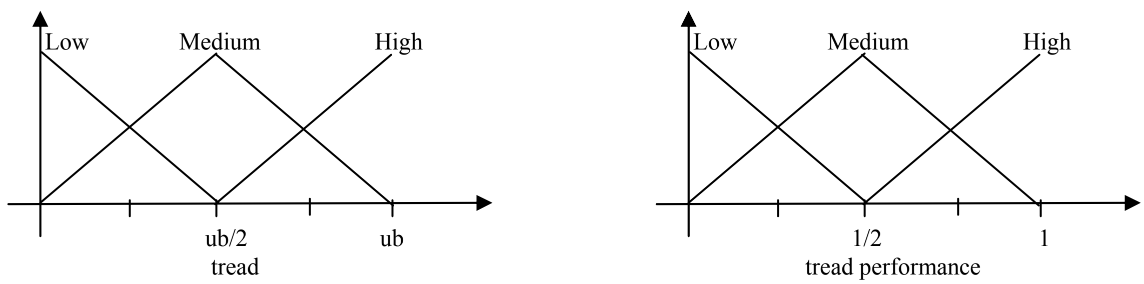
[0 0 0 0 0]

14.11

Air FIS



Tread FIS



$$wf * air\ performance + (1 - wf) * tread\ performance = total\ performance$$

If the amount of air in the tires is medium and the amount of tread in the tires is high:

$$air\ performance = \ 0.5$$

$$tread\ performance = \ 1$$

$$wf = 0.5$$

$$total\ performance = 0.5 * 0.5 + 0.5 * 1 = 0.75$$

14.12

Footings of building foundations classification		Bearing Capacity		
		W	M	G
Loads On Columns	L	O	S	S
	M	O	S	S
	H	--	O	O

Loads on columns: Low (L), Medium (M), High (H)
Bearing Capacity: Weak (W), Moderate (M), Good (G)

Footing Design: Standard (S), Overdesign (O)

Typical Rule:
IF Load is L and Bearing Capacity is W, THEN Footing Design is O

IF Load is L and Bearing Capacity is M, THEN Footing Design is S
IF Load is L and Bearing Capacity is G, THEN Footing Design is S

IF Load is M and Bearing Capacity is W, THEN Footing Design is O
IF Load is M and Bearing Capacity is M, THEN Footing Design is S
IF Load is M and Bearing Capacity is G, THEN Footing Design is S

IF Load is H and Bearing Capacity is W, THEN cannot build
IF Load is H and Bearing Capacity is M, THEN Footing Design is O
IF Load is H and Bearing Capacity is G, THEN Footing Design is O

CHAPTER 15

Monotone Measures: Belief, Plausibility, Probability and Possibility

15.1

Focal Elements	m_1	m_2	bel_1	bel_2	pl_1	pl_2
F	.3	.3	.3	.2	.5	.4
NF	.5	.6	.5	.6	.7	.7
$F \cup NF$	.2	.1	1.0	1.0	1.0	1.0

$bel_1(F) = m_1(F) = 0.3$

$bel_1(NF) = m_1(NF) = 0.5$

$bel_1(F \cup NF) = m_1(F) + m_1(NF) + m_1(F \cup NF)$
$= 0.3 + 0.5 + 0.2 = 1$

$pl_1(F) = m_1(F) + m_1(F \cup NF) = 0.3 + 0.2 = 0.5$

$pl(NF) = m_1(NF) + m_1(F \cup NF) = 0.5 + 0.2 = 0.7$

$pl_1(F \cup NF) = m_1(F) + m_1(NF) + m_1(F \cup NF)$
$= 0.3 + 0.5 + 0.2 = 1.0$

Similarly, other values can be determined.

15.2

FE	m_1	bel_1	pl_1	m_2
R	0.05	0.05	0.8	0.15
D	0.1	0.1	0.9	0.1
W	0	0	0.65	0
$R \cup D$	0.2	0.35	1.0	0.25
$R \cup W$	0.05	0.1	0.9	0.05
$D \cup W$	0.1	0.2	0.95	0.05
$R \cup D \cup W$	0.5	1.0	1.0	0.4

FE	pl_2	m_{12}	bel_{12}	pl_{12}
R	0.85	0.1969	0.1969	0.6795
D	0.80	0.2047	0.2047	0.7559
W	0.5	0.0079	0.0079	0.2963
$R \cup D$	1.0	0.2677	0.6693	0.9577
$R \cup W$	0.9	0.0049	0.2097	0.7609
$D \cup W$	0.85	0.0735	0.2861	0.7687
$R \cup D \cup W$	0.5	1.0	1.0	0.4

$\mathrm{bel}_1(\mathrm{R} \cup \mathrm{W}) = \mathrm{m}_1(\mathrm{R}) + \mathrm{m}_1(\mathrm{W}) + \mathrm{m}_2(\mathrm{R} \cup \mathrm{W})$
$= 0.05 + 0 + 0.05 = 0.1$

$pl_1(R \cup W) = m_1(R) + m_1(W) + m_1(R \cup W)$
$+ m_1(R \cup D) + m_1(D \cup W) + m_1(R \cup D \cup W) = 0.9$

Other values of $bel_1(), bel_2(), pl_1(), pl_2()$ can be determined as above.

$K = m_1(R)m_2(D) + m_1(R)m_2(W) + m_1(R)m_2(D \cup W)$
$+ m_1(D)m_2(R) + m_1(D)m_2(W) + m_1(D)m2(R \cup W)$
$+ m_1(W)m_2(R) + m_1(W)m_2(D) + m_1(W)m_2(R \cup D)$
$+ m_1(R \cup D)m_2(W) + m_1(R \cup W)m_2(D) + m_1(D \cup W)m_2(R)$
$= 0.005 + 0.0025 + 0 + 0.015 + 0 + 0.005$
$+ 0 + 0 + 0 + 0 + 0.005 + 0.15 = 0.0475$
$1 - K = 0.9525$

$m_{12}(D) = [m_1(D)m_2(R \cup D) + m_1(D)m_2(D \cup W)$
$+ m_1(D)m_2(R \cup D \cup W) + m_1(D)m_2(D)$
$+ m_1(R \cup D)m_2(D) + m_1(D \cup W)m_2(D)$
$+ m_1(R \cup D \cup W)m_2(D) + m_1(R \cup D)m_2(D \cup W)$
$+ m_1(D \cup W)m_2(R \cup D)]/(1-k) = 0.2047$

Similarly, other values of $m_{12}()$ can be determined

$\mathrm{bel}_{12}(\mathrm{R}) = \mathrm{m}_{12}(\mathrm{R}) = 0.1969$
$\mathrm{bel}_{12}(\mathrm{R} \cup \mathrm{W}) = \mathrm{m}_{12}(\mathrm{R}) + \mathrm{m}_{12}(\mathrm{W}) + \mathrm{m}_{12}(\mathrm{R} \cup \mathrm{W}) = 0.2097$

Other combined belief values can be computed accordingly.

$pl_{12}(R) = m_{12}(R) + m_{12}(R \cup D) + m_{12}(R \cup W)$
$+ m_{12}(R \cup D \cup W) = 0.1969 + 0.2671 + 0.0049 + 0.21$
$= 0.6795$

Similarly, other combined plausibility values can be determined.

15.3

This problem is similar to 15.2 and can be solved similarly.

FE	m_1	bel_1	pl_1	m_2
C	0.03	0.3	0.85	0.2
I	0.05	0.05	0.6	0.1
L	0.05	0.05	0.45	0.05
C⋃I	0.2	0.55	0.95	0.15
$C \cup L$	0.05	0.4	0.95	0.05
$I \cup L$	0.05	0.5	0.7	0.15
$C \cup I \cup L$	0.3	1	1	0.3

FE	bel_2	pl_2	m_{12}	bel_{12}
C	0.2	0.7	0.4	0.4
I	0.1	0.7	0.15	0.15
L	0.05	0.55	0.04	0.04
C⋃I	0.45	0.95	0.16	0.71
$C \cup L$	0.3	0.9	0	0.44
$I \cup L$	0.3	0.8	0.07	0.26
$C \cup I \cup L$	1	1	0.11	0.93

FE	pl_{12}
C	0.67
I	0.49
L	0.22
C⋃I	0.89
$C \cup L$	0.78
$I \cup L$	0.53
$C \cup I \cup L$	0.93

$$\mathrm{K} = 0.03 + 0.015 + 0.045 + 0.01 + 0.0025 + 0.0025 + 0.01 + 0.05 + 0.0075 + 0.01 + 0.005 + 0.01 = 0.1975$$

$$1 - \mathrm{K} = 0.8025$$

15.4

F.E.	m_1	bel_1
A_1	0.1	0.1
A_2	0.05	0.05
A_3	0.05	0.05
$A_1 \cup A_2$	0.05	0.2
$A_1 \cup A_3$	0.05	0.2
$A_2 \cup A_3$	**0.1**	0.2
$A_1 \cup A_2 \cup A_3$	0.6	1

F.E.	m_2	bel_2
A_1	0	0
A_2	0.05	0.05
A_3	0.1	0.1
$A_1 \cup A_2$	0.05	0.1
$A_1 \cup A_3$	0.15	0.25
$A_2 \cup A_3$	0.05	0.2
$A_1 \cup A_2 \cup A_3$	0.6	1

F.E.	m_{12}	bel_{12}
A_1	0.09	0.09
A_2	0.09	0.09
A_3	0.14	0.14
$A_1 \cup A_2$	0.07	0.25
$A_1 \cup A_3$	0.13	0.36
$A_2 \cup A_3$	0.1	0.33
$A_1 \cup A_2 \cup A_3$	0.38	1

$$\mathrm{K} = 0.005 + 0.01 + 0.005 + 0 + 0.005 + 0.0075 + 0 + 0.025 + 0.005 + 0.0025 + 0 = 0.045$$

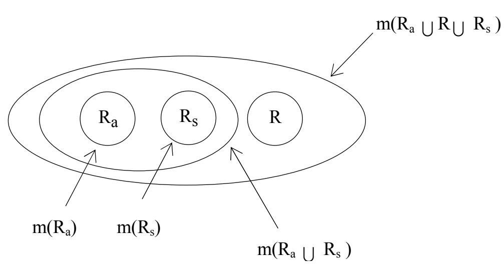
$$1 - \mathrm{K} = 0.955$$

$$m_{12}(A_2) = [0.0025 + 0.0025 + 0.0025 + 0.03 + 0.0025 + 0.0025 + 0.005 + 0.005 + 0.03]/(1 - K) = 0.09$$

15.5

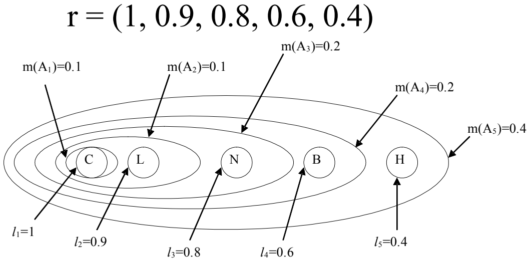
F.E.	m_1	m_2
R_a	0.1	0.2
R	0.1	0
R_s	0	0.4
$R_a \cup R_s$	0.3	0.3
$R \cup R_a$	0.1	0
$R_s \cup R$	0.3	0
$R_a \cup R \cup R_s$	0.1	0.1

a.) m_1 has inter-nesting, so it does not represent a possibility measure.
m_2 is nested and represents a possibility measure.

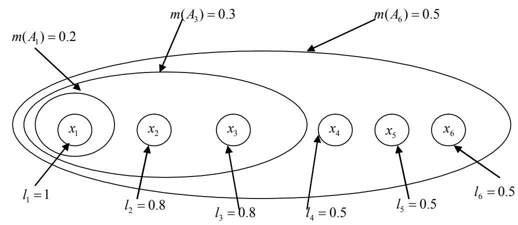


b.)Possibilities distribution for m_2:
r = {1, 0.8, 0.8, 0.4, 0.1, 0.1, 0.1}

15.6
m = (0.1, 0.1, 0.2, 0.2, 0.4) $\Rightarrow$
r = (1, 0.9, 0.8, 0.6, 0.4)



15.7
m = (0.2, 0, 0.3, 0, 0, 0.5) $\Rightarrow$ r = (1.0, 0.8, 0.8, 0.5, 0.5, 0.5)



15.8
By inspection we can determine that m_3 and m_4 would result in an ordered possibility distribution as the sets in the power set are nested.

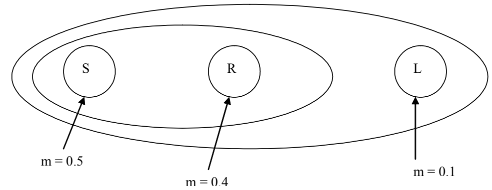
m_3 = (0.2, 0, 0, 0.3, 0.5) basic equation
r_3 = (1, 0.8, 0.8, 0.8, 0.5) ordered possibility distribution

m_4 = (0.1, 0, 0, 0.4, 0.5)
r_4 = (1, 0.9, 0.9, 0.9, 0.5)
Nesting shows that the evidence all tends to agree there can't be any contradicting evidence.

15.9
a)
Only the link (m_2) has basic assignments that result in rested subsets
r = {1, 1, 1, 1, 1, 0.5, 0.5, 0.1}

b and c)
In this case the nesting indicates that there is high possibility of multiple contributions to the error occurrence and while it is possible that either the sender or the receiver and the link are involved (since the link cannot correct errors but only propagates them). It is not very possible that the sources are contributions. More information would be

available if the joint assignments were taken and a possibility distribution examined for that joint assignment.



15.10

INTERSECTIONS

Intervals	Wt.	Normalized Wt
[0.03, 0.06]	0.25	0.125
[0.01, 0.06]	0.25	0.125
[0.03, 0.1]	0.25	0.125
[0.03, 0.15]	0.25	0.125
[0.01, 0.12]	0.25	0.125
[0.03, 0.24]	0.25	0.125
[0.008, 0.06]	0.25	0.125
Support- [0.008, 0.24]	0.25	0.125
	2	1

REDISTRIBUTION

Non-Consonant Intervals	Wt.	Consonant Intervals	β	κ	ρ
[0.01, 0.06]	0.125	[0.03, 0.06]	1	0.472711	0.059089
		[0.03, 0.1]	0.428571	0.20259	0.025324
		[0.03, 0.15]	0.25	0.118178	0.014772
		[0.03, 0.2]	0.176471	0.08342	0.010427
		[0.008, 0.2]	0.260417	0.123102	0.015388
			2.115459		
[0.01, 0.12]	0.125	[0.03, 0.06]	1	0.288696	0.036087
		[0.03, 0.1]	1	0.288696	0.036087
		[0.03, 0.15]	0.583333	0.168406	0.021051
		[0.03, 0.2]	0.411765	0.118875	0.014859
		[0.008, 0.2]	0.46875	0.135326	0.016916
			3.463848		
[0.008, 0.06]	0.125	[0.03, 0.06]	1	0.470395	0.058799
		[0.03, 0.1]	0.428571	0.201598	0.0252
		[0.03, 0.15]	0.25	0.117599	0.0147
		[0.03, 0.2]	0.176471	0.083011	0.010376
		[0.008, 0.2]	0.270833	0.127399	0.015925
			2.125875		

FINAL WEIGHTS

Interval	Wt	Final Wt
[0.03, 0.06]	0.125	0.278975
[0.03, 0.12]	0.125	0.211611
[0.03, 0.15]	0.125	0.175523
[0.03, 0.24]	0.125	0.160663
[0.008, 0.24]	0.125	0.173228
		1

15.11

INTERSECTIONS

Intervals ($x10^{-10}$)		Wt.	Normalized Wt
8	9	0.25	0.125
6	9	0.25	0.125
5	9	0.25	0.125
8	9	0.25	0.125
8	11	0.25	0.125
9	11	0.25	0.125
9	20	0.25	0.125
Support - 5	20	0.25	0.125
		2	1

REDISTRIBUTION

Non-Consonant Intervals ($x10^{-10}$)		Wt.	Consonant Intervals ($x10^{-10}$)		β	κ	ρ
8	8.5	0.125	8	9	0.5	0.75	0.09375
8	8.5	0.125	8	11	0.166667	0.25	0.03125
8	8.5	0.125	9	11	0	0	0
8	8.5	0.125	9	20	0	0	0
					0.666667		
6	8.5	0.125	8	9	0.5	0.75	0.09375
6	8.5	0.125	8	11	0.166667	0.25	0.03125
6	8.5	0.125	9	11	0	0	0
6	8.5	0.125	9	20	0	0	0
					0.666667		
5	9	0.125	8	9	1	0.75	0.09375
5	9	0.125	8	11	0.333333	0.25	0.03125
5	9	0.125	9	11	0	0	0
5	9	0.125	9	20	0	0	0
					1.333333		
5	20	0.125	8	9	1	0.25	0.03125
5	20	0.125	8	11	1	0.25	0.03125
5	20	0.125	9	11	1	0.25	0.03125
5	20	0.125	9	20	1	0.25	0.03125
					4		

FINAL WEIGHTS

Interval ($x10^{-10}$)		Wt	Final Wt
8	9	0.125	0.4375
6	8.5	0.125	0.25
5	9	0.125	0.15625
5	20	0.125	0.15625

15.12

INTERSECTIONS

Intervals	Wt.	Normalized Wt
[1.0, 1.25]	0.2	0.142857
[0.75, 1.25]	0.2	0.142857
[0.75, 1.5]	0.2	0.142857
[1.5, 2.0]	0.2	0.142857
[1.75, 2.0]	0.2	0.142857
[1.75, 2.25]	0.2	0.142857
Support – [0.75, 2.25]	0.2	0.142857
	1.4	

REDISTRIBUTION

Non-Consonant Interval	Wt.	Consonant Interval	β	κ	ρ
[1.75, 2.0]	0.143	[1.0, 1.25]	0	0	0
		[1.0, 0.75]	0	0	0
		[0.75, 1.5]	0	0	0
		[0.75, 2.25]	0.166667	1	0.143
			0.166667		
[1.5, 2.0]	0.143	[1.0, 1.25]	0	0	0
		[1.0, 0.75]	0	0	0
		[0.75, 1.5]	0	0	0
		[0.75, 2.25]	0.333333	1	0.143
[1.75, 2.25]	0.143	[1.0, 1.25]	0	0	0
		[1.0, 0.75]	0	0	0
		[0.75, 1.5]	0	0	0
		[0.75, 2.25]	0.333333	1	0.143

FINAL WEIGHTS

Interval	Wt	Final Wt
[1.0, 1.25]	0.143	0.143
[0.75, 1.25]	0.143	0.143
[0.75, 1.5]	0.143	0.143
[0.75, 2.25]	0.143	0.573
		1.000

a) Given the solution above, the possibility that the interest rates will be greater than 2% - 0.573

b) Generation of consonant intervals depends on the characteristics of the original data intervals and whether the expert desires

a pessimistic or an optimistic estimate. In this case, the expert is assumed to lean more towards the pessimistic attitude and hence a more conservative estimate is generated. Therefore, intervals with lower values are assigned more weight.

c) Degree of confirmation: $C(A) = 0 + 0.573 - 1 = -0.427$

9.17

First scenario: $P = \{0.8, 0.4, 0.8, 0.6\}$

$$\bar{b}_1 = 0.2, \bar{b}_2 = 0.6, \bar{b}_3 = 0.2, \bar{b}_4 = 0.4$$

$$D(a_1) = (0.2 \vee 0.2) \wedge (0.6 \vee 0.6) \wedge (0.2 \vee 1.0) \wedge (0.4 \vee 0.7)$$

$$D(a_1) = (0.2) \wedge (0.6) \wedge (1.0) \wedge (0.7) = 0.2$$
$$D(a_2) = (0.2 \vee 0.9) \wedge (0.6 \vee 1.0) \wedge 2(0.4 \vee 0.6) \wedge (0.4 \vee 0.7)$$

$$D(a_2) = (0.9) \wedge (1.0) \wedge (0.6) \wedge (0.7) = 0.6$$
$$D(a_3) = (0.2 \vee 0.4) \wedge (0.6 \vee 0.2) \wedge (0.2 \vee 0.8) \wedge (0.4 \vee 0.2)$$

$$D(a_3) = (0.4) \wedge (0.6) \wedge (0.8) \wedge (0.4) = 0.4$$
$$D^* = \vee\{D(a_1), D(a_2), D(a_3)\} = 0.6$$

Therefore, the best choice for the first senario is a_2, GOES.

Second scenario: $P = \{0.4, 0.6, 0.4, 0.7\}$

$$\bar{b}_1 = 0.6, \bar{b}_2 = 0.4, \bar{b}_3 = 0.6, \bar{b}_4 = 0.3$$

$$D(a_1) = (0.6 \vee 0.2) \wedge (0.4 \vee 0.6) \wedge (0.6 \vee 1.0) \wedge (0.3 \vee 0.7)$$

$$D(a_1) = (0.6) \wedge (0.6) \wedge (1.0) \wedge (0.7) = 0.6$$
$$D(a_2) = (0.6 \vee 0.9) \wedge (0.4 \vee 1.0) \wedge 2(0.6 \vee 0.6) \wedge (0.3 \vee 0.7)$$

$$D(a_2) = (0.9) \wedge (1.0) \wedge (0.6) \wedge (0.7) = 0.6$$
$$D(a_3) = (0.6 \vee 0.4) \wedge (0.4 \vee 0.2) \wedge (0.6 \vee 0.8) \wedge (0.3 \vee 0.2)$$

$$D(a_3) = (0.6) \wedge (0.4) \wedge (0.8) \wedge (0.3) = 0.3$$
$$D^* = \vee\{D(a_1), D(a_2), D(a_3)\} = 0.6$$

There is a tie between alternatives 1 and 2.

$$\hat{D}(a_1) = (1.0) \wedge (0.7) = 0.7$$
$$\hat{D}(a_2) = (0.9) \wedge (1.0) \wedge (0.7) = 0.7$$

There is still a tie between alternatives 1 and 2.

$$\hat{\hat{D}}(a_1) = (1.0) = 1.0$$
$$\hat{\hat{D}}(a_2) = (0.9) \wedge (1.0) = 0.9$$
$$D^* = \vee\{D(a_1), D(a_2)\} = 1.0$$

The tie is now broken; thus, the best choice for the second scenario is a_1, LANTSANT7.

9.18

$$P = \{0.6, 0.5, 0.6, 0.8, 0.6\}$$

$$\bar{b}_1 = 0.4, \bar{b}_2 = 0.5, \bar{b}_3 = 0.4, \bar{b}_4 = 0.2, \bar{b}_5 = 0.4$$

$$D(Pipe) = (\bar{b}_1 \vee O_1) \wedge (\bar{b}_2 \vee O_2) \wedge (\bar{b}_3 \vee O_3) \wedge (\bar{b}_4 \vee O_4) \wedge (\bar{b}_5 \vee O_5)$$

$$D(Pipe) = (0.4 \vee 0.8) \wedge (0.5 \vee 0.9) \wedge (0.4 \vee 0.6) \wedge (0.2 \vee 0.4) \wedge (0.4 \vee 0.7)$$
$$= (0.8) \wedge (0.9) \wedge (0.6) \wedge (0.4) \wedge (0.7) = 0.4$$

$$D(Pond) = (0.4 \vee 0.5) \wedge (0.5 \vee 0.4) \wedge (0.4 \vee 0.8) \wedge (0.2 \vee 0.9) \wedge (0.4 \vee 0.4)$$
$$= (0.5) \wedge (0.5) \wedge (0.8) \wedge (0.9) \wedge (0.4) = 0.4$$

There is a tie between alternatives 1 and 2.

$$\hat{D}(a_1) = (0.8) \wedge (0.9) \wedge (0.6) \wedge (0.7) = 0.6$$
$$\hat{D}(a_2) = (0.5) \wedge (0.5) \wedge (0.8) \wedge (0.9) = 0.5$$
$$D^* = \vee\{D(a_1), D(a_2)\} = 0.6$$

The tie is broken now; thus the first choice, pipe, is preferred.

9.19 To find: $P(\underset{\sim}{D}_2 \mid \underset{\sim}{M}_3)$ use Equation 9.31 to determine $p(x_k)$.

$$p(x_k) = \sum_{i=1}^{n} p(x_k \mid s_i) \cdot p(s_i)$$
$$p(x_1) = 0.12, p(x_2) = 0.18,$$
$$p(x_3) = 0.29, p(x_4) = 0.15$$
$$p(x_5) = 0.15, p(x_6) = 0.11,$$

Now, using Equation 9.52b for imperfect information.

$$p(\underset{\sim}{D}_2 \mid M_3) = 0.477$$

For perfect information

$$p(x_1) = 0.1, p(x_2) = 0.1,$$
$$p(x_3) = 0.2, p(x_4) = 0.1$$
$$p(x_5) = 0.4, p(x_6) = 0.1,$$
$$p(\underset{\sim}{D}_2 \mid \underset{\sim}{M}_3) = 0.691$$

b) To find: $E(U_1|M_2)$, for imperfect info

$$p(\underset{\sim}{D}_1 \mid \underset{\sim}{M}_2) = 0.361$$
$$p(\underset{\sim}{D}_2 \mid \underset{\sim}{M}_2) = 0.512$$
$$p(\underset{\sim}{D}_3 \mid \underset{\sim}{M}_2) = 0.129$$

Expected Utility using eq. 9.53b

$E\left(u_1 \mid \underset{\sim}{M}_2\right) = 2.340$,

For perfect information,

$p\left(\underset{\sim}{D}_1 \mid \underset{\sim}{M}_2\right) = 0.559$

$p\left(\underset{\sim}{D}_2 \mid \underset{\sim}{M}_2\right) = 0.332$

$p\left(\underset{\sim}{D}_3 \mid \underset{\sim}{M}_2\right) = 0.108$

$E\left(u_1 \mid \underset{\sim}{M}_2\right) = 2.795$

9.20

Using Equation 9.31:

$$p(x_k) = \sum_{i=1}^{n} p(x_k \mid s_i) \cdot p(s_i)$$

$p(x_1) = 0.11,\ p(x_2) = 0.235,$

$p(x_3) = 0.305,\ p(x_4) = 0.35$

a) $P(F_2|M_1) = 0.211$
$P(F_3|M_3) = 0.751$

b) $P(F_1|M_3) = 0.010$
$P(F_2|M_3) = 0.238$
$P(F_3|M_3) = 0.751$

$E(U_1|M_3) = 9.950$

9.21 a)

$p(x_1) = 0.22,\ p(x_2) = 0.18,$

$p(x_3) = 0.165,\ p(x_4) = 0.155,$

$p(x_5) = 0.28$

$P(F_1|x_1) = 0.164,\ P(F_1|x_2) = 0.306$
$P(F_1|x_3) = 0.221,\ P(F_1|x_4) = 0.506$
$P(F_1|x_5) = 0.193,\ P(F_2|x_1) = 0.623$
$P(F_2|x_2) = 0.325,\ P(F_2|x_3) = 0.409$
$P(F_2|x_4) = 0.461,\ P(F_2|x_5) = 0.698$
$P(F_3|x_1) = 0.214,\ P(F_3|x_2) = 0.369$
$P(F_3|x_3) = 0.370,\ P(F_3|x_4) = 0.032$
$P(F_3|x_5) = 0.109$

$E(U_1|x_1) = 0.250, E(U_1|x_2) = 0.319$
$E(U_1|x_3) = 0.742, E(U_1|x_4) = -2.371$
$E(U_1|x_5) = -0.420, E(U_2|x_1) = 0.745$
$E(U_2|x_2) = 0.011, E(U_2|x_3) = -0.667$
$E(U_2|x_4) = 5.665, E(U_2|x_5) = 2.236$

$E(U^* | x_1) = 0.745, E(U^*|x_2) = 0.319$
$E(U^*|x_3) = 0.742, E(U^*|x_4) = 5.665$
$E(U^*|x_5) = 2.236$

$E(U_{\Phi}^*) = 1.848$

$P(F_1) = 0.26, P(F_2) = 0.53, P(F_3) = 0.21$

$E(U^*) = 0.53$

V(x) = 1.848 – 0.53 = 1.318

b) $p(x_1) = 0.15,\ p(x_2) = 0.12,$

$p(x_3) = 0.08,\ p(x_4) = 0.3,$

$p(x_5) = 0.35$

$P(F_1|x_1) = 0.1,\ P(F_1|x_2) = 0$
$P(F_1|x_3) = 0,\ P(F_1|x_4) = 0.7$
$P(F_1|x_5) = 0.1,\ P(F_2|x_1) = 0.8$
$P(F_2|x_2) = 0.2,\ P(F_2|x_3) = 0.2$
$P(F_2|x_4) = 0.3,\ P(F_2|x_5) = 0.8$
$P(F_3|x_1) = 0.1,\ P(F_3|x_2) = 0.8$
$P(F_3|x_3) = 0.8,\ P(F_3|x_4) = 0$
$P(F_3|x_5) = 0.1$

$E(U_1|x_1) = 0, E(U_1|x_2) = 4$
$E(U_1|x_3) = 4, E(U_1|x_4) = -3.5$
$E(U_1|x_5) = 0, E(U_2|x_1) = 1.6$
$E(U_2|x_2) = -7.6, E(U_2|x_3) = -7.6$
$E(U_2|x_4) = 7.6, E(U_2|x_5) = 1.6$

$E(U_{x_1}^*) = 1.6, E(U_{x_2}^*) = 4$
$E(U_{x_3}^*) = 4, E(U_{x_4}^*) = 7.6$
$E(U_{x_5}^*) = 1.6$

$E(U_{xp}^*) = 3.88$

$E(U^*) = 0.53$

V(x_p) = 3.88 – 0.53 = 3.35

Note: $V(x_p) = 3.35 > V(x) = 1.318$
indicates that computations are correct.

