Chapter 4
Properties of Membership Functions, Fuzzification and Defuzzification

Dr. Wafa’ H. AlAlaween
wafa.alalaween@gmail.com

Membership Function: Definition

- A membership function: The values assigned to elements of a universal set fall within a specified range.
- It describes the information contained in a fuzzy set.
- Larger values indicate higher degrees of membership.
- The core: The region (element) of a universe that is characterized by full membership in a set.
- The support: The region (element) of a universe that is characterized by nonzero membership in a set.

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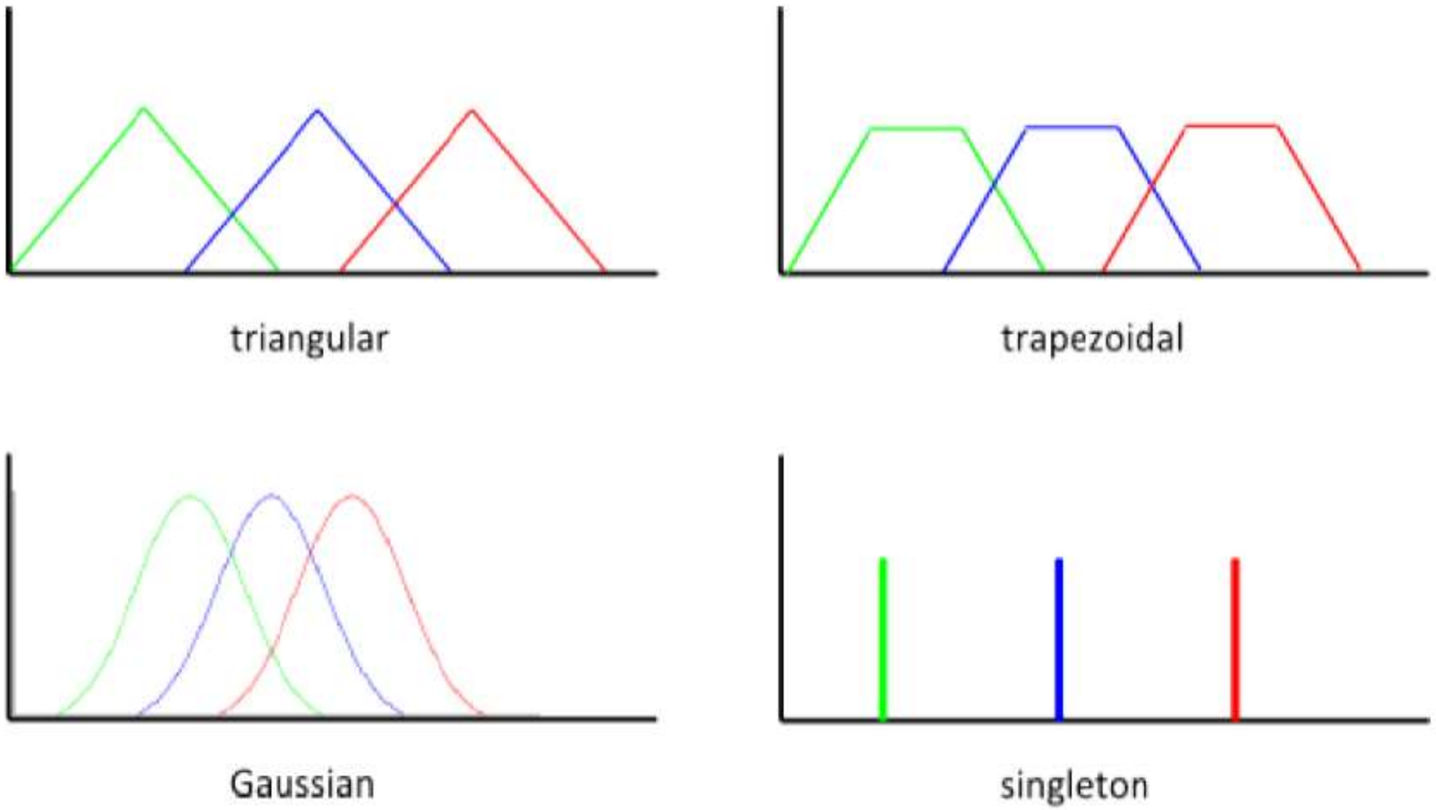
- A normal fuzzy set: A set whose membership function has at least one element whose membership value is unity.
- Prototype: It is an element (only one element) that has a membership value that is equal to one.
- A convex fuzzy set: It is a set which is described by a membership function whose values are (1) strictly monotonically increasing, (2) strictly monotonically decreasing, or (3) monotonically increasing then decreasing with increasing the elements values.
- The height of a fuzzy set is the maximum value of the membership function.

