

## (Final) Exam Second Semester 2017/2018

**Course Title:** Special Topics **Course No.:** IE0946501 **Instructor:** Dr. Wafa' H. AlAlaween **Date**: 13 May 2018 **Time:** 12:00-14:00





## **Exam policy and Ethics**:

# Exam is closed book; electronic devices are forbidden except for calculators.

# During the exam, an absolute silence is required. Also, all kinds of communications with other students are strictly forbidden.

# Clarity, accuracy, and justification of your answers are key elements in the evaluation.

# Students are not allowed to attend the exam if arrived after a single student is leaving the classroom.

*By signing this form, the student recognizes that he/she understands and accepts the exam policy and ethics. He also recognizes that if he does not respect these ethical rules, the professor will take the appropriate measures including exclusion from the exam.*

## **Feedback summary:**

.. .. **Signature ……………** Question 1: Using the truth table, show that:

a. The dual of equivalence  $((P \vee Q) \vee ((\overline{P}) \wedge (\overline{Q})))$  is also true.

b. The following expression is a tautology:  $((P \rightarrow Q) \land P) \rightarrow Q$ 

Question 2: The calculation of the vibration of an elastic structure depends on knowing the material properties of the structure as well as its support conditions. Suppose we have an elastic structure, such as a bar of known material, with properties like wave speed (C), modulus of elasticity (E), and cross-sectional area (A). However, the support stiffness is not well-known; hence the fundamental natural frequency of the system is not precise either. A relationship does exist between them, though, as illustrated in the figure below:



Let us define two fuzzy sets:

- *K* : support stiffness,
- *F* : first natural frequency.

with membership functions:

$$
\underline{K} = \left\{ \frac{0}{10^3}, \frac{0.2}{10^4}, \frac{0.5}{10^5}, \frac{0.8}{5 \times 10^5}, \frac{1}{10^6}, \frac{0.8}{5 \times 10^6}, \frac{0.2}{10^7} \right\}
$$

$$
\underline{F} = \left\{ \frac{0}{100}, \frac{0}{200}, \frac{0.2}{500}, \frac{0.5}{800}, \frac{1}{1000}, \frac{0.8}{2000}, \frac{0.2}{5000} \right\}
$$

a. Using the proposition, IF x is  $K$ , THEN y is  $F$ , find such a relation using the Mamdani (Min) implication:

b. Assume that another antecedent, damaged support  $(K')$  has been defined as follows:

$$
\underline{K}' = \left\{ \frac{0}{10^3}, \ \frac{0.8}{10^4}, \ \frac{0.2}{10^5} \right\}
$$

using max-product composition, find the system's natural frequency due to the damaged support.

Question 3: From thermodynamics it is known that for an ideal gas in an adiabatic reversible process

$$
\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma}{\gamma}}.
$$

where  $T_1$  and  $T_2$  are temperatures in kelvin (K) and  $P_1$  and  $P_2$  are pressures in bars and, for an ideal gas. For the Sugeno solution, use the following functions for the consequents of the three rules:

Rule 1 :  $T_1 = 320$  K and  $\gamma = 1.5$ Rule 2 : T<sub>1</sub> = 300 K and  $\gamma$  = 1.4 Rule 3 :  $T_1 = 300$  K and  $\gamma = 1.3$ 

For this problem,  $T_1$  will be fixed and the fuzzy model will predict  $P_2$  for the given input variables  $P_1$  and  $T_2$ . In other words, we are interested in finding the final pressure,  $P_2$ , of the system if the temperature of the system is changed to  $T_2$  from an original pressure equal to  $P_1$ . A real application could use a similar model built from experimental data to do a prediction on nonideal gases.

The rules used are:

Rule 1: IF  $P_1 = \text{atmP}$  AND  $T_2 = \text{lowT}$  THEN  $P_2 = \text{lowP}$ .

Rule 2: IF  $P_1 = \text{atmP}$  AND  $T_2 = \text{midT}$  THEN  $P_2 = \text{lowP}$ .

Rule 3: IF  $P_1 = \text{lowP}$  AND  $T_2 = \text{lowT}$  THEN  $P_2 = \text{very highP}$ .

Given the rule-base, the membership functions shown in the figure below, and the following pair of input values,  $P_1=1.6$  bar and  $T_2=415$  K, conduct a simulation to determine  $P_2$  for the inference methods of Sugeno and Tsukamoto. For the Sugeno consequents use the ideal gas formula, given above.



Question 4: For the data presented in the data table, show the first two iterations using a genetic algorithm in trying to find the optimum membership functions (use right-triangle functions) for the input variable x and output variable y in the rule table. For the rule table, the symbols ZE, S, and LG mean zero, small, and large, respectively. The functional mapping tells us that a LG x maps to a ZE y, and a small x maps to a small y. Assume that the range of the variable x is [0, 100] and that of y is [0, 1].

Hint: For the first iteration, use three strings with binary values: [18, 20, 7, 6]; [42, 50, 8, 3] and [45, 36, 9, 7]. For the second iteration, make all the assumptions clear.



Question 5: In the city of Calgary, Alberta, subdivisions constructed before 1970 were not required to retain overland storm-water flow on a site during major storm events to the level that has been accepted under current design criteria. In order to properly mitigate flooding and property damage in older subdivisions prone to flooding, they are being upgraded based on technical feasibility and public acceptance of the work. Presently, a subdivision is being considered for an upgrade of its storm-water sewer. It has been determined that there are two different methods to achieve the mitigation, either larger storm sewers have to be installed through the affected neighborhoods (pipe network) or storm-water retention facilities (pond) have to be built close enough to the neighborhood to reduce the flood threat. The mitigation alternatives (A) and the considered impacts or objectives (O) are described below:

Alternatively: 
$$
A = \{pipe, pond\}
$$
.