CHAPTER 10

Fuzzy Classification and Pattern Recognition

10.1

$$\underset{\sim 1}{R} = \begin{bmatrix} 1 & 0.7 & 0 & 0.2 & 0.1 \\ 0.7 & 1 & 0.9 & 0 & 0.4 \\ 0 & 0.9 & 1 & 0 & 0.3 \\ 0.2 & 0 & 0 & 1 & 0.5 \\ 0.1 & 0.4 & 0.3 & 0.5 & 1 \end{bmatrix}$$

$$\underset{\sim e}{R} = \underset{\sim 3}{R} = \begin{bmatrix} 1 & 0.7 & 0.7 & 0.4 & 0.4 \\ 0.7 & 1 & 0.9 & 0.4 & 0.4 \\ 0.7 & 0.9 & 1 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 1 & 0.5 \\ 0.4 & 0.4 & 0.4 & 0.5 & 1 \end{bmatrix}$$

$$\lambda = 0.9 \quad \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

4 classes $x_1, \{x_2, x_3\}, x_4, x_5$

$$\lambda = 0.7 \quad \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

3 *classes* $\{x_1, x_2, x_3\}, x_4, x_5$

$$\lambda = 0.5 \quad \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

2 *classes* $\{x_1, x_2, x_3\}, \{x_4, x_5\}$

$$\lambda = 0.4 \quad \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

1 *class* $\{x_1, x_2, x_3, x_4, x_5\}$

10.2 $$\mathrm{r_{ij}} = \frac{\sum_{R=1}^{3} \min(x_{ik}, x_{jk})}{\sum_{R=1}^{m} \max(x_{ik}, x_{jk})}$$

$$R(1,2) = \frac{0.2+0.3+0.1}{0.6+0.3+0.5} = 0.429$$

$$R(1,3) = \frac{0+0.3+0.1}{0.6+0.8+0.2} = 0.250$$

$$R(1,4) = \frac{0.6+0.1+0.1}{0.8+0.3+0.1} = 0.667$$

$$R(2,3) = \frac{0+0.3+0.2}{0.2+0.8+0.5} = 0.333$$

$$R(2,4) = \frac{0.2+0.1+0.1}{0.8+0.3+0.5} = 0.250$$

$$R(3,4) = \frac{0+0.1+0.1}{0.8+0.8+0.2} = 0.111$$

$$\underset{\sim 1}{R} = \begin{bmatrix} 1 & 0.429 & 0.250 & 0.667 \\ 0.429 & 1 & 0.333 & 0.250 \\ 0.250 & 0.333 & 1 & 0.11 \\ 0.667 & 0.250 & 0.111 & 1 \end{bmatrix}$$

$$\underset{\sim e}{R} = \underset{\sim 2}{R} = \begin{bmatrix} 1 & 0.429 & 0.333 & 0.667 \\ 0.429 & 1 & 0.333 & 0.429 \\ 0.333 & 0.333 & 1 & 0.333 \\ 0.667 & 0.429 & 0.333 & 1 \end{bmatrix}$$

$$\quad x_1 \quad x_2 \quad x_3 \quad x_4$$

$$\lambda = 0.5 \quad \begin{matrix} x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

3 classes $\{x_1, x_4\}, x_2, x_3$

10.3 a) It is a tolerance relation.
b) Equivalence relation

$$\underset{\sim e}{R} = \underset{\sim 2}{R} = \begin{bmatrix} 1 & 0.7 & 0.5 & 0.7 \\ 0.7 & 1 & 0.5 & 0.8 \\ 0.5 & 0.5 & 1 & 0.5 \\ 0.7 & 0.8 & 0.5 & 1 \end{bmatrix}$$

$$\lambda = 0.4 \quad \underset{\sim 0.4}{R} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$1\, class\ \{x_1, x_2, x_3, x_4\}$

$$\lambda = 0.6 \quad \underset{\sim 0.6}{R} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$2\, classes\ \{x_1, x_2, x_4\}, x_3$

$$\lambda = 0.7 \quad \underset{\sim 0.7}{R} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$2\, classes\ \{x_1, x_2, x_4\}, x_3$

10.4 By using m=2 and $\varepsilon_L \le 0.01$ and assuming $U^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

$$U^{(25)} = \begin{bmatrix} 0.911 & 0.824 & 0.002 & 0.906 \\ 0.089 & 0.176 & 0.998 & 0.094 \end{bmatrix}$$

$V_1 = \{8.458 \quad 1.634\}; V_2 = \{7.346 \quad 1.792\}$

10.5 After 4 cycles :

$$U^{(4)} = \begin{bmatrix} 0.963 & 0.820 & 0.054 & 0.029 \\ 0.037 & 0.180 & 0.946 & 0.971 \end{bmatrix}$$

$V_1 = \{5.431 \quad 6.282\}; V_2 = \{8.470 \quad 10.974\}$

$\varepsilon_L = 0.005$

10.6 After 7 cycles :

$$U^{(7)} = \begin{bmatrix} 0.145 & 0.979 & 0.950 & 0.199 & 0.300 & 0.105 & 0.862 \\ 0.855 & 0.021 & 0.050 & 0.801 & 0.700 & 0.895 & 0.138 \end{bmatrix}$$

$V_1 = \{8.029 \quad 3.047\}; V_2 = \{4.757 \quad 8.094\}$

$\varepsilon_L = 0.006$

10.7 After 4 cycles :

$$U^{(4)} = \begin{bmatrix} 0.999 & 0.175 & 0.055 \\ 0.001 & 0.825 & 0.945 \end{bmatrix}$$

$V_1 = \{2.034 \quad 1.530\}; V_2 = \{3.563 \quad 2.218\}$

$\varepsilon_L = 0.005$

10.8 After 3 cycles: (not converged)

$$U^{(3)} = \begin{bmatrix} 0.941 & 0.969 & 0.731 & 0.003 \\ 0.059 & 0.031 & 0.289 & 0.997 \end{bmatrix}$$

$V_1 = \{-2.619 \quad 1.337\}; V_2 = \{-0.106 \quad 1.767\}$

$\varepsilon_L = 0.125 > 0.01$

After 11 cycles: (converged)

$$U^{(11)} = \begin{bmatrix} 0.941 & 0.969 & 0.731 & 0.003 \\ 0.059 & 0.031 & 0.289 & 0.997 \end{bmatrix}$$

$V_1 = \{-2.153 \quad 1.511\}; V_2 = \{0.845 \quad 1.962\}$

$\varepsilon_L = 0.005$

10.9 After 2 cycles: (not converged)

$$U^{(2)} = \begin{bmatrix} 0.469 & 0.999 & 0.959 & 0.007 & 0.0002 \\ 0.531 & 0.001 & 0.041 & 0.993 & 0.9998 \end{bmatrix}$$

$V_1 = \{233.62 \quad 57.22\}; V_2 = \{753.98 \quad 15.41\}$

$\varepsilon_L = 0.141 > 0.01$

After 6 cycles: (converged)

$$U^{(6)} = \begin{bmatrix} 0.351 & 0.987 & 0.976 & 0.014 & 0.002 \\ 0.649 & 0.013 & 0.024 & 0.986 & 0.998 \end{bmatrix}$$

$V_1 = \{196.15 \quad 63.84\}; V_2 = \{762.92 \quad 14.97\}$

$\varepsilon_L = 0.006$

10.10 Iteration 1

$$v_{1j} = \frac{\sum_{k=1}^{5} \mu_{1k}^{2} x_{kj}}{\mu_{1k}^{2}}, v_{2j} = \frac{\sum_{k=1}^{5} \mu_{2k}^{2} x_{kj}}{\mu_{2k}^{2}}$$

$$v_{11} = \frac{1\times 3.5 + 1\times 4}{1+1} = 3.75$$

$$v_{12} = \frac{1\times 35 + 1\times 25}{1+1} = 30$$

$$v_1 = \{3.75 \quad 30\}$$

$$v_{21} = \frac{1\times 5 + 1\times 7 + 1\times 8}{1+1+1} = 6.667$$

$$v_{22} = \frac{1\times 20 + 1\times 10 + 1\times 22}{1+1+1} = 17.333$$

$$v_2 = \{6.667 \quad 17.333\}$$

$$d_{11} = \sqrt{(5-3.75)^2 + (20-30)^2} = 10.078$$

$$d_{21} = \sqrt{(5-6.667)^2 + (20-17.333)^2} = 3.145$$

$$d_{12} = \sqrt{(3.5-3.75)^2 + (35-30)^2} = 5.006$$

$$d_{22} = \sqrt{(3.5-6.667)^2 + (35-17.333)^2} = 17.949$$

$$d_{13} = \sqrt{(4-3.75)^2 + (25-30)^2} = 5.006$$

$$d_{23} = \sqrt{(4-6.667)^2 + (25-17.333)^2} = 8.118$$

$$d_{14} = \sqrt{(7-3.75)^2 + (10-30)^2} = 20.262$$

$$d_{24} = \sqrt{(7-6.667)^2 + (10-17.333)^2} = 7.341$$

$$d_{15} = \sqrt{(8-3.75)^2 + (22-30)^2} = 9.059$$

$$d_{25} = \sqrt{(8-6.667)^2 + (22-17.333)^2} = 4.854$$

$$\mu_{11} = \left[\left(\frac{d_{11}}{d_{11}}\right)^2 + \left(\frac{d_{11}}{d_{21}}\right)^2\right]^{-1} = \left[1 + \left(\frac{10.078}{3.145}\right)^2\right]^{-1} = 0.089$$

$$\mu_{21} = 1 - \mu_{11} = 1 - 0.089 = 0.911$$

$$\mu_{12} = \left[\left(\frac{d_{12}}{d_{12}}\right)^2 + \left(\frac{d_{12}}{d_{22}}\right)^2\right]^{-1} = \left[1 + \left(\frac{5.005}{17.949}\right)^2\right]^{-1} = 0.928$$

$$\mu_{22} = 1 - \mu_{12} = 1 - 0.928 = 0.072$$

$$\mu_{13} = \left[\left(\frac{d_{13}}{d_{13}}\right)^2 + \left(\frac{d_{13}}{d_{23}}\right)^2\right]^{-1} = \left[1 + \left(\frac{5.006}{8.118}\right)^2\right]^{-1} = 0.724$$

$$\mu_{23} = 1 - \mu_{13} = 1 - 0.724 = 0.276$$

$$\mu_{14} = \left[\left(\frac{d_{14}}{d_{14}}\right)^2 + \left(\frac{d_{14}}{d_{24}}\right)^2\right]^{-1} = \left[1 + \left(\frac{20.262}{7.341}\right)^2\right]^{-1} = 0.116$$

$$\mu_{24} = 1 - \mu_{14} = 1 - 0.116 = 0.884$$

$$\mu_{15} = \left[\left(\frac{d_{15}}{d_{15}}\right)^2 + \left(\frac{d_{15}}{d_{25}}\right)^2\right]^{-1} = \left[1 + \left(\frac{9.059}{4.854}\right)^2\right]^{-1} = 0.223$$

$$\mu_{25} = 1 - \mu_{15} = 1 - 0.223 = 0.777$$

$$\underset{\sim}{U}^{(1)} = \begin{bmatrix} 0.089 & 0.928 & 0.724 & 0.116 & 0.223 \\ 0.911 & 0.072 & 0.276 & 0.884 & 0.777 \end{bmatrix}$$

$$v_{11} = \frac{0.089^2 \times 5 + 0.928^2 \times 3.5 + 0.724^2 \times 4 + 0.116^2 \times 7 + 0.223^2 \times 8}{0.089^2 + 0.928^2 + 0.724^2 + 0.116^2 + 0.223^2} = 3.874$$

$$v_{12} = \frac{0.089^2 \times 20 + 0.928^2 \times 35 + 0.724^2 \times 25 + 0.116^2 \times 10 + 0.223^2 \times 22}{0.089^2 + 0.928^2 + 0.724^2 + 0.116^2 + 0.223^2} = 30.641$$

v_{21}
$= \frac{0.911^2 \times 5 + 0.072^2 \times 3.5 + 0.276^2 \times 4 + 0.884^2 \times 7 + 0.777^2 \times 8}{0.911^2 + 0.072^2 + 0.276^2 + 0.884^2 + 0.777^2}$
$= 6.433$
v_{22}
$= \frac{0.911^2 \times 20 + 0.072^2 \times 35 + 0.276^2 \times 25 + 0.884^2 \times 10 + 0.777^2 \times 22}{0.911^2 + 0.072^2 + 0.276^2 + 0.884^2 + 0.777^2}$
$= 17.323$

$$v_1 = \{3.874 \quad 30.641\};\ v_2 = \{6.433 \quad 17.323\}$$

Iteration 2

$$d_{11} = \sqrt{(5-3.874)^2 + (20-30.641)^2} = 10.701$$
$$d_{21} = \sqrt{(5-6.433)^2 + (20-17.323)^2} = 3.037$$
$$d_{12} = \sqrt{(3.5-3.874)^2 + (35-30.641)^2} = 5.643$$
$$d_{22} = \sqrt{(3.5-6.433)^2 + (35-17323)^2} = 17.919$$
$$d_{13} = \sqrt{(4-3.874)^2 + (25-30.641)^2} = 5.643$$
$$d_{23} = \sqrt{(4-6.433)^2 + (25-17.323)^2} = 8.054$$
$$d_{14} = \sqrt{(7-3.874)^2 + (10-30.641)^2} = 20.877$$
$$d_{24} = \sqrt{(7-6.433)^2 + (10-17.323)^2} = 7.345$$
$$d_{15} = \sqrt{(8-3.874)^2 + (22-30.641)^2} = 9.576$$
$$d_{25} = \sqrt{(8-6.433)^2 + (22-17.323)^2} = 4.933$$

$$\mu_{11} = \left[\left(\frac{d_{11}}{d_{11}}\right)^2 + \left(\frac{d_{11}}{d_{21}}\right)^2\right]^{-1} = \left[1 + \left(\frac{10.701}{3.037}\right)^2\right]^{-1} = 0.076$$
$$\mu_{21} = 1 - \mu_{11} = 1 - 0.076 = 0.925$$
$$\mu_{12} = \left[\left(\frac{d_{12}}{d_{12}}\right)^2 + \left(\frac{d_{12}}{d_{22}}\right)^2\right]^{-1} = \left[1 + \left(\frac{4.375}{17.919}\right)^2\right]^{-1} = 0.944$$
$$\mu_{22} = 1 - \mu_{12} = 1 - 0.944 = 0.056$$
$$\mu_{13} = \left[\left(\frac{d_{13}}{d_{13}}\right)^2 + \left(\frac{d_{13}}{d_{23}}\right)^2\right]^{-1} = \left[1 + \left(\frac{5.643}{8.054}\right)^2\right]^{-1} = 0.671$$
$$\mu_{23} = 1 - \mu_{13} = 1 - 0.671 = 0.329$$
$$\mu_{14} = \left[\left(\frac{d_{14}}{d_{14}}\right)^2 + \left(\frac{d_{14}}{d_{24}}\right)^2\right]^{-1} = \left[1 + \left(\frac{20.877}{8.054}\right)^2\right]^{-1} = 0.110$$
$$\mu_{24} = 1 - \mu_{14} = 1 - 0.110 = 0.890$$
$$\mu_{15} = \left[\left(\frac{d_{15}}{d_{15}}\right)^2 + \left(\frac{d_{15}}{d_{25}}\right)^2\right]^{-1} = \left[1 + \left(\frac{9.576}{4.933}\right)^2\right]^{-1} = 0.210$$
$$\mu_{25} = 1 - \mu_{15} = 1 - 0.210 = 0.790$$

$$\underset{\sim}{U}^{(1)} = \begin{bmatrix} 0.076 & 0.944 & 0.671 & 0.110 & 0.210 \\ 0.924 & 0.056 & 0.329 & 0.890 & 0.790 \end{bmatrix}$$

$$\varepsilon_L = \|U^{(2)} - U^{(1)}\| = 0.054 \geq 0.01$$

not converged

After 11 cycles converges:

$$\underset{\sim}{U}^{(2)} = \begin{bmatrix} 0.026 & 0.984 & 0.447 & 0.115 & 0.108 \\ 0.974 & 0.016 & 0.553 & 0.885 & 0.892 \end{bmatrix}$$
$$v_1 = \{3.672 \quad 32.856\};\ v_2 = \{6.293 \quad 18.309\}$$
$$\varepsilon_L = 0.008$$

10.10

$$\underset{\sim}{U}^{(0)} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$v_{ij} = \frac{\sum_{k=1}^{n} (\mu_{ik})^{m'} \cdot x_{kj}}{\sum_{k=1}^{n} (\mu_{ik})^{m'}}$$

where for c =1,

$$v_{1j} = \frac{\mu_1^2 x_{1j} + \mu_2^2 x_{2j} + \mu_3^2 x_{3j}}{\mu_1^2 + \mu_2^2 + \mu_3^2}$$
$$v_{1j} = \frac{1x_{1j} + 1x_{2j} + 0x_{3j}}{\mu_1^2 + \mu_2^2 + \mu_3^2}$$
$$v_{11} = \frac{50+40}{2} = 45$$
$$v_{11} = \frac{10+12}{2} = 11$$
$$v_1 = \{45, 11\}$$

and for c =2,

$$v_{21} = \frac{20}{1} = 20$$
$$v_{22} = \frac{5}{1} = 5$$
$$v_2 = \{20, 5\}$$

Now calculate the distance measures using Eq (11.29).

$$d_{11} = \sqrt{(50-45)^2 + (10-11)^2} = 5.0990$$
$$d_{12} = \sqrt{(40-45)^2 + (12-11)^2} = 5.0990$$
$$d_{13} = \sqrt{(20-45)^2 + (5-11)^2} = 25.7099$$
$$d_{21} = \sqrt{(50-20)^2 + (10-5)^2} = 30.4138$$
$$d_{22} = \sqrt{(40-20)^2 + (12-5)^2} = 21.1896$$

$$d_{23} = \sqrt{(20-20)^2 + (5-5)^2} = 0$$

$$\mu_{11} = \left[\left(\frac{d_{11}}{d_{11}}\right)^2 + \left(\frac{d_{11}}{d_{21}}\right)^2\right]^{-1} = \left[\left(\frac{5.099}{5.099}\right)^2 + \left(\frac{5.099}{30.4138}\right)^2\right]^{-1} = 0.9727$$

$$\mu_{12} = \left[\left(\frac{d_{12}}{d_{12}}\right)^2 + \left(\frac{d_{12}}{d_{22}}\right)^2\right]^{-1} = \left[1^2 + \left(\frac{5.099}{21.1896}\right)^2\right]^{-1} = 0.9453$$

$$\mu_{13} = \left[\left(\frac{d_{13}}{d_{13}}\right)^2 + \left(\frac{d_{13}}{d_{23}}\right)^2\right]^{-1} = \left[1^2 + \left(\frac{25.7099}{0}\right)^2\right]^{-1} = 0$$

$$\mu_{21} = \left[\left(\frac{d_{21}}{d_{11}}\right)^2 + \left(\frac{d_{21}}{d_{21}}\right)^2\right]^{-1} = \left[\left(\frac{30.4138}{5.099}\right)^2 + (1)^2\right]^{-1} = 0.0273$$

$$\mu_{22} = \left[\left(\frac{d_{22}}{d_{12}}\right)^2 + \left(\frac{d_{22}}{d_{22}}\right)^2\right]^{-1} = \left[\left(\frac{21.1896}{5.099}\right)^2 + (1)^2\right]^{-1} = 0.05474$$

$$\mu_{23} = \left[\left(\frac{d_{23}}{d_{13}}\right)^2 + \left(\frac{d_{23}}{d_{23}}\right)^2\right]^{-1} = \left[\left(\frac{0}{25.7099}\right)^2 + 1^2\right]^{-1} = 1$$

$$\underset{\sim}{U}^{(1)} = \begin{bmatrix} 0.9727 & 0.9453 & 0 \\ 0.0273 & 0.0547 & 1 \end{bmatrix}$$

where for c =1,

$$v_{1j} = \frac{\mu_1^2 x_{1j} + \mu_2^2 x_{2j} + \mu_3^2 x_{3j}}{\mu_1^2 + \mu_2^2 + \mu_3^2}$$

$$v_{1j} = \frac{0.9727 x_{1j} + 0.9453 x_{2j} + 0.x_{3j}}{\mu_1^2 + \mu_2^2 + \mu_3^2}$$

$$v_{11} = 46.9888$$

$$v_{12} = 11.4530$$

$$v_1 = \{46.9888, 11.4530\}$$

and for c =2,

$$v_{21} = 23.4653$$

$$v_{22} = 5.9073$$

$$v_2 = \{23.4653, 5.9073\}$$

Now calculate the distance measures using Eq (11.29).

$$d_{11} = \sqrt{(50-46.9888)^2 + (10-11.4530)^2} = 3.3434$$

$$d_{12} = \sqrt{(40-46.9888)^2 + (12-11.4530)^2} = 7.010$$

$$d_{13} = \sqrt{(20-46.9888)^2 + (5-11.4530)^2} = 27.7495$$

$$d_{21} = \sqrt{(50-23.4653)^2 + (10-5.9073)^2} = 26.8485$$

$$d_{22} = \sqrt{(40-23.4653)^2 + (12-5.9073)^2} = 17.6215$$

$$d_{23} = \sqrt{(20-23.4653)^2 + (5-5.9073)^2} = 3.5821$$

$$\mu_{11} = \left[\left(\frac{d_{11}}{d_{11}}\right)^2 + \left(\frac{d_{11}}{d_{21}}\right)^2\right]^{-1} = \left[(1)^2 + \left(\frac{3.1219}{26.8485}\right)^2\right]^{-1} = 0.984$$

$$\mu_{12} = \left[\left(\frac{d_{12}}{d_{12}}\right)^2 + \left(\frac{d_{12}}{d_{22}}\right)^2\right]^{-1} = \left[1^2 + \left(\frac{7.010}{17.6215}\right)^2\right]^{-1} = 0.8634$$

$$\mu_{13} = \left[\left(\frac{d_{13}}{d_{13}}\right)^2 + \left(\frac{d_{13}}{d_{23}}\right)^2\right]^{-1} = \left[1^2 + \left(\frac{28.0455}{3.5821}\right)^2\right]^{-1} = 0.01605$$

$$\mu_{21} = \left[\left(\frac{d_{21}}{d_{11}}\right)^2 + \left(\frac{d_{21}}{d_{21}}\right)^2\right]^{-1} = \left[\left(\frac{26.8485}{3.3434}\right)^2 + (1)^2\right]^{-1} = 0.01527$$

$$\mu_{22} = \left[\left(\frac{d_{22}}{d_{12}}\right)^2 + \left(\frac{d_{22}}{d_{22}}\right)^2\right]^{-1} = \left[\left(\frac{17.6215}{7.010}\right)^2 + (1)^2\right]^{-1} = 0.1366$$

$$\mu_{23} = \left[\left(\frac{d_{23}}{d_{13}}\right)^2 + \left(\frac{d_{23}}{d_{23}}\right)^2\right]^{-1} = \left[\left(\frac{3.5821}{27.7495}\right)^2 + 1^2\right]^{-1} = 0.9836$$

$$\underset{\sim}{U}^{(2)} = \begin{bmatrix} 0.984 & 0.8643 & 0.01605 \\ 0.0152 & 0.1366 & 0.9836 \end{bmatrix}$$

10.11 a)

$$U^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$V_1(0.679, 85.5, 2.25)$

$V_2\left(\frac{0.7056+0.701+0.718}{3}, \frac{93+91+98.3}{3}, \frac{2.177+2.253+2.177}{3}\right)$

$= V_2(0.7082, 94.1, 2.202)$

$d_{11} = 0$

$d_{12} = \sqrt{(0.679-0.7082)^2 + (85.3-94.1)^2 + (2.25-2.202)^2} = 8.6$

$d_{21} = \sqrt{(0.7056-0.679)^2 + (93-85.5)^2 + (2.177-2.25)^2} = 7.5$

$d_{22} = \sqrt{(0.7056-0.7082)^2 + (93-94.1)^2 + (2.177-2.202)^2} = 1.1$

$d_{31} = \sqrt{(0.701-0.679)^2 + (91-85.5)^2 + (2.253-2.25)^2} = 5.5$

$d_{32} = \sqrt{(0.701-0.7082)^2 + (91-94.1)^2 + (2.253-2.202)^2} = 3.1$

$d_{41} = \sqrt{(0.718-0.679)^2 + (98.3-85.5)^2 + (2.177-2.25)^2} = 12.8$

$d_{42} = \sqrt{(0.718-0.7082)^2 + (98.3-94.1)^2 + (2.177-2.202)^2} = 4.2$

Therefore $U^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

$U^{(1)} = U^{(0)}$ converged

b) Assuming $U^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

$$U^{(7)} = \begin{bmatrix} 0.992 & 0.065 & 0.392 & 0.080 \\ 0.008 & 0.935 & 0.608 & 0.920 \end{bmatrix}$$

$V_1(0.682, 86.309, 2.250), V_2(0.710, 94.769, 2.191)$

$\varepsilon_L = 0.008 < 0.01$ converged

c) Using U in part b

$$F_c(U) = \frac{tr(U * U^T)}{4} = 0.810$$

d) Using equation 10.41 and U in part b

$r_{11} = 1$

$r_{12} = \min(0.992,0.065) + \min(0.008,0.935)$
$= 0.065 + 0.008 = 0.073$

$r_{13} = \min(0.992,0.392) + \min(0.008,0.608)$
$= 0.392 + 0.008 = 0.400$

$r_{14} = \min(0.992,0.008) + \min(0.008,0.920)$
$= 0.080 + 0.008 = 0.088$

$r_{22} = 1$

$r_{23} = \min(0.065,0.392) + \min(0.935,0.608)$
$= 0.065 + 0.608 = 0.673$

$r_{24} = \min(0.065,0.080) + \min(0.935,0.920)$
$= 0.065 + 0.920 = 0.985$

$r_{33} = 1$

$r_{34} = \min(0.392,0.080) + \min(0.608,0.920)$
$= 0.080 + 0.608 = 0.688$

$r_{44} = 1$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.073 & 0.400 & 0.088 \\ & 1 & 0.673 & 0.985 \\ & & 1 & 0.688 \\ sym. & & & 1 \end{bmatrix}$$

10.12 a)

$$\underset{\sim}{U} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$V_1(1.35, 395), V_2(6.5, 278.33)$

b) Assuming $\underset{\sim}{U}^{(0)} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

$$U^{(3)} = \begin{bmatrix} 0.971 & 0.934 & 0.015 & 0.138 & 0.008 \\ 0.029 & 0.066 & 0.985 & 0.862 & 0.992 \end{bmatrix}$$

$V_1(1.412, 395.14), V_2(6.303,275.87)$

$\varepsilon_L = 0.003 < 0.01$ converged

c) Using U in part b

$$F_c(U) = \frac{tr(U*U^T)}{5} = 0.907$$

d) Using equation 10.41 and U in part b

$r_{11} = 1$

$r_{12} = 0.934 + 0.029 = 0.963$

$r_{13} = 0.015 + 0.029 = 0.044$

$r_{14} = 0.138 + 0.029 = 0.167$

$r_{15} = 0.008 + 0.029 = 0.037$

$r_{22} = 1$

$r_{23} = 0.015 + 0.066 = 0.081$

$r_{24} = 0.138 + 0.066 = 0.204$

$r_{25} = 0.008 + 0.066 = 0.074$

$r_{33} = 1$

$r_{34} = 0.015 + 0.862 = 0.877$

$r_{35} = 0.008 + 0.985 = 0.993$

$r_{44} = 1$

$r_{45} = 0.008 + 0.862 = 0.870$

$r_{55} = 1$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.963 & 0.044 & 0.167 & 0.037 \\ & 1 & 0.081 & 0.204 & 0.074 \\ & & 1 & 0.877 & 0.993 \\ & sym. & & 1 & 0.870 \\ & & & & 1 \end{bmatrix}$$

10.13 a)

$$\underset{\sim}{U} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$V_1(-55,3.125), V_2(1.667,3.083)$

b) Assuming $\underset{\sim}{U}^{(0)} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

$$U^{(3)} = \begin{bmatrix} 0.929 & 0.986 & 0.790 & 0.088 & 0.039 \\ 0.071 & 0.014 & 0.230 & 0.912 & 0.961 \end{bmatrix}$$

$V_1(-46.61,3.222), V_2(14.50,2.899)$

$\varepsilon_L = 0.0096 < 0.01$ converged

c) Using U in part b

$$F_c(U) = \frac{tr(U*U^T)}{5} = 0.850$$

d) Using equation 10.40 and U in part b

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 0.943 & 0.841 & 0.160 & 0.111 \\ & 1 & 0.784 & 0.103 & 0.054 \\ & & 1 & 0.319 & 0.270 \\ & sym. & & 1 & 0.951 \\ & & & & 1 \end{bmatrix}$$

CHAPTER 11

Fuzzy Classification and Pattern Recognition

11.1

$\underset{\sim}{a} = \underset{\sim}{b}$

$\underset{\sim}{a} \cdot \underset{\sim}{b}^T = \underset{\sim}{a} \cdot \underset{\sim}{a}^T$

$\overset{n}{\underset{i=1}{\vee}} (a_i \wedge a_i) = \overset{n}{\underset{i=1}{\max}}\, a_i$

$\underset{\sim}{a} \oplus \underset{\sim}{b}^T = \underset{\sim}{a} \oplus \underset{\sim}{a}^T$

$\overset{n}{\underset{i=1}{\wedge}} (a_i \vee a_i) = \overset{n}{\underset{i=1}{\min}}\, a_i$

11.2

$\hat{a} = \hat{b} = 1; \quad \underset{\wedge}{a} = \underset{\wedge}{b} = 0; \quad \underset{\sim}{a} = \underset{\sim}{b}$

i) $\underset{\sim}{a} \cdot \underset{\sim}{b}^T = \underset{\sim}{a} \cdot \underset{\sim}{a}^T$

$\overset{n}{\underset{i=1}{\vee}} (a_i \wedge a_i) = \overset{n}{\underset{i=1}{\max}}\, a_i = \hat{a} = 1$

ii) $\underset{\sim}{a} \oplus \underset{\sim}{b}^T = \underset{\sim}{a} \oplus \underset{\sim}{a}^T$

$= \overset{n}{\underset{i=1}{\wedge}} (a_i \vee a_i) = \overset{n}{\underset{i=1}{\min}}\, a_i = \underset{\wedge}{a} = 0$

11.3

a) $a = \{a_1, a_2, \ldots, a_n\}$ and

$b = \{b_1, b_2, \ldots, b_n\}$

$\overline{a \cdot b} = 1 - a \cdot b = 1 - \{a_1, a_2, \ldots, a_n\} \cdot$

$\{b_1, b_2, \ldots, b_n\}$

$= 1 - \vee\{a_1 \wedge b_1, a_2 \wedge b_2, \ldots, a_n \wedge b_n\}$

$= \wedge \begin{Bmatrix} 1 - (a_1 \wedge b_1), 1 - (a_2 \wedge b_2), \ldots, \\ 1 - (a_n \wedge b_n) \end{Bmatrix}$

but $1 - (a_i \wedge b_i) = (1 - a_i) \vee (1 - b_i)$

thus $\overline{a \cdot b} =$

$= \wedge \begin{Bmatrix} (1 - a_1) \vee (1 - b_1), (1 - a_2) \vee \\ (1 - b_2), \ldots, (1 - a_n) \vee (1 - b_n) \end{Bmatrix}$

$= \{1 - a_1, 1 - a_2, \ldots 1 - a_n\} \oplus$

$\{1 - b_1, 1 - b_2, \ldots 1 - b_n\}$

$= \{\overline{a_1}, \overline{a_2}, \ldots, \overline{a_n}\} \oplus \{\overline{b_1}, \overline{b_2}, \ldots, \overline{b_n}\}$

$= \overline{a} \oplus \overline{b}$

b)

$a \cdot \overline{a} = \{a_1, a_2, \ldots, a_n\} \cdot$

$\{1 - a_1, 1 - a_2, \ldots, 1 - a_n\}$

$= \vee \begin{Bmatrix} a_1 \wedge (1 - a_1), a_2 \wedge (1 - a_2), \ldots, \\ a_n \wedge (1 - a_n) \end{Bmatrix}$

therefore $0 \le a_i \le 1$.

If $a_i < 0.5, \rightarrow (1 - a_i) > 0.5$

therefore,

$a_i \wedge 1 - a_i = \min[a_i, (1 - a_i)] < 0.5$.

If $a_i > 0.5, \rightarrow (1 - a_i) < 0.5$

therefore, $a_i \wedge 1 - a_i < 0.5$.

If $a_i = 0.5, \rightarrow (1 - a_i) = 0.5$

therefore, $a_i \wedge 1 - a_i = 0.5$.

We showed that whatever a_i is, the expression $a \cdot \overline{a} \le 0.5$ is correct.