Membership Functions: Types

- Membership functions can be symmetrical or asymmetrical.
- They can also be defined as 1D or nD membership functions.

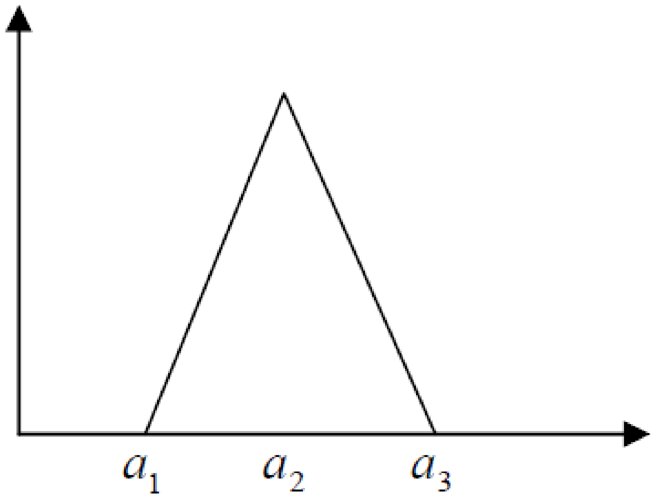
In Matlab, the fuzzy logic toolbox includes 11 built-in membership function types.



Fuzzification

- Fuzzification represents the process of mapping crisp values to fuzzy sets.

a) Triangular Membership Function:



$$\mu_A(u) = \begin{cases} 0 & u < a_1 \\ \dfrac{u - a_1}{a_2 - a_1} & a_1 \le u < a_2 \\ \dfrac{a_3 - u}{a_3 - a_2} & a_2 \le u < a_3 \\ 0 & u \ge a_3 \end{cases}$$

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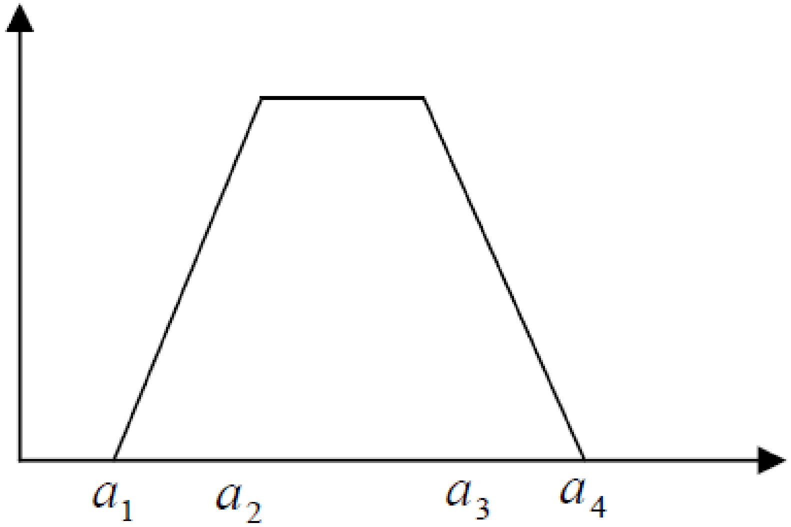
b) Trapezoidal Membership Function:



$$\mu_A(u) = \begin{cases} 0 & u < a_1 \\ \dfrac{u - a_1}{a_2 - a_1} & a_1 \le u < a_2 \\ 1 & a_2 \le u < a_3 \\ \dfrac{a_4 - u}{a_4 - a_3} & a_3 \le u < a_4 \\ 0 & u \ge a_4 \end{cases}$$