Objectives: Additional land required  $(O_1)$ , cost  $(O_2)$ , flood damage  $(O_3)$ , public acceptance  $(O_4)$ , and environmental constraints  $(O_5)$ :

$$
O = \{Q_1, Q_2, Q_3, Q_4, Q_5\}.
$$

On the basis of previous experience with other subdivisions, the city design engineer has determined the following ratings for this subdivision:

$$
Q_1 = \left\{ \frac{0.8}{\text{pipe}}, \frac{0.5}{\text{pond}} \right\}, \quad Q_2 = \left\{ \frac{0.9}{\text{pipe}}, \frac{0.4}{\text{pond}} \right\}, \quad Q_3 = \left\{ \frac{0.6}{\text{pipe}}, \frac{0.8}{\text{pond}} \right\},
$$
  

$$
Q_4 = \left\{ \frac{0.4}{\text{pipe}}, \frac{0.9}{\text{pond}} \right\}, \quad Q_5 = \left\{ \frac{0.7}{\text{pipe}}, \frac{0.4}{\text{pond}} \right\}.
$$

The city council has given the administration the following preference values for each objective. Using the above objectives and preferences determine which system to use for this subdivision.

$$
P = \{b_1, b_2, b_3, b_4, b_5\} = \{0.6, 0.5, 0.6, 0.8, 0.6\}.
$$

Question 6: In a particular region, a water authority must decide whether to build dikes to prevent flooding in case of excess rainfall. Three fuzzy courses of action may be considered:

- 1. build a permanent dike  $(A_1)$
- 2. build a temporary dike  $(A_2)$
- 3. do not build a dike  $(A_3)$ .

The sets  $A_1$ ,  $A_2$ , and  $A_3$  are fuzzy sets depending on the type and size of the dike to be built. The utility from each of these investments depends on the rainfall in the region. The crisp states of nature,  $S = \{s_1, s_2, s_3, s_4, s_5\}$ , are the amount of total rainfall in millimeters in the region. The utility for each of the alternatives has been developed for three levels of rainfall; (1) low ( $F_1$ ), (2) medium  $(F_2)$ , and (3) heavy  $(F_3)$ , which are defined by fuzzy sets on S. The utility matrix may be given as follows:

| $u_{ij}$   | $E_1$ | $E_2$ | $E_3$ |
|--|-------|-------|-------|
|  | $-2$  |       | 12    |
|  |       |       | $-10$ |
| $\begin{array}{c}\nA_1 \\ A_2 \\ A_3\n\end{array}$ | 13    | $-5$  | $-20$ |

The membership functions of  $F_1$ ,  $F_2$  and  $F_3$ , and the prior probabilities are given here:

|                           | 81       | $s_2$    | $s_3$ | $S_4$    | S5   |
|---------------------------|----------|----------|-------|----------|------|
| $\mu_{F_1}(s_i)$          |          | 0.3      | 0.1   | $\Omega$ | 0    |
| $\mu_{\mathrm{F}_2}(s_i)$ | $\bf{0}$ | 0.7      | 0.80  | 0.3      | 0    |
| $\mu_{F_3}(s_i)$          | 0        | $\Omega$ | 0.1   | 0.7      |      |
| $P(s_i)$                  | 0.1      | 0.2      | 0.2   | 0.35     | 0.15 |

Let  $X = \{x_1, x_2, x_3, x_4\}$  be the set of amount of rainfall in the next year. This represents the new information. The conditional probabilities  $p(x_i|s_i)$  for probabilistic uncertain information are as given in the following table:



Consider a fuzzy information system,

$$
\underline{M} = \{\underline{M}_1, \underline{M}_2, \underline{M}_3\},
$$

where

 $M_1$  = rainfall is less than approximately 35 mm  $M_2$  = rainfall is equal to approximately 35 mm  $M_3$  = rainfall is greater than approximately 35 mm.

|                           | $x_1$   | x <sub>2</sub> | $x_3$ | $x_4$ |
|---------------------------|---------|----------------|-------|-------|
| $\mu_{\mathbf{M}_1}(x_i)$ | $1.0\,$ | 0.3            | 0.1   | 0.0   |
| $\mu_{\mathbf{M}_2}(x_i)$ | 0.0     | 0.7            | 0.8   | 0.1   |
| $\mu_{\rm M_3}(x_i)$      | 0.0     | 0.0            | 0.1   | 0.9   |

The membership functions for the new fuzzy information that satisfy the orthogonality condition are given here:

Determine the following:

(a) Posterior probabilities for fuzzy state  $F_2$  and fuzzy information  $M_1$ , and for fuzzy state  $F_3$ and fuzzy information  $M_3$ .

(b) Conditional expected utility of building a permanent dike  $(A_1)$  when fuzzy information  $M_3$ is given.

Question 7: Using your own intuition and your own definitions of the universe of discourse, plot fuzzy membership functions for the following variables:

(a) Age of people

- 1. Very young
- 2. Young
- 3. Middle-aged
- 4. Old
- 5. Very old

(b) Education of people

- 1. Fairly educated
- 2. Educated
- 3. Highly educated
- 4. Not highly educated
- 5. More or less educated.

Question 8: Design a fuzzy system for developing and introducing a new orange juice product to the Jordanian market.

Hint: Discuss the factors that will affect the product, the outputs that you are interested in, how are you going to define the membership functions, why such a process could be considered as a fuzzy systems, etc.