11.4 a) The expression posed in this problem is wrong. If, for example, A is a normal fuzzy set, then we get the following:

$A = [a_1, a_2, \ldots, a_n]$

$(A, A)_1 = [(A.A) \wedge \overline{(A \oplus A)}]$

$(A, A)_2 = \frac{1}{2}[(A.A) + \overline{(A \oplus A)}]$

$$A.A = \max\left[\min(a_1, a_1), \min(a_2, a_2), \ldots, \min(a_n, a_n)\right] = \max[a_1, a_2, \ldots, a_n] = \hat{a} = 1$$

$$A \oplus A = \min\left[\min(a_1, a_1), \max(a_2, a_2), \ldots, \max(a_n, a_n)\right] = \min[a_1, a_2, \ldots, a_n] = \underset{\wedge}{a}$$

$$\overline{A \oplus A} = 1 - \underset{\wedge}{a}$$

$$(A, A)_1 = 1 \wedge \left(1 - \underset{\wedge}{a}\right) = 1 - \underset{\wedge}{a}$$

$$(A, A)_2 = \frac{1}{2}\left[1 + 1 - \underset{\wedge}{a}\right] = 1 - \underset{\wedge}{a} / 2$$

(

b)

$\mu_1 = \underset{\sim}{A} \cdot \overline{\underset{\sim}{A}}$ $\quad \mu_2 = \underset{\sim}{A} \oplus \overline{\underset{\sim}{A}}$ $\quad \overline{\mu_2} = \overline{\underset{\sim}{A} \oplus \overline{\underset{\sim}{A}}}$

From equation 11.11 we know that $\mu_1 \le 0.5$ and $\mu_2 \le 0.5$ thus $\overline{\mu_2} \ge 0.5$.

$$\left(\underset{\sim}{A}, \overline{\underset{\sim}{A}}\right)_1 = \underset{\sim}{A} \cdot \overline{\underset{\sim}{A}} \wedge \overline{\underset{\sim}{A} \oplus \overline{\underset{\sim}{A}}} = \mu_1 \wedge \overline{\mu}_2$$

$$= \mu_1 \le 0.5$$

The second part of part b is wrong. The following expression must be replaced:

$$\left(\underset{\sim}{A}, \overline{\underset{\sim}{A}}\right)_2 \ge \frac{1}{4}$$

The proof:

$$\left(\underset{\sim}{A}, \overline{\underset{\sim}{A}}\right)_2 = \frac{1}{2}\left[\underset{\sim}{A} \cdot \overline{\underset{\sim}{A}} + \overline{\underset{\sim}{A} \oplus \overline{\underset{\sim}{A}}}\right] = \frac{1}{2}\left[\mu_1 + \overline{\mu}_2\right]$$

$$\ge \frac{1}{2}[0 + 0.5] \ge \frac{1}{4}$$

11.5

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1 = \left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) \wedge \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 = \frac{1}{2}\left[\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) \wedge \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)\right]$$

case I: Let $\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) = \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$

then $\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1 = \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 = \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$$

case II: Let $\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) < \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1 = \left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right)$$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 > \frac{1}{2}\left[\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) + \left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right)\right] =$$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 \ge \left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right)$$

case II: Let $\left(\underset{\sim}{A} \cdot \underset{\sim}{B}\right) > \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$

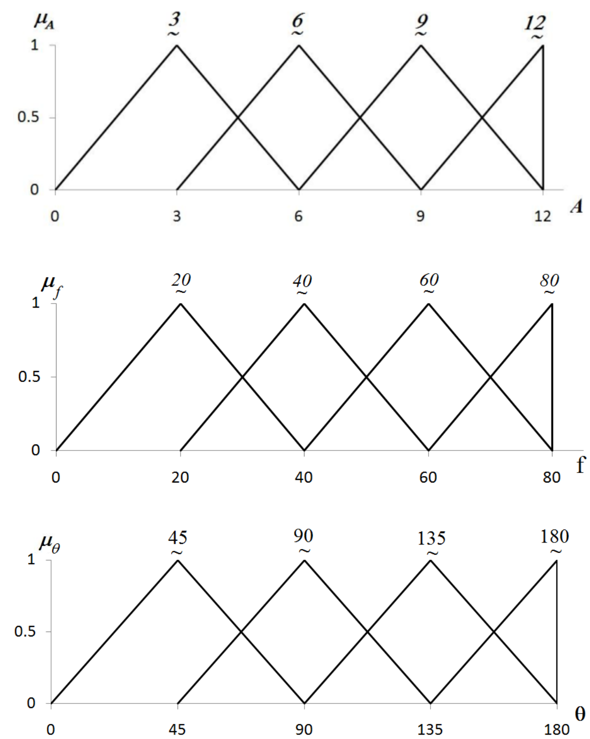
$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1 = \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)$$

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 > \frac{1}{2}\left[\left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right) + \left(\overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}\right)\right] = \overline{\underset{\sim}{A} \oplus \underset{\sim}{B}}$$

In all three cases,

$$\left(\underset{\sim}{A}, \underset{\sim}{B}\right)_2 \ge \left(\underset{\sim}{A}, \underset{\sim}{B}\right)_1$$

11.6



C_1=[3V, 20Hz, 45°]
C_2=[6V, 40Hz, 90°]
C_3=[9V, 60Hz, 135°]
C_4=[12V, 80Hz, 180°]

$A_{11}=0$ $\quad A_{21}=0$ $\quad A_{31}=0.78$
$A_1 = 0.5\times0 + 0.25\times0 + 0.25\times0.78 = 0.19$

$A_{12}=0.67$ $\quad A_{22}=0.5$ $\quad A_{32}=0.22$
$A_2 = 0.5\times0.67 + 0.25\times05 + 0.25\times0.22 = 0.52$

A_{13}=0.33 A_{23}=0.5 A_{33}=0
A_3= 0.5×0.33 + 0.25×0.5 + 0.25×0 = 0.29

A_{14}=0 A_{24}=0 A_{34}=0
A_4= 0.5×0 + 0.25×0 + 0.25×0 = 0

Therefore, the best match is C_2.

11.7
Voltage

$$\underset{\sim}{3} = \left\{\frac{0}{0} + \frac{0.33}{1} + \frac{0.67}{2} + \frac{1}{3} + \frac{0.67}{4} + \frac{0.33}{5} + \frac{0}{6}\right\}$$

$$\underset{\sim}{6} = \left\{\frac{0}{3} + \frac{0.33}{4} + \frac{0.67}{5} + \frac{1}{6} + \frac{0.67}{7} + \frac{0.33}{8} + \frac{0}{9}\right\}$$

$$\underset{\sim}{9} = \left\{\frac{0}{6} + \frac{0.33}{7} + \frac{0.67}{8} + \frac{1}{9} + \frac{0.67}{10} + \frac{0.33}{11} + \frac{0}{12}\right\}$$

$$\underset{\sim}{12} = \left\{\frac{0}{9} + \frac{0.33}{10} + \frac{0.67}{11} + \frac{1}{12}\right\}$$

Frequency

$$\underset{\sim}{20} = \left\{\frac{0}{0} + \frac{0.5}{10} + \frac{1}{20} + \frac{0.5}{30} + \frac{0}{40}\right\}$$

$$\underset{\sim}{40} = \left\{\frac{0}{20} + \frac{0.5}{30} + \frac{1}{40} + \frac{0.5}{50} + \frac{0}{60}\right\}$$

$$\underset{\sim}{60} = \left\{\frac{0}{40} + \frac{0.5}{50} + \frac{1}{60} + \frac{0.5}{70} + \frac{0}{80}\right\}$$

$$\underset{\sim}{80} = \left\{\frac{0}{60} + \frac{0.5}{70} + \frac{1}{80}\right\}$$

Phase

$$\underset{\sim}{45} = \left\{\frac{0}{0} + \frac{0.50}{22.5} + \frac{1}{45} + \frac{0.89}{50} + \frac{0.44}{70} + \frac{0}{90}\right\}$$

$$\underset{\sim}{90} = \left\{\frac{0}{45} + \frac{0.11}{50} + \frac{0.56}{70} + \frac{1}{90} + \frac{0.56}{110} + \frac{0.33}{120} + \frac{0.11}{130}\right\}$$

$$\underset{\sim}{135} = \left\{\frac{0}{90} + \frac{0.44}{110} + \frac{0.67}{120} + \frac{0.89}{130} + \frac{1}{135} + \frac{0}{180}\right\}$$

$$\underset{\sim}{180} = \left\{\frac{0}{135} + \frac{0.5}{158.5} + \frac{1}{180}\right\}$$

$$A_{11=}(B_{voltage}, \underset{\sim}{3})_{1\ or\ 2}$$

$$(A_{11})_1 = (B_{voltage} \cdot \underset{\sim}{3}) \wedge \overline{(B_{voltage} \oplus \underset{\sim}{3})}$$

$$B_{voltage} \cdot \underset{\sim}{3} = \vee\left\{\frac{0}{1} + \frac{0.3}{2} + \frac{0.7}{3} + \frac{0.67}{4} + \frac{0.33}{5} + \frac{0}{6}\right\} = 0.7$$

$$B_{voltage} \oplus \underset{\sim}{3} = \wedge\left\{\frac{0.33}{1} + \frac{0.67}{2} + \frac{1}{3} + \frac{0.85}{4} + \frac{0.1}{5} + \frac{0}{6}\right\} = 0$$

$$\overline{(B_{voltage} \oplus \underset{\sim}{3})} = 1 \qquad (A_{11})_1 = 0.7$$

$$(A_{11})_2 = \frac{1}{2}\left[(B_{voltage} \cdot \underset{\sim}{3}) + \overline{(B_{voltage} \oplus \underset{\sim}{3})}\right] = 0.85$$

$$A_{21=}(B_{frequency}, \underset{\sim}{20})_{1\ or\ 2}$$

$$(A_{21})_1 = (B_{frequency} \cdot \underset{\sim}{20}) \wedge \overline{(B_{frequency} \oplus \underset{\sim}{20})}$$

$$B_{frequency} \cdot \underset{\sim}{20} = \vee(0, 0.5) = 0.5$$

$$B_{frequency} \oplus 20 = 0 \qquad \overline{(B_{frequency} \oplus 20)} = 1$$

$$(A_{21})_1 = 0.5$$

$$(A_{21})_2 = \frac{1}{2}\left[(B_{frequency} \cdot \underset{\sim}{20}) + \overline{(B_{frequency} \oplus \underset{\sim}{20})}\right] = 0.75$$

$$A_{31=}(B_{phase}, \underset{\sim}{45})_{1\ or\ 2}$$

$$(A_{31})_1 = (B_{phase} \cdot \underset{\sim}{45}) \wedge \overline{(B_{phase} \oplus \underset{\sim}{45})}$$

$$B_{phase} \cdot \underset{\sim}{45} = \vee(0, 0.3, 0) = 0.3$$

$$B_{phase} \oplus \underset{\sim}{45} = 0 \qquad \overline{(B_{phase} \oplus \underset{\sim}{45})} = 1$$

$$(A_{31})_1 = 0.3$$

$$(A_{31})_2 = \frac{1}{2}\left[(B_{phase} \cdot \underset{\sim}{45}) + \overline{(B_{phase} \oplus \underset{\sim}{45})}\right] = 0.65$$

$$(A_1)_1 = 0.5 \times 0.7 + 0.25 \times 0.5 + 0.25 \times 0.3 = 0.55$$
$$(A_1)_2 = 0.5 \times 0.85 + 0.25 \times 0.75 + 0.25 \times 0.65 = 0.78$$

$$A_{12=}(B_{voltage}, \underset{\sim}{6})_{1\ or\ 2}$$

$$(A_{12})_1 = (B_{voltage} \cdot \underset{\sim}{6}) \wedge \overline{(B_{voltage} \oplus \underset{\sim}{6})}$$

$$B_{voltage} \cdot \underset{\sim}{6} = 0.67$$

$$B_{voltage} \oplus 6 = 0 \qquad \overline{(B_{voltage} \oplus 6)} = 1$$

$$(A_{12})_1 = 0.67$$

$$(A_{12})_2 = \frac{1}{2}\left[(B_{voltage} \cdot \underset{\sim}{6}) + \overline{(B_{voltage} \oplus \underset{\sim}{6})}\right] = 0.83$$

$$A_{22=}(B_{frequency}, \underset{\sim}{40})_{1\ or\ 2}$$

$$(A_{22})_1 = (B_{frequency} \cdot \underset{\sim}{40}) \wedge \overline{(B_{frequency} \oplus \underset{\sim}{40})}$$

$$B_{frequency} \cdot \underset{\sim}{40} = 1$$

$$B_{frequency} \oplus 40 = 0 \qquad \overline{(B_{frequency} \oplus 40)} = 1$$

$$(A_{22})_1 = 1$$

$$(A_{22})_2 = \frac{1}{2}\left[(B_{frequency} \cdot \underset{\sim}{40}) + \overline{(B_{frequency} \oplus \underset{\sim}{40})}\right] = 1$$

$$A_{32=}(B_{phase}, \underset{\sim}{90})_{1\ or\ 2}$$

$$(A_{32})_1 = (B_{phase} \cdot \underset{\sim}{90}) \wedge \overline{(B_{phase} \oplus \underset{\sim}{90})}$$

$$B_{phase} \cdot \underset{\sim}{90} = 0.6$$

$$B_{phase} \oplus \underset{\sim}{90} = 0 \qquad \overline{(B_{phase} \oplus \underset{\sim}{90})} = 1$$

$$(A_{32})_1 = 0.6$$

$$(A_{32})_2 = \frac{1}{2}\left[(B_{phase} \cdot \underset{\sim}{90}) + \overline{(B_{phase} \oplus \underset{\sim}{90})}\right] = 0.8$$

$$(A_2)_1 = 0.5 \times 0.67 + 0.25 \times 1 + 0.25 \times 0.6 = 0.74$$
$$(A_2)_2 = 0.5 \times 0.83 + 0.25 \times 1 + 0.25 \times 0.8 = 0.87$$

$$A_{13=}(B_{voltage}, \underset{\sim}{9})_{1\ or\ 2}$$

$$(A_{13})_1 = (B_{voltage} \cdot \underset{\sim}{9}) \wedge \overline{(B_{voltage} \oplus \underset{\sim}{9})}$$

$$B_{voltage} \cdot \underset{\sim}{9} = 0$$

$B_{voltage} \oplus 9 = 0 \qquad \overline{(B_{voltage} \oplus 9)} = 1$

$(A_{13})_1 = 0$

$(A_{13})_2 = \frac{1}{2}\left[(B_{voltage} \cdot \underset{\sim}{9}) + \overline{(B_{voltage} \oplus \underset{\sim}{9})}\right] = 0.5$

$A_{23} = (B_{frequency}, \underset{\sim}{60})_{1\ or\ 2}$

$(A_{23})_1 = (B_{frequency} \cdot \underset{\sim}{60}) \wedge \overline{(B_{frequency} \oplus \underset{\sim}{60})}$

$B_{frequency} \cdot \underset{\sim}{60} = 0.4$

$B_{frequency} \oplus 60 = 0 \qquad \overline{(B_{frequency} \oplus 60)} = 1$

$(A_{23})_1 = 0.4$

$(A_{23})_2 = \frac{1}{2}\left[(B_{frequency} \cdot \underset{\sim}{60}) + \overline{(B_{frequency} \oplus \underset{\sim}{60})}\right] = 0.7$

$A_{33} = (B_{phase}, \underset{\sim}{135})_{1\ or\ 2}$

$(A_{33})_1 = (B_{phase} \cdot \underset{\sim}{135}) \wedge \overline{(B_{phase} \oplus \underset{\sim}{135})}$

$B_{phase} \cdot \underset{\sim}{135} = 0.67$

$B_{phase} \oplus \underset{\sim}{135} = 0 \qquad \overline{(B_{phase} \oplus \underset{\sim}{135})} = 1$

$(A_{33})_1 = 0.67$

$(A_{33})_2 = \frac{1}{2}\left[(B_{phase} \cdot \underset{\sim}{135}) + \overline{(B_{phase} \oplus \underset{\sim}{135})}\right] = 0.83$

$(A_3)_1 = 0.5 \times 0 + 0.25 \times 0.4 + 0.25 \times 0.67 = 0.27$

$(A_3)_2 = 0.5 \times 0.5 + 0.25 \times 0.7 + 0.25 \times 0.83 = 0.63$

$A_{14} = (B_{voltage}, \underset{\sim}{12})_{1\ or\ 2}$

$(A_{14})_1 = (B_{voltage} \cdot \underset{\sim}{12}) \wedge \overline{(B_{voltage} \oplus \underset{\sim}{12})}$

$B_{voltage} \cdot \underset{\sim}{12} = 0$

$B_{voltage} \oplus 12 = 0 \qquad \overline{(B_{voltage} \oplus 12)} = 1$

$(A_{14})_1 = 0$

$(A_{14})_2 = \frac{1}{2}\left[(B_{voltage} \cdot \underset{\sim}{12}) + \overline{(B_{voltage} \oplus \underset{\sim}{12})}\right] = 0.5$

$A_{24} = (B_{frequency}, \underset{\sim}{80})_{1\ or\ 2}$

$(A_{24})_1 = (B_{frequency} \cdot \underset{\sim}{80}) \wedge \overline{(B_{frequency} \oplus \underset{\sim}{80})}$

$B_{frequency} \cdot \underset{\sim}{80} = 0$

$B_{frequency} \oplus 80 = 0 \qquad \overline{(B_{frequency} \oplus 80)} = 1$

$(A_{24})_1 = 0$

$(A_{24})_2 = \frac{1}{2}\left[(B_{frequency} \cdot \underset{\sim}{80}) + \overline{(B_{frequency} \oplus \underset{\sim}{80})}\right] = 0.5$

$A_{34} = (B_{phase}, \underset{\sim}{180})_{1\ or\ 2}$

$(A_{34})_1 = (B_{phase} \cdot \underset{\sim}{180}) \wedge \overline{(B_{phase} \oplus \underset{\sim}{180})}$

$B_{phase} \cdot \underset{\sim}{180} = 0$

$B_{phase} \oplus \underset{\sim}{180} = 0 \qquad \overline{(B_{phase} \oplus \underset{\sim}{180})} = 1$

$(A_{34})_1 = 0$

$(A_{34})_2 = \frac{1}{2}\left[(B_{phase} \cdot \underset{\sim}{180}) + \overline{(B_{phase} \oplus \underset{\sim}{180})}\right] = 0.5$

$(A_4)_1 = 0.5 \times 0 + 0.25 \times 0 + 0.25 \times 0 = 0$

$(A_4)_2 = 0.5 \times 0.5 + 0.25 \times 0.5 + 0.25 \times 0.5 = 0.5$

Therefore, the best match is C_2.

11.8

C_1=[Depth=*very low*, Clay=*very low*, moisture=*very low*]

C_2=[Depth=*low*, Clay= *low*, moisture=*low*]

C_3=[Depth=*medium*, Clay=*medium*, moisture=*medium*]

C_4=[Depth=*high*, Clay=*high*, moisture=*high*]

$A_{11}=0 \quad A_{21}=0 \quad A_{31}=0$

$A_1=0.4\times0 + 0.4\times0 + 0.2\times0 = 0$

$A_{12}=0 \quad A_{22}=0.704 \quad A_{32}=0.3$

$A_2=0.4\times0 + 0.4\times0.704 + 0.2\times0.3 = 0.34$

$A_{13}=0.5 \quad A_{23}=0.296 \quad A_{33}=0.7$

$A_3=0.4\times0.5 + 0.4\times0.296 + 0.2\times0.7 = 0.46$

$A_{14}=0.5 \quad A_{24}=0 \quad A_{34}=0$

$A_4=0.4\times0.5 + 0.4\times0 + 0.2\times0 = 0.20$

Therefore, based on the metric 1 and 2 the best match is C_3.

11.9

$A_1=0.4(A_{11}) + 0.4(A_{21}) + 0.2(A_{31})$

$(A_{11})_1= (B_{depth}, Depth_{very\ low})_1$

$=(B_{depth} \bullet Depth_{very\ low}) \wedge \overline{(B_{depth} \oplus Depth_{very\ low})}$

$(A_{11})_2= (B_{depth}, Depth_{very\ low})_2$

$= \frac{1}{2}[(B_{depth} \bullet Depth_{very\ low}) + \overline{(B_{depth} \oplus Depth_{very\ low})}]$

$B_{depth} \bullet Depth_{very\ low}$

$= \vee \{[0, 0.4, 1, 0.6, 0] \wedge [0, 0, 0, 0, 0]\} = 0$

$B_{depth} \oplus Depth_{very\ low}$

$= \wedge\{[0, 0.4, 1, 0.6, 0] \vee [0, 0, 0, 0, 0]\} = 0$

$\overline{B_{depth} \oplus Depth_{very\ low}}=1$

$(A_{11})_1=0 \qquad (A_{11})_2=0.5$

$(A_{21})_1= (B_{clay}, Clay_{very\ low})_1$

$=(B_{Clay} \bullet Clay_{very\ low}) \wedge \overline{(B_{Clay} \oplus Clay_{very\ low})}$

$(A_{21})_2 = (B_{clay}, Clay_{very\ low})_2$
$= \frac{1}{2}[(B_{clay} \bullet Clay_{very\ low}) + (\overline{B_{Clay} \oplus Clay_{very\ low}})]$
$B_{Clay} \bullet Clay_{very\ low}$
$= \vee \{[0, 0.7, 1, 0.5, 0] \wedge [0, 0, 0, 0, 0]\} = 0$
$B_{Clay} \oplus Clay_{very\ low}$
$= \wedge\{[0, 0.7, 1, 0.5, 0] \vee [0, 0, 0, 0, 0]\} = 0$
$\overline{B_{Clay} \oplus Clay_{very\ low}} = 1$
$(A_{21})_1 = 0 \quad (A_{21})_2 = 0.5$

$(A_{31})_1 = (B_{moisture}, Moisture_{very\ low})_1$
$= (B_{Moisture} \bullet Moisture_{very\ low}) \wedge (\overline{B_{Moisture} \oplus Moisture_{very\ low}})$
$(A_{31})_2 = (B_{moisture}, Moisture_{very\ low})_2$
$= \frac{1}{2}[(B_{clay} \bullet Moisture_{very\ low}) + (\overline{B_{Moisture} \oplus Moisture_{very\ low}})]$
$B_{Moisture} \bullet Moisture_{very\ low}$
$= \vee \{[0, 0.5, 1, 0.8, 0] \wedge [0, 0, 0, 0, 0]\} = 0$
$B_{Moisture} \oplus Moisture_{very\ low}$
$= \wedge\{[0, 0.5, 1, 0.8, 0] \vee [0, 0, 0, 0, 0]\} = 0$
$\overline{B_{Clay} \oplus Moisture_{very\ low}} = 1$
$(A_{31})_1 = 0 \quad (A_{31})_2 = 0.5$

$(A_1)_1 = 0.4 \times 0 + 0.4 \times 0 + 0.2 \times 0 = 0$
$(A_1)_2 = 0.4 \times 0.5 + 0.4 \times 0.5 + 0.2 \times 0.5 = 0.5$

$A_2 = 0.4(A_{12}) + 0.4(A_{22}) + 0.2(A_{32})$
$(A_{12})_1 = (B_{depth}, Depth_{low})_1$
$= (B_{depth} \bullet Depth_{low}) \wedge (\overline{B_{depth} \oplus Depth_{low}})$
$(A_{12})_2 = (B_{depth}, Depth_{low})_2$
$= \frac{1}{2}[(B_{depth} \bullet Depth_{low}) + (\overline{B_{depth} \oplus Depth_{low}})]$
$B_{depth} \bullet Depth_{low}$
$= \vee \{[0, 0.4, 1, 0.6, 0] \wedge [0, 0, 0, 0, 0]\} = 0$
$B_{depth} \oplus Depth_{low}$
$= \wedge\{[0, 0.4, 1, 0.6, 0] \vee [0, 0, 0, 0, 0]\} = 0$
$\overline{B_{depth} \oplus Depth_{low}} = 1$
$(A_{12})_1 = 0 \quad (A_{12})_2 = 0.5$

$(A_{22})_1 = (B_{clay}, Clay_{low})_1$
$= (B_{Clay} \bullet Clay_{low}) \wedge (\overline{B_{Clay} \oplus Clay_{low}})$
$(A_{22})_2 = (B_{clay}, Clay_{low})_2$
$= \frac{1}{2}[(B_{clay} \bullet Clay_{low}) + (\overline{B_{Clay} \oplus Clay_{low}})]$

$B_{Clay} \bullet Clay_{low}$
$= \vee \{[0, 0.7, 1, 0.5, 0] \wedge [0.8, 0.68, 0.47, 0.42, 0.3]\}$
$= \vee [0, 0.68, 0.47, 0.42, 0] = 0.68$
$B_{Clay} \oplus Clay_{low}$
$= \wedge\{[0, 0.7, 1, 0.5, 0] \vee [0.8, 0.68, 0.47, 0.42, 0]\}$
$= \wedge[0.8, 0.7, 1, 0.5, 0.3] = 0.3$
$\overline{B_{Clay} \oplus Clay_{low}} = 0.7$
$(A_{22})_1 = 0.3 \quad (A_{22})_2 = 0.69$

$(A_{32})_1 = (B_{moisture}, Moisture_{low})_1$
$= (B_{Moisture} \bullet Moisture_{low}) \wedge (\overline{B_{Moisture} \oplus Moisture_{low}})$
$(A_{32})_2 = (B_{moisture}, Moisture_{low})_2$
$= \frac{1}{2}[(B_{clay} \bullet Moisture_{low}) + (\overline{B_{Moisture} \oplus Moisture_{low}})]$
$B_{Moisture} \bullet Moisture_{low} =$
$\vee \{[0, 0.5, 1, 0.8, 0] \wedge [0.8, 0.65, 0.5, 0.35, 0.2]\}$
$= 0.5$
$B_{Moisture} \oplus Moisture_{low}$
$= \wedge\{[0, 0.5, 1, 0.8, 0] \vee [0.8, 0.65, 0.5, 0.35, 0.2]\}$
$= 0.2$
$\overline{B_{Clay} \oplus Moisture_{low}} = 0.8$
$(A_{32})_1 = 0.5 \quad (A_{32})_2 = 0.65$

$(A_2)_1 = 0.4 \times 0 + 0.4 \times 0.3 + 0.2 \times 0.5 = 0.22$
$(A_2)_2 = 0.4 \times 0.5 + 0.4 \times 0.69 + 0.2 \times 0.65 = 0.61$

$A_3 = 0.4(A_{13}) + 0.4(A_{23}) + 0.2(A_{33})$
$(A_{13})_1 = (B_{depth}, Depth_{medium})_1$
$= (B_{depth} \bullet Depth_{medium}) \wedge (\overline{B_{depth} \oplus Depth_{medium}})$
$(A_{13})_2 = (B_{depth}, Depth_{medium})_2$
$= \frac{1}{2}[(B_{depth} \bullet Depth_{medium}) + (\overline{B_{depth} \oplus Depth_{medium}})]$
$B_{depth} \bullet Depth_{medium}$
$= \vee \{[0, 0.4, 1, 0.6, 0] \wedge [0.6, 0.55, 0.5, 0.45, 0.4]\}$
$= 0.5$
$B_{depth} \oplus Depth_{medium}$
$= \wedge\{[0, 0.4, 1, 0.6, 0] \vee [0.6, 0.55, 0.5, 0.45, 0.4]\}$
$= 0.4$
$\overline{B_{depth} \oplus Depth_{medium}} = 0.6$
$(A_{13})_1 = 0.5 \quad (A_{13})_2 = 0.55$

$(A_{23})_1 = (B_{clay}, Clay_{medium})_1$
$=(B_{Clay} \bullet Clay_{medium}) \wedge (\overline{B_{Clay} \oplus Clay_{medium}})$
$(A_{23})_2 = (B_{clay}, Clay_{medium})_2$
$= \frac{1}{2}[(B_{clay} \bullet Clay_{medium}) + (\overline{B_{Clay} \oplus Clay_{medium}})]$

$B_{Clay} \bullet Clay_{medium} = \vee\{[0, 0.7, 1, 0.5, 0] \wedge [0.2, 0.32, 0.53, 0.58, 0.7]\}$
$= 0.53$
$B_{Clay} \oplus Clay_{medium} = \wedge\{[0, 0.7, 1, 0.5, 0] \vee [0.2, 0.32, 0.53, 0.58, 0.7]\}$
$= 0.2$
$\overline{B_{Clay} \oplus Clay_{medium}} = 0.8$
$(A_{23})_1 = 0.53 \quad (A_{23})_2 = 0.67$

$(A_{33})_1 = (B_{moisture}, Moisture_{medium})_1$
$=(B_{Moisture} \bullet Moisture_{medium}) \wedge (\overline{B_{Moisture} \oplus Moisture_{medium}})$
$(A_{33})_2 = (B_{moisture}, Moisture_{medium})_2$
$= \frac{1}{2}[(B_{clay} \bullet Moisture_{medium}) + (\overline{B_{Moisture} \oplus Moisture_{medium}})]$
$B_{Moisture} \bullet Moisture_{medium} = \vee\{[0, 0.5, 1, 0.8, 0] \wedge [0.2, 0.35, 0.5, 0.65, 0.8]\}$
$= 0.65$
$B_{Moisture} \oplus Moisture_{medium} = \wedge\{[0, 0.5, 1, 0.8, 0] \vee [0.2, 0.35, 0.5, 0.65, 0.8]\}$
$= 0.2$
$\overline{B_{Clay} \oplus Moisture_{medium}} = 0.8$
$(A_{33})_1 = 0.65 \quad (A_{33})_2 = 0.73$

$(A_3)_1 = 0.4 \times 0.5 + 0.4 \times 0.53 + 0.2 \times 0.65 = 0.22$
$(A_3)_2 = 0.4 \times 0.55 + 0.4 \times 0.67 + 0.2 \times 0.73 = 0.63$

$A_4 = 0.4(A_{14}) + 0.4(A_{24}) + 0.2(A_{34})$
$(A_{14})_1 = (B_{depth}, Depth_{high})_1$
$=(B_{depth} \bullet Depth_{high}) \wedge (\overline{B_{depth} \oplus Depth_{high}})$
$(A_{14})_2 = (B_{depth}, Depth_{high})_2$
$= \frac{1}{2}[(B_{depth} \bullet Depth_{high}) + (\overline{B_{depth} \oplus Depth_{high}})]$
$B_{depth} \bullet Depth_{high} = \vee\{[0, 0.4, 1, 0.6, 0] \wedge [0.4, 0.45, 0.5, 0.55, 0.6]\}$
$= 0.55$
$B_{depth} \oplus Depth_{high} = \wedge\{[0, 0.4, 1, 0.6, 0] \vee [0.4, 0.45, 0.5, 0.55, 0.6]\}$
$= 0.4$
$\overline{B_{depth} \oplus Depth_{high}} = 0.6$
$(A_{14})_1 = 0.55 \quad (A_{14})_2 = 0.58$

$(A_{24})_1 = (B_{clay}, Clay_{high})_1$
$=(B_{Clay} \bullet Clay_{high}) \wedge (\overline{B_{Clay} \oplus Clay_{high}})$
$(A_{24})_2 = (B_{clay}, Clay_{high})_2$
$= \frac{1}{2}[(B_{clay} \bullet Clay_{high}) + (\overline{B_{Clay} \oplus Clay_{high}})]$
$B_{Clay} \bullet Clay_{high} = \vee\{[0, 0.7, 1, 0.5, 0] \wedge [0, 0, 0, 0, 0]\} = 0$
$B_{Clay} \oplus Clay_{high} = \wedge\{[0, 0.7, 1, 0.5, 0] \vee [0, 0, 0, 0, 0]\} = 0$
$\overline{B_{Clay} \oplus Clay_{high}} = 1$
$(A_{24})_1 = 0 \quad (A_{24})_2 = 0.5$

$(A_{34})_1 = (B_{moisture}, Moisture_{high})_1$
$=(B_{Moisture} \bullet Moisture_{high}) \wedge (\overline{B_{Moisture} \oplus Moisture_{high}})$
$(A_{34})_2 = (B_{moisture}, Moisture_{high})_2$
$= \frac{1}{2}[(B_{clay} \bullet Moisture_{high}) + (\overline{B_{Moisture} \oplus Moisture_{high}})]$
$B_{Moisture} \bullet Moisture_{high} = \vee\{[0, 0.5, 1, 0.8, 0] \wedge [0, 0, 0, 0, 0]\} = 0$
$B_{Moisture} \oplus Moisture_{high} = \wedge\{[0, 0.5, 1, 0.8, 0] \vee [0, 0, 0, 0, 0]\} = 0$
$\overline{B_{Clay} \oplus Moisture_{high}} = 1$
$(A_{34})_1 = 0 \quad (A_{34})_2 = 0.5$

$(A_4)_1 = 0.4 \times 0.55 + 0.4 \times 0 + 0.2 \times 0 = 0.22$
$(A_4)_2 = 0.4 \times 0.58 + 0.4 \times 0.5 + 0.2 \times 0.5 = 0.53$

Therefore, based on metric 1 and 2 the best match is C_3.