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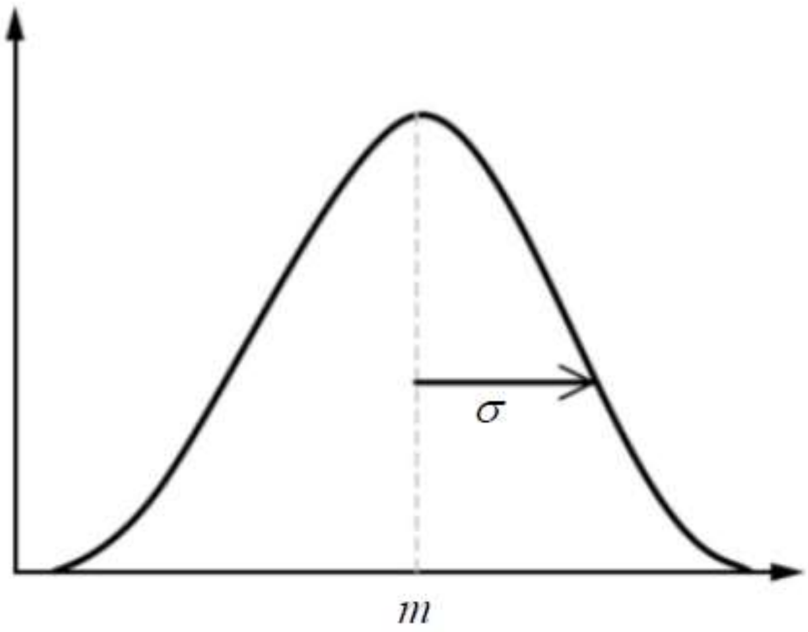
c) Gaussian Membership Function:



$$\mu_A(u) = e^{-\left(\frac{u-m}{\sigma}\right)^2}$$

Defuzzification

- Defuzzification represents the process of mapping fuzzy values to crisp ones.
- Methods (common):

a) Max membership (the height method): $\mu_{\underset{\sim}{C}}(z^*) \geq \mu_{\underset{\sim}{C}}(z)$

b) Centroid method (centre of area/gravity): $$z^* = \frac{\int \mu_{\underset{\sim}{C}}(z) \cdot z \, dz}{\int \mu_{\underset{\sim}{C}}(z) \, dz}$$

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c) Weighted average method: $z^* = \dfrac{\sum \mu_{\underset{\sim}{C}}(\overline{z}) \cdot \overline{z}}{\sum \mu_{\underset{\sim}{C}}(\overline{z})}$

d) Mean max membership: This method is similar to the maximum membership method, except that the locations of the maximum membership can be non-unique.

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e) Centre of sum: $z^* = \dfrac{\sum_{k=1}^{n} \mu_{\underset{\sim}{C}_k}(z) \int_z \bar{z}\, dz}{\sum_{k=1}^{n} \mu_{\underset{\sim}{C}_k}(z) \int_z dz}$

Note: Two drawbacks to this method are that the intersecting areas are added twice, and the method also involves finding the centroids of the individual membership functions.

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f) Centre of largest area: $z^* = \dfrac{\int \mu_{\underset{\sim}{C}_m}(z) z \, dz}{\int \mu_{\underset{\sim}{C}_m}(z) \, dz}$

It is a CONVEX sub region

g) First (or last) of maxima: The value of the domain with maximized membership degree.

$$\mathrm{hgt}(\underset{\sim}{C}_k) = \sup_{z \in Z} \mu_{\underset{\sim}{C}_k}(z) \longrightarrow z^* = \inf_{z \in Z}\{z \in Z \mid \mu_{\underset{\sim}{C}_k}(z) = \mathrm{hgt}(\underset{\sim}{C}_k)\}$$

Example

A railroad company intends to lay a new rail line in a particular part of a county. The whole area through which the new line is passing must be purchased for right-of-way considerations. It is surveyed in three stretches, and the data are collected for analysis. The surveyed data for the road are given by the sets: B1, B2 and B3, where the sets are defined on the universe of right-of-way widths, in meters. For the railroad to purchase the land, it must have an assessment of the amount of land to be bought. The three surveys on right-of-way width are ambiguous, however, because some of the land along the proposed railway route is already public domain and will not need to be purchased. Additionally, the original surveys are so old (circa 1860) that some ambiguity exists on boundaries and public right-of-way for old utility lines and old roads. The three fuzzy sets shown in the Figures below, represent the uncertainty in each survey as to the membership of right-of-way width, in meters, in privately owned land.