11.10 B=[8, 120] W=[0.4, 0.6]

(B, Low flow rate) = 0.4 (B_V, L-vel) + 0.6 (B_A, S-area)
(B, Moderate flow rate) = 0.4 (B_V, M-vel) + 0.6 (B_A, M-area)
(B, High flow rate) = 0.4 (B_V, H-vel) + 0.6 (B_A, B-area)

$(B_V, \text{L-vel}) = \exp\left(-\frac{(8-5)^2}{3^2}\right) = 0.368$
$(B_V, \text{M-vel}) = \exp\left(-\frac{(8-10)^2}{5^2}\right) = 0.852$
$(B_V, \text{H-vel}) = \exp\left(-\frac{(8-15)^2}{3^2}\right) = 0.004$

$(B_A, \text{S-area}) = \exp\left(-\frac{(120-50)^2}{26^2}\right) = 0.001$
$(B_A, \text{M- area}) = \exp\left(-\frac{(120-100)^2}{18^2}\right) = 0.291$
$(B_A, \text{B- area}) = \exp\left(-\frac{(120-150)^2}{4^2}\right) = 0$

(B, Low flow rate) = 0.4×0.368 + 0.6×0.001 = 0.148
(B, Moderate flow rate) = 0.4×0.852 + 0.6×0.291 = 0.515
(B, High flow rate) = 0.4×0.004 + 0.6×0 = 0.002

Therefore, the best match is Moderate flow rate.

11.11

This problem is the same as problem 11.10 but B = [11, 130]

(B, Low flow rate) = 0.4 (B_V, L-vel) + 0.6 (B_A, S-area)
(B, Moderate flow rate) = 0.4 (B_V, M-vel) + 0.6 (B_A, M-area)
(B, High flow rate) = 0.4 (B_V, H-vel) + 0.6 (B_A, B-area)
(B_V, L-vel) = 0.018
(B_V, M-vel) = 0.961
(B_V, H-vel) = 0.169

(B_A, S-area) = 0
(B_A, M- area) = 0.062
(B_A, B- area) = 0

(B, Low flow rate) = 0.4×0.018 + 0.6×0 = 0.007
(B, Moderate flow rate) = 0.4×0.961 + 0.6×0.062 = 0.422
(B, High flow rate) = 0.4×0.169 + 0.6×0 = 0.068
Therefore, the best match is Moderate flow rate.

11.12

$(B, A_{i1})_1 = (B \bullet A_{i1}) \wedge (\overline{B \oplus A_{11}})$
$(B, A_{i1})_2 = ½[(B \bullet A_{i1}) + (\overline{B \oplus A_{11}})]$

$B \bullet A_{11} = \vee\{[1] \wedge [0.1]\} = \vee[0.1] = 0.1$
$B \oplus A_{11} = \wedge\{[1] \vee [0.1]\} = \wedge[1] = 1$
$\overline{B \oplus A_{11}} = 0$
$(B, A_{11})_1 = 0$ $(B, A_{11})_2 = 0.05$
$B \bullet A_{21} = \vee\{[1] \wedge [0.2]\} = \vee[0.2] = 0.2$
$B \oplus A_{21} = \wedge\{[1] \vee [0.2]\} = \wedge[1] = 1$
$\overline{B \oplus A_{21}} = 0$
$(B, A_{21})_1 = 0$ $(B, A_{21})_2 = 0.1$
$B \bullet A_{31} = \vee\{[1] \wedge [0.7]\} = \vee[0.7] = 0.7$
$B \oplus A_{31} = \wedge\{[1] \vee [0.7]\} = \wedge[1] = 1$
$\overline{B \oplus A_{31}} = 0$
$(B, A_{31})_1 = 0$ $(B, A_{31})_2 = 0.35$

$(A_1)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_1)_2 = 0.3\times0.05 + 0.3\times0.1 + 0.4\times0.35 = 0.185$

$(B, A_{i2})_1 = (B \bullet A_{i2}) \wedge (\overline{B \oplus A_{12}})$
$(B, A_{i2})_2 = ½[(B \bullet A_{i2}) + (\overline{B \oplus A_{12}})]$

$B \bullet A_{12} = \vee\{[1] \wedge [0.3]\} = \vee[0.3] = 0.3$
$B \oplus A_{12} = \wedge\{[1] \vee [0.3]\} = \wedge[1] = 1$
$\overline{B \oplus A_{12}} = 0$
$(B, A_{12})_1 = 0$ $(B, A_{12})_2 = 0.15$

$B \bullet A_{22} = \vee\{[1] \wedge [0.4]\} = \vee[0.4] = 0.4$
$B \oplus A_{22} = \wedge\{[1] \vee [0.4]\} = \wedge[1] = 1$
$\overline{B \oplus A_{22}} = 0$
$(B, A_{22})_1 = 0$ $(B, A_{22})_2 = 0.2$

$B \bullet A_{32} = \vee\{[1] \wedge [0.8]\} = \vee[0.8] = 0.8$
$B \oplus A_{32} = \wedge\{[1] \vee [0.8]\} = \wedge[1] = 1$
$\overline{B \oplus A_{32}} = 0$
$(B, A_{32})_1 = 0$ $(B, A_{32})_2 = 0.4$

$(A_2)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_2)_2 = 0.3\times0.15 + 0.3\times0.2 + 0.4\times0.4 = 0.265$

$(B, A_{i3})_1 = (B \bullet A_{i3}) \wedge (\overline{B \oplus A_{13}})$
$(B, A_{i3})_2 = ½[(B \bullet A_{i3}) + (\overline{B \oplus A_{13}})]$

$B \bullet A_{13} = \vee\{[1] \wedge [0.3]\} = \vee[0.3] = 0.3$
$B \oplus A_{13} = \wedge\{[1] \vee [0.3]\} = \wedge[1] = 1$
$\overline{B \oplus A_{13}} = 0$
$(B, A_{13})_1 = 0$ $(B, A_{13})_2 = 0.15$

$B \bullet A_{23} = \vee\{[1] \wedge [0.6]\} = \vee[0.6] = 0.6$
$B \oplus A_{23} = \wedge\{[1] \vee [0.6]\} = \wedge[1] = 1$
$\overline{B \oplus A_{23}} = 0$
$(B, A_{23})_1 = 0$ $(B, A_{23})_2 = 0.3$

$B \bullet A_{33} = \vee\{[1] \wedge [0.8]\} = \vee[0.8] = 0.8$
$B \oplus A_{33} = \wedge\{[1] \vee [0.8]\} = \wedge[1] = 1$
$\overline{B \oplus A_{33}} = 0$
$(B, A_{33})_1 = 0$ $(B, A_{33})_2 = 0.4$

$(A_3)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_3)_2 = 0.3\times0.15 + 0.3\times0.3 + 0.4\times0.4$
$= 0.295$

$(B, A_{i4})_1 = (B \bullet A_{i4}) \wedge (\overline{B \oplus A_{14}})$
$(B, A_{i4})_2 = ½[(B \bullet A_{i4}) + (\overline{B \oplus A_{14}})]$

$B \bullet A_{14} = \vee\{[1] \wedge [0.4]\} = \vee[0.4] = 0.4$
$B \oplus A_{14} = \wedge\{[1] \vee [0.4]\} = \wedge[1] = 1$
$\overline{B \oplus A_{14}} = 0$
$(B, A_{14})_1 = 0$ $(B, A_{14})_2 = 0.2$

$B \bullet A_{24} = \vee\{[1] \wedge [0.8]\} = \vee[0.8] = 0.8$
$B \oplus A_{24} = \wedge\{[1] \vee [0.8]\} = \wedge[1] = 1$
$\overline{B \oplus A_{24}} = 0$
$(B, A_{24})_1 = 0$ $(B, A_{24})_2 = 0.4$

$B \bullet A_{34} = \vee\{[1] \wedge [0.9]\} = \vee[0.9] = 0.9$
$B \oplus A_{34} = \wedge\{[1] \vee [0.9]\} = \wedge[1] = 1$
$\overline{B \oplus A_{34}} = 0$
$(B, A_{34})_1 = 0$ $(B, A_{34})_2 = 0.45$

$(A_4)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_4)_2 = 0.3\times0.2 + 0.3\times0.4 + 0.4\times0.45$
$= 0.36$

$(B, A_{i5})_1 = (B \bullet A_{i5}) \wedge (\overline{B \oplus A_{15}})$
$(B, A_{i5})_2 = ½[(B \bullet A_{i5}) + (\overline{B \oplus A_{15}})]$

$B \bullet A_{15} = \vee\{[1] \wedge [0.6]\} = \vee[0.6] = 0.6$
$B \oplus A_{15} = \wedge\{[1] \vee [0.6]\} = \wedge[1] = 1$
$\overline{B \oplus A_{15}} = 0$
$(B, A_{15})_1 = 0$ $(B, A_{15})_2 = 0.5$

$B \bullet A_{25} = \vee\{[1] \wedge [0.9]\} = \vee[0.9] = 0.9$
$B \oplus A_{25} = \wedge\{[1] \vee [0.9]\} = \wedge[1] = 1$
$\overline{B \oplus A_{25}} = 0$
$(B, A_{25})_1 = 0$ $(B, A_{25})_2 = 0.45$

$B \bullet A_{35} = \vee\{[1] \wedge [1]\} = \vee[1] = 1$
$B \oplus A_{35} = \wedge\{[1] \vee [1]\} = \wedge[1] = 1$
$\overline{B \oplus A_{35}} = 0$
$(B, A_{35})_1 = 0$ $(B, A_{35})_2 = 0.5$

$(A_5)_1 = 0.3\times0 + 0.3\times0 + 0.4\times0 = 0$
$(A_5)_2 = 0.3\times0.5 + 0.3\times0.45 + 0.4\times0.5$
$= 0.485$

Based on metric 1 it is not possible to choose any match. But based on the metric 2 the best match is A_5.

11.13
$B = [6.5, 825, 750]$ $W = [0.5, 0.3, 0.2]$

$(B, PGO) = 0.5\ (B_{color}, Color_{PGO}) + 0.3\ (B_{viscosity}, Viscosity_{PGO}) + 0.2\ (B_{flash\ point}, Flash\ point_{PGO}) = 0.5\times0 + 0.3\times0 + 0.2\times0 = 0$

$(B, 100N) = 0.5\ (B_{color}, Color_{100N}) + 0.3\ (B_{viscosity}, Viscosity_{100N}) + 0.2\ (B_{flash\ point}, Flash\ point_{100N}) = 0.5\times0 + 0.3\times0 + 0.2\times0 = 0$

$(B, 150N) = 0.5\ (B_{color}, Color_{150N}) + 0.3\ (B_{viscosity}, Viscosity_{150N}) + 0.2\ (B_{flash\ point}, Flash\ point_{150N}) = 0.5\times0 + 0.3\times0.375 + 0.2\times0.25 = 0.16$

$(B, HSN) = 0.5\ (B_{color}, Color_{HSN}) + 0.3\ (B_{viscosity}, Viscosity_{HSN}) + 0.2\ (B_{flash\ point}, Flash\ point_{HSN}) = 0.5\times0.75 + 0.3\times0.625 + 0.2\times0.75 = 0.71$

$(B, 500N) = 0.5\ (B_{color}, Color_{500N}) + 0.3\ (B_{viscosity}, Viscosity_{500N}) + 0.2\ (B_{flash\ point}, Flash\ point_{500N}) = 0.5\times0.25 + 0.3\times0 + 0.2\times0 = 0.13$

Therefore, the best match is HSN.

11.14

$B = [0.3, 0.3, 0.3]$ $W = [0.4, 0.4, 0.2]$

$(B, A_{i1})_1 = (B \bullet A_{i1}) \wedge (\overline{B \oplus A_{11}})$

$(B, A_{i1})_2 = ½[(B \bullet A_{i1}) + (\overline{B \oplus A_{11}})]$

$B \bullet A_{11} = \vee\{[0.3] \wedge [0.1]\} = \vee[0.1] = 0.1$

$\underline{B \oplus A_{11}} = \wedge\{[0.3] \vee [0.1]\} = \wedge[0.3] = 0.3$

$\overline{B \oplus A_{11}} = 0.7$

$(B, A_{11})_1 = 0.1$ $(B, A_{11})_2 = 0.4$

$B \bullet A_{21} = \vee\{[0.3] \wedge [0.15]\} = \vee[0.15] = 0.15$

$\underline{B \oplus A_{21}} = \wedge\{[0.3] \vee [0.15]\} = \wedge[0.3] = 0.3$

$\overline{B \oplus A_{21}} = 0.7$

$(B, A_{21})_1 = 0.15$ $(B, A_{21})_2 = 0.425$

$B \bullet A_{31} = \vee\{[0.3] \wedge [0.2]\} = \vee[0.2] = 0.2$

$\underline{B \oplus A_{31}} = \wedge\{[0.3] \vee [0.2]\} = \wedge[0.3] = 0.3$

$\overline{B \oplus A_{31}} = 0.7$

$(B, A_{31})_1 = 0.2$ $(B, A_{31})_2 = 0.45$

$(A_1)_1 = 0.3 \times 0.1 + 0.3 \times 0.15 + 0.4 \times 0.2 = 0.155$

$(A_1)_2 = 0.3 \times 0.4 + 0.3 \times 0.425 + 0.4 \times 0.45$
$= 0.4275$

$(B, A_{i2})_1 = (B \bullet A_{i2}) \wedge (\overline{B \oplus A_{12}})$

$(B, A_{i2})_2 = ½[(B \bullet A_{i2}) + (\overline{B \oplus A_{12}})]$

$B \bullet A_{12} = \vee\{[0.3] \wedge [0.2]\} = \vee[0.2] = 0.2$

$\underline{B \oplus A_{12}} = \wedge\{[0.3] \vee [0.2]\} = \wedge[0.3] = 0.3$

$\overline{B \oplus A_{12}} = 0.7$

$(B, A_{12})_1 = 0.2$ $(B, A_{12})_2 = 0.45$

$B \bullet A_{22} = \vee\{[0.3] \wedge [0.2]\} = \vee[0.2] = 0.2$

$\underline{B \oplus A_{22}} = \wedge\{[0.3] \vee [0.2]\} = \wedge[0.3] = 0.3$

$\overline{B \oplus A_{22}} = 0.7$

$(B, A_{22})_1 = 0.2$ $(B, A_{22})_2 = 0.45$

$B \bullet A_{32} = \vee\{[0.3] \wedge [0.3]\} = \vee[0.3] = 0.3$

$\underline{B \oplus A_{32}} = \wedge\{[0.3] \vee [0.3]\} = \wedge[0.3] = 0.3$

$\overline{B \oplus A_{32}} = 0.7$

$(B, A_{32})_1 = 0.3$ $(B, A_{32})_2 = 0.5$

$(A_2)_1 = 0.3 \times 0.2 + 0.3 \times 0.2 + 0.4 \times 0.3 = 0.24$

$(A_2)_2 = 0.3 \times 0.45 + 0.3 \times 0.45 + 0.4 \times 0.5 = 0.47$

$(B, A_{i3})_1 = (B \bullet A_{i3}) \wedge (\overline{B \oplus A_{13}})$

$(B, A_{i3})_2 = ½[(B \bullet A_{i3}) + (\overline{B \oplus A_{13}})]$

$B \bullet A_{13} = \vee\{[0.3] \wedge [0.5]\} = \vee[0.3] = 0.3$

$\underline{B \oplus A_{13}} = \wedge\{[0.3] \vee [0.5]\} = \wedge[0.5] = 0.5$

$\overline{B \oplus A_{13}} = 0.5$

$(B, A_{13})_1 = 0.3$ $(B, A_{13})_2 = 0.4$

$B \bullet A_{23} = \vee\{[0.3] \wedge [0.7]\} = \vee[0.3] = 0.3$

$\underline{B \oplus A_{23}} = \wedge\{[0.3] \vee [0.7]\} = \wedge[0.7] = 0.7$

$\overline{B \oplus A_{23}} = 0.3$

$(B, A_{23})_1 = 0.3$ $(B, A_{23})_2 = 0.3$

$B \bullet A_{33} = \vee\{[0.3] \wedge [0.5]\} = \vee[0.3] = 0.3$

$\underline{B \oplus A_{33}} = \wedge\{[0.3] \vee [0.5]\} = \wedge[0.5] = 0.5$

$\overline{B \oplus A_{33}} = 0.5$

$(B, A_{33})_1 = 0.3$ $(B, A_{33})_2 = 0.4$

$(A_3)_1 = 0.3 \times 0.3 + 0.3 \times 0.3 + 0.4 \times 0.3 = 0.3$

$(A_3)_2 = 0.3 \times 0.4 + 0.3 \times 0.3 + 0.4 \times 0.4 = 0.37$

$(B, A_{i4})_1 = (B \bullet A_{i4}) \wedge (\overline{B \oplus A_{14}})$

$(B, A_{i4})_2 = ½[(B \bullet A_{i4}) + (\overline{B \oplus A_{14}})]$

$B \bullet A_{14} = \vee\{[0.3] \wedge [0.9]\} = \vee[0.3] = 0.3$

$\underline{B \oplus A_{14}} = \wedge\{[0.3] \vee [0.9]\} = \wedge[0.9] = 0.9$

$\overline{B \oplus A_{14}} = 0.1$

$(B, A_{14})_1 = 0.1$ $(B, A_{14})_2 = 0.2$

$B \bullet A_{24} = \vee\{[0.3] \wedge [0.9]\} = \vee[0.3] = 0.3$

$\underline{B \oplus A_{24}} = \wedge\{[0.3] \vee [0.9]\} = \wedge[0.9] = 0.9$

$\overline{B \oplus A_{24}} = 0.1$

$(B, A_{24})_1 = 0.1$ $(B, A_{24})_2 = 0.2$

$B \bullet A_{34} = \vee\{[0.3] \wedge [0.9]\} = \vee[0.3] = 0.3$

$\underline{B \oplus A_{34}} = \wedge\{[0.3] \vee [0.9]\} = \wedge[0.9] = 0.9$

$\overline{B \oplus A_{34}} = 0.1$

$(B, A_{34})_1 = 0.1$ $(B, A_{34})_2 = 0.2$

$(A_4)_1 = 0.3 \times 0.1 + 0.3 \times 0.1 + 0.4 \times 0.1 = 0.1$

$(A_4)_2 = 0.3 \times 0.2 + 0.3 \times 0.2 + 0.4 \times 0.2 = 0.2$

Based on metric 1, the best match is A_3 (SEASAT) whereas, based on metric 2, the best match is A_2 (DMSP).

11.15

The Gaussian distribution given in this problem is incorrect. It should be in the following form:

$$\mu_{A_{ij}}(x) = \exp\left[-\left(\frac{x_j - a_{ij}}{\sigma_{a_{ij}}}\right)^2\right]$$

(B, Light) = 0.5 ($B_{Quality}$, $Quality_{Light}$) + 0.3 ($B_{Visibility}$, $Visibility_{Light}$) + 0.2 ($B_{Geometery}$, $Geometery_{Light}$)

($B_{Quality}$, $Quality_{Light}$) =

$$\exp\left(-\left(\frac{45-30}{15}\right)^2\right) = 0.368$$

($B_{Visibility}$, $Visibility_{Light}$) =

$$\exp\left(-\left(\frac{55-40}{12}\right)^2\right) = 0.210$$

($B_{Geometery}$, $Geometery_{Light}$) =

$$\exp\left(-\left(\frac{35-20}{7}\right)^2\right) = 0.010$$

(B, Light) = 0.5×0.368 + 0.3×0.210 + 0.2×0.010 = 0.249

(B, Moderate) = 0.5 ($B_{Quality}$, $Quality_{Moderate}$) + 0.3 ($B_{Visibility}$, $Visibility_{Moderate}$) + 0.2 ($B_{Geometery}$, $Geometery_{Moderate}$)

($B_{Quality}$, $Quality_{Moderate}$) =

$$\exp\left(-\left(\frac{45-40}{20}\right)^2\right) = 0.939$$

($B_{Visibility}$, $Visibility_{Moderate}$) =

$$\exp\left(-\left(\frac{55-40}{5}\right)^2\right) = 0.000$$

($B_{Geometery}$, $Geometery_{Moderate}$) =

$$\exp\left(-\left(\frac{35-35}{10}\right)^2\right) = 1$$

(B, Moderate) = 0.5×0.939 + 0.3×0.000 + 0.2×1 = 0.670

(B, Good) = 0.5 ($B_{Quality}$, $Quality_{Good}$) + 0.3 ($B_{Visibility}$, $Visibility_{Good}$) + 0.2 ($B_{Geometery}$, $Geometery_{Good}$)

($B_{Quality}$, $Quality_{Good}$) =

$$\exp\left(-\left(\frac{45-50}{15}\right)^2\right) = 0.895$$

($B_{Visibility}$, $Visibility_{Good}$) =

$$\exp\left(-\left(\frac{55-50}{10}\right)^2\right) = 0.779$$

($B_{Geometery}$, $Geometery_{Good}$) =

$$\exp\left(-\left(\frac{35-60}{10}\right)^2\right) = 0.002$$

(B, Good) = 0.5×0.895 + 0.3×0.779 + 0.2×0.002 = 0.682

(B, Rush-hour)=0.5($B_{Quality}$, $Quality_{Rush\text{-}hour}$) + 0.3 ($B_{Visibility}$, $Visibility_{Rush\text{-}hour}$) + 0.2 ($B_{Geometery}$, $Geometery_{Rush\text{-}hour}$)

($B_{Quality}$, $Quality_{Rush\text{-}hour}$) =

$$\exp\left(-\left(\frac{45-60}{10}\right)^2\right) = 0.105$$

($B_{Visibility}$, $Visibility_{Rush\text{-}hour}$) =

$$\exp\left(-\left(\frac{55-60}{6}\right)^2\right) = 0.499$$

($B_{Geometery}$, $Geometery_{Rush\text{-}hour}$) =

$$\exp\left(-\left(\frac{35-70}{15}\right)^2\right) = 0.004$$

(B, Rush-hour) = 0.5×0.105 + 0.3×0.499 + 0.2×0.004 = 0.203

The best match is Good capacity.

11.16

(B, Light) = 0.5 ($B_{Quality}$, $Quality_{Light}$) + 0.3 ($B_{Visibility}$, $Visibility_{Light}$) + 0.2 ($B_{Geometery}$, $Geometery_{Light}$)

($B_{Quality}$, $Quality_{Light}$)$_1$ =

$$\exp\left(-\left(\frac{45-30}{10+15}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Quality}, Quality_{Light})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{45-30}{10+15}\right)^2\right)+1\right] = 0.500$$

$(B_{Visibility}, Visibility_{Light})_1 =$

$$\exp\left(-\left(\frac{55-40}{12-12}\right)^2\right) \wedge 1 = 0$$

$(B_{Visibility}, Visibility_{Light})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{55-40}{12-12}\right)^2\right)+1\right] = 0.500$$

$(B_{Geometery}, Geometery_{Light})_1 =$

$$\exp\left(-\left(\frac{35-20}{8-7}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Geometery}, Geometery_{Light})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{35-20}{8-7}\right)^2\right)+1\right] = 0.500$$

$(B, Light)_1 = 0.5\times0 + 0.3\times0 + 0.2\times0 = 0$
$(B, Light)_2 = 0.5\times0.5+0.3\times0.5+0.2\times0.5 = 0.5$

$(B, Moderate) = 0.5\ (B_{Quality}, Quality_{Moderate}) + 0.3\ (B_{Visibility}, Visibility_{Moderate}) + 0.2\ (B_{Geometery}, Geometery_{Moderate})$

$(B_{Quality}, Quality_{Moderate})_1 =$

$$\exp\left(-\left(\frac{45-40}{10-20}\right)^2\right) \wedge 1 = 0.779$$

$(B_{Quality}, Quality_{Moderate})_2 =$

$$\frac{1}{2}\left[exp\left(-\left(\frac{45-40}{10-20}\right)^2\right)+1\right] = 0.889$$

$(B_{Visibility}, Visibility_{Moderate})_1 =$

$$\exp\left(-\left(\frac{55-40}{12-5}\right)^2\right) \wedge 1 = 0.010$$

$(B_{Visibility}, Visibility_{Moderate})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{55-40}{12-5}\right)^2\right)+1\right] = 0.505$$

$(B_{Geometery}, Geometery_{Moderate})_1 =$

$$\exp\left(-\left(\frac{35-35}{8-10}\right)^2\right) \wedge 1 = 1$$

$(B_{Geometery}, Geometery_{Moderate})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{35-35}{8-10}\right)^2\right)+1\right] = 1$$

$(B, Moderate)_1 = 0.5\times0.779 + 0.3\times0.010 + 0.2\times1 = 0.593$
$(B, Moderate)_2 = 0.5\times0.889 + 0.3\times0.505 + 0.2\times1 = 0.593$

$(B, Good) = 0.5\ (B_{Quality}, Quality_{Good}) + 0.3\ (B_{Visibility}, Visibility_{Good}) + 0.2\ (B_{Geometery}, Geometery_{Good})$

$(B_{Quality}, Quality_{Good})_1 =$

$$\exp\left(-\left(\frac{45-50}{10-15}\right)^2\right) \wedge 1 = 0.368$$

$(B_{Quality}, Quality_{Good})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{45-50}{10-15}\right)^2\right)+1\right] = 0.684$$

$(B_{Visibility}, Visibility_{Good})_1 =$

$$\exp\left(-\left(\frac{55-50}{12-10}\right)^2\right) \wedge 1 = 0.002$$

$(B_{Visibility}, Visibility_{Good})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{55-50}{12-10}\right)^2\right)+1\right] = 0.501$$

$(B_{Geometery}, Geometery_{Good})_1 =$

$$\exp\left(-\left(\frac{35-60}{8-10}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Geometery}, Geometery_{Good})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{35-60}{8-10}\right)^2\right)+1\right] = 0.500$$

$(B, Good)_1 = 0.5\times0.368 + 0.3\times0.002 + 0.2\times0 = 0.185$
$(B, Good)_2 = 0.5\times0.684 + 0.3\times0.501 + 0.2\times0 = 0.492$

$(B, Rush\text{-}hour)=0.5(B_{Quality}, Quality_{Rush\text{-}hour}) + 0.3\ (B_{Visibility}, Visibility_{Rush\text{-}hour}) + 0.2\ (B_{Geometery}, Geometery_{Rush\text{-}hour})$

$(B_{Quality}, Quality_{Rush\text{-}hour})_1 =$

$$\exp\left(-\left(\frac{45-60}{10-10}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Quality}, Quality_{Rush\text{-}hour})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{45-60}{10-10}\right)^2\right)+1\right] = 0.500$$

$(B_{Visibility}, Visibility_{Rush\text{-}hour})_1 =$

$$\exp\left(-\left(\frac{55-60}{12-6}\right)^2\right) \wedge 1 = 0.499$$

$(B_{Visibility}, Visibility_{Rush\text{-}hour})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{55-60}{12-6}\right)^2\right)+1\right] = 0.750$$

$(B_{Geometery}, Geometery_{Rush\text{-}hour})_1 =$

$$\exp\left(-\left(\frac{35-70}{8-15}\right)^2\right) \wedge 1 = 0.000$$

$(B_{Geometery}, Geometery_{Rush\text{-}hour})_2 =$

$$\frac{1}{2}\left[\exp\left(-\left(\frac{35-70}{8-15}\right)^2\right)+1\right] = 0.500$$

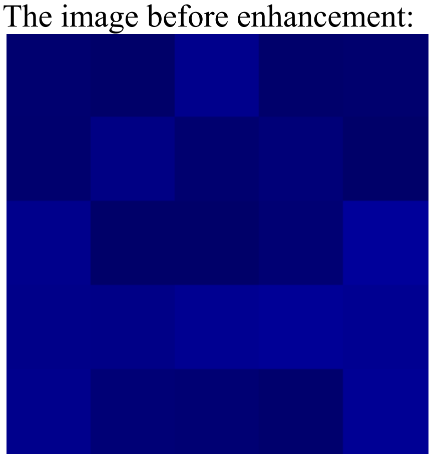
$(B, \text{Rush-hour})_1 = 0.5\times 0 + 0.3\times 0.499 + 0.2\times 0 = 0.150$

$(B, \text{Rush-hour})_2 = 0.5\times 0.500 + 0.3\times 0.750 + 0.2\times 0.500 = 0.575$

Therefore, based on both metric 1 and 2, the best match is Good capacity.

11.17

The image before enhancement:



Scaling the pixel values between 0 and 1 by dividing by 255 we get:

0.44	0.41	0.55	0.42	0.43
0.43	0.52	0.44	0.47	0.41
0.55	0.41	0.41	0.45	0.60
0.54	0.53	0.57	0.59	0.57
0.55	0.46	0.45	0.43	0.58

using the algorithm explained in the book.

$$\mu = \begin{cases} 2(x)^2 & \forall x \le 0.5 \\ 1-2(1-x)^2 & \forall x > 0.5 \end{cases}$$

where x = scaled values

(1)

0.38	0.34	0.59	0.35	0.37
0.37	0.53	0.38	0.44	0.34
0.59	0.34	0.34	0.41	0.69
0.57	0.56	0.63	0.66	0.63
0.59	0.43	0.41	0.37	0.65

(2)

0.29	0.23	0.67	0.25	0.28
0.28	0.57	0.29	0.39	0.23
0.67	0.23	0.23	0.33	0.80
0.63	0.61	0.72	0.77	0.72
0.67	0.37	0.33	0.27	0.75

(3)

0.16	0.11	0.78	0.12	0.15
0.15	0.62	0.16	0.31	0.11
0.78	0.11	0.11	0.22	0.92
0.73	0.69	0.85	0.89	0.85
0.78	0.27	0.22	0.14	0.88

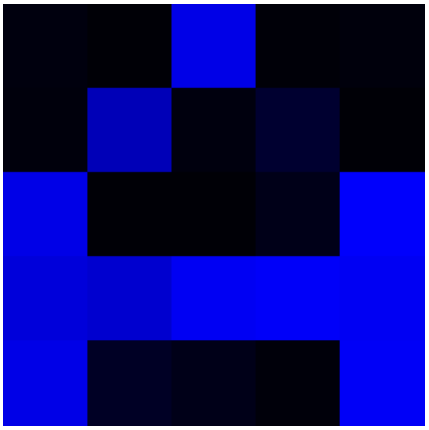
(4)

0.05	0.02	0.90	0.03	0.05
0.05	0.72	0.05	0.19	0.02
0.90	0.02	0.02	0.10	0.99
0.86	0.81	0.95	0.98	0.95
0.90	0.14	0.10	0.04	0.97

Restoring the original scale of the pixel values by multiplying by 255 we get:

14	6	231	8	12
12	183	14	48	6
231	6	6	24	252
218	207	243	249	243
231	37	24	10	247

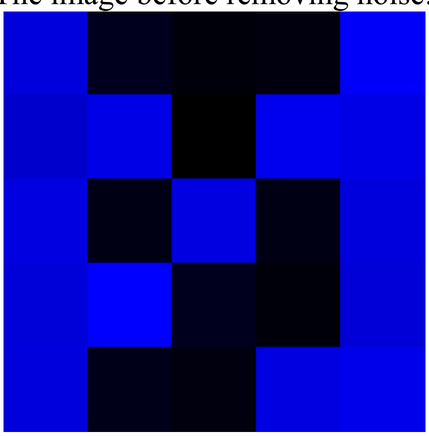
The achieved image after enhancement:



This is the letter A.

11.18

The image before removing noise:



Scaling the pixel values between 0 and 1 by dividing by 255 we get:

0.86	0.12	0.04	0.06	0.98
0.8	0.9	0	0.94	0.9
0.88	0.08	0.88	0.08	0.86
0.85	1	0.12	0.04	0.84
0.86	0.1	0.06	1	0.92

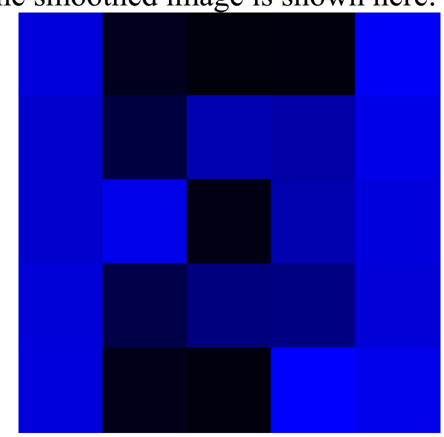
Using the algorithm explained in the text, i.e. $\mu_{00} = ¼(\mu_{-1,0}+\mu_{1,0}+\mu_{0,1}+\mu_{0,-1})$, we get:

0.86	0.12	0.04	0.06	0.98
0.8	0.25	0.69	0.26	0.9
0.88	0.92	0.07	0.68	0.86
0.85	0.29	0.5	0.51	0.84
0.86	0.1	0.06	1	0.92

Note: the pixels on the edges of the pixel graph (screen) would not be affected by the algorithm. Restoring the original scale of the pixel values by multiplying by 255 we get:

220	30	10	15	250
205	63.8	176	66.3	230
225	234	17.5	174	220
217	73	126	130	215
220	25	15	255	235

The smoothed image is shown here:



CHAPTER 12

Fuzzy Arithmetic and Extension Principle

12.1

a) $[2,3]+[3,4]=[5,7]$

b) $[1,2]*[2,3]=$
$[\min(1,2,3,6),\max(1,2,3,6)]=[1,6]$

c) $[3,6]\div[1,3]=[3,6]\div[\tfrac{1}{3},1]$
$=[\min(3,6,1,2),\max(3,6,1,2)]$
$=[1,6]$

d)
$[2,5]-[4,6]=[2-6,5-4]=[-4,1]$

12.2

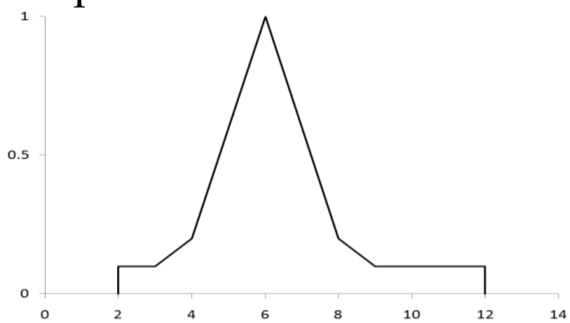
$$\underset{\sim}{I}=\underset{\sim}{3}=\frac{0.2}{2}+\frac{1}{3}+\frac{0.2}{4}$$

$$\underset{\sim}{J}=\underset{\sim}{2}=\frac{0.1}{1}+\frac{1}{2}+\frac{0.1}{3}$$

$$\underset{\sim}{K}=\underset{\sim}{6}=\underset{\sim}{I}\times\underset{\sim}{J}=$$

$$\frac{\min(0.1,0.2)}{2}+\frac{\min(1,0.1)}{3}+$$
$$\frac{\max(\min(0.2,0.1),\min(0.2,1))}{4}+$$
$$\frac{\max(\min(0.2,0.1),\min(1,1))}{6}+$$
$$\frac{\min(0.2,1)}{8}+\frac{\min(0.1,1)}{9}+$$
$$\frac{\min(0.2,0.1)}{12}$$
$$=\left\{\frac{0.1}{2}+\frac{0.1}{3}+\frac{0.2}{4}+\frac{1}{6}+\frac{0.2}{8}+\frac{0.1}{9}+\frac{0.1}{12}\right\}$$

By plotting the above points it can be shown that it is non-convex as show hereafter.



12.3

a) $z=3x-2$

$$=\frac{0.0}{3(0)-2}+\frac{0.3}{3(1)-2}+\frac{0.1}{3(2)-2}+$$
$$\frac{0.8}{3(3)-2}+\frac{1.0}{3(4)-2}+\frac{0.7}{3(5)-2}+$$
$$\frac{0.2}{3(6)-2}+\frac{0.0}{3(7)-2}$$
$$=\frac{0}{-2}+\frac{0.3}{1}+\frac{0.6}{4}+\frac{0.8}{7}+\frac{1.0}{10}+\frac{0.7}{13}+$$
$$\frac{0.2}{16}+\frac{0}{19}$$

b) $$z=\frac{0}{3}+\frac{0.3}{7}+\frac{0.6}{19}+\frac{0.8}{39}+\frac{1.0}{67}+\frac{0.7}{103}+$$
$$\frac{0.2}{147}+\frac{0}{199}$$

c)
$$z=\frac{0}{0}+\frac{0}{1}+\frac{0.3}{2}+\frac{0}{4}+\frac{0.6}{5}+\frac{0.6}{8}+\frac{0}{9}+$$
$$\frac{0.8}{10}+\frac{0.8}{13}+\frac{0}{16}+\frac{1.0}{17}+\frac{0.5}{18}+\frac{0.9}{20}+$$
$$\frac{0.5}{25}+\frac{0.7}{26}+\frac{0.7}{29}+\frac{0.2}{32}+\frac{0.5}{34}+\frac{0}{36}+$$
$$\frac{0.2}{37}+\frac{0.2}{40}+\frac{0.2}{41}+\frac{0.2}{45}+\frac{0}{49}+\frac{0.7}{50}+$$
$$\frac{0.2}{52}+\frac{0}{53}+\frac{0}{58}+\frac{0.1}{61}+\frac{0}{65}+\frac{0}{72}+$$
$$\frac{0}{74}+\frac{0}{85}+\frac{0}{98}$$

d) $z = \frac{0.0}{0-0} + \frac{0.3}{1-1} + \frac{0.6}{2-2} + \frac{0.8}{3-3} + \frac{1.0}{4-4} + \frac{0.7}{5-5} + \frac{0.1}{6-6} + \frac{0.0}{7-7} = \frac{1}{0}$

e)

$$\frac{0}{0} + \frac{0.3}{1} + \frac{0.6}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.1}{5} + \frac{0}{6} + \frac{0}{7}$$

12.4

a) $\left\{\frac{0}{-11} + \frac{0}{-10} + \frac{0}{-8} + \frac{0}{-7} + \frac{0}{-6} + \frac{0.5}{-4} + \frac{0}{-3} + \frac{0}{-2} + \frac{0.5}{0} + \frac{1}{1} + \frac{0}{2} + \frac{0}{4} + \frac{0.5}{5} + \frac{0}{6} + \frac{0}{8} + \frac{0}{9} + \frac{0}{10} + \frac{0}{12} + \frac{0}{13} + \frac{0}{16} + \frac{0}{17}\right\}$

b) $\frac{\partial f}{\partial x_1} = 2x_1 - 4 = 0 \qquad x_1 = 2$

$\frac{\partial f}{\partial x_2} = 2x_2 = 0 \qquad x_2 = 0$

$E(2,0) \qquad f(E) = 0$

$I_{0^+} \quad x_1[1,4] \qquad x_2[-1,1]$

$(a) \quad x_1 = 1 \quad x_2 = -1 \quad f(a) = 2$

$(b) \quad x_1 = 1 \quad x_2 = 1 \quad f(b) = 2$

$(c) \quad x_1 = 4 \quad x_2 = -1 \quad f(c) = 5$

$(d) \quad x_1 = 4 \quad x_2 = 1 \quad f(d) = 5$

$B_{0^+} = [\min(2,2,5,5,0), \max(2,2,5,5,0)] = [0,5]$

$I_{0.5} \quad x_1[2,3.5] \qquad x_2[-0.5,0.5]$

$(a) \quad x_1 = 2 \quad x_2 = -0.5 \quad f(a) = 0.25$

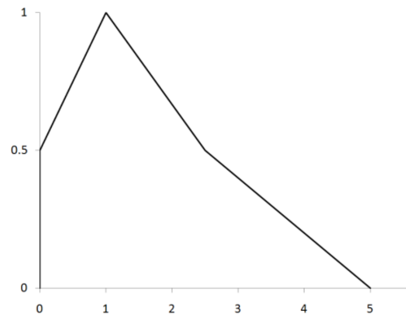
$(b) \quad x_1 = 2 \quad x_2 = 0.5 \quad f(b) = 0.25$

$(c) \quad x_1 = 3.5 \quad x_2 = -0.5 \quad f(c) = 2.5$

$(d) \quad x_1 = 3.5 \quad x_2 = 0.5 \quad f(d) = 2.5$

$B_{0.5} = [\min(0.25, 0.25, 2.5, 2.5, 0), \max(0.25, 0.25, 2.5, 2.5, 0)] = [0,5]$

$I_1 \quad (a)\ x_1 = 3 \quad x_2 = 0 \quad f(a) = 1 \quad B_1 = 1$



c) $I_{0^+} \quad x_1[1,4] \qquad x_2[-1,1]$

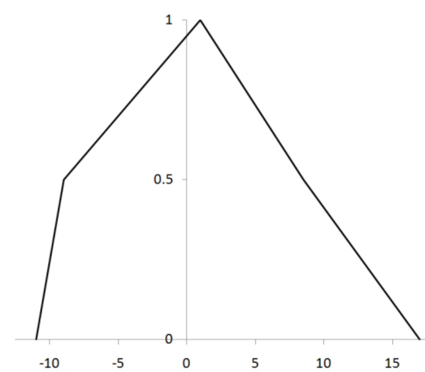
$B_{0^+} = [1^2, 4^2] + [0, 1^2] - 4[1,4] + 4$
$= [1,16] + [0,1] + [-16,-4] + 4$
$= [1 + 0 - 16 + 4, 16 + 1 - 4 + 4] = [-11, 17]$

$I_{0.5} \quad x_1[2,3.5] \qquad x_2[-0.5,0.5]$

$B_{0^+} = [2^2, 3.5^2] + [0, 0.5^2] - 4[2,3.5] + 4$
$= [4 + 0 - 17 + 4, 12.25 + 0.25 - 8 + 4] = [-9, 8.5]$

$I_1 \quad x_1 = 3 \quad x_2 = 0$

$B_1 = 3^2 + 0 - 4 \times 3 + 4 = 1$



12.5

$$\left\{\frac{0}{0} + \frac{0.7}{0.5} + \frac{1}{1} + \frac{0.7}{1.5} + \frac{0}{2}\right\} \times \left\{\frac{0.5}{500} + \frac{0.8}{750} + \frac{1}{1000} + \frac{0.8}{1250} + \frac{0.5}{1500}\right\}$$

$$= \{\frac{0.8}{0} + \frac{0.5}{250} + \frac{0.7}{375} + \frac{0.7}{500} + \frac{0.7}{625} + \frac{0.8}{750} + \frac{1}{1000} + \frac{0.7}{1125} + \frac{0.8}{1250} + \frac{0.7}{1500} + \frac{0.7}{1875} + \frac{0}{2000} + \frac{0.5}{2250} + \frac{0}{2500} + \frac{0}{3000}\}$$

12.6

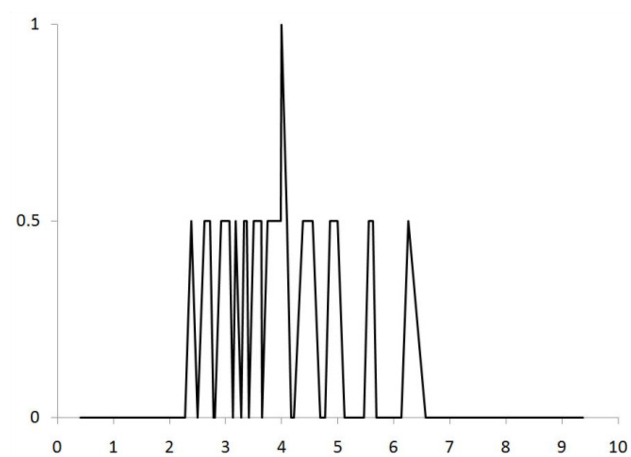
$$\left\{\frac{0.5}{3}+\frac{1}{4}+\frac{0.6}{5}\right\}+\left\{\frac{0.4}{8}+\frac{1}{9}+\frac{0.3}{10}\right\}$$
$$=\frac{0.4}{11}+\frac{0.5}{12}+\frac{0.3}{13}+\frac{0.4}{12}+\frac{1}{13}+\frac{0.3}{14}+\frac{0.4}{13}+\frac{0.6}{14}+\frac{0.3}{15}$$
$$=\left\{\frac{0.4}{11}+\frac{0.5}{12}+\frac{1}{13}+\frac{0.6}{14}+\frac{0.3}{15}\right\}$$

12.7

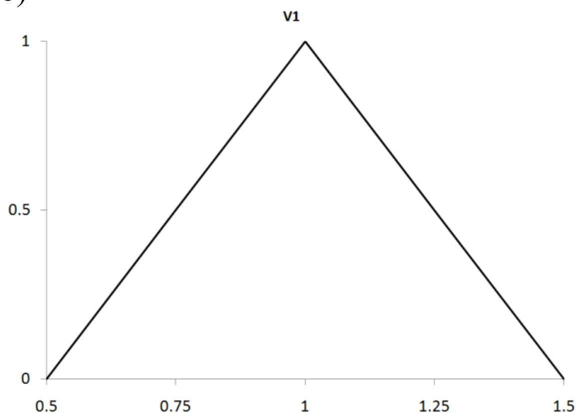
a) $$P_1V_1=\left\{\frac{0}{0.5}+\frac{0.5}{0.75}+\frac{1}{1}+\frac{0.5}{1.25}+\frac{0}{1.5}\right\}\times\left\{\frac{0}{0.5}+\frac{0.5}{1.75}+\frac{1}{2}+\frac{0.5}{2.25}+\frac{0}{2.5}\right\}$$
$$=\{\frac{0}{0.25}+\frac{0}{0.375}+\frac{0}{0.5}+\frac{0}{0.625}+\frac{0}{0.75}+\frac{0}{0.875}+\frac{0}{1}+\frac{0}{1.125}+\frac{0}{1.25}+\frac{0.5}{1.3125}+\frac{0.5}{1.5}+\frac{0.5}{1.6875}+\frac{0.5}{1.75}+\frac{0}{1.875}+\frac{1}{2}+\frac{0.5}{2.1875}+\frac{0.5}{2.25}+\frac{0.5}{2.5}+\frac{0}{2.625}+\frac{0.5}{2.8125}+\frac{0}{3}+\frac{0}{3.125}+\frac{0}{3.375}+\frac{0}{3.75}\}$$

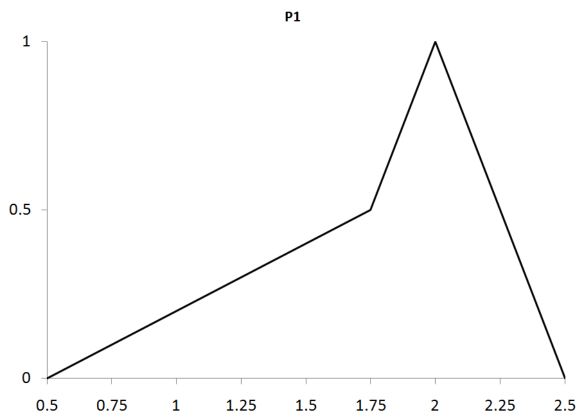
$$P_2=P_1V_1\div V_2=$$
$$P_1V_1\div\left\{\frac{0}{0.4}+\frac{0.5}{0.45}+\frac{1}{0.5}+\frac{0.5}{0.55}+\frac{0}{0.6}\right\}$$

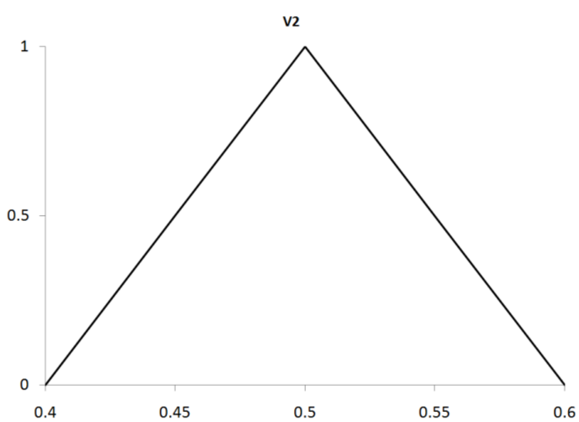
$$=\{\frac{0}{0.417}+\frac{0}{0.455}+\frac{0}{0.5}+\frac{0}{0.555}+\frac{0}{0.625}+\frac{0}{0.682}+\frac{0}{0.75}$$
$$\frac{0}{0.833}+\frac{0}{0.909}+\frac{0}{0.938}+\frac{0}{1}+\frac{0}{1.042}+\frac{0}{1.111}+\frac{0}{1.136}$$
$$\frac{0}{1.25}+\frac{0}{1.364}+\frac{0}{1.389}+\frac{0}{1.458}+\frac{0}{1.5}+\frac{0}{1.563}+\frac{0}{1.591}$$
$$\frac{0}{1.667}+\frac{0}{1.75}+\frac{0}{1.818}+\frac{0}{1.875}+\frac{0}{1.944}+\frac{0}{2}+\frac{0}{2.045}$$
$$\frac{0}{2.083}+\frac{0}{2.188}+\frac{0}{2.222}+\frac{0}{2.25}+\frac{0}{2.273}+\frac{0.5}{2.386}+\frac{0}{2.5}$$
$$\frac{0.5}{2.625}+\frac{0.5}{2.727}+\frac{0}{2.778}+\frac{0}{2.813}+\frac{0.5}{2.917}+\frac{0.5}{3}+\frac{0.5}{3.068}$$
$$\frac{0}{3.125}+\frac{0.5}{3.182}+\frac{0}{3.281}+\frac{0.5}{3.333}+\frac{0.5}{3.375}+\frac{0}{3.409}+\frac{0.5}{3.5}$$
$$\frac{0.5}{3.636}+\frac{0}{3.646}+\frac{0.5}{3.75}+\frac{0.5}{3.889}+\frac{0.5}{3.977}+\frac{1}{4}+\frac{0.5}{4.091}$$
$$\frac{0}{4.167}+\frac{0}{4.219}+\frac{0.5}{4.375}+\frac{0.5}{4.444}+\frac{0.5}{4.5}+\frac{0.5}{4.545}+\frac{0}{4.688}$$
$$\frac{0}{4.773}+\frac{0.5}{4.861}+\frac{0.5}{5}+\frac{0}{5.114}+\frac{0}{5.208}+\frac{0}{5.25}+\frac{0}{5.455}$$
$$\frac{0}{5.469}+\frac{0.5}{5.556}+\frac{0.5}{5.625}+\frac{0}{5.682}+\frac{0}{5.833}+\frac{0}{6}+\frac{0}{6.136}$$
$$\frac{0.5}{6.25}+\frac{0}{6.563}+\frac{0}{6.667}+\frac{0}{6.75}+\frac{0}{6.818}+\frac{0}{6.944}+\frac{0}{7.031}$$
$$\frac{0}{7.5}+\frac{0}{7.813}+\frac{0}{8.333}+\frac{0}{8.438}+\frac{0}{9.375}\}$$



b)







b-i)

I_{0^+} $V_1[0.5, 1.5]$ $P_1[0.5, 2.5]$ $V_2[0.4, 0.6]$

$V_1 = 0.5$ $P_1 = 0.5$ $V_2 = 0.4$ $f(a) = 0.625$

$V_1 = 0.5$ $P_1 = 0.5$ $V_2 = 0.6$ $f(b) = 0.417$

$V_1 = 0.5$ $P_1 = 2.5$ $V_2 = 0.4$ $f(c) = 3.125$

$V_1 = 0.5$ $P_1 = 2.5$ $V_2 = 0.6$ $f(d) = 2.083$

$V_1 = 1.5$ $P_1 = 0.5$ $V_2 = 0.4$

$f(e) = 1.875$

$V_1 = 1.5$ $P_1 = 0.5$ $V_2 = 0.6$

$f(f) = 1.25$

$V_1 = 1.5$ $P_1 = 2.5$ $V_2 = 0.4$

$f(g) = 9.375$

$V_1 = 1.5$ $P_1 = 2.5$ $V_2 = 0.6$

$f(h) = 6.25$

$$f(V_1, P_1, V_2) = \frac{V_1 P_1}{P_2}$$

$$\frac{\partial f}{\partial V_1} = \frac{P_1}{P_2} = 0 \quad P_1 = 0$$

$$\frac{\partial f}{\partial P_1} = \frac{V_1}{P_2} = 0 \quad V_1 = 0$$

$$\frac{\partial f}{\partial V_1} = -\frac{V_1 P_1}{{P_2}^2} = 0 \quad P_2 \neq 0$$

$E(0, 0, P_2) \notin \{[0.5, 1.5], [0.5, 2.5], [0.4, 0.6]\}$

B_{0^+}
$= [\min(0.625, 0.417, 2.083, 1.875, 1.25, 9.375, 6.25)$
$, \max(0.625, 0.417, 2.083, 1.875, 1.25, 9.375, 6.25)]$

$B_{0^+} = [0.417, 9.375]$

$I_{0.5}$

$V_1[0.75, 1.25]$ $P_1[1.75, 2.25]$ $V_2[0.45, 0.55]$

$V_1 = 0.75$ $P_1 = 1.75$ $V_2 = 0.45$

$f(a) = 2.917$

$V_1 = 0.75$ $P_1 = 1.75$ $V_2 = 0.55$

$f(b) = 2.386$

$V_1 = 0.75$ $P_1 = 2.25$ $V_2 = 0.45$

$f(c) = 3.75$

$V_1 = 0.75$ $P_1 = 2.25$ $V_2 = 0.55$

$f(d) = 3.068$

$V_1 = 1.25$ $P_1 = 1.75$ $V_2 = 0.45$

$f(e) = 4.861$

$V_1 = 1.25$ $P_1 = 1.75$ $V_2 = 0.55$

$f(f) = 3.977$

$V_1 = 1.25$ $P_1 = 2.25$ $V_2 = 0.45$

$f(g) = 6.25$

$V_1 = 1.25$ $P_1 = 2.25$ $V_2 = 0.55$

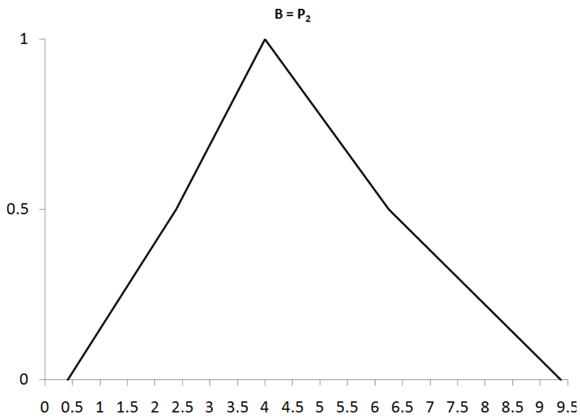
$f(h) = 5.114$

$B_{0.5}$
$= [\min(2.917, 2.386, 3.75, 3.068, 4.861, 3.977, 6.25, 5.114)$
$, \max(2.917, 2.386, 3.75, 3.068, 4.861, 3.977, 6.25, 5.114)]$

$B_{0.5} = [2.386, 6.25]$

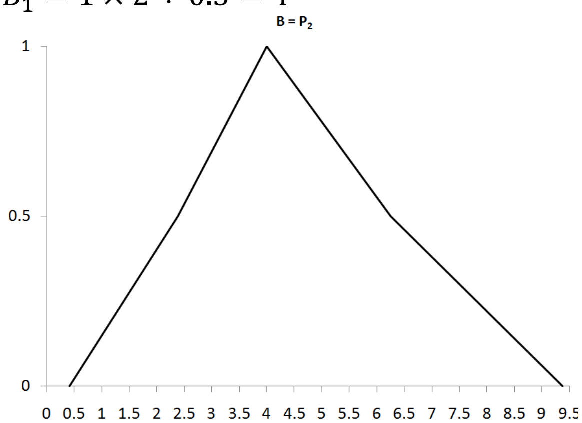
I_1 $V_1 = 1$ $P_1 = 2$ $V_2 = 0.5$

$f(a) = 4$ $B_1 = 4$



b-ii)

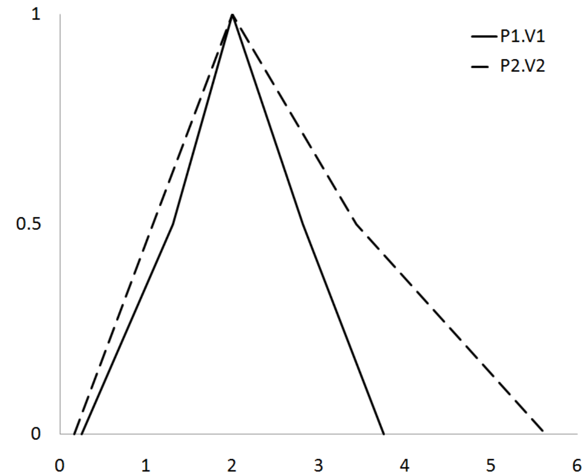
$B_{0^+} = [0.5,1.5].[0.5,2.5] \div [0.4,0.6]$
$= [0.25,3.75] \div [0.4,0.6] = [0.417,9.375]$
$B_{0.5} = [0.75,1.25].[1.75,2.25] \div [0.45,0.55]$
$= [1.313,2.813] \div \{0.45,0.55]$
$= [2.386,6.25]$
$B_1 = 1 \times 2 \div 0.5 = 4$



c) We will use DSW method in this part of the exercise to achieve more comparable results:

P_1V_1: $B_{0^+} = [0.5,1.5].[0.5,2.5]$
$= [0.25,0.375]$
$B_{0.5} = [0.75,1.25].[1.75,2.25]$
$= [1.313,2.813]$
$B_1 = 1 \times 2 = 2$

P_2V_2: (Using P_2 calculated in part b-ii)
$B_{0^+} = [0.4,0.6].[0.417,9.937]$
$= [0.167,5.625]$
$B_{0.5} = [0.45,0.55].[2.386,6.25]$
$= [1.074,3.438]$
$B_1 = 0.5 \times 4 = 2$



We can use restricted DSW algorithm in this case: $P_1[a,b]$ $V_1[c,d]$ $V_2[e,f]$
For example, when we calculated I_λ for P_1V_1 we had:

$B_\lambda = [a,b].[c,d] = [ac,bd]$

Also, we calculated P_2 using the equation $P_2=P_1V_1 \div V_2$, thus for P_2 we obtained the following:

$B_\lambda = [ac,bd] \div [e,f] = \left[\frac{ac}{f},\frac{bd}{e}\right]$

And then we calculated P_2V_2 as follows:

$B_\lambda = \left[\frac{ac}{f},\frac{bd}{e}\right] \times [e,f] = \left[\frac{ace}{f},\frac{bdf}{e}\right]$;

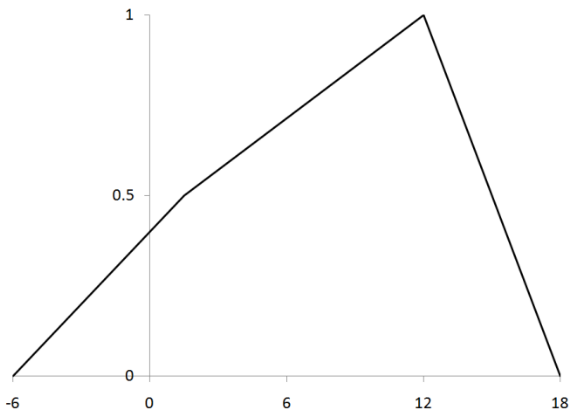
while P_1V_1 we have $B_\lambda = [ac,bd]$.
Therefore, that is evident that we would not have the same P_1V_1 and P_2V_2 due to the property of the interval dividing operation.