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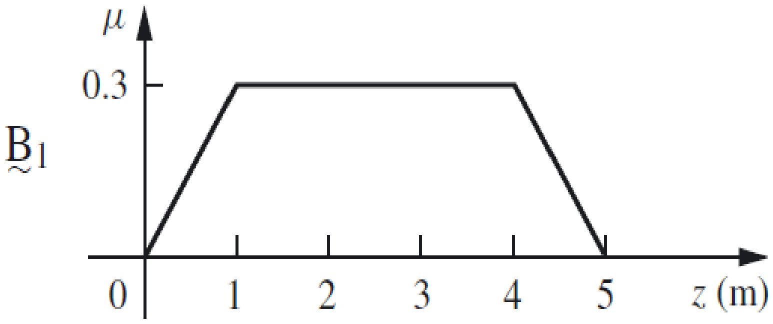
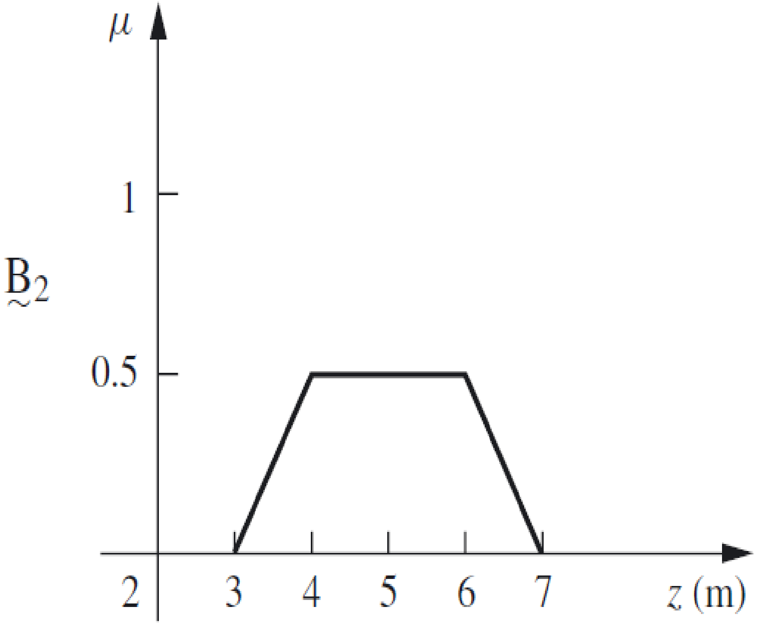
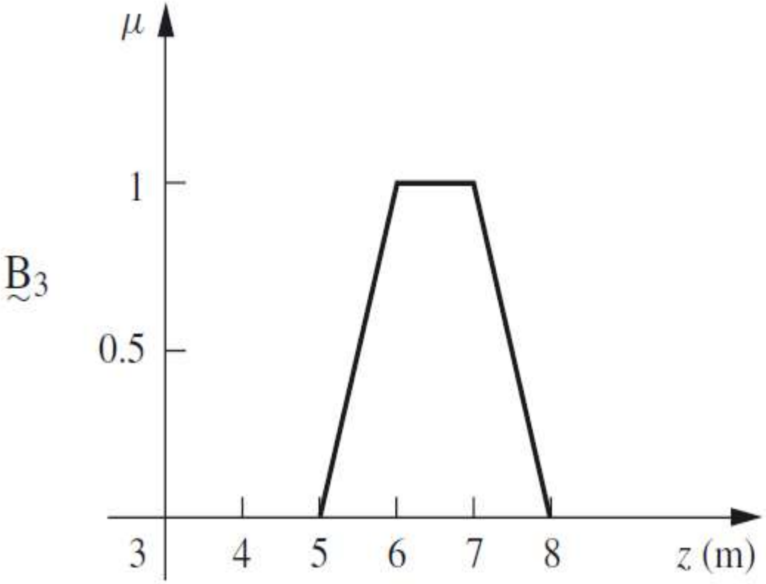
The fuzzy sets:







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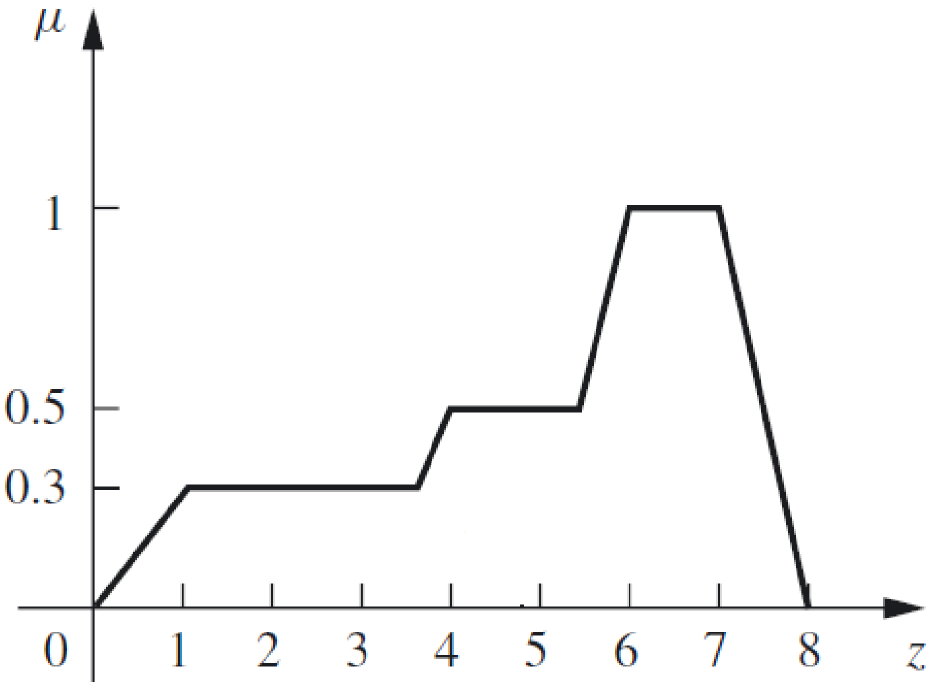
Find the single most nearly representative right-of-way width (z).



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Using the centroid method:

$$z^* = \frac{\int \mu_{\underset{\sim}{B}}(z) \cdot z\,dz}{\int \mu_{\underset{\sim}{B}}(z)\,dz}$$

$$= \left[\int_0^1 (0.3z)z\,dz + \int_1^{3.6} (0.3)z\,dz + \int_{3.6}^4 \left(\frac{z-3.0}{2}\right) z\,dz + \int_4^{5.5} (0.5)z\,dz \right.$$

$$\left. + \int_{5.5}^6 (z-5)z\,dz + \int_6^7 z\,dz + \int_7^8 (8-z)\,z\,dz \right]$$

$$\div \left[\int_0^1 (0.3z)\,dz + \int_1^{3.6} (0.3)\,dz + \int_{3.6}^4 \left(\frac{z-3.6}{2}\right) dz + \int_4^{5.5} (0.5)\,dz \right.$$

$$\left. + \int_{5.5}^6 \left(\frac{z-5.5}{2}\right) dz + \int_6^7 dz + \int_7^8 \left(\frac{7-z}{2}\right) dz \right]$$

$$= 4.9\,\mathrm{m},$$

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Using the weighted average method:

$$z^* = \frac{(0.3 \times 2.5) + (0.5 \times 5) + (1 \times 6.5)}{0.3 + 0.5 + 1} = 5.41\,\text{m},$$

Using the centre of sum method:

$$z^* = \frac{[2.5 \times 0.5 \times 0.3(3+5) + 5 \times 0.5 \times 0.5(2+4) + 6.5 \times 0.5 \times 1(3+1)]}{[0.5 \times 0.3(3+5) + 0.5 \times 0.5(2+4) + 0.5 \times 1(3+1)]}$$
$$= 5.0\,\text{m},$$

The centre of largest area method provides the same result as the centroid method (z*=4.9).

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Using the first of maxima method $z^*= 6$.

Using the last of maxima method $z*=7$.