12.8 a-i)

I_{0^+} $x_1[1,3]$ $x_2[1,3]$
$x_1 = 1$ $x_2 = 1$ $f(a) = -2$
$x_1 = 1$ $x_2 = 3$ $f(b) = -6$
$x_1 = 3$ $x_2 = 1$ $f(c) = 6$
$x_1 = 3$ $x_2 = 3$ $f(d) = 18$
$B_{0^+} = [\min(-2,-6,6,18), \max(-2,-6,6,18)]$
$= [-6,18]$

$I_{0.5}$ $x_1[2,3]$ $x_2[1.5,2.5]$
$x_1 = 2$ $x_2 = 1.5$ $f(a) = 1.5$
$x_1 = 2$ $x_2 = 2.5$ $f(b) = 2.5$
$x_1 = 3$ $x_2 = 1.5$ $f(c) = 9$
$x_1 = 3$ $x_2 = 2.5$ $f(d) = 15$
$B_{0.5} = [\min(1.5,2.5,9,15), \max(1.5,2.5,9,15)]$
$= [1.5,15]$

$I_1 \quad x_1 = 3 \quad x_2 = 2 \quad f(a) = 12 \quad B_1 = 12$



a-ii) $\frac{\partial y}{\partial x_1} = 2x_1x_2 = 0 \qquad x_1x_2 = 0$

$\frac{\partial y}{\partial x_2} = {x_1}^2 - 3 = 0 \qquad x_1 = \pm\sqrt{3}$

$\rightarrow x_2 = 0$

$E_1(-\sqrt{3},0) \rightarrow f(E_1) = 0$

$E_1(+\sqrt{3},0) \rightarrow f(E_2) = 0$

$I_{0^+} \quad E_1 \notin \{x_1[1,3], x_2[1,3]\}$

$\quad E_2 \notin \{x_1[1,3], x_2[1,3]\}$

$I_{0.5} \quad E_1 \notin \{x_1[2,3], x_2[1.5,2.5]\}$

$\quad E_2 \notin \{x_1[2,3], x_2[1.5,2.5]\}$

$I_1 \quad E_1 \neq (3,2) \quad , \quad E_2 \neq (3,2)$

Thus, consideration of extreme points does not change the previous result in which we ignored the extreme points.

b) $B_{0^+} = ([1,3])^2.[1,3] - 3[1,3]$

$= [1,9].[1,3] - [3,9]$

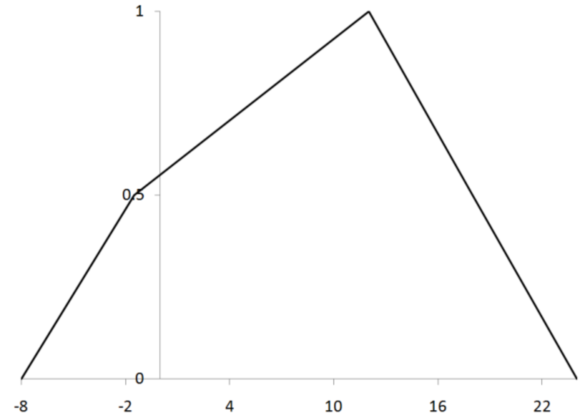
$= [1,27] - [3,9] = [-8,24]$

$B_{0.5} = ([2,3])^2.[1.5,2.5] - 3[1.5,2.5]$

$= [4,9].[1.5,2.5] - [4.5,7.5]$

$= [6,22.5] - [4.5,7.5] = [-1.5,18]$

$B_1 = (3)^2 \times 2 - 3 \times 2 = 12$



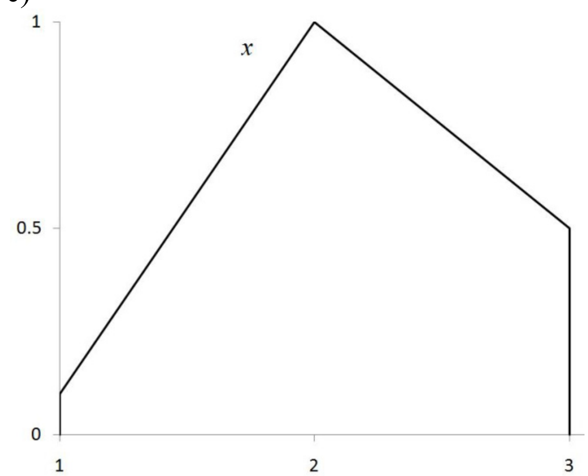
12.9 a) $z = \left\{\frac{0.1}{1} + \frac{1}{4} + \frac{0.5}{9}\right\}$

b) $z = \left\{\frac{0.1}{1} + \frac{1}{2} + \frac{0.5}{3}\right\} \times \left\{\frac{0.1}{1} + \frac{1}{2} + \frac{0.5}{3}\right\}$

$= \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.1}{3} + \frac{0.1}{2} + \frac{1}{4} + \frac{0.5}{6} + \frac{0.1}{3} + \frac{0.5}{6} + \frac{0.5}{9}$

$= \left\{\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.1}{3} + \frac{1}{4} + \frac{0.5}{6} + \frac{0.5}{9}\right\}$

c)



$z = x^2$

$I_{0^+} \quad x[1,3]$

x = 1 z = 1

x = 3 z = 9

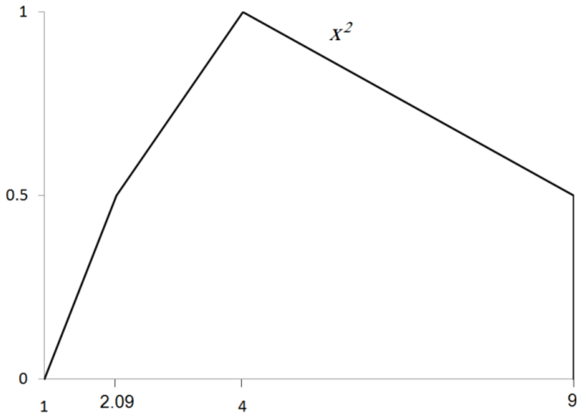
$B_{0^+} = [\min(1,9), \max(1,9)] = [1,9]$

$I_{0.5} \quad x[1.44,3]$

x = 1.44 z = 2.09

x = 3 z = 9

$B_{0.5} = [\min(2.09,9), \max(2.09,9)]$
$= [2.09,9]$
$I_1 \quad x = 2 \quad z = 4 \quad B_1 = 4$



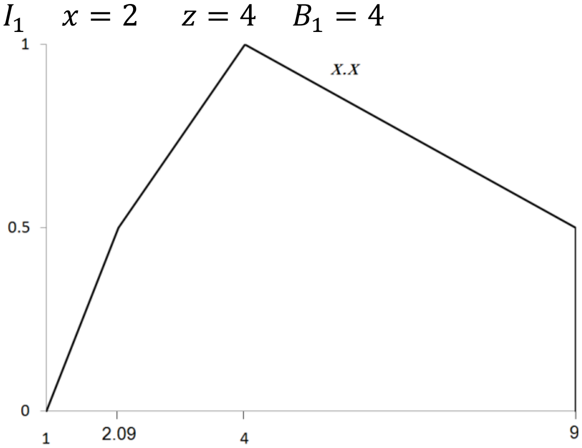
$z = x.x$
$I_{0^+} \quad x[1,3] \quad x[1,3]$

x = 1	x = 1	z = 1
x = 1	x = 3	z = 3
x = 3	x = 1	z = 3
x = 3	x = 3	z = 9

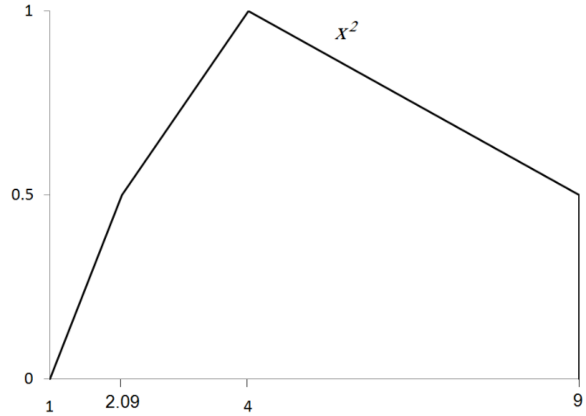
$B_{0^+} = [\min(1,3,9), \max(1,3,9)]$
$= [1,9]$
$I_{0.5} \quad x[1.44,3] \quad x[1.44,3]$

x = 1.44	x = 1.44	z = 2.09
x = 1.44	x = 3	z = 4.32
x = 3	x = 1.44	z = 4.32
x = 3	x = 3	z = 9

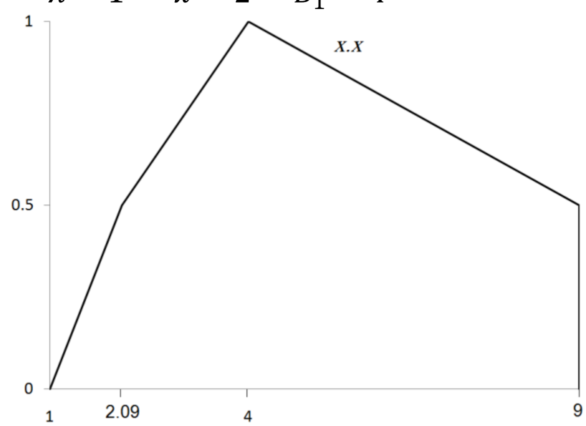
$B_{0.5}$
$= [\min(2.09,4.32,9), \max(2.09,4.32,9)]$
$= [2.0,9]$
$I_1 \quad x = 2 \quad z = 4 \quad B_1 = 4$



d)
$z = x^2$
$\lambda = 0^+ \quad x[1,3]$
$B_{0^+} = ([1,3])^2 = [1,9]$
$\lambda = 0.5 \quad x[1.44,3]$
$B_{0.5} = ([1.44,3])^2 = [2.09,9]$
$\lambda = 1 \quad x = 2 \quad B_1 = 4$



$z = x.x$
$\lambda = 0^+ \quad x[1,3]$
$B_{0^+} = [1,3].[1,3] = [1,9]$
$\lambda = 0.5 \quad x[1.44,3]$
$B_{0.5} = [1.44,3].[1.44,3] = [2.09,9]$
$\lambda = 1 \quad x = 2 \quad B_1 = 4$



e) The answers in the first part (a) and (b) are different because in a) x is a single variable, and when it is squared its value (the crisp value) is squared too. In (b) 'x' and 'x' are treated as two separate variables, i.e. for all practical purpose, they could be 'x' and 'y' with the same membership function.
For both vertex and DSW methods in this exercise there is no difference between $z = x^2$ and $z = x.x$. Also,

outcomes from vertex and DSW are the same.

12.10

a) $z = \left\{\frac{1}{0} + \frac{0.7}{1} + \frac{0.3}{4} + \frac{0.1}{9}\right\}$

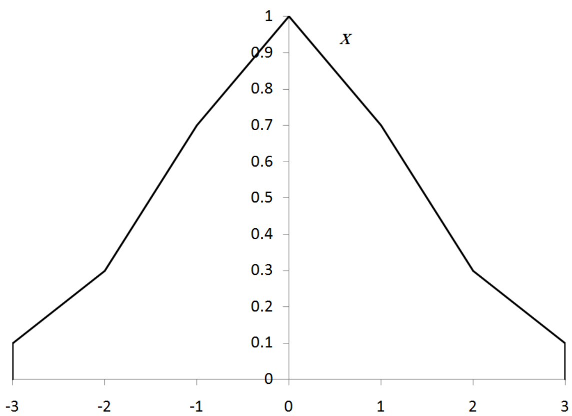
b) $z = \left\{\frac{0.1}{-3} + \frac{0.3}{-2} + \frac{0.7}{-1} + \frac{1}{0} + \frac{0.7}{1} + \frac{0.3}{2} + \frac{0.1}{3}\right\}$

$\times \left\{\frac{0.1}{-3} + \frac{0.3}{-2} + \frac{0.7}{-1} + \frac{1}{0} + \frac{0.7}{1} + \frac{0.3}{2} + \frac{0.1}{3}\right\}$

$= \{\frac{0.1}{-9} + \frac{0.1}{-6} + \frac{0.3}{-4} + \frac{0.1}{-3} + \frac{0.3}{-2} + \frac{0.7}{-1} + \frac{1}{0} + \frac{0.7}{1}$

$+ \frac{0.3}{2} + \frac{0.1}{3} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.1}{9}\}$

c)



$$\frac{\partial z}{\partial x} = 2x = 0 \rightarrow x = 0 \rightarrow z = 0$$

$z = x^2$

I_{0^+} x[-3, 3]

x = -3 z = 9

x = 3 z = 9

x = 0 z = 0

$B_{0^+} = [\min(0,9), \max(0,9)] = [0,9]$

$I_{0.5}$ x[-1.5, 1.5]

x = -1.5 z = 2.25

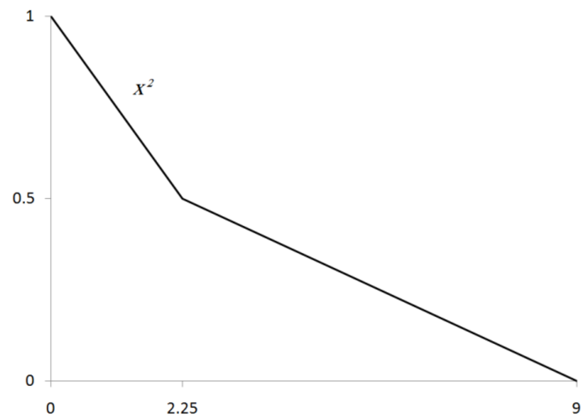
x = 1.5 z = 2.25

x = 0 z = 0

$B_{0.5} = [\min(0,2.25), \max(0,2.25)]$

$= [0,2.25]$

I_1 x = 0 z = 0 $B_1 = 0$



z = x.x

I_{0^+} x[-3, 3] x[-3, 3]

x = -3 x = -3 z = 9

x = -3 x = 3 z = -9

x = 3 x = -3 z = -9

x = 3 x = 3 z = 9

$B_{0^+} = [\min(-9,9), \max(-9,9)]$

$= [-9,9]$

$I_{0.5}$ x[-1.5, 1.5] x[-1.5, 1.5]

x = -1.5 x = -1.5 z = 2.25

x = -1.5 x = 1.5 z = -2.25

x = 1.5 x = -1.5 z = -2.25

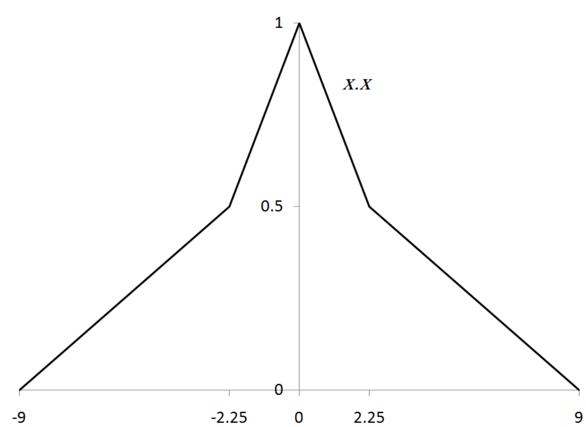
x = 1.5 x = 1.5 z = 2.25

$B_{0.5} =$

$[\min(-2.25,2.25), \max(-2.25,2.25)]$

$= [0,2.25]$

I_1 x = 0 z = 0 $B_1 = 0$



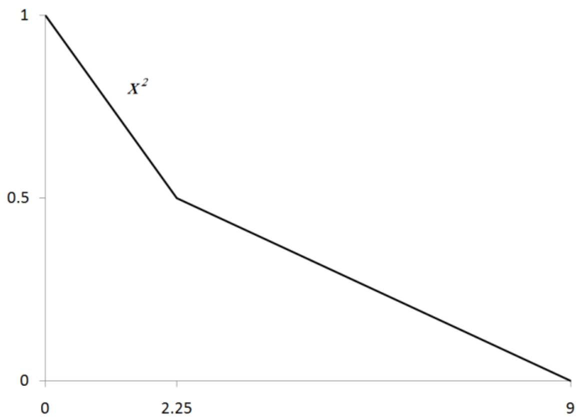
d) $z = x^2$

$\lambda=0^+$ x[-3, 3] $B_{0^+} = ([-3,3])^2 = [0,9]$

$\lambda=0.5$ x[-1.5, 1.5]

$B_{0.5} = ([-1.5,1.5])^2 = [0,2.25]$

$\lambda=1$ x=0 $B_1=0$



z = x.x

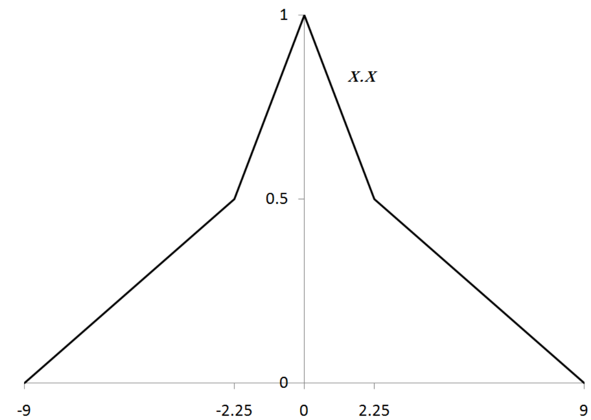
$\lambda=0^+$ x[-3, 3] x[-3, 3]

$B_{0^+} = [-3,3].[-3,3] = [-9,9]$

$\lambda=0.5$ x[-1.5, 1.5] x[-1.5, 1.5]

$B_{0.5} = [-1.5,1.5].[-1.5,1.5] = [-2.25,2.25]$

$\lambda=1$ x=0 $B_1=0$



e) Despite the problem 12.9 there is a difference between $z = x^2$ and $z = x\,.\,x$ because the extreme point (x = 0 , z = 0) is located in the internal. The outcomes from both vertex and DSW methods are still the same.

12.11

$$\underset{\sim}{t} = \{\frac{\min(1.0,1.0)}{30} + \frac{\max(\min(1.0,0.6),\min(0.8,1.0))}{40} + \frac{\max(\min(1.0,0.3)\min(0.8,0.6),\min(0.5,1.0))}{50} + \frac{\max(\min(0.8,0.3),\min(0.5,0.6))}{60} + \frac{\min(0.5,0.3)}{70}\}$$

$$\underset{\sim}{t} = \left\{\frac{1.0}{30} + \frac{0.8}{40} + \frac{0.6}{50} + \frac{0.5}{60} + \frac{0.3}{70}\right\}$$

CHAPTER 13

Fuzzy Control Systems

13.1

The membership functions can be expressed as:

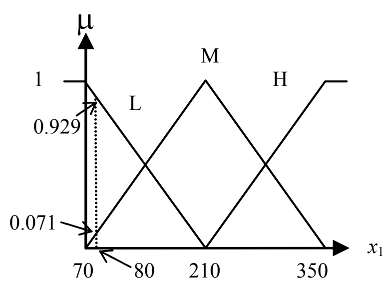
$$\mu_{L(x)} = \begin{cases} 1 & if \;\; x \le 70 \\ 1 - \dfrac{1}{140}(x-70), & if \;\; x \in (70,210) \\ 0, & otherwise \end{cases}$$

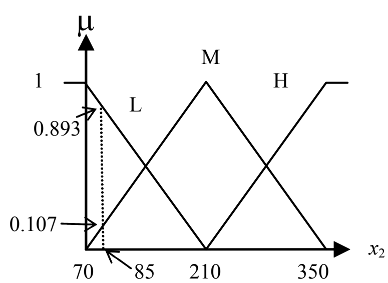
$$\mu_{m(x)} = \begin{cases} \dfrac{1}{140}(x-70), & if \;\; x \in (70,\, 210) \\ 1 - \dfrac{1}{140}(x-210), & if \;\; x \in (210,\, 350) \\ 0, & otherwise \end{cases}$$

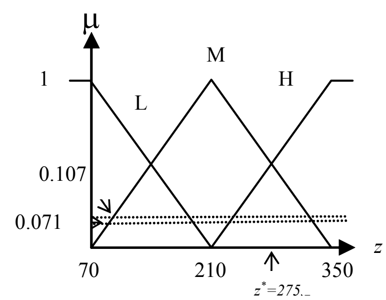
$$\mu_{H(x)} = \begin{cases} 1 & \text{if } x \ge 350 \\ \dfrac{1}{140}(x-210), & \text{if } x \in (210,\, 350) \\ 0, & \text{otherwise} \end{cases}$$

cycle 1: k =0

$x_1(0) = 80^o$, $x_2(0) = 85^o$







From FAM Table, the following rules fire:

R_1 : $\min(0.929, 0.893) = 0.893(\mathrm{H})$

R_2 : $\min(0.071, 0.893) = 0.071(\mathrm{H})$

R_3 : $\min(0.929, 0.107) = 0.107(\mathrm{M})$

$$z^* = \frac{\int_{70}^{85} \frac{1}{40}(x-70)x\mathrm{d}x + \int_{85}^{225} 0.107x\mathrm{d}x + \int_{225}^{335} \frac{1}{140}(x-210)x\mathrm{d}x + \int_{70}^{85} 0.893x\mathrm{d}x}{\int_{70}^{85} \frac{1}{140}(x-70)\mathrm{d}x + \int_{85}^{225} 0.107\mathrm{d}x + \int_{225}^{335} \frac{1}{140}(x-210)\mathrm{d}x + \int_{70}^{85} 0.893\mathrm{d}x}$$

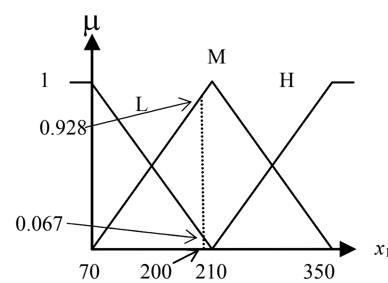
$z^* = 275.20 = \mathrm{U}(0)$

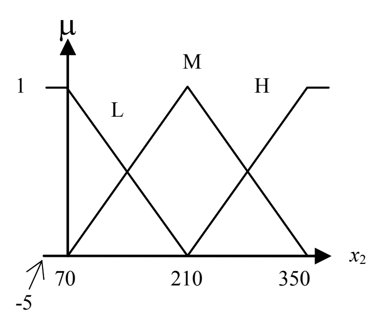
To begin the next cycle (k=1) we find $x_1(1)$ and $x_2(1)$:

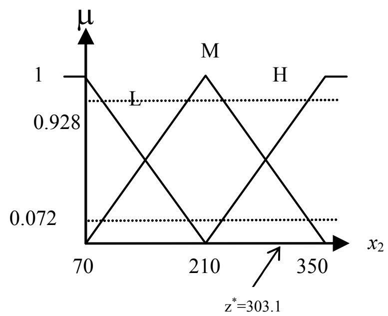
$x_1(1) = -2x_1(0) + x_2(0) + \mathrm{U}(0) = 200.2$

$x_2(1) = x_1(0) - x_2(0) = -5$

cycle 2







From FAM Table, the following rules fire:

$R_1: \quad \min(0.928, 1) = 0.928(H)$

$R_2: \quad \min(0.072, 1.0) = 0.072(H)$

$z^* = 303.10 = U(1)$

Performing a third cycle (k = 2) the initial points would be.

$x_1(2) = -2x_1(1) + x_2(1) + U(1) = -102.3$

$x_2(2) = x_1(1) - x_2(1) = 205.2$

From FAM Table, the following rules fire:

$R_1: \quad \min(1, 0.964) = 0.964(H)$

$R_2: \quad \min(1, 0.0357) = 0.0357(M)$

$z^* = 292.55 = U(2)$

Performing a fourth cycle (k = 3) the initial points would be.

$x_1(3) = -102.3$

$x_2(3) = -307.5$

From FAM Table:

$R_3 : \min(1, 1) = 1(H), \ z^* = 303.33 = U3$

$x_1(3) = -1409.1$

$x_2 4 = -1009.8$

13.2

Cycle 1:

$s(0) = 52.5 \rightarrow \text{High} \,\&\, \text{OK}$

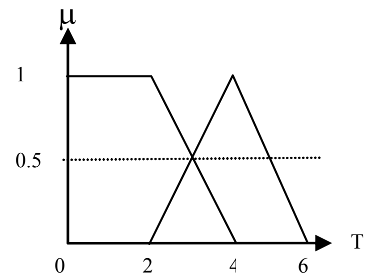
$\theta(0) = -5 \rightarrow \text{Down}$

$\text{IF } (s = H \wedge \theta = \text{Down}) \text{ THEN}$

$(T = \text{Low}) \Rightarrow \min(0.5, 1) = 0.5 \text{ Low}$

$\text{IF } (s = OK \wedge \theta = \text{Down}) \text{ THEN}$

$(T = LM) \Rightarrow \min(0.5, 1) = 0.5 \text{ LM}$



T^* =3, Therefore:

$s_1 = 0.9(52.5) + 3 - 0.1(-5) = 50.75$

Cycle 2

$s_1 = 50.75 \rightarrow \text{OK} \,\&\, \text{High}$

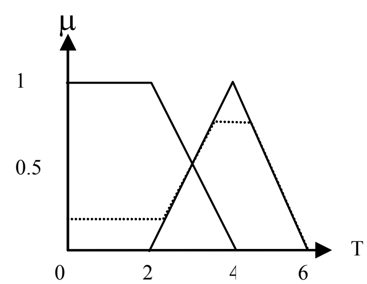
$\theta_1 = -5 \rightarrow \text{Down}$

$\text{IF } (s = H \wedge \theta = \text{Down}) \text{ THEN}$

$(T = \text{Low}) \Rightarrow \min(0.2, 1) = 0.2 \text{ Low}$

$\text{IF } (s = OK \wedge \theta = \text{Down}) \text{ THEN}$

$(T = LM) \Rightarrow \min(0.8, 1) = 0.8 \text{ LM}$



T^* =3.8, Therefore: $s_2 = 49.975$

Cycle 3

$s_2 = 49.975 \rightarrow \text{OK} \,\&\, \text{Low}$

$\theta_2 = -2.5 \rightarrow \text{Level} \,\&\, \text{Down}$

$\text{IF } (s = \text{Low} \wedge \theta = \text{Down}) \text{ THEN}$

$(T = HM) \Rightarrow \min(.005, 0.5) = 0.05 \text{ HM}$

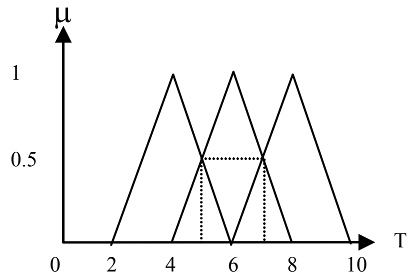
$\text{IF } (s = \text{Low} \wedge \theta = \text{Level}) \text{ THEN}$

$(T = HM) \Rightarrow \min(0.005, 0.5) = 0.005 \text{ HM}$

$\text{IF } (s = OK \wedge \theta = \text{Down}) \text{ THEN}$

$(T = LM) \Rightarrow \min(.995, 0.5) = 0.5 \text{ LM}$

$\text{IF}\left(s = \text{OK} \wedge \theta = \text{Level}\right)\text{THEN}$
$\left(T = M\right) \Rightarrow \min\left(0.995, 0.5\right) = 0.5\text{ M}$

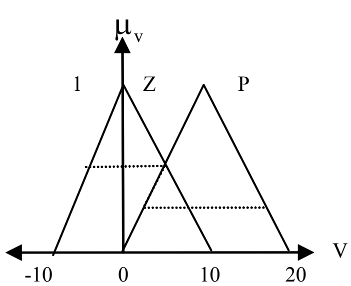


T^* =5, Therefore: $s_3 = 50.228$

Cycle 43
$s_3 = 50.228 \rightarrow \text{OK \& HIGH}$
$\theta_3 = -2.5 \rightarrow \text{Level \& Down}$
$\text{IF HIGH} \wedge \text{DOWN} \Rightarrow \min\left(0.46, 0.5\right)\text{Low}$
$\text{IF HIGH} \wedge \text{Level} \Rightarrow \min\left(0.46, 0.5\right)\text{LM}$
$\text{IF OK} \wedge \text{DOWN} \Rightarrow \min\left(0.954, 0.5\right)\text{ LM}$
$\text{IF OK} \wedge \text{Level} \Rightarrow \min\left(0.954, 0.5\right)\text{ M}$
T^* =5, Therefore: $s_4 = 50.455$

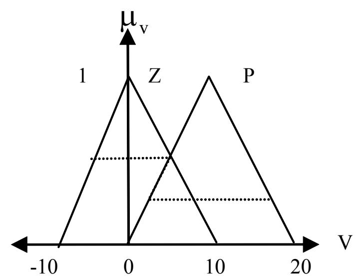
13.3

Cycle 1:
$\text{IF}\left(x_1 = Z \wedge x_2 = N\right)\text{THEN}$
$\left(v = P\right) \Rightarrow \min\left(.25, 0.5\right) = 0.25\text{ P}$
$\text{IF}\left(x_1 = P \wedge x_2 = N\right)\text{THEN}$
$\left(v = Z\right) \Rightarrow \min\left(0.75, 0.5\right) = 0.5\text{ Z}$
$\text{IF}\left(x_1 = Z \wedge x_2 = Z\right)\text{THEN}$
$\left(v = Z\right) \Rightarrow \min\left(0.25, 0.5\right) = 0.25\text{ Z}$
$\text{IF}\left(x_1 = P \wedge x_2 = Z\right)\text{THEN}$
$\left(v = Z\right) \Rightarrow \min\left(0.75, 0.5\right) = 0.5\text{ Z}$



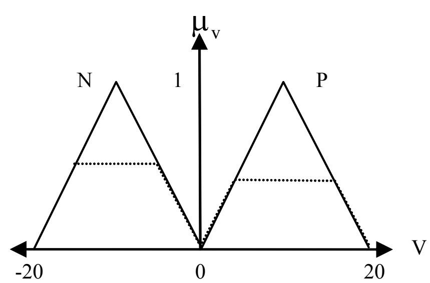
$v^* = 3.44,\ \because x_1\left(k+1\right) = x_2\left(k\right) + x_1\left(k\right)$
$\because x_2\left(k+1\right) = 17.5v + 0.36x_2\left(k\right)$
$\therefore x_1\left(1\right) = -16.4, \quad x_2 = 6.2$

Cycle 2:
$\text{IF}\left(x_1 = N \wedge x_2 = Z\right)\text{THEN}$
$\left(v = P\right) \Rightarrow \min\left(1, 0.98\right) = 0.98\text{ P}$
$\text{IF}\left(x_1 = N \wedge x_2 = P\right)\text{THEN}$
$\left(v = N\right) \Rightarrow \min\left(1, 0.02\right) = 0.02\text{ N}$



$v^* = 9.6,$
$\therefore x_1\left(1\right) = -15.4, \quad x_2 = 170.2$

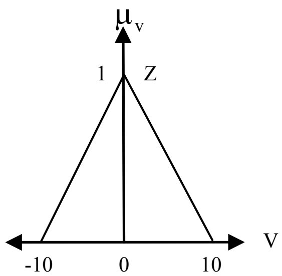
Cycle 3:
$\text{IF}\left(x_1 = N \wedge x_2 = Z\right)\text{THEN}$
$\left(v = P\right) \Rightarrow \min\left(1, 0.43\right) = 0.43\text{ P}$
$\text{IF}\left(x_1 = N \wedge x_2 = P\right)\text{THEN}$
$\left(v = N\right) \Rightarrow \min\left(1, 0.57\right) = 0.57\text{ N}$



v^* = -0.09, $\therefore x_1\left(1\right) = 11.7, \quad x_2 = 0.59$

Cycle 4:
$\text{IF}\left(x_1 = P \wedge x_2 = Z\right)\text{THEN}$
$\left(v = Z\right) \Rightarrow \min\left(1, 1\right) = 1\text{ Z}$

$$\text{IF } (x_1 = P \wedge x_2 = P) \text{ THEN}$$
$$(v = NB) \Rightarrow \min(1, 0) = 0 \text{ NB}$$



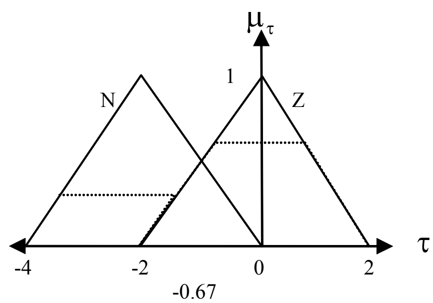
$v^* = 0.0,$

13.4

$$\theta(0) = 0.7^\circ, \quad \dot{\theta}(0) = -0.2\ {}^\circ\!/_{sec}$$

Cycle 1

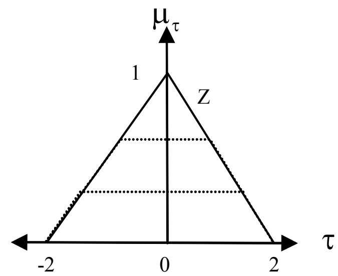
$$\text{IF } (\theta = P \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.8, 0.7) = 0.7 \text{ Z}$$
$$\text{IF } (\theta = P \wedge \dot{\theta} = Z) \text{ THEN}$$
$$(\tau = N) \Rightarrow \min(0.8, 0.3) = 0.3 \text{ Z}$$



$$\tau^* = -0.67 \text{ N} \cdot \text{m},$$
$$\therefore \theta(1) = 0.5, \quad \dot{\theta}(1) = -3.97$$

Cycle 2

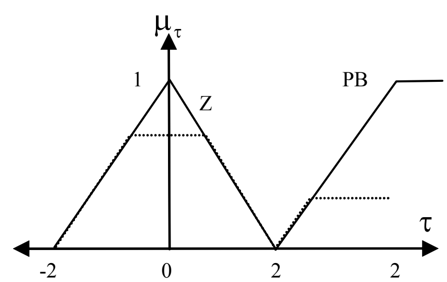
$$\text{IF } (\theta = P \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.62, 1) = 0.62 \text{ Z}$$
$$\text{IF } (\theta = Z \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.37, 1) = 0.37 \text{ Z}$$



$$\tau^* = 0.0 \text{ N} \cdot \text{m},$$
$$\therefore \theta(2) = -0.19, \quad \dot{\theta}(2) = -12.4$$

Cycle 3

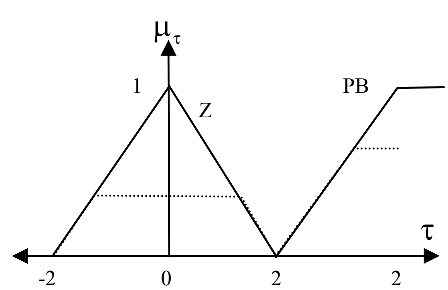
$$\text{IF } (\theta = Z \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.7, 1) = 0.7 \text{ Z}$$
$$\text{IF } (\theta = N \wedge \dot{\theta} = N) \text{ THEN}$$
$$(\tau = PB) \Rightarrow \min(0.25, 1) = 0.25 \text{ PB}$$



$$\tau^* = 1.2 \text{ N} \cdot \text{m},$$
$$\therefore \theta(3) = -0.5, \quad \dot{\theta}(3) = 2.46$$

Cycle 4

$$\text{IF } (\theta = N \wedge \dot{\theta} = P) \text{ THEN}$$
$$(\tau = PB) \Rightarrow \min(0.65, 1) = 0.65 \text{ PB}$$
$$\text{IF } (\theta = Z \wedge \dot{\theta} = P) \text{ THEN}$$
$$(\tau = Z) \Rightarrow \min(0.35, 1) = 0.35 \text{ Z}$$



$\tau^* = 1.4056 \text{ N} \cdot \text{m},$

$\therefore \theta(4) = -0.19, \quad \dot{\theta}(4) = 3.63$

13.5

Cycle 1:

$IF\ (I_1 = P \wedge I_2 = N)\ THEN$

$(v = Z) \Rightarrow \min(0.5, 0.5) = 0.25\ Z$

$IF\ (I_1 = P \wedge I_2 = Z)\ THEN$

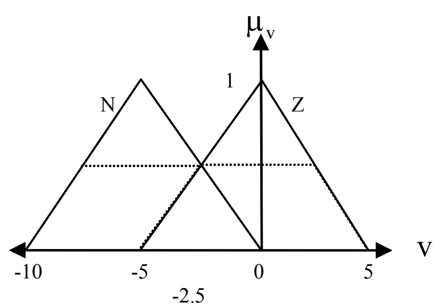
$(v = N) \Rightarrow \min(0.5, 0.5) = 0.5\ N$

$IF\ (I_1 = Z \wedge I_2 = Z)\ THEN$

$(v = Z) \Rightarrow \min(0.5, 0.5) = 0.5\ Z$

$IF\ (I_1 = Z \wedge I_2 = Z)\ THEN$

$(v = Z) \Rightarrow \min(0.5, 0.5) = 0.5\ Z$



$v^* = 2.5,$

$\therefore I_1(1) = -8, \quad I_2(1) = 1.5$

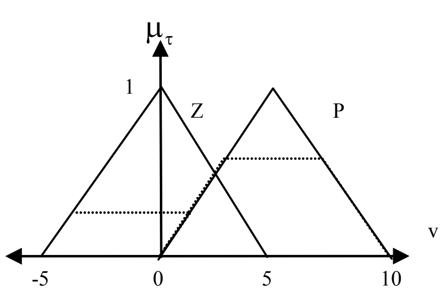
Cycle 2

$IF\ (I_1 = N \wedge I_2 = Z)\ THEN$

$(v = P) \Rightarrow \min(1.0, 0.75) = 0.75\ P$

$IF\ (I_1 = N \wedge I_2 = P)\ THEN$

$(v = Z) \Rightarrow \min(1, 0.25) = 0.25\ Z$



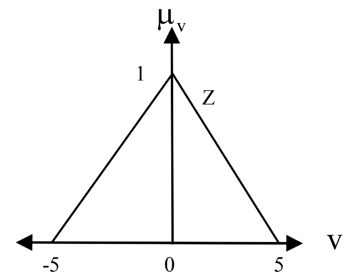
$v^* = 3.08,$

$\therefore I_1(2) = 2.0, \quad I_2(2) = -2.0$

Cycle 3

$IF\ (I_1 = P \wedge I_2 = N)\ THEN$

$(v = Z) \Rightarrow \min(1, 1) = 1.0\ Z$



$v^* = 0,$

$\therefore I_1(2) = -2.0, \quad I_2(2) = 2.0$

Cycle 4

$IF\ (I_1 = N \wedge I_2 = P)\ THEN$

$(v = Z) \Rightarrow \min(1, 1) = 1\ Z$

$v^* = 0,$

$\therefore I_1(2) = 2.0, \quad I_2(2) = 2.0$

13.6

We have a cylindrical tank with cross sectional area, A_c. Liquid flows in at a rate F_i and liquid flows out at a constant rate F_o. We want to control the tank liquid level h using a level controller to change the liquid level set height h_s. The available tank liquid height is H_T. The flow rate in the tank (F_i) is proportional to the percentage that the value is opened. We call this set Flow into the tank F_{is}.

$$F_i - F_o = A_c \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{F_i - F_o}{A_c}$$

The difference between the liquid level set point and the actual tank liquid level is "e"

$e = h - h_s$

and the percent difference is:

$$e = \frac{h - h_s}{h}$$

The percent difference is used to govern the flow into the system with the following rules:

If $e = 0\%$ then $\Delta F_i = 0$
If $e > 10\%$ then $\Delta F_i = 4\%$
If $e < -10\%$ then $\Delta F_i = -4\%$

The percent change in flow into the system (ΔF_i) is:

$$\Delta F_i \% = \frac{F_{is} - F_i}{F_{is}}$$

The initial values are:

$F_{is} = 0.3\ m^3/s$
$F_{is} = 0.3\ m^3/s$
$H_T = 2\ m$
$A_c = 3\ m^2$
$h_s = 1\ m$

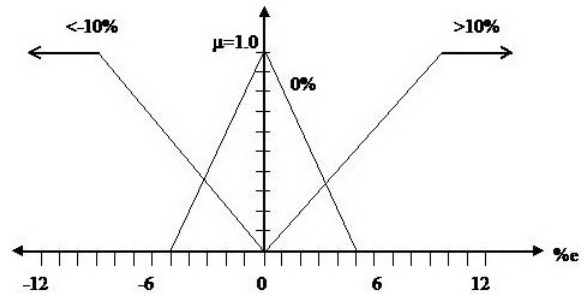
At t = 0 the disturbance in the inlet flow is:

$F_i = 0.4\ m^3/s$
$F_o = 0.3\ m^3/s$
$e = 0$

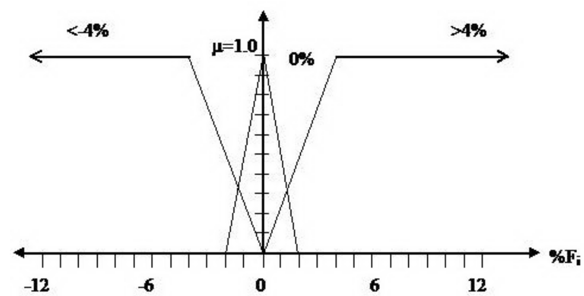
At t = 0.5s $\Delta h = \frac{0.4 - 0.3}{3} = 0.03\ m$, thus

h = 1.03 m and $\%e = \frac{1.03 - 1}{1} = 3\%$.

We now make use of the fuzzy controller. The input membership function is as follows:



While the output membership function is as follows:



The value for percent error is a member of the fuzzy set 0% and >10% and triggers the first and second rule.

$$\mu_{e_{0\%}}(3) = -\frac{1}{5}(3) + 1 = 0.4$$

$$\mu_{e_{10\%}}(3) = -\frac{1}{10}(3) + 1 = 0.3$$

Using the weighted average method we determine the percent change in flow.

$$\Delta F_i \% = \frac{(0.4)(0) + 0.3(4)}{0.7} = 1.7$$

Now let's move on to the system after another 0.5 seconds.

At t = 1 s

$$0.017 = \frac{0.3 - F_i}{0.3} = 0.29$$

$$\Delta h = \frac{0.29 - 0.3}{3} = -0.0033$$

$$h = 1.03 - 0.0033 = 1.027\ m$$

$$\%e = \frac{1.027 - 1}{1} = 2.7$$

Now we find that the percent error is a member of the fuzzy set 0 and >10%.

$$\mu_{e_{0\%}}(3) = -\frac{1}{5}(2.7) + 1 = 0.46$$
$$\mu_{e_{10\%}}(3) = -\frac{1}{10}(2.7) + 1 = 0.27$$

Using the weighted average method we determine the percent change in flow.

$$\Delta F_i\,\% = \frac{(0.46)(0) + 0.27(4)}{0.73} = 1.5$$

Now we move on to the system after another 0.5 seconds.

At t = 1.5 s

$$0.015 = \frac{0.3 - F_i}{0.3} = 0.296$$
$$\Delta h = \frac{0.296 - 0.3}{3} = -0.0013$$
$$h = 1.0257\text{ m}$$
$$\%e = \frac{1.0257 - 1}{1} = 2.6$$

13.7

The transport of toxic chemicals in water principally depends on two phenomena: advection and dispersion. In advection, the mathematical expression for time-variable diffusion is a partial differential equation accounting for concentration difference in space and time, which is derived from Ficks First Law.

$$J = -DA \cdot \frac{\Delta C}{\Delta x}$$
$$V \cdot \frac{\Delta C}{\Delta t} = -DA \cdot \frac{\Delta C}{\Delta x}, \text{ and } V = A \cdot \Delta\alpha$$

$$\frac{\Delta C}{\Delta t} = -D \cdot \frac{\Delta C}{\Delta x \Delta x} \quad \Rightarrow \quad \frac{\partial C}{\partial t} = -D\frac{\partial^2 C}{\partial x^2}$$

Where J = the mass flux rate due to molecular diffusion, mg/s
D = the molecular diffusion coefficient cm^2/s
A = the area of the cross section, cm2
$\frac{\Delta C}{\Delta t}$ = the concentration gradient $mg/cm^3/s$
Δx = movement distance, cm

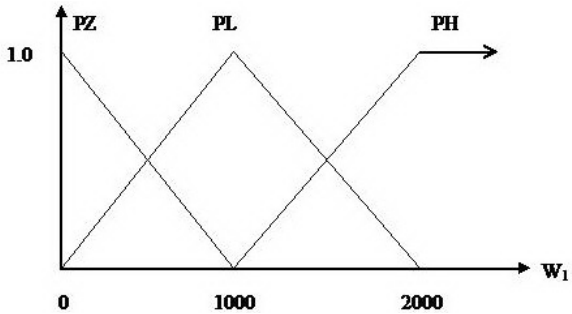
So if we want to control the $\frac{\Delta C}{\Delta t}$, we can set a control as follows:
W_1 = C (concentration, mg/cm^3)
$W_2 = \frac{\Delta C}{\Delta t}$ (concentration gradient, $mg/cm^3/s$)

So $\frac{dW_1}{dx} = W_2$ and
$\frac{dW_1}{dx} = -\alpha\left(if\ \ D = 1.0cm^2/s\right)$.

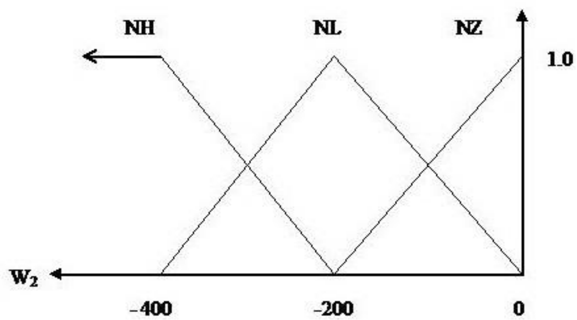
Therefore $W_1(k+1) = W_1(k) + W_2(k)$ and $W_2(k+1) = W_2(k) - \mu(k)$

For this problem, we assume
$0 \le W_1 \le 2000\,mg/cm^3$
$-400 \le W_2 \le 0\ \ mg/cm^3$
$0 \le \alpha \le 80\,mg/cm^3$
(W_2 is negative because flow direction is from high concentration to concentration)

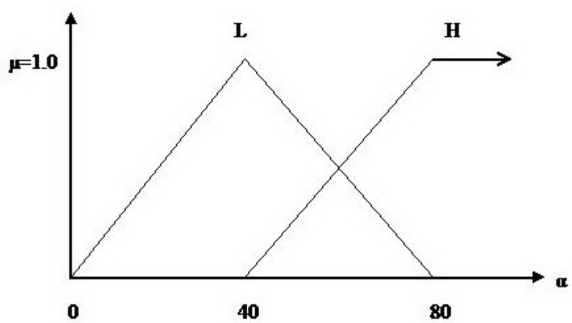
Step 1: Partition W_1 to Zero (PZ), Low (PL), High (PH).



Partition W2 to Zero (Z), Low (NL), High (NH)



Step 2: Partition α to low (L) and High (H).



Step 3: Construct Rules based on experience

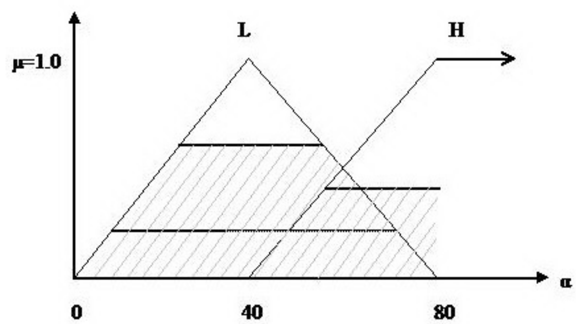
FAM Table

x_1 \ x_2	**NZ**	**NL**	**NH**
PZ	L	L	L
PL	L	L	H
PH	L	H	H

Step 4: Initial Conditions:

$W_1(0) = 800\ \text{mg/cm}^3$,
$W_2(0) = -280\ \text{mg/cm}^3 \cdot \text{cm}$

If $W_1 = PZ$, $W_2 = NL \rightarrow \alpha = L$
and $\wedge(0.2, 0.6) = 0.2(L)$
If $W_1 = PZ$, $W_2 = NH \rightarrow \alpha = L$
and $\wedge(0.2, 0.4) = 0.2(L)$
If $W_1 = PL$, $W_2 = NL \rightarrow \alpha = L$
and $\wedge(0.8, 0.6) = 0.6(L)$
If $W_1 = PL$, $W_2 = NL \rightarrow \alpha = H$
and $\wedge(0.8, 0.4) = 0.4(H)$



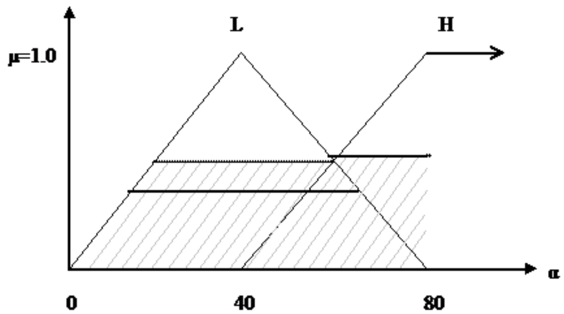
$\alpha^*(0) = 47\ \text{mg/cm}^3 \cdot \text{s}$
$W_1(1) = W_1(0) + W_2(0) =$
$800 - 280 = 520\ \text{mg/cm}^3$

$W_2(1) = W_2(0) + \alpha(0) =$
$-280 - 47 = 327\ \text{mg/cm}^3 \cdot \text{cm}$

Step 5: Second Cycle

If $W_1 = PZ$, $W_2 = NH \rightarrow \alpha = L$
and $\wedge(0.48, 0.64) = 0.48(L)$

If $W_1 = PZ$, $W_2 = NL \rightarrow \alpha = L$
and $\wedge(0.48, 0.36) = 0.36(L)$
If $W_1 = PL$, $W_2 = NH \rightarrow \alpha = H$
and $\wedge(0.52, 0.64) = 0.52(H)$
If $W_1 = PL$, $W_2 = NL \rightarrow \alpha = L$
and $\wedge(0.52, 0.36) = 0.36(L)$



$\alpha^*(1) = 48\ \text{mg/cm}^3 \cdot \text{s}$

$W_1(2) = W_1(1) + W_2(1) = 193\ \text{mg/cm}^3$

$W_2(1) = W_2(0) + \alpha(0) =$

$-375\ \text{mg/cm}^3 \cdot \text{cm}$

13.8

GIS (Global Information System) is a powerful tool in Environmental Modeling. It integrates geographical information with data stored in databases. The main issue in using GIS is selecting the appropriate spatial resolution. If the spatial resolution selected is low, then the mapping tool cannot fully represent the true topograph. On the other hand, if the spatial resolution selected is too high, then the data base size will be larger than necessary thus increasing storage requirement and processing speed.

Two parameters can be used in a fuzzy control system, to govern the GIS. The first one is the digital elevation value (DE). This value is the difference between the highest and lowest elevation in that certain area. The second parameter is the area of coverage (AC). The update equation is defined as follows:

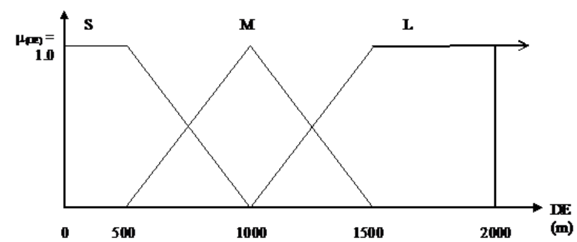
$$DE_{new} = \frac{SR^2}{AC} DE_{old} + DE_{old}$$

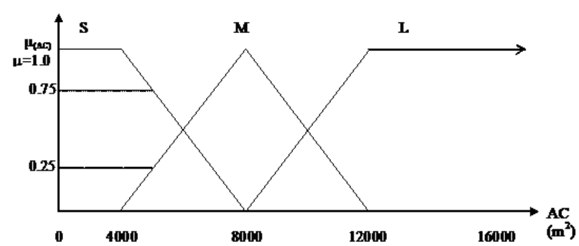
In the above equation SR represents spatial resolution (meters).

The first input is DE and can be either {small, medium, large} (in meters), the second input is AC and can be either {small, medium, large} (in meters^2) and SR is the output, which can either be {increase (I), decrease (D)}.

FAM Table

x_2 \ x_1	L	M	S
L	D	I	I
M	I	D	I
S	D	D	D





Initial condition for DE is DE(0) = 3000 m
Initial condition for AC is AC(0) = 5000 m^2

The specified rules used to govern the system are:

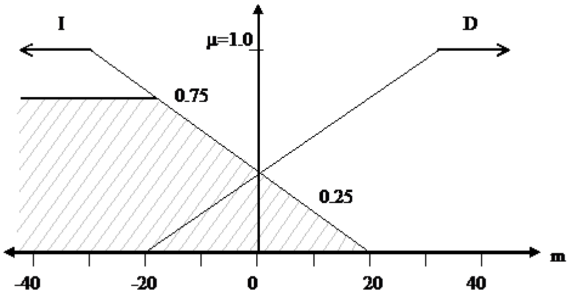
1) If DE is large and AC is small then increase SR.

2) If DE is large and AC is medium then increase SR.

In these rules the initial conditions have the following values:

For rule one, $\wedge(1, 0.75) = 0.75(\mathrm{I})$

For rule two, $\wedge(1, 0.25) = 0.25(\mathrm{I})$



Using the weighted average C = -14.06 meters.

$$DE(1) = \frac{(-14.06)^2}{5000} \cdot 2000 + 2000 = 2079m$$

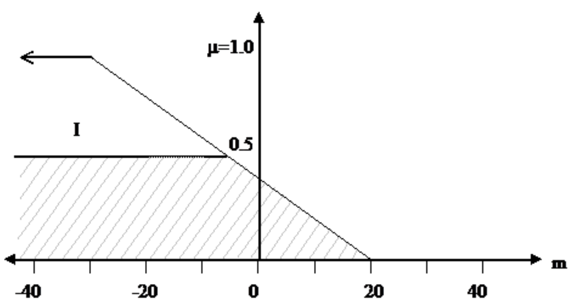
With a measured value of AC(1) = 6000m

Now back to the rule base

1) If DE is large and AC is small then increase SR.
2) If DE is large and AC is medium then increase SR.

In these rules the initial conditions have the following values:

For rule one, $\wedge(1, 0.5) = 0.5(\mathrm{I})$
For rule two, $\wedge(1, 0.5) = 0.5(\mathrm{I})$



The weighted average is C = -17.1 m

13.9

Input membership functions – with input from worker D on day 20.

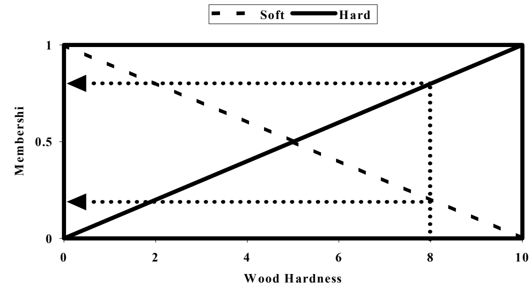


Figure 1. Membership functions for Wood Hardness- example- Wood Hardness = 8, membership in Hard = 0.8 and membership in Soft = 0.2

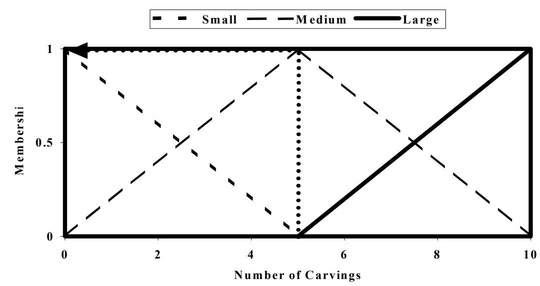


Figure 2. Membership functions for Number of Carvings- example- Number of Carvings = 5, membership in Small =0.0, membership in Medium = 1.0, and membership in Large = 0.0.

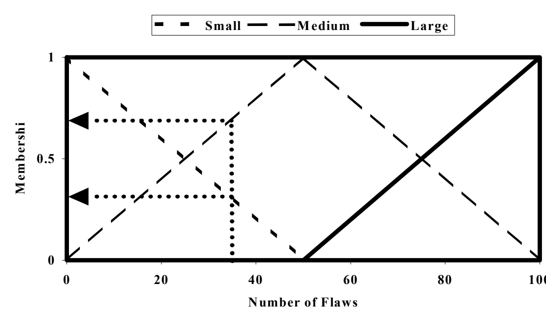


Figure 3. Membership functions for Number of Flaws- example- Number of Flaws = 35, membership in Small =0.3, membership in Medium = 0.7, and membership in Large = 0.0.

Table I. Woodcutters rules fired – with strengths and resolution

Rule Number	Wood Hardness	Number of Carvings	Number of Flaws	Type of Day
4	Soft (0.2)	Medium (1.0)	Small (0.3)	OK (0.2)
5	Soft (0.2)	Medium (1.0)	Medium (0.7)	OK (0.2)
13	Hard (0.8)	Medium (1.0)	Small (0.3)	Fair (0.3)
14	Hard (0.8)	Medium (1.0)	Medium (0.7)	OK (0.7)

On day 20 Worker D made 5 carvings from wood with an average hardness (for all 5 carvings) of 8, containing a total of 35 flaws. The membership values for the rules to be fired are obtained from figures 1-3, as shown above. These values are applied to the rules and four rules are fired as shown in Table I. The Max-Min rule is applied to the rules to find the strength at which each rule is fired (Min-value) and resolve the final output membership, as shown in Table I. The final values for the output membership functions are 0.3 for Fair and 0.7 for OK (Max-value). The output membership functions are "clipped" at these values as shown in figure 4. We used the centroid technique here to defuzzify the "clipped" output membership function. The centroid of this figure is 0.42. This is the type of day for worker D on Day 20. This number is added to the Type of Day for each of the other workers (A= 0.45, B= 0.45, C = 0.43). The average or X bar for Day 20 is then 0.4585and the Range, R, is 0.03. These numbers are added to Table II in the homework problem. The Grand Average (all 20 X bars) is $\overline{\overline{X}} = 0.4585$ and the average Range is $\overline{R} = 0.0615$. If we use the parameters A_2 and D_4 from our abbreviated parameter Table, Table II, we find A_2 = 0.729 and D_4 = 2.282 for n = 4 (The number of workers).

Table II. Abbreviated Parameter Table

N	A_2	D_3	D_4
2	1.880	-	3.268
3	1.023	-	2.574
4	0.729	-	2.282

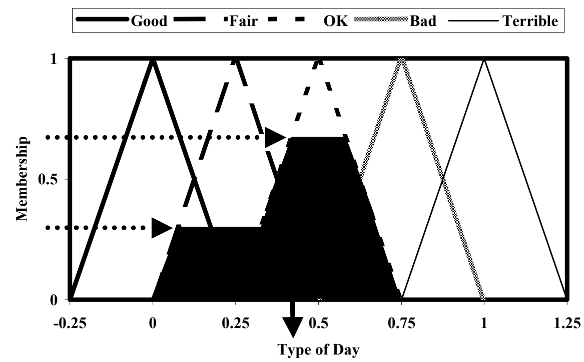


Figure 4. Output membership functions for Type of Day with a centroid at approximately 0.42 for Worker D on Day 20.

We can compute our upper control limit for X bar from equation (1), as = 0.5033 and the lower control limit from equation (2) as = 0.4137. The upper control limit for the range is found from equation (3) to be = 0.140. Recall that there is no lower control limit for the Range for n less than or equal to 6.

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} \quad \textbf{(1)}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} \quad \textbf{(2)}$$

$$UCL_R = D_4\overline{R} \quad \textbf{(3)}$$

The X bar chart for this operation is shown in figure 5. The R chart is shown in figure 6. The X bar chart indicates that the wood carving process was out of control on days 5, 6, and 7. These were most likely the rainy days. Note that there are several points that fall below the lower control limit. This is because there are only 20 data sets and the three out of control sets raised the Grand Average significantly. If data were collected continuously over a long period of time, this would not happen.

Also the out of control situation would be detected immediately. The R chart indicates that there is no significant difference between the workers on a daily basis. The R chart would show, for example if a worker came to work with a hangover and produced more poorly than normal on that day.

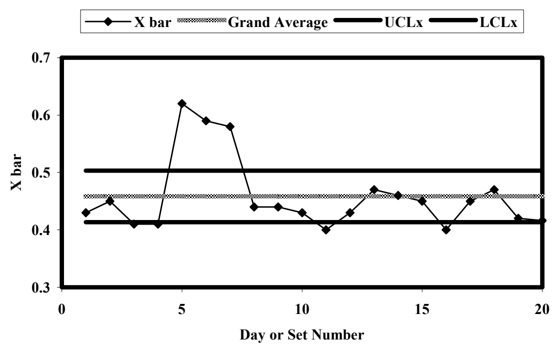


Figure 5. X bar chart for the wood carver problem.

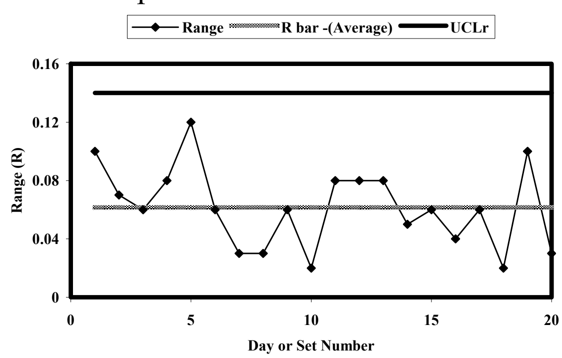


Figure 6. R chart for the wood carver problem.

13.10

Input membership functions – with input for part a.

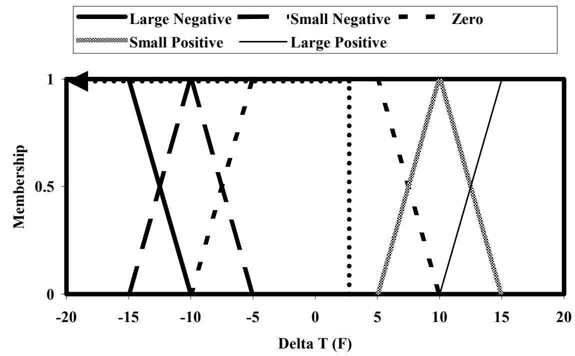


Figure 7. Membership functions for Delta T- For part a) the membership in Zero is 1.0, the membership in all other functions is 0.0.

For part a) with T = 113 F and T_s = 110 F, ΔT = +3. Figure 7. shows that ΔT = +3 means that ΔT has a

membership of 1.0 in the function Zero and a membership of 0.0 in all other functions. Only rule 3 is fired.

If ΔT = Zero (1.0) Then f = Zero (1.0).

The output for part a) is shown in the output membership functions, in figure 8. The centroid is zero and therefore in this case the new valve position is the same as the old one, 0.6. This is an example of using a "dead band" in a control problem.

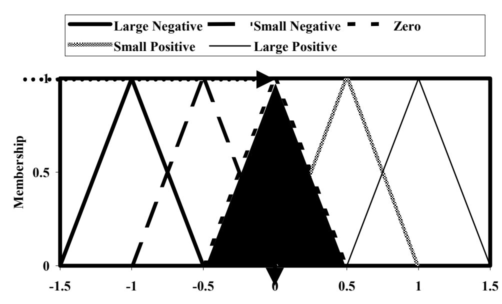


Figure 8. Output membership functions for Valve Fractional Change, f, with a centroid at 0.0 for part a).

Part b) T = 122 F, ΔT = +12. This produces a membership of 0.6 in the set Small

Positive and a membership of 0.4 in the set Large Positive for ΔT, as shown in figure 9.

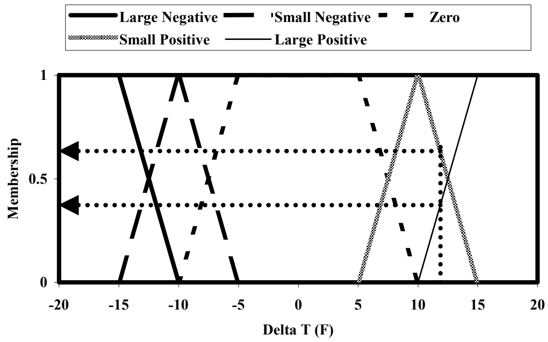


Figure 9. Membership functions for Delta T- For part b) the membership in Small Positive is 0.6, the membership in Large Positive is 0.4, and the membership in all other functions is 0.0.

This causes rules 1 and 2 to be fired with their relative strengths as shown below.

1- If ΔT Large Positive (0.4) Then f is Large Positive (0.4)

2- If ΔT Small Positive (0.6) Then f is Small Positive (0.6)

The output results are shown in figure 10. The centroid of the "clipped" figure is 0.71, the Valve Fractional change, f. Since f is greater than 0.0, the Range is defined as 1.0 – the current valve position (0.6). The Range is 0.4.

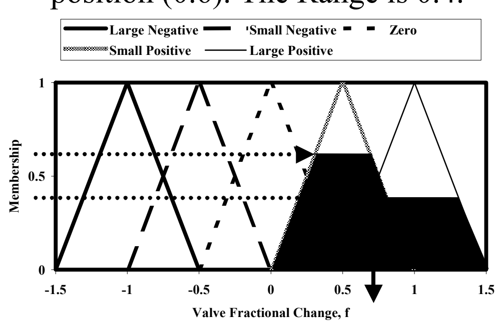


Figure 10. Output membership functions for Valve Fractional Change, f, with a centroid at 0.71 for part b).

The new valve position is equal to the old valve position plus f times the Range (0.6 + 0.71*0.4).
For part b) the new valve position is adjusted to approximately 0.8.

Part c) T = 98 F, ΔT = -12. This produces a membership of 0.6 in the set Small Negative and a membership of 0.4 in the set Large Negative for ΔT, as shown in figure 11.

This causes rules 4 and 5 to be fired with their relative strengths as shown below.

4- If ΔT Small Negative (0.6) Then f is Small Negative (0.6)

5- If ΔT Large Negative (0.4) Then f is Large Negative (0.4)

The output results are shown in figure 12. The centroid of the "clipped" figure is -0.71, the Valve Fractional change, f. Since f is less than 0.0, the Range is defined as the current valve position (0.6). The Range is 0.6. (if we are working from the original valve position)

The new valve position is equal to the old valve position plus f times the Range (0.6 - 0.71*0.6).
For part c) the new valve position is adjusted to approximately 0.174.

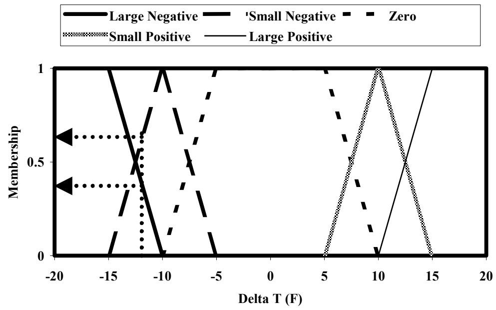


Figure 11. Membership functions for Delta T- For part c) the membership in Small Negative is 0.6, the membership in Large Negative is 0.4, and the membership in all other functions is 0.0.

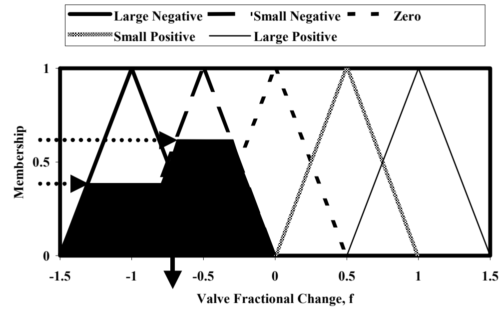


Figure 12. Output membership functions for Valve Fractional Change, f, with a centroid at -0.71 for part c).

CHAPTER 14

Miscellaneous Topics

14.1

The error function for the closed loop system is:

$$\frac{E(s)}{R(s)} = \frac{1}{1+H(s)} = \frac{1}{1+\frac{1}{s+1}} = \frac{s+1}{s+2}$$

Where; E(s) is the error function
R(s) is the input function
H(s) is the system forward transfer function

Now, the input signal is a step function.
So, R(s)= 1/s

$$E(s) = \left(\frac{s+1}{s+2}\right)\frac{1}{s} = \frac{0.5}{s+2} + \frac{0.5}{s}$$

Using Laplace inverse transform, we get

$$e(t) = 0.5e^{-2t} + 0.5$$

Therefore,

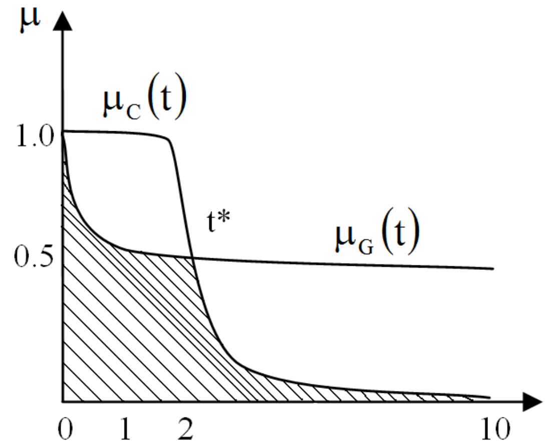
$$\mu_{\underset{\sim}{G}}(t) = \frac{e(t)-0}{1-0} = e(t) = 0.5e^{-2t} + 0.5$$

$$\mu_{\underset{\sim}{D}}(t) = \mu_{\underset{\sim}{C}}(t) \cap \mu_{\underset{\sim}{G}}(t)$$

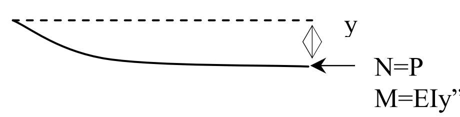
$$\begin{cases} 0.5e^{-2t} + 0.5, & 0 \le t \le t^* \\ e^{1-t}, & t > t^* \end{cases}$$

$$0.5e^{-2t} + 0.5 = e^{1-t}$$

The optimal solution for the minimum error is shown in the figure below.



14.2



$$-Py = M = EIy''$$

$$y'' + \frac{P}{EI}y = 0$$

(1)

So, $y = C_1 \sin(kx) + C_2 \cos(kx)$ (2)

Consider the boundary conditions for C_1 and C_2:

$x = 0,\quad y = 0,\quad$ so $C_2 = 0$

$x = l,\quad y = 0,\quad$ so $C_1 \sin(kl) = 0$

Because equation (2) exists, C_1 will not be zero, so $kl = n\pi$, $\quad n = 1, 2, \ldots, m$

That is: $k^2 = \frac{P}{EI} = \frac{(\pi n)^2}{l^2}$

$$P = \frac{n^2\pi^2 EI}{l^2}$$

Find $\mu_{\underset{\sim}{G}}(n)$:

$0 \le n \le 2$, $\quad P(s) = \dfrac{4\pi^2 EI}{l^2}$

$P(0)=0$

$$\mu_{\underset{\sim}{G}} = \frac{P(n)-P(0)}{P(2)-P(0)} = \frac{n^2}{4}$$

Find the optimal solution n^*

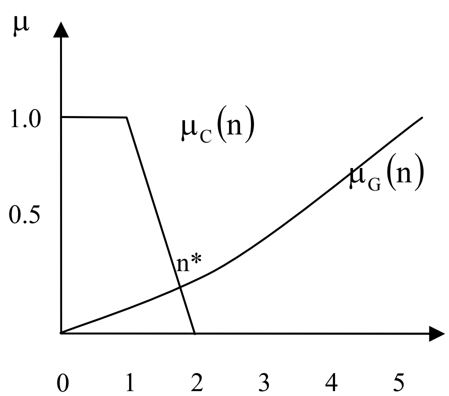
$$\mu_{\underset{\sim}{C}}(n) = \begin{cases} 1-n, & 1 \le n \le 2 \\ 0, & n > 1 \end{cases}$$

$$\mu_{\underset{\sim}{D}}(n) = \begin{cases} \dfrac{n^2}{4}, & 0 \le n \le n^* \\ 1-n, & n > n^* \end{cases}$$

$$\frac{n^2}{4} = 1-n$$

$$n^* = 1.86$$

Therefore: $P(n^*) = \dfrac{(0.8284)^2 \pi^2 EI}{l^2}$



14.3

Find $\mu_{\underset{\sim}{G}}(\sigma_b)$

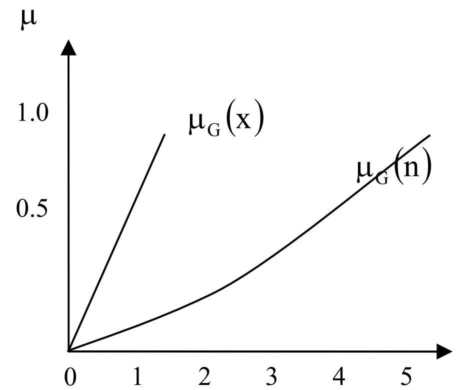
P(0) = 0, $\quad$ P(60M)=$\dfrac{4Wz60M}{l}$

$$\mu_{\underset{\sim}{G}} = \frac{\sigma_b}{60M}$$

$\because \mu_{\underset{\sim}{G}}$ is unit less, $\therefore$ let $\dfrac{\sigma_b}{60M} = x$

$$\mu_{\underset{\sim}{G}}(x) = x$$

Find μ_G for $\bigcap_{j=1,2} \underset{\sim}{\sigma}_j$

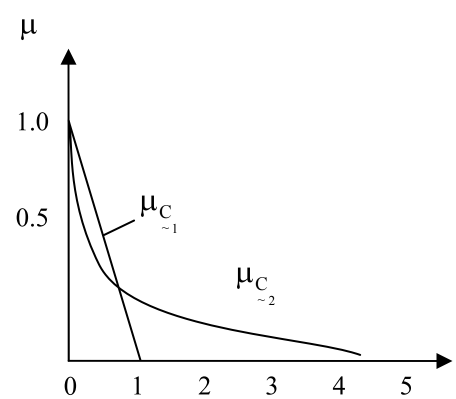


Therfore, $\mu_{\underset{\sim}{G}_{\min}} = \mu_{\underset{\sim}{G}}(n) = \dfrac{n^2}{25}$

Find, $\mu_{C\min}$:

$$\mu_{C_1}(n) = \begin{cases} 1-n, & 0 \le n \le 1 \\ 0, & n > 1 \end{cases}$$

$$\mu_{C_1}(n) = \begin{cases} \dfrac{1-n}{(1+x)^2}, & 0 \le n \le 1 \\ x, & x > 1 \end{cases}$$



$$\mu_{\underset{\sim}{C}} = \bigcap_{j=1,2} \mu_{C_j}$$

$$\frac{1}{(1+x)^2} = 1-x \Rightarrow x = 0.62$$

$$\mu_{\tilde{c}}(x) = \begin{cases} \frac{1}{(1+x)^2}, 0 \le x \le 0.62 \\ 1-x, 0.62 \le x \le 1 \end{cases}$$

Find optimal x^* for *P*:

$$\mu_{\tilde{D}} = \mu_{\tilde{C}} \cap \mu_{\tilde{G}} = \begin{cases} \frac{x^2}{25}, & 0 \le x \le x^* \\ 1 - x, & x > x^* \end{cases}$$

$$\therefore x^* = 0.96$$

$$\sigma_b = 0.96 * 60\text{MPa} = 57.6\text{MPa}$$

The minimum *P* is:

$$P_{\min} = \min\left(\frac{(0.96)^2 \pi^2 \text{EI}}{l^2}, \frac{4Wz(57.6\text{M})}{l^2} \right)$$

14.4

Find out $\mu_{\tilde{G}}(R)$:

$$\sigma_H(R) = 0.564\sqrt{\frac{PE}{LR}}, \; 10 \le R \le 30$$

So,

$$\sigma_H(10) = 0.564\sqrt{\frac{PE}{L}}\sqrt{\frac{1}{10}}$$

$$\sigma_H(30) = 0.564\sqrt{\frac{PE}{L}}\sqrt{\frac{1}{30}}$$

$$\mu_{\tilde{G}}(R) = \frac{\sigma_H(R) - \sigma_H(30)}{\sigma_H(10) - \sigma_H(30)} = \frac{\sqrt{\frac{1}{R}} - 0.183}{0.134}$$

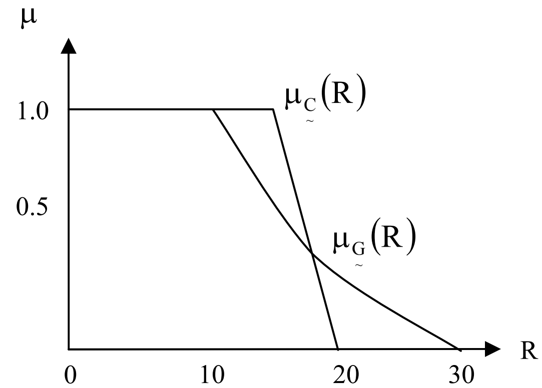
Determine optimal R^*:

$$\mu_{\tilde{C}}(R) = \begin{cases} 1, & 10 \le R \le 15 \\ 4 - \frac{R}{5}, & 15 < R \le 20 \end{cases}$$

$$\mu_{\tilde{D}}(R) = \begin{cases} \left(\sqrt{\frac{1}{R}} - 0.183 \right) 0.75, & 10 \le R \le R^* \\ 4 - \frac{R}{5}, & R > R^* \end{cases}$$

$$7.5\left(\sqrt{\frac{1}{R}} - 0.183 \right) = 4 - \frac{R}{5}$$

$$R^* \approx 18\text{cm}$$



14.5

$[0, 1, 1, 1, -1]$ stabilized in 3 iterations

14.6

$I_1(C_1, C_5) = \min(\text{much}, \text{a lot}) = \text{much};$

$I_2(C_1, C_5) = \min(\text{much}, \text{a lot}, \text{much}, \text{some})$

$I_2(C_1, C_5) = \text{some}$

$T(C_1, C_5) = \max(I_1, I_2) = \max(\text{much}, \text{some})$

$= \text{much}$

Therefore, an increase in "required natural gas" for this facility results in "much economic gain."

14.7 $[0, 1, 1, 1, -1]$ stabilized in 3 iterations

$I_1 = \min(\text{a lot}, \text{a lot}) = \text{a lot};$

$I_2 = \min(\text{a lot}, \text{some}, \text{much}, \text{some})$

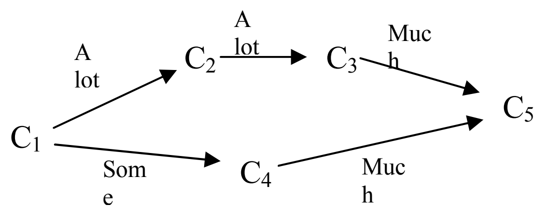
$I_2 = \text{some}$

$T = \max(I_1, I_2) = \max(\text{a lot}, \text{some}) = \text{a lot}$

Therefore, an increase in "CO_2 emissions" for this facility results in "a lot of economic gain"

14.8

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

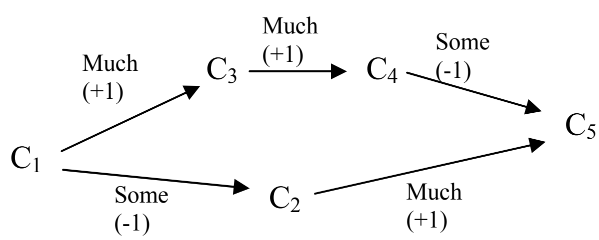


The resulting state vector stabilized after 3 iterations:

[0 0 0 0 -1]

14.9

$$\begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

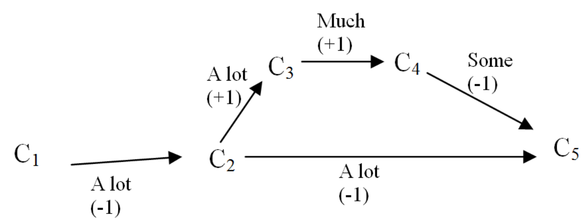


The resulting state vector stabilized after 4 iterations:

[0 0 0 0 0]

14.10

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

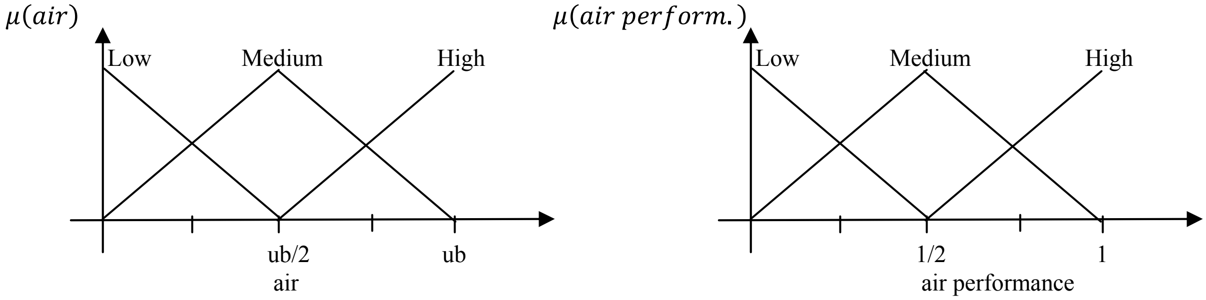


The resulting state vector stabilized after 5 iterations:

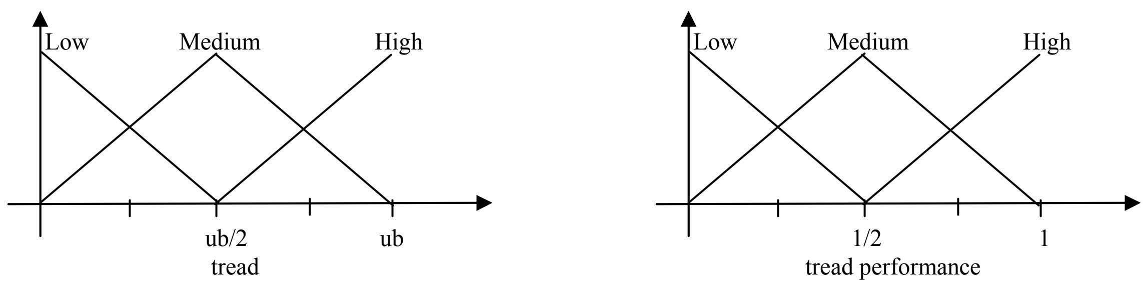
[0 0 0 0 0]

14.11

Air FIS



Tread FIS



$$wf * air\ performance + (1 - wf) * tread\ performance = total\ performance$$

If the amount of air in the tires is medium and the amount of tread in the tires is high:

$$air\ performance = \ 0.5$$

$$tread\ performance = \ 1$$

$$wf = 0.5$$

$$total\ performance = 0.5 * 0.5 + 0.5 * 1 = 0.75$$

14.12

Footings of building foundations classification		Bearing Capacity		
		W	M	G
Loads On Columns	L	O	S	S
	M	O	S	S
	H	--	O	O

Loads on columns: Low (L), Medium (M), High (H)
Bearing Capacity: Weak (W), Moderate (M), Good (G)

Footing Design: Standard (S), Overdesign (O)

Typical Rule:
IF Load is L and Bearing Capacity is W, THEN Footing Design is O

IF Load is L and Bearing Capacity is M, THEN Footing Design is S
IF Load is L and Bearing Capacity is G, THEN Footing Design is S

IF Load is M and Bearing Capacity is W, THEN Footing Design is O
IF Load is M and Bearing Capacity is M, THEN Footing Design is S
IF Load is M and Bearing Capacity is G, THEN Footing Design is S

IF Load is H and Bearing Capacity is W, THEN cannot build
IF Load is H and Bearing Capacity is M, THEN Footing Design is O
IF Load is H and Bearing Capacity is G, THEN Footing Design is O

CHAPTER 15

Monotone Measures: Belief, Plausibility, Probability and Possibility

15.1

Focal Elements	m_1	m_2	bel_1	bel_2	pl_1	pl_2
F	.3	.3	.3	.2	.5	.4
NF	.5	.6	.5	.6	.7	.7
$F \cup NF$	.2	.1	1.0	1.0	1.0	1.0

$$bel_1(F) = m_1(F) = 0.3$$
$$bel_1(NF) = m_1(NF) = 0.5$$
$$bel_1(F \cup NF) = m_1(F) + m_1(NF) + m_1(F \cup NF)$$
$$= 0.3 + 0.5 + 0.2 = 1$$
$$pl_1(F) = m_1(F) + m_1(F \cup NF) = 0.3 + 0.2 = 0.5$$
$$pl(NF) = m_1(NF) + m_1(F \cup NF) = 0.5 + 0.2 = 0.7$$
$$pl_1(F \cup NF) = m_1(F) + m_1(NF) + m_1(F \cup NF)$$
$$= 0.3 + 0.5 + 0.2 = 1.0$$

Similarly, other values can be determined.

15.2

FE	m_1	bel_1	pl_1	m_2
R	0.05	0.05	0.8	0.15
D	0.1	0.1	0.9	0.1
W	0	0	0.65	0
$R \cup D$	0.2	0.35	1.0	0.25
$R \cup W$	0.05	0.1	0.9	0.05
$D \cup W$	0.1	0.2	0.95	0.05
$R \cup D \cup W$	0.5	1.0	1.0	0.4

FE	pl_2	m_{12}	bel_{12}	pl_{12}
R	0.85	0.1969	0.1969	0.6795
D	0.80	0.2047	0.2047	0.7559
W	0.5	0.0079	0.0079	0.2963
$R \cup D$	1.0	0.2677	0.6693	0.9577
$R \cup W$	0.9	0.0049	0.2097	0.7609
$D \cup W$	0.85	0.0735	0.2861	0.7687
$R \cup D \cup W$	0.5	1.0	1.0	0.4

$$bel_1(R \cup W) = m_1(R) + m_1(W) + m_2(R \cup W)$$
$$= 0.05 + 0 + 0.05 = 0.1$$
$$pl_1(R \cup W) = m_1(R) + m_1(W) + m_1(R \cup W)$$
$$+ m_1(R \cup D) + m_1(D \cup W) + m_1(R \cup D \cup W) = 0.9$$

Other values of $bel_1(), bel_2(), pl_1(), pl_2()$ can be determined as above.

$$K = m_1(R)m_2(D) + m_1(R)m_2(W) + m_1(R)m_2(D \cup W)$$
$$+ m_1(D)m_2(R) + m_1(D)m_2(W) + m_1(D)m2(R \cup W)$$
$$+ m_1(W)m_2(R) + m_1(W)m_2(D) + m_1(W)m_2(R \cup D)$$
$$+ m_1(R \cup D)m_2(W) + m_1(R \cup W)m_2(D) + m_1(D \cup W)m_2(R)$$
$$= 0.005 + 0.0025 + 0 + 0.015 + 0 + 0.005$$
$$+ 0 + 0 + 0 + 0 + 0.005 + 0.15 = 0.0475$$
$$1 - K = 0.9525$$

$$m_{12}(D) = [m_1(D)m_2(R \cup D) + m_1(D)m_2(D \cup W)$$
$$+ m_1(D)m_2(R \cup D \cup W) + m_1(D)m_2(D)$$
$$+ m_1(R \cup D)m_2(D) + m_1(D \cup W)m_2(D)$$
$$+ m_1(R \cup D \cup W)m_2(D) + m_1(R \cup D)m_2(D \cup W)$$
$$+ m_1(D \cup W)m_2(R \cup D)]/(1 - k) = 0.2047$$

Similarly, other values of $m_{12}()$ can be determined

$$bel_{12}(R) = m_{12}(R) = 0.1969$$
$$bel_{12}(R \cup W) = m_{12}(R) + m_{12}(W) + m_{12}(R \cup W) = 0.2097$$

Other combined belief values can be computed accordingly.

$$pl_{12}(R) = m_{12}(R) + m_{12}(R \cup D) + m_{12}(R \cup W)$$
$$+ m_{12}(R \cup D \cup W) = 0.1969 + 0.2671 + 0.0049 + 0.21$$
$$= 0.6795$$

Similarly, other combined plausibility values can be determined.

15.3

This problem is similar to 15.2 and can be solved similarly.

FE	m_1	bel_1	pl_1	m_2
C	0.03	0.3	0.85	0.2
I	0.05	0.05	0.6	0.1
L	0.05	0.05	0.45	0.05
C⋃I	0.2	0.55	0.95	0.15
$C \cup L$	0.05	0.4	0.95	0.05
$I \cup L$	0.05	0.5	0.7	0.15
$C \cup I \cup L$	0.3	1	1	0.3

FE	bel_2	pl_2	m_{12}	bel_{12}
C	0.2	0.7	0.4	0.4
I	0.1	0.7	0.15	0.15
L	0.05	0.55	0.04	0.04
C⋃I	0.45	0.95	0.16	0.71
$C \cup L$	0.3	0.9	0	0.44
$I \cup L$	0.3	0.8	0.07	0.26
$C \cup I \cup L$	1	1	0.11	0.93

FE	pl_{12}
C	0.67
I	0.49
L	0.22
C⋃I	0.89
$C \cup L$	0.78
$I \cup L$	0.53
$C \cup I \cup L$	0.93

$$\mathrm{K} = 0.03 + 0.015 + 0.045 + 0.01 + 0.0025 + 0.0025 + 0.01 + 0.05 + 0.0075 + 0.01 + 0.005 + 0.01 = 0.1975$$

$$1 - \mathrm{K} = 0.8025$$

15.4

F.E.	m_1	bel_1
A_1	0.1	0.1
A_2	0.05	0.05
A_3	0.05	0.05
$A_1 \cup A_2$	0.05	0.2
$A_1 \cup A_3$	0.05	0.2
$A_2 \cup A_3$	**0.1**	0.2
$A_1 \cup A_2 \cup A_3$	0.6	1

F.E.	m_2	bel_2
A_1	0	0
A_2	0.05	0.05
A_3	0.1	0.1
$A_1 \cup A_2$	0.05	0.1
$A_1 \cup A_3$	0.15	0.25
$A_2 \cup A_3$	0.05	0.2
$A_1 \cup A_2 \cup A_3$	0.6	1

F.E.	m_{12}	bel_{12}
A_1	0.09	0.09
A_2	0.09	0.09
A_3	0.14	0.14
$A_1 \cup A_2$	0.07	0.25
$A_1 \cup A_3$	0.13	0.36
$A_2 \cup A_3$	0.1	0.33
$A_1 \cup A_2 \cup A_3$	0.38	1

$$\mathrm{K} = 0.005 + 0.01 + 0.005 + 0 + 0.005 + 0.0075 + 0 + 0.025 + 0.005 + 0.0025 + 0 = 0.045$$

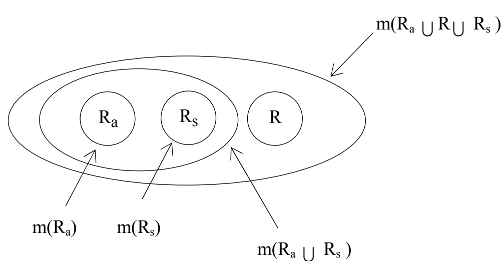
$$1 - \mathrm{K} = 0.955$$

$$m_{12}(A_2) = [0.0025 + 0.0025 + 0.0025 + 0.03 + 0.0025 + 0.0025 + 0.005 + 0.005 + 0.03]/(1 - K) = 0.09$$

15.5

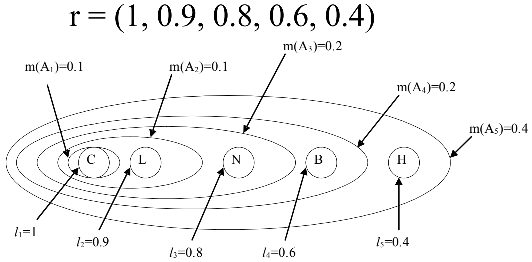
F.E.	m_1	m_2
R_a	0.1	0.2
R	0.1	0
R_s	0	0.4
$R_a \cup R_s$	0.3	0.3
$R \cup R_a$	0.1	0
$R_s \cup R$	0.3	0
$R_a \cup R \cup R_s$	0.1	0.1

a.) m_1 has inter-nesting, so it does not represent a possibility measure.
m_2 is nested and represents a possibility measure.

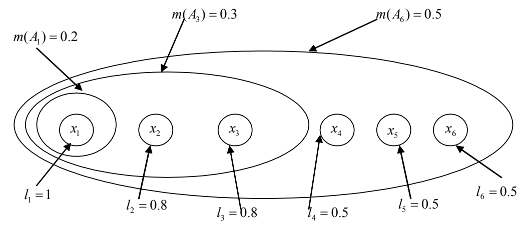


b.)Possibilities distribution for m_2:
r = {1, 0.8, 0.8, 0.4, 0.1, 0.1, 0.1}

15.6
m = (0.1, 0.1, 0.2, 0.2, 0.4) $\Rightarrow$
r = (1, 0.9, 0.8, 0.6, 0.4)



15.7
m = (0.2, 0, 0.3, 0, 0, 0.5) $\Rightarrow$ r = (1.0, 0.8, 0.8, 0.5, 0.5, 0.5)



15.8
By inspection we can determine that m_3 and m_4 would result in an ordered possibility distribution as the sets in the power set are nested.

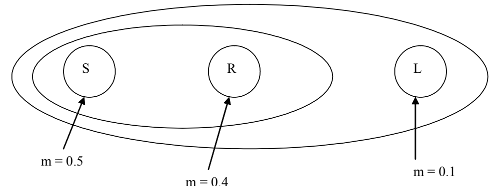
m_3 = (0.2, 0, 0, 0.3, 0.5) basic equation
r_3 = (1, 0.8, 0.8, 0.8, 0.5) ordered possibility distribution

m_4 = (0.1, 0, 0, 0.4, 0.5)
r_4 = (1, 0.9, 0.9, 0.9, 0.5)
Nesting shows that the evidence all tends to agree there can't be any contradicting evidence.

15.9
a)
Only the link (m_2) has basic assignments that result in rested subsets
r = {1, 1, 1, 1, 1, 0.5, 0.5, 0.1}

b and c)
In this case the nesting indicates that there is high possibility of multiple contributions to the error occurrence and while it is possible that either the sender or the receiver and the link are involved (since the link cannot correct errors but only propagates them). It is not very possible that the sources are contributions. More information would be

available if the joint assignments were taken and a possibility distribution examined for that joint assignment.



15.10

INTERSECTIONS

Intervals	Wt.	Normalized Wt
[0.03, 0.06]	0.25	0.125
[0.01, 0.06]	0.25	0.125
[0.03, 0.1]	0.25	0.125
[0.03, 0.15]	0.25	0.125
[0.01, 0.12]	0.25	0.125
[0.03, 0.24]	0.25	0.125
[0.008, 0.06]	0.25	0.125
Support- [0.008, 0.24]	0.25	0.125
	2	1

REDISTRIBUTION

Non-Consonant Intervals	Wt.	Consonant Intervals	β	κ	ρ
[0.01, 0.06]	0.125	[0.03, 0.06]	1	0.472711	0.059089
		[0.03, 0.1]	0.428571	0.20259	0.025324
		[0.03, 0.15]	0.25	0.118178	0.014772
		[0.03, 0.2]	0.176471	0.08342	0.010427
		[0.008, 0.2]	0.260417	0.123102	0.015388
			2.115459		
[0.01, 0.12]	0.125	[0.03, 0.06]	1	0.288696	0.036087
		[0.03, 0.1]	1	0.288696	0.036087
		[0.03, 0.15]	0.583333	0.168406	0.021051
		[0.03, 0.2]	0.411765	0.118875	0.014859
		[0.008, 0.2]	0.46875	0.135326	0.016916
			3.463848		
[0.008, 0.06]	0.125	[0.03, 0.06]	1	0.470395	0.058799
		[0.03, 0.1]	0.428571	0.201598	0.0252
		[0.03, 0.15]	0.25	0.117599	0.0147
		[0.03, 0.2]	0.176471	0.083011	0.010376
		[0.008, 0.2]	0.270833	0.127399	0.015925
			2.125875		

FINAL WEIGHTS

Interval	Wt	Final Wt
[0.03, 0.06]	0.125	0.278975
[0.03, 0.12]	0.125	0.211611
[0.03, 0.15]	0.125	0.175523
[0.03, 0.24]	0.125	0.160663
[0.008, 0.24]	0.125	0.173228
		1

15.11

INTERSECTIONS

Intervals ($x10^{-10}$)		Wt.	Normalized Wt
8	9	0.25	0.125
6	9	0.25	0.125
5	9	0.25	0.125
8	9	0.25	0.125
8	11	0.25	0.125
9	11	0.25	0.125
9	20	0.25	0.125
Support - 5	20	0.25	0.125
		2	1

REDISTRIBUTION

Non-Consonant Intervals ($x10^{-10}$)		Wt.	Consonant Intervals ($x10^{-10}$)		β	κ	ρ
8	8.5	0.125	8	9	0.5	0.75	0.09375
8	8.5	0.125	8	11	0.166667	0.25	0.03125
8	8.5	0.125	9	11	0	0	0
8	8.5	0.125	9	20	0	0	0
					0.666667		
6	8.5	0.125	8	9	0.5	0.75	0.09375
6	8.5	0.125	8	11	0.166667	0.25	0.03125
6	8.5	0.125	9	11	0	0	0
6	8.5	0.125	9	20	0	0	0
					0.666667		
5	9	0.125	8	9	1	0.75	0.09375
5	9	0.125	8	11	0.333333	0.25	0.03125
5	9	0.125	9	11	0	0	0
5	9	0.125	9	20	0	0	0
					1.333333		
5	20	0.125	8	9	1	0.25	0.03125
5	20	0.125	8	11	1	0.25	0.03125
5	20	0.125	9	11	1	0.25	0.03125
5	20	0.125	9	20	1	0.25	0.03125
					4		

FINAL WEIGHTS

Interval ($x10^{-10}$)		Wt	Final Wt
8	9	0.125	0.4375
6	8.5	0.125	0.25
5	9	0.125	0.15625
5	20	0.125	0.15625

15.12

INTERSECTIONS

Intervals	Wt.	Normalized Wt
[1.0, 1.25]	0.2	0.142857
[0.75, 1.25]	0.2	0.142857
[0.75, 1.5]	0.2	0.142857
[1.5, 2.0]	0.2	0.142857
[1.75, 2.0]	0.2	0.142857
[1.75, 2.25]	0.2	0.142857
Support – [0.75, 2.25]	0.2	0.142857
	1.4	

REDISTRIBUTION

Non-Consonant Interval	Wt.	Consonant Interval	β	κ	ρ
[1.75, 2.0]	0.143	[1.0, 1.25]	0	0	0
		[1.0, 0.75]	0	0	0
		[0.75, 1.5]	0	0	0
		[0.75, 2.25]	0.166667	1	0.143
			0.166667		
[1.5, 2.0]	0.143	[1.0, 1.25]	0	0	0
		[1.0, 0.75]	0	0	0
		[0.75, 1.5]	0	0	0
		[0.75, 2.25]	0.333333	1	0.143
[1.75, 2.25]	0.143	[1.0, 1.25]	0	0	0
		[1.0, 0.75]	0	0	0
		[0.75, 1.5]	0	0	0
		[0.75, 2.25]	0.333333	1	0.143

FINAL WEIGHTS

Interval	Wt	Final Wt
[1.0, 1.25]	0.143	0.143
[0.75, 1.25]	0.143	0.143
[0.75, 1.5]	0.143	0.143
[0.75, 2.25]	0.143	0.573
		1.000

a) Given the solution above, the possibility that the interest rates will be greater than 2% - 0.573

b) Generation of consonant intervals depends on the characteristics of the original data intervals and whether the expert desires

a pessimistic or an optimistic estimate. In this case, the expert is assumed to lean more towards the pessimistic attitude and hence a more conservative estimate is generated. Therefore, intervals with lower values are assigned more weight.

c) Degree of confirmation: $C(A) = 0 + 0.573 - 1 = -0.